

AI1103–Assignment-3

Name: Aravinda Kumar Reddy Thippareddy

Roll.No.:CS20BTECH11053

Download all python codes from

https://github.com/AravindCSEiith/Probability-and-Random-variables_AI1103_Assignment-3/blob/main/ASSIGNMENT_3_AI1103.py

and latex-tikz codes from

https://github.com/AravindCSEiith/Probability-and-Random-variables_AI1103_Assignment-3/blob/main/Assignment_3_AI1103.tex

The expectation value of 'X', E(X) is,

$$E(X) = \bar{X} = \sum x_i \cdot P(X = x_i) \quad (0.0.5)$$

$$= (0)(p) + (1)(1 - 3p) + (3)(p) + (k)(p) \quad (0.0.6)$$

$$= 1 + kp \quad (0.0.7)$$

Now the expression for variance,

$$V(X) = \sum P(X = x_i)(x_i - \bar{X})^2 \quad (0.0.8)$$

$$V(X) = \sum P(X = x_i)(x_i - (1 + kp))^2 \quad (0.0.9)$$

$$(0.0.10)$$

QUESTION

Let the random variable X have the distribution $P(X = 0) = P(X = 3) = p$, $P(X = 1) = 1 - 3p$ for $0 \leq p \leq \frac{1}{2}$. What is the maximum value of V(X)?

- A) 3
- B) 4
- C) 5
- D) 6
- E) none

Let $V(X) = f(k)$.

$$V(X) = f(k) = (p - p^2)k^2 + (-2p)k + (6p) \quad (0.0.11)$$

$$V(X) = f(k) = (p - p^2)k^2 - 2pk + 6p \quad (0.0.12)$$

Compare the above quadratic equation with the general standard form of quadratic equation ' $ax^2 + bx + c$ '. We get,

$$a = p - p^2 \quad (0.0.13)$$

$$b = -2p \quad (0.0.14)$$

$$c = 6p \quad (0.0.15)$$

SOLUTION

Value of X	0	1	3	k
Probability of 'X'	p	$1 - 3p$	p	?

TABLE 5: Distribution of 'X'

We know that "sum of all probabilities= 1". Hence,

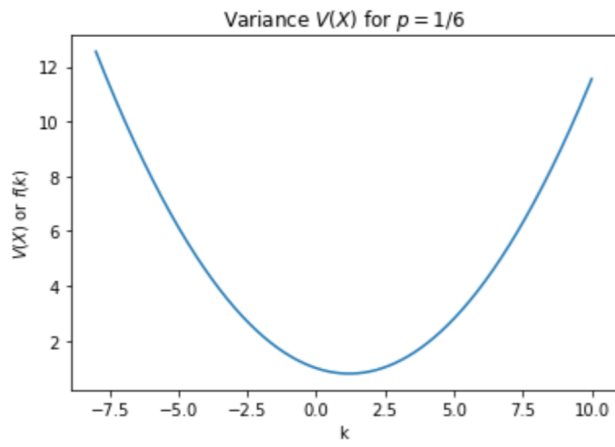
$$P(X = 0) + P(X = 1) + P(X = 3) + P(X = k) = 1 \quad (0.0.1)$$

$$(p) + (1 - 3p) + (p) + P(X = k) = 1 \quad (0.0.2)$$

$$1 - p + P(X = k) = 1 \quad (0.0.3)$$

$$P(X = k) = p \quad (0.0.4)$$

$a > 0$, $\forall p \in (0, 1)$. It is given in question that $0 \leq p \leq \frac{1}{2}$. Hence $a \geq 0$. Hence maximum value of quadratic expression, V(X) is $+\infty$, as $k \rightarrow \pm\infty$.



Hence maximum value of $V(X)$ is $+\infty$.

Answer : Option E

Maximum value of $V(X)=+\infty$