

AI1103–Assignment-3

Name: Aravinda Kumar Reddy Thippareddy

Roll.No.:CS20BTECH11053

Download all latex-tikz codes from

https://github.com/AravindCSEiith/Probability-and-Random-variables_AI1103_Assignment-3/blob/main/Assignment_3_AI1103.tex

QUESTION

Let the random variable X have the distribution $P(X = 0) = P(X = 3) = p$, $P(X = 1) = 1 - 3p$ for $0 \leq p \leq \frac{1}{2}$. What is the maximum value of $V(X)$?

- A) 3
- B) 4
- C) 5
- D) 6
- E) none

SOLUTION

Value of X	0	1	3	k
Probability of 'X'	p	$1 - 3p$	p	?

TABLE 5: Distribution of 'X'

We know that "sum of all probabilities= 1". Hence,

$$P(X = 0) + P(X = 1) + P(X = 3) + P(X = k) = 1 \quad (0.0.1)$$

$$(p) + (1 - 3p) + (p) + P(X = k) = 1 \quad (0.0.2)$$

$$1 - p + P(X = k) = 1 \quad (0.0.3)$$

$$P(X = k) = p \quad (0.0.4)$$

The expectation value of 'X', $E(X)$ is,

$$E(X) = \bar{X} = \sum x_i \cdot P(X = x_i) \quad (0.0.5)$$

$$= (0)(p) + (1)(1 - 3p) + (3)(p) + (k)(p) \quad (0.0.6)$$

$$= 1 + kp \quad (0.0.7)$$

Now the expression for variance,

$$V(X) = \sum P(X = x_i)(x_i - \bar{X})^2 \quad (0.0.8)$$

$$= \sum P(X = x_i)(x_i - (1 + kp))^2 \quad (0.0.9)$$

$$= (p)(0 - 1 - kp)^2 + (1 - 3p)(1 - 1 - kp)^2 + (p)(3 - 1 - kp)^2 \quad (0.0.10)$$

Now consider the first term of equation(0.0.10). Let it be T_1 For the variance to be maximum consider the following cases.

Case-1:

$p \neq 0$ and $k \rightarrow +\infty$

$$\lim_{k \rightarrow +\infty} T_1 = \lim_{k \rightarrow +\infty} (p)(-1 - kp)^2 \quad (0.0.11)$$

$$= T_1 \rightarrow +\infty \quad (0.0.12)$$

$$V(X) \rightarrow +\infty \quad (0.0.13)$$

Case-2:

$p \neq 0$ and $k \rightarrow -\infty$

$$\lim_{k \rightarrow -\infty} T_1 = \lim_{k \rightarrow -\infty} (p)(-1 - kp)^2 \quad (0.0.14)$$

$$= T_1 \rightarrow +\infty \quad (0.0.15)$$

$$V(X) \rightarrow +\infty \quad (0.0.16)$$

In both the cases $V(X) \rightarrow +\infty$. And nothing is greater than infinity.

Hence maximum value of $V(X)$ is $+\infty$.

Maximum value of $V(X) = +\infty$