

AI1103–Assignment-3

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QUESTION

Let X be a random variable such that $E(X) = E(X^2) = 1$. Then $E(X^{100}) = ?$

- (A) 0
- (B) 1
- (C) 2^{100}
- (D) $2^{100} + 1$

SOLUTION

Let $x_1, x_2, x_3, \dots, x_n$ be the random values that 'X' take.

$$Var(X) = E(X^2) - (E(X))^2 \quad (0.0.1)$$

$$= 1 - (1)^2 \quad (0.0.2)$$

$$= 0 \quad (0.0.3)$$

$$Var(X) = \frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n} \quad (0.0.4)$$

$$0 = \frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n} \quad (0.0.5)$$

$$0 = \sum_{i=1}^n (x_i - \bar{X})^2 \quad (0.0.6)$$

$$x_1 = x_2 = x_3 = \dots = x_{n-1} = x_n = \bar{X} = E(X) = 1 \quad (0.0.7)$$

$$E(X^{100}) = \frac{\sum_{i=1}^n (x_i)^{100}}{n} \quad (0.0.8)$$

$$= \frac{\sum_{i=1}^n (1)^{100}}{n} \quad (0.0.9)$$

$$= \frac{\sum_{i=1}^n 1}{n} \quad (0.0.10)$$

$$= \frac{n}{n} \quad (0.0.11)$$

$$= 1 \quad (0.0.12)$$

Answer : Option B

Therefore, $E(X^{100}) = 1$