

# AI1103–Assignment-3

Name: Aravinda Kumar Reddy Thippareddy

Roll.No.:CS20BTECH11053

Download all latex-tikz codes from

[https://github.com/AravindCSEiith/Probability-and-Random-variables\\_AI1103\\_Assignment-3/blob/main/Assignment\\_3\\_AI1103.tex](https://github.com/AravindCSEiith/Probability-and-Random-variables_AI1103_Assignment-3/blob/main/Assignment_3_AI1103.tex)

## QUESTION

Prove that for  $r = 1, 2, 3, \dots, n$

$$\frac{1}{\Gamma(r)} \int_{\mu}^{\infty} t^{r-1} e^{-t} dt = \sum_{x=0}^{r-1} \frac{e^{-\mu} \mu^x}{x!} \quad (0.0.1)$$

## SOLUTION

The gamma function of a positive integer 'm' is given by,

$$\Gamma(m) = (m-1)! \quad (0.0.2)$$

Similarly if we go on applying IBP, we will get;

$$L.H.S = \frac{(e^{-\mu} \mu^{r-1})}{(r-1)!} + \frac{(e^{-\mu} \mu^{r-1})}{(r-2)!} + \dots + \frac{1}{(1)!} \left( \int_{\mu}^{\infty} t^1 d(-e^{-t}) \right) \quad (0.0.8)$$

$$= \frac{(e^{-\mu} \mu^{r-1})}{(r-1)!} + \frac{(e^{-\mu} \mu^{r-1})}{(r-2)!} + \dots + \frac{(e^{-\mu} \mu^1)}{(1)!} + \frac{(e^{-\mu} \mu^0)}{(0)!} \quad (0.0.9)$$

$$= \sum_{x=0}^{r-1} \frac{e^{-\mu} \mu^x}{x!} = R.H.S \quad (0.0.10)$$

Hence, it is proved.

Therefore, for  $r = 1, 2, 3, \dots, n$

$$\frac{1}{\Gamma(r)} \int_{\mu}^{\infty} t^{r-1} e^{-t} dt = \sum_{x=0}^{r-1} \frac{e^{-\mu} \mu^x}{x!} \quad (0.0.11)$$

## Integration By Parts(IBP):

This is a special method of integration used for integrating product of two functions. Consider two real valued integrable functions 'u' and 'dv'. Now using Integration By Parts we can write;

$$\int u dv = uv - \int v du \quad (0.0.3)$$

Applying IBP to the L.H.S of the equation 0.0.1;

$$L.H.S = \frac{1}{\Gamma(r)} \int_{\mu}^{\infty} t^{r-1} e^{-t} dt \quad (0.0.4)$$

$$= \frac{1}{\Gamma(r)} \int_{\mu}^{\infty} t^{r-1} d(-e^{-t}) \quad (0.0.5)$$

$$= \frac{1}{\Gamma(r)} \left( t^{r-1} (-e^{-t}) \Big|_{\mu}^{\infty} - \int_{\mu}^{\infty} (-e^{-t}) (r-1) t^{r-2} dt \right) \quad (0.0.6)$$

$$= \frac{1}{(r-1)!} (e^{-\mu} \mu^{r-1}) + \frac{1}{(r-2)!} \left( \int_{\mu}^{\infty} t^{r-2} d(-e^{-t}) \right) \quad (0.0.7)$$