Development of Extended Kalman Filter EKF

Given and Assumptions:

$$\delta(k) = \delta(k-1) + T(v(k-1) - v_L(k-1))$$

$$\begin{aligned} v(k) &= v(k-1) + \frac{T}{m(k-1)}\left(-A_\rho v^2(k-1) - d + f(k-1)\right) \\ f(k) &= f(k-1) + \frac{T}{\tau}(-f(k-1) + u(k-1)) \\ m(k) &= m(k-1) \end{aligned}$$

Where, T = 0.01
Ap = 0.3
Ti (tau) = 0.2
d = 100

The measurement of states (Interspace distance and velocity) is given by

$$z(k+1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x(k+1) + \begin{bmatrix} v_1(k+1) \\ v_2(k+1) \end{bmatrix}$$

Where, v1 and v2 are noise (given by randn in MatLab code)

The control action taken such that y tends to zero, in other words, if y tends to zero with lamba value equal to 0.9, it means that interspacing will be one vehicle length per 10mph (Given).

$$y_i = -\left(\delta_i + \lambda v_i\right) + noise$$

The Vehicle is controled by PD Controller of follwing configuration,

$$y(k) = -((\delta(k) + v_1(k)) + \lambda(v(k) + v_2(k)))$$

$$u(k) = K_p y(k) + \frac{K_d}{T}(y(k) - y(k-1))$$

Where, Lamba =0.9
kp =1088
kd = 1525.9

EKF Working:

The EFF is same as Linear KF, but EKF linear the system before estimate the states. For linearizing the system, Jocobian Matrix (F, H) are calculated and used as system for updating and predicting using linear KF. The derivation for predicting and updating step in shown below.

Model:

$$x_k = f(x_{k-1}, u_k) + w_k$$

$$z_k = h(x_k) + v_k$$

Predict:

$$\hat{x}_k = f(\hat{x}_{k-1}, u_k)$$

$$P_k = F_{k-1} P_{k-1} F_{k-1}^T + Q_{k-1}$$

Update:

$$G_k = P_k H_k^T (H_k P_k H_k^T + R)^{-1}$$

$$\hat{x}_k \leftarrow \hat{x}_k + G_k (z_k - h(\hat{x}_k))$$

$$P_k \leftarrow (I - G_k H_k) P_k$$

Where, f(…) = non-linear system equation
F = Jacobian of f()
H = Jacobian of h()

RESULTS:

The result are shown as below;

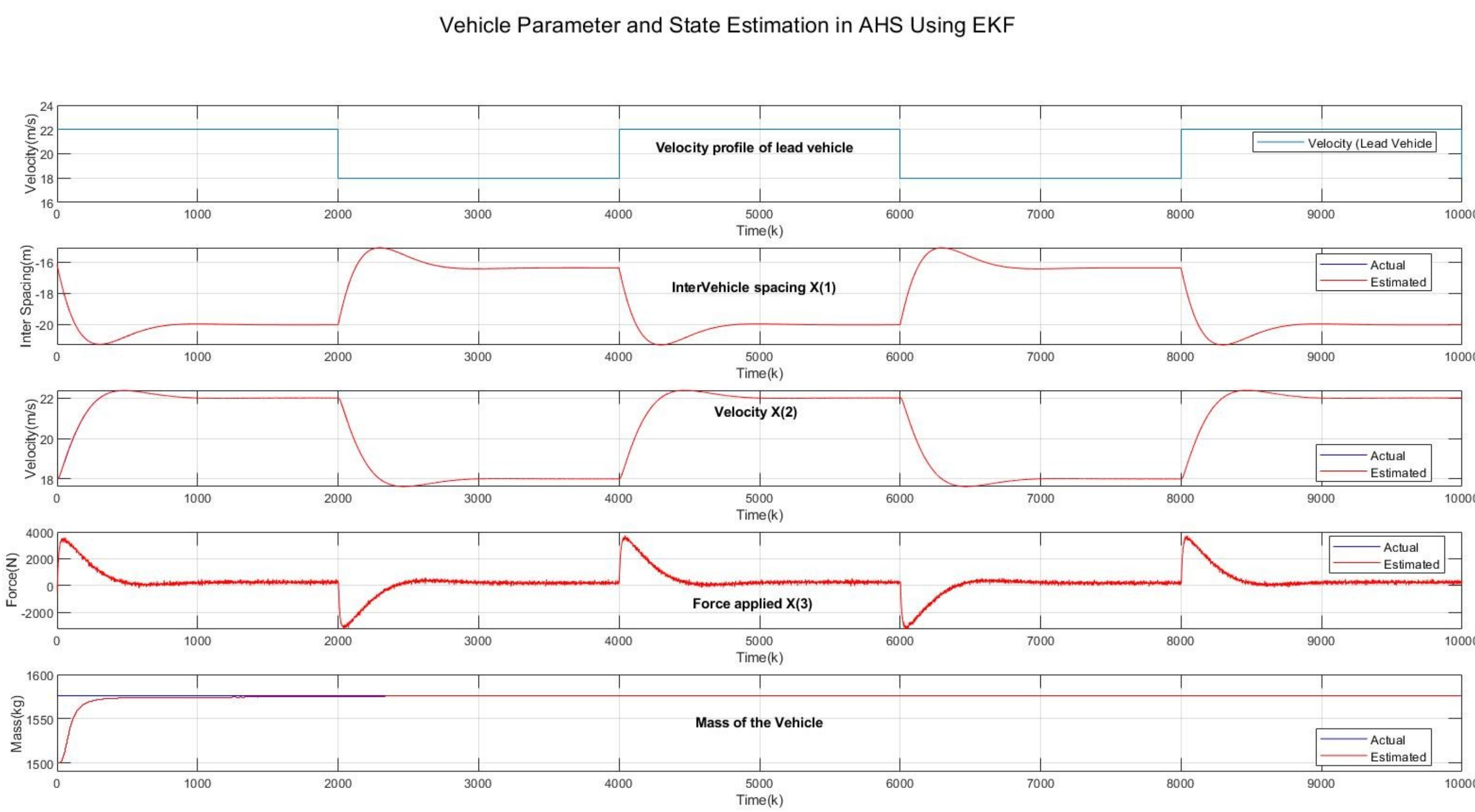

Fig 1: Response for speed change (Lead Vehicle) of 4m/s every 20sec

Comparing the relation between the states:

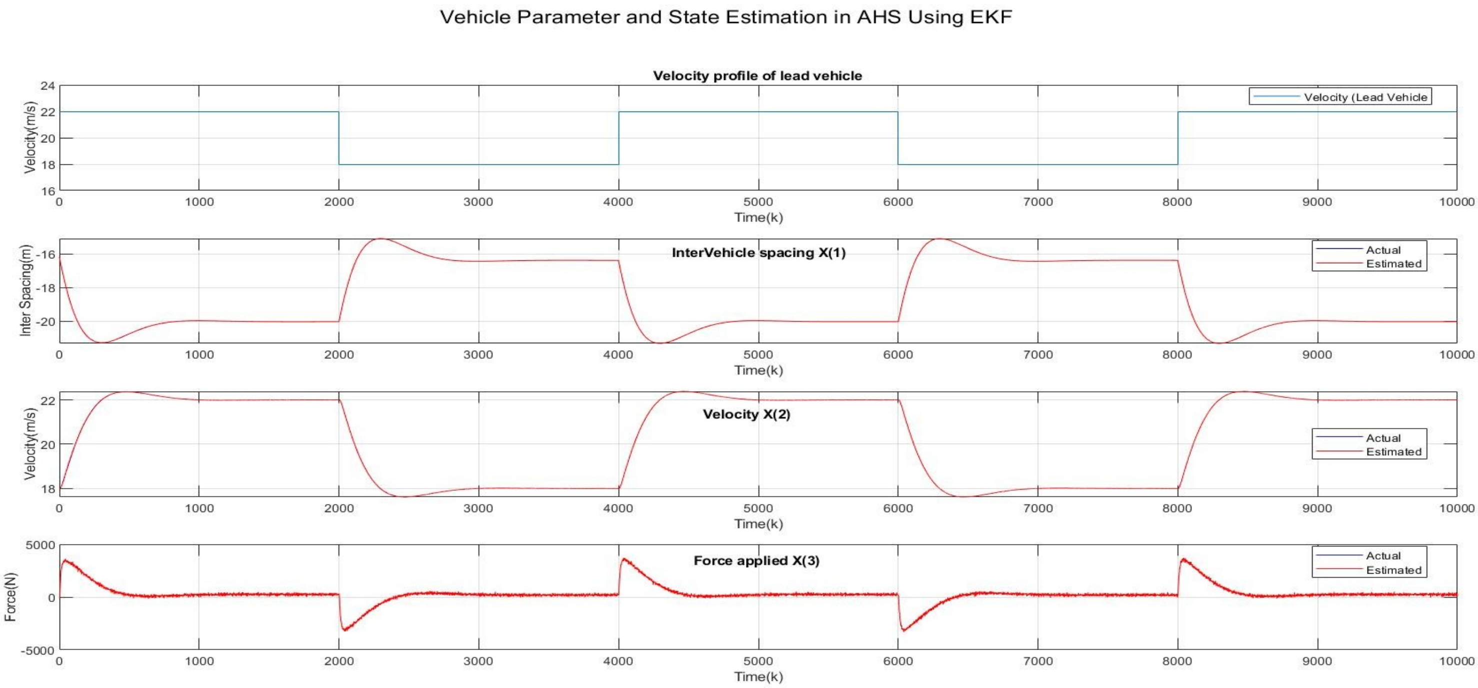

Fig 2: Comparison of State Variables

Inference:

We can infer that, as the speed of vehicle change (say decreases after 20sec), the interspacing is also reduced follows the same pattern (i.e. decreases, consider absolute value of the interspacing distance). Eventually, control action is applied as force (reduces), as a result the speed is reduced (reduced). Thus, the follow_vehicle always follows the velocity pattern of lead vehicle.

Comparison of response for different velocity profile of lead Vehicle (Change the step TIME)

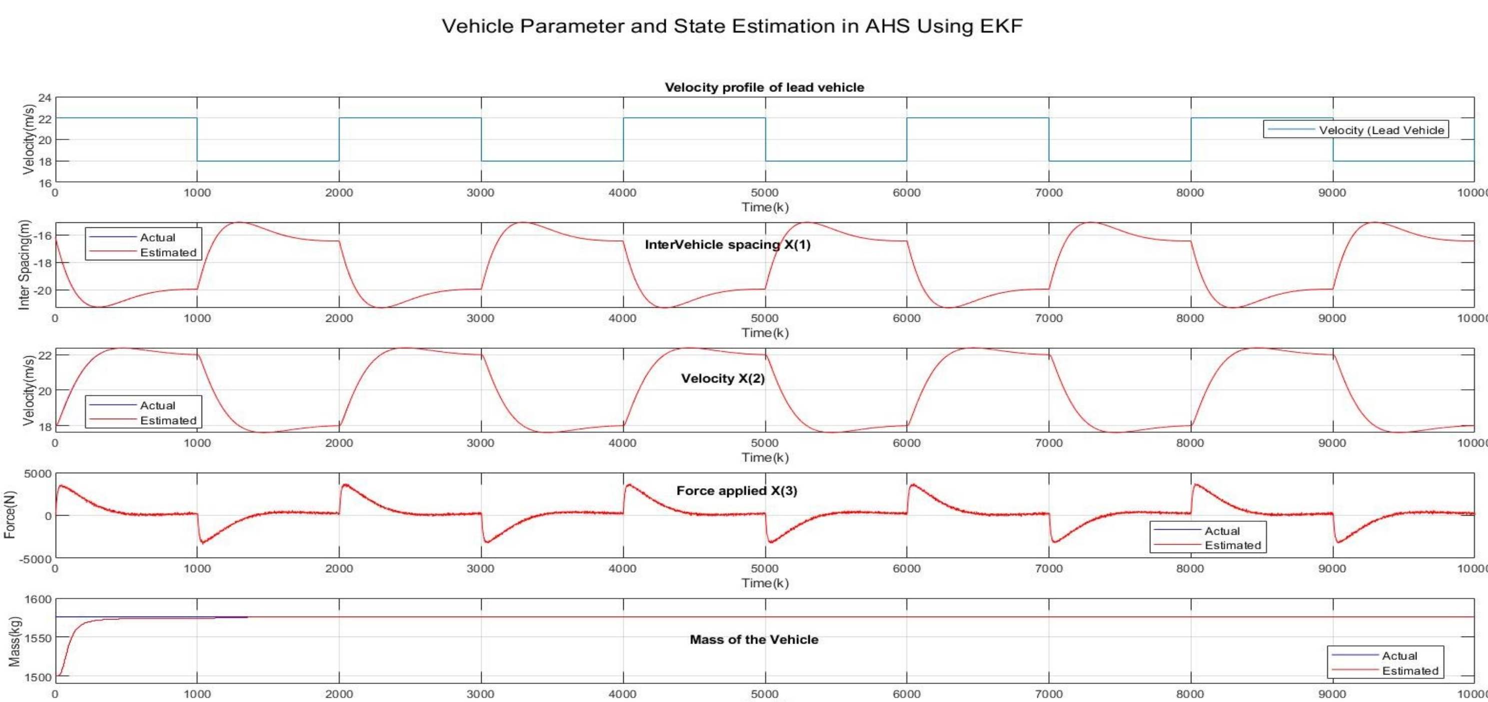

Fig 3: Response for speed change (Lead Vehicle) of 4m/s every 10sec

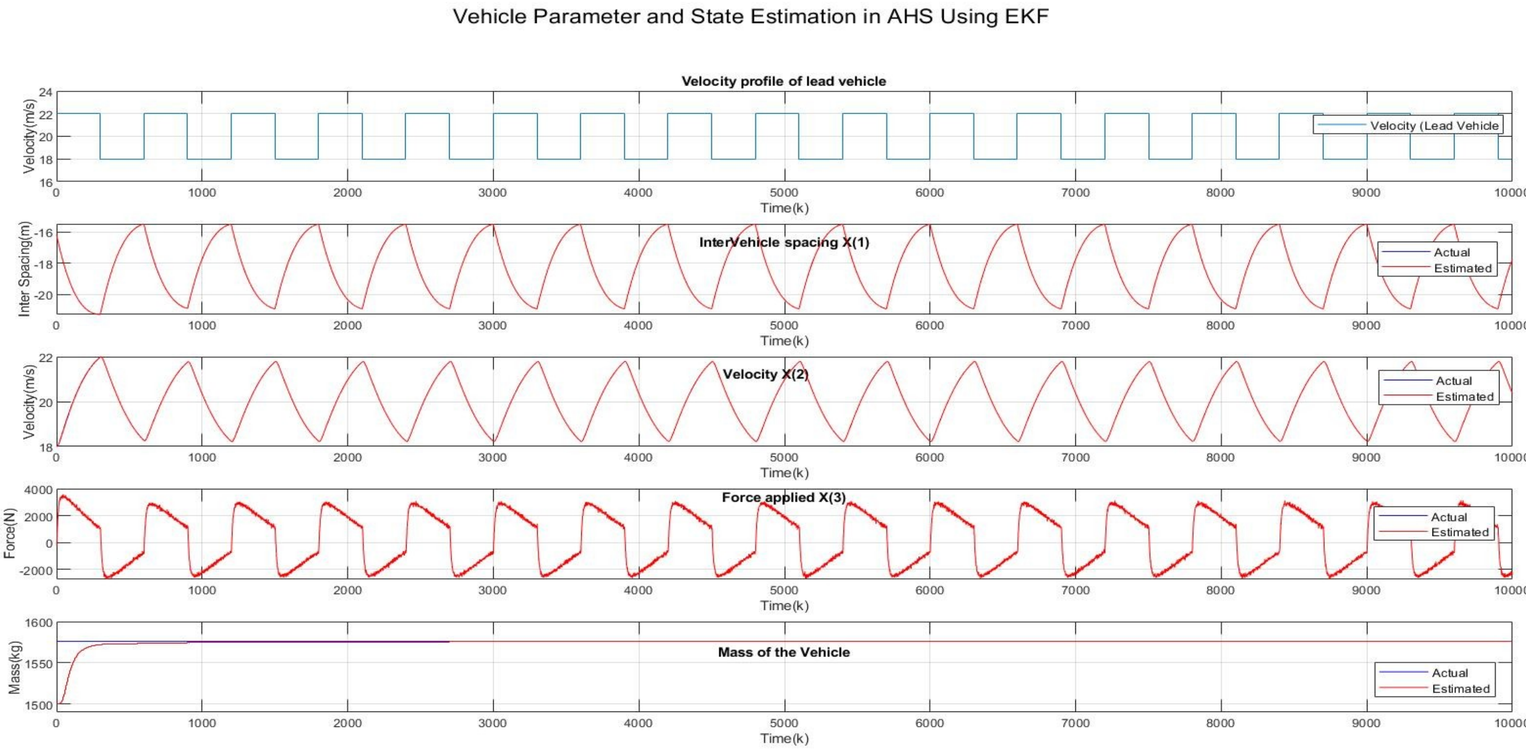

Fig 4: Response for speed change (Lead Vehicle) of 4m/s every 3sec

Inference:

From figure 1, 3, 4, we can infer that the rate of change of velocity of lead vehicle has huge impact on follow-vehicle. In fig 4, we can see that the velocity and force applied on the follow-vehicle is oscillatory in nature, which is practically undesirable. However, in figure 1, we can see a stable response, i.e. the follow-vehicle reaches a steady state before the speed changes. Thus, we can infer that slower change in velocity will leads to desirable response.

Comparison of response for different velocity profile of lead Vehicle (Change the step SIZE)

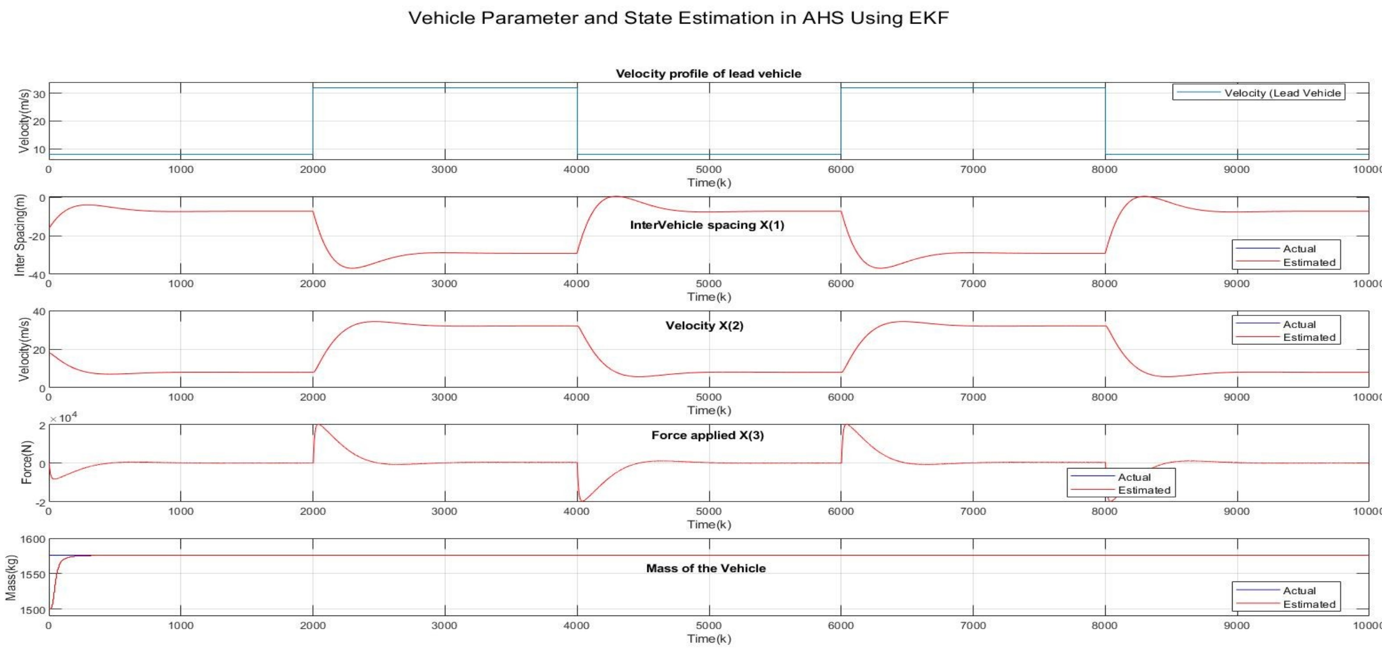

Fig 5: Response for speed change (Lead Vehicle) of 24 m/s every 20sec

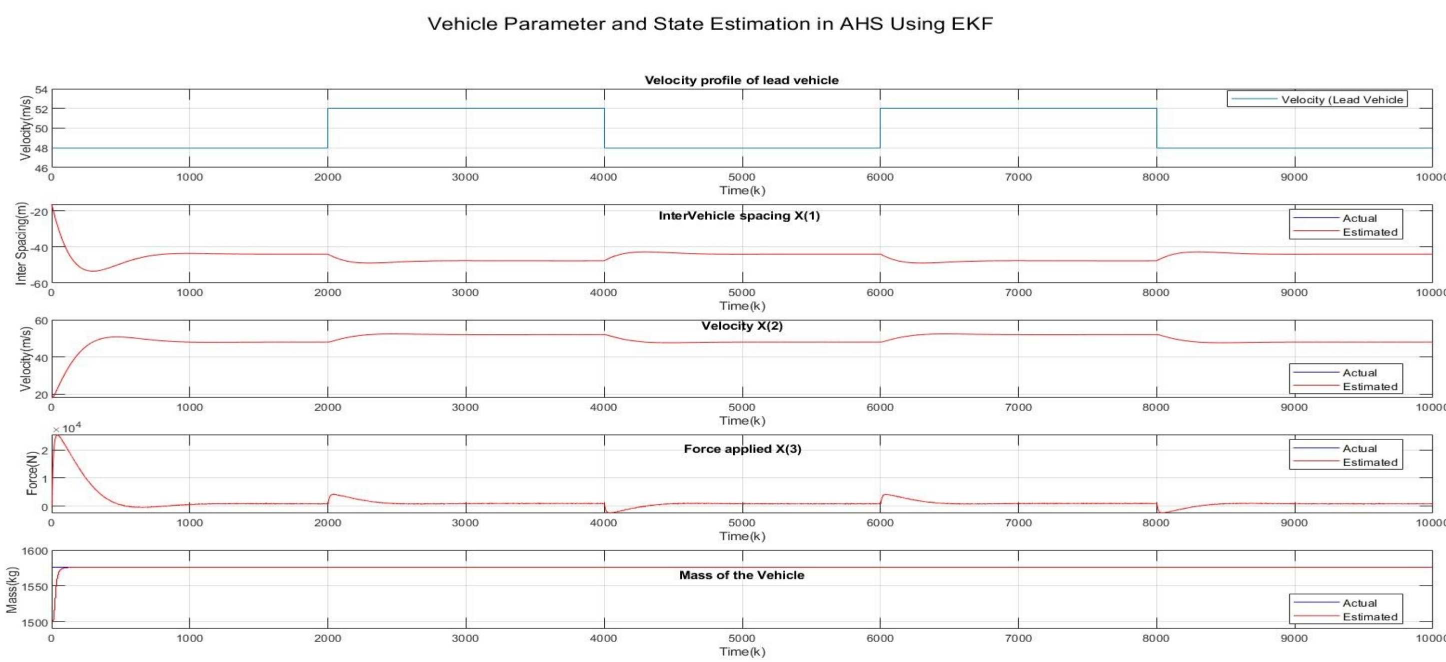

Fig 6: Response for speed change (Lead Vehicle) of 4 m/s every 20sec (But at higher average velocity)

Inference:

From figure 1, 5, 6, we can infer that for larger step size (24 m/s), the force (because of control action) goes to high values, which is pratically undesirable. However, when the step size is less (4 m/s) the force acting is in desirable range. NOTE: in this work, I did not include any constrain on Force, as a result, it systems look like it is stable. But in reality, we can not provide such high control action to bring our system to desire point.

Results:

For desirable respone,

- the velocity step size has to as small as possible.
- the velocity step time has to as long as possible.

For poor reponse

We can use large step size with short time period.

Reference:

https://home.wlu.edu/~levys/kalman_tutorial/