AI1103–Assignment-2

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Question

Probability density function $p(x)$ of random variable x is as shown below. The value of a is

A) $\frac{2}{c}$
B) $\frac{1}{c}$
C) $\frac{2}{(b+c)}$
D) $\frac{1}{(b+c)}$

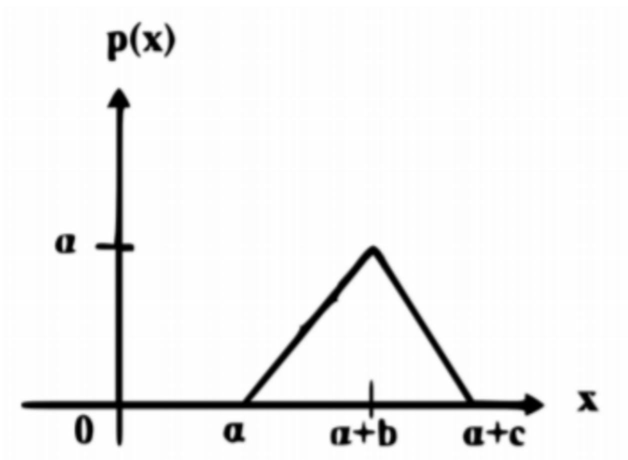

Fig. 4: PDF

Solution

Let Y_1 and Y_2 be two independent and identically distributed (IID) uniform random variables.
Let X be a random variable such that

$$X = Y_1 + Y_2 \tag{0.0.1}$$

Let

$$p_{Y_1}(y) = \Pr(Y_1 = y) \tag{0.0.2}$$
$$p_{Y_2}(y) = \Pr(Y_2 = y) \tag{0.0.3}$$
$$p_X(x) = \Pr(X = x) \tag{0.0.4}$$

be the probability densities of random variables Y_1, Y_2 and X.

Y_1 and Y_2 lie in the range $\left(\frac{-c}{4}, \frac{c}{4}\right)$, therefore, the PDF for Y_1 and Y_2,

$$p_{Y_1}(y) = p_{Y_2}(y) = \begin{cases} \frac{2}{c} & \frac{-c}{4} \leq y \leq \frac{c}{4} \\ 0 & \text{otherwise} \end{cases} \tag{0.0.5}$$

The density of X is obtained by convolution of Y_1 and Y_2

$$p_X(x) = p_{Y_1}(x) * p_{Y_2}(x) \tag{0.0.6}$$

where $*$ denotes the convolution operation. Since convolution operation is time invariant,

$$\begin{aligned} p_X(x-t) &= p_{Y_1}(x-t) * p_{Y_2}(x) \\ &= p_{Y_1}(x) * p_{Y_2}(x-t) \end{aligned} \tag{0.0.7}$$

On time shifting Y_1 by shifting factor $t = a + \frac{c}{2}$,

$$p_X\left(x - \left(a + \frac{c}{2}\right)\right) = p_{Y_1}\left(x - \left(a + \frac{c}{2}\right)\right) * p_{Y_2}(x) \tag{0.0.8}$$

Thus, the PDF of time shifted X obtained by convolution is,

$$p_x = \begin{cases} \frac{4}{c^2}(x-a) & a \leq x \leq a + \frac{c}{2} \\ \frac{4}{c^2}(a+c-x) & a + \frac{c}{2} \leq x \leq a + c \\ 0 & \text{otherwise} \end{cases} \tag{0.0.9}$$

On comparing the parameters of PDF of time shifted X with that in the question, we have

$$b = \frac{c}{2} \tag{0.0.10}$$
$$a = \frac{2}{c} \tag{0.0.11}$$

Answer : Option A

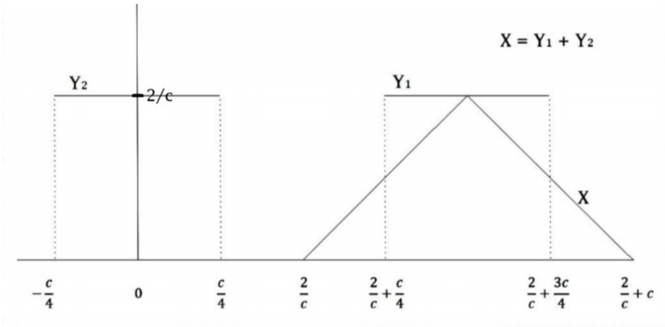

Fig. 4: PDF of time shifted X

The following are some observations:

1) The sum of two equally distributed random variables will lead to a triangular probability density
2) The two uniformly distributed random variables lie in the range $\left(\frac{-c}{4}, \frac{c}{4}\right)$ and $\left(\frac{2}{c} + \frac{c}{4}, \frac{2}{c} + \frac{3c}{4}\right)$. $\because$X = $Y_1 + Y_2$ the range of X is thus $\left(\frac{2}{c}, \frac{2}{c} + c\right)$
3) On time shifting Y_1 to the right by a factor $a+\frac{c}{2}$, the convoluted PDF of X also shifts by the same factor without any change in it's width.

Fig 3 and Fig 3 are the plots of PDF and CDF obtained by taking c=2

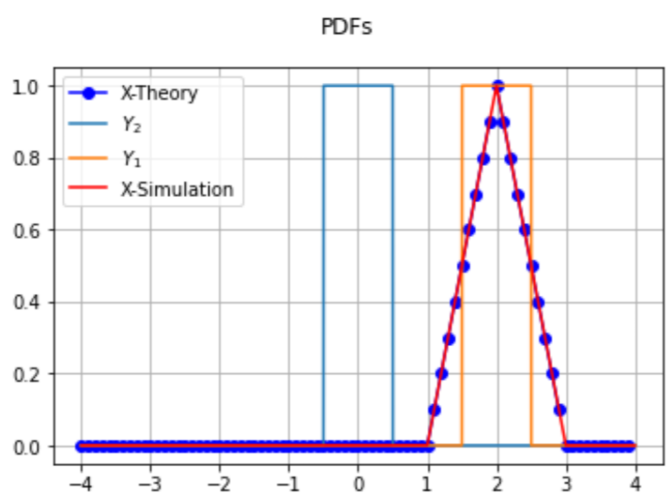

Fig. 3: PDF of Y_1, Y_2 and X

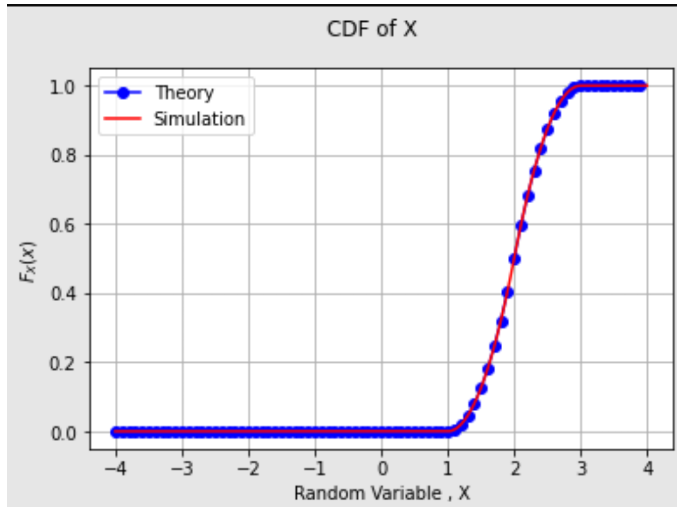

Fig. 3: CDF of X