AI1103–Assignment-3

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Download all python codes from

```
https://github.com/AravindCSEiith/Probability−and
    −Random−variables_AI1103_Asignment−3/
    blob/main/Assignment−3−−AI1103.py
```

and latex-tikz codes from

```
https://github.com/AravindCSEiith/Probability−and
    −Random−variables_AI1103_Asignment−3/
    blob/main/Assignment−3−−AI1103.tex
```

Question

Let X be a random variable such that $E(X) = E(X^2) = 1$. Then $E(X^{100}) =$?
(A)0
(B)1
(C)2^{100}
(D)$2^{100} + 1$

Solution

Let $x_1, x_2, x_3, ..., x_n$ be the random values that 'X' take.

$$Var(X) = E(X^2) - (E(X))^2 \tag{0.0.1}$$
$$= 1 - (1)^2 \tag{0.0.2}$$
$$= 0 \tag{0.0.3}$$

Markov's Inequality:

Markov's Inequality states that for a random variable X and any positive real number a, the probability that X is greater than or equal to a is less than or equal to the expectation value of X divided by a.

$$p(X \geq a) \leq \frac{E(X)}{a} \tag{0.0.4}$$

Now replace 'X' with $(X - E(X))^2$ and 'a' with a^2.

$$p((X - E(X))^2 \geq a^2) \leq \frac{E((X - E(X))^2)}{a^2} \tag{0.0.5}$$
$$p(|X - E(X)| \geq a) \leq \frac{Var(X)}{a^2} \tag{0.0.6}$$

The above equation represents the Chebyshev's Inequality.

$$p(|X - E(X)| \geq a) \leq \frac{Var(X)}{a^2} \tag{0.0.7}$$
$$a^2 \times p(|X - E(X)| \geq a) \leq Var(X) \tag{0.0.8}$$

From equation (0.0.3) $Var(X) = 0$.

$$a^2 \times p(|X - E(X)| \geq a) \leq 0 \tag{0.0.9}$$

Hence for $a > 0$ $p(|X - E(X)| \geq a) = 0$. And for $a = 0$, $p(|X - E(X)| \geq a) = 1$. Therefore,

$$X = E(X) = 1 \tag{0.0.10}$$

$$E(X^{100}) = \frac{\sum_{i=1}^{n} (x_i)^{100}}{n} \tag{0.0.11}$$
$$= \frac{\sum_{i=1}^{n} (1)^{100}}{n} \tag{0.0.12}$$
$$= \frac{\sum_{i=1}^{n} 1}{n} \tag{0.0.13}$$
$$= \frac{n}{n} \tag{0.0.14}$$
$$= 1 \tag{0.0.15}$$

Answer : Option B

$\boxed{\text{Therefore, } E(X^{100}) = 1}$