AI1103–Assignment-3

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Download all latex-tikz codes from

https://github.com/AravindCSEiith/Probability−and−Random−variables_AI1103_Asignment−3/blob/main/Assignment−3−−AI1103.tex

Question

Prove that for $r = 1, 2, 3, ..., n$

$$\frac{1}{\Gamma(r)}\int_{\mu}^{\infty} t^{r-1}e^{-t}dt = \sum_{x=0}^{r-1}\frac{e^{-\mu}\mu^{x}}{x!} \tag{0.0.1}$$

Solution

The gamma function of a positive integer 'm' is given by,

$$\Gamma(m) = (m-1)! \tag{0.0.2}$$

Integration By Parts(IBP):

This is a special method of integration used for integrating product of two functions. Consider two real valued integrable functions 'u' and 'dv'. Now using Integration By Parts we can write;

$$\int udv = uv - \int vdu \tag{0.0.3}$$

Applying IBP to the L.H.S of the equation 0.0.1;

$$L.H.S = \frac{1}{\Gamma(r)}\int_{\mu}^{\infty} t^{r-1}e^{-t}dt \tag{0.0.4}$$

$$= \frac{1}{\Gamma(r)}\int_{\mu}^{\infty} t^{r-1}d(-e^{-t}) \tag{0.0.5}$$

$$= \frac{1}{\Gamma(r)}\left(t^{r-1}(-e^{-t})|_{\mu}^{\infty} - \int_{\mu}^{\infty}(-e^{-t})\,(r-1)t^{r-2}dt\right) \tag{0.0.6}$$

$$= \frac{1}{(r-1)!}\left(e^{-\mu}\mu^{r-1}\right) + \frac{1}{(r-2)!}\left(\int_{\mu}^{\infty} t^{r-2}d(-e^{-t})\right) \tag{0.0.7}$$

Similarly if we go on applying IBP, we will get;

$$L.H.S = \frac{\left(e^{-\mu}\mu^{r-1}\right)}{(r-1)!} + \frac{\left(e^{-\mu}\mu^{r-1}\right)}{(r-2)!} + .. + \frac{1}{(1)!}\left(\int_{\mu}^{\infty} t^{1}d(-e^{-t})\right) \tag{0.0.8}$$

$$= \frac{\left(e^{-\mu}\mu^{r-1}\right)}{(r-1)!} + \frac{\left(e^{-\mu}\mu^{r-1}\right)}{(r-2)!} + .. + \frac{\left(e^{-\mu}\mu^{1}\right)}{(1)!} + \frac{\left(e^{-\mu}\mu^{0}\right)}{(0)!} \tag{0.0.9}$$

$$= \sum_{x=0}^{r-1}\frac{e^{-\mu}\mu^{x}}{x!} = R.H.S \tag{0.0.10}$$

Hence, it is proved.

Therefore, for $r = 1, 2, 3, ..., n$

$$\boxed{\frac{1}{\Gamma(r)}\int_{\mu}^{\infty} t^{r-1}e^{-t}dt = \sum_{x=0}^{r-1}\frac{e^{-\mu}\mu^{x}}{x!}} \tag{0.0.11}$$