ECE 5554 - PowerTrain Control
Home Work 2

Aravind Chandradoss

1. Solution

1.1 Given

Following equations represent the linerized crank-angle domain equation for the idle speed control:

$$\frac{\mathrm{d}\delta p(\theta)}{\mathrm{d}\theta} = -K_3\delta p - \frac{K_2}{\omega_0^2}\delta w + \frac{K_1}{\omega_0}\delta\alpha \tag{1.1}$$

$$\frac{\mathrm{d}\delta\omega}{\mathrm{d}\theta} = -\frac{K_f}{J\omega_0^2}\delta\omega + \frac{1}{J\omega_0}\delta T - \frac{1}{J\omega_0}\delta T_l \tag{1.2}$$

$$\delta T = K_p\delta p(\theta-\theta_d) + K_v\delta_v \tag{1.3}$$

where:

- δin the equations represents the perturbation from the nominal values: (δ_0, v_0, T_l) and (p_o, ω_0, T_0);
- θ is the crank angle in radians;
- θ_d is the delay in radians;
- p is the manifold pressure in Pa;
- ω is the engine speed in rad/sec;
- α is the throttle angle in degrees;
- v is the spark angle advance in degrees;
- T is the engine indicated torque in Nm;
- T_l is the load torque (disturbance) in Nm;
- the values of K_* is given in table below.

Expression	**Value**
$K_1 = K_{p1}K_{thr}(2a_2\alpha_0 + a_1)$	$7584Pa/s^2/deg$
$K_2 = K_{p1}K_{thr}(a_2\alpha_0^2 + a_1\alpha_0 + a_0)$	$41790Pa/s$
$K_3 = K_{p2}$	$0.02671/deg$
$K_p = K_T b_1(c_2 v_0^2 + c_1 v_0 + c_0)$	$0.00233Nm/Pa$
$K_f = 2K_{fr1}\omega_0^2 + Kfr2\omega_0$	$0.553Nm/s$
$K_v = K_T(b_1 p_0 - b_0)(2c_2 v_0 + c_1)$	$1.2373Nm/deg$

1.2 Problem 1

Discretization of linearized crank angle domain equations (1.1), (1.2), (1.3) using bilinear transformation,

Bilinear Transformation: For Bilinear Transformation, we have to substitute (1.4) and (1.5) in (1.1), (1.2), and (1.3) and then, $Z-transform$ the resultant equations.

$$\frac{dx}{d\theta} = \frac{x(K\tau_0+\tau_0)-x(K\tau_0)}{\tau_0} = \frac{x(K+1)-x(K)}{\tau_0} \tag{1.4}$$

$$x = \frac{x(K\tau_0+\tau_0)+x(K)}{2} = \frac{x(K+1)+x(K)}{2} \tag{1.5}$$

Substituting (1.4) and (1.5) in (1.1),

$$\frac{p(k+1)-p(k)}{\tau_0} = -K_3\frac{p(k+1)+p(k)}{2} - \frac{K_2}{\omega_0^2}\frac{\omega(k+1)+\omega(k)}{2} + \frac{K_1}{\omega_0}\frac{\alpha(k+1)+\alpha(k)}{2} \tag{1.6}$$

Rearranging,

$$\left(\frac{1}{\tau_0}+\frac{K_3}{2}\right)p(k+1) + \left(\frac{-1}{\tau_0}+\frac{K_3}{2}\right)p(k) = -\frac{K_2}{2\omega_0^2}(\omega(k+1)+\omega(k)) + \frac{K_1}{2\omega_0}(\alpha(k+1)+\alpha(k)) \tag{1.7}$$

Similarly for (1.2)

$$\frac{\omega(k+1)-\omega(k)}{\tau_0} = -\frac{K_f}{J\omega_0^2}\frac{\omega(k+1)+\omega(k)}{2} + \frac{1}{J\omega_0}\frac{T(k+1)+T(k)}{2} - \frac{1}{J\omega_0}\frac{T_l(k+1)+T_l(k)}{2} \tag{1.8}$$

Rearranging,

$$\left(\frac{1}{\tau_0}+\frac{K_f}{2J\omega_0^2}\right)\omega(k+1) + \left(\frac{-1}{\tau_0}+\frac{K_f}{2J\omega_0^2}\right)\omega(k) = \frac{1}{2J\omega_0}(T(k+1)-T(k)) - \frac{1}{2J\omega_0}(T_l(k+1)+T_l(k)) \tag{1.9}$$

Similarly for (1.3), after rearranging,

$$T(k) = K_p p(k-n) + K_v v(k) \tag{1.10}$$

We know that (1.7) and (1.9) will be in form of,

$$C_{T1}p(k+1) + C_{T2}p(k) = -C_{T3}[\omega(k+1)+\omega(k)] + C_{T4}[\alpha(k+1)+\alpha(k)] \tag{1.11}$$

$$C_{T5}\omega(k+1) + C_{T6}\omega(k) = C_{T7}[T(k+1)+T(k)] - C_{T7}[T_l(k+1)+T_l(k)] \tag{1.12}$$

On Comparing, (1.7) and (1.9) with (1.11) and (1.12),

$$C_{T1} = \left(\frac{1}{\tau_0} + \frac{K_3}{2}\right) \qquad C_{T2} = \left(\frac{-1}{\tau_0} + \frac{K_3}{2}\right) \qquad C_{T3} = \frac{K_2}{2\omega_0^2}$$

$$C_{T4} = \frac{K_1}{2\omega_0} \qquad C_{T5} = \left(\frac{1}{\tau_0} + \frac{K_f}{2J\omega_0^2}\right) \qquad C_{T6} = \left(\frac{-1}{\tau_0} + \frac{K_f}{2J\omega_0^2}\right)$$

$$C_{T7} = \frac{1}{2J\omega_0}$$

The above C_i represents the needed coefficients.

2. Problem 2

2.1 Transfer Function

A.Applying $Z - Transform$ on equations (1.10), (1.11) and (1.12),

$$(C_{T1}Z + C_{T2})p(z) = -C_{T3}(Z+1)\omega(z) + C_{T4}(Z+1)\alpha(z) \tag{2.13}$$

$$(C_{T5}Z + C_{T6})\omega(z) = C_{T7}(Z+1)T(z) - C_{T7}(Z+1)T_l(z) \tag{2.14}$$

$$T(z) = K_p Z^{-n} p(z) + K_v v(z) \tag{2.15}$$

Since, we assumed the system is linear, we can remove T_l and v from equation, (2.14), (2.15) will become

$$(C_{T5}Z + C_{T6})\omega(z) = C_{T7}(Z+1)T(z) \tag{2.16}$$

$$T(z) = K_p Z^{-n} p(z) \tag{2.17}$$

Substituting (2.17) in (2.16)

$$(C_{T5}Z + C_{T6})\omega(z) = C_{T7}K_p(Z+1)Z^{-n}\ p(z) \tag{2.18}$$

Substituting $p(z)$ from (2.18) in (2.13),

$$(C_{T1}Z + C_{T2})\frac{(C_{T5}Z + C_{T6})\omega(z)}{C_{T7}K_p(Z+1)Z^{-n}} = -C_{T3}(Z+1)\omega(z) + C_{T4}(Z+1)\alpha(z) \tag{2.19}$$

Rearranging,

$$\frac{\omega(z)}{\alpha(z)} = \frac{C_{T4}C_{T7}K_p(Z^2+2Z+1)}{C_{T5}C_{T1}Z^{n+2} + (C_{T5}C_{T2} + C_{T1}C_{T6})Z^{n+1} + C_{T6}C_{T2}Z^n + C_{T3}C_{T7}K_p(Z^2+2Z+1)} \tag{2.20}$$

Taking n=3, as $\tau_0 = 60degree$ and torque delay $\theta_d = 180degree$, $n = \frac{\tau_0}{\theta_0} = 3$
Thus, we get,

$$\frac{\omega(z)}{\alpha(z)} = \frac{C_{T4}C_{T7}K_p(Z^2+2Z+1)}{C_{T5}C_{T1}Z^5 + (C_{T5}C_{T2}+C_{T1}C_{T6})Z^4 + C_{T6}C_{T2}Z^3 + C_{T3}C_{T7}K_p(Z^2+2Z+1)} \tag{2.21}$$

The above equation represents the transfer function from throttle angle α to speed ω.

2.2 B. Numerical Values

On substituting the values of C_is in MatLab we get,

$$\frac{\omega(z)}{\alpha(z)} = \frac{0.007955Z^2 + 0.01591Z + 0.007955}{0.9251Z^5 - 1.824Z^4 + 0.8987Z^3 + 0.0005224Z^2 + 0.001045Z + 0.0005224} \tag{2.22}$$

2.3 Z-plane

From matlab function **pzmap,**the poles were found to be inside the unit circle. The locations of poles are as follows,

$$\mathbf{2-Zeros}: (-1+0i), (-1+0i) \tag{2.23}$$

$$\mathbf{5-Poles}: (-0.0754+0i), (0.0354 \pm 0.08i), (0.988 \pm 0.046i) \tag{2.24}$$

The plot representing the poles and zeros is shown below 1.

3. Problem 3

3.1 Open Loop System Block Diagram

Using the following equations, the transfer function blocks are created in simulink. The system is treated as **SISO** system, that is, effect of spark is not considered. Once, the equations are rearragned, we get the needed transfer function as follows,

$$(C_{T1}Z + C_{T2})p(z) = -C_{T3}(Z+1)\omega(z) + C_{T4}(Z+1)\alpha(z) \tag{3.25}$$

$$(C_{T5}Z + C_{T6})\omega(z) = C_{T7}(Z+1)T(z) - C_{T7}(Z+1)T_l(z) \tag{3.26}$$

$$T(z) = K_pZ^{-n}p(z) \tag{3.27}$$

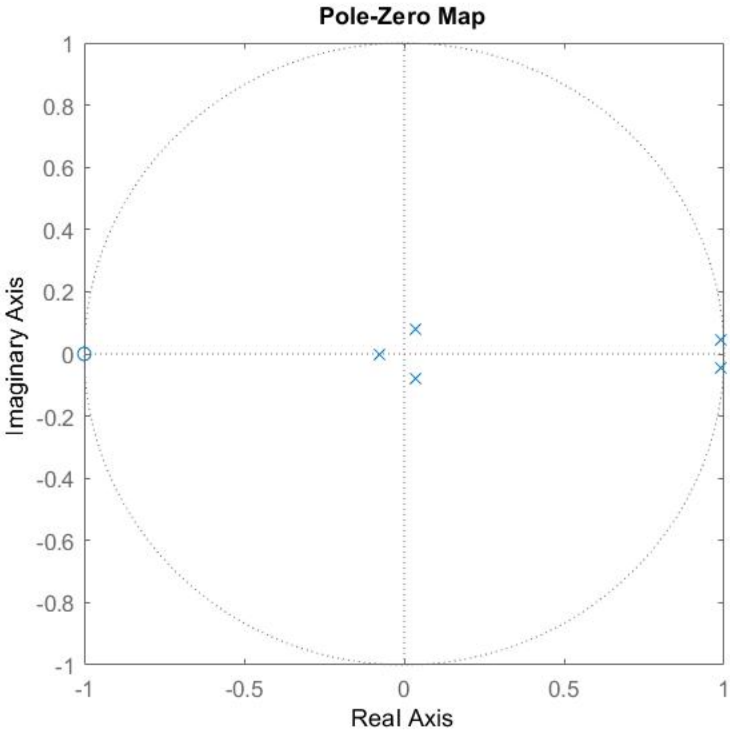

Figure 1: Zeros and Poles in Z-plane

The transfer function (Asked) blocks are listed below,

$$TF1 = TF2 = \frac{Z+1}{C_{T1}Z + C_{T2}}$$
$$TF3 = TF4 = \frac{Z+1}{C_{T5}Z + C_{T6}}$$

The completed simulink block diagram is shown below 2,

4. Problem 4

4.1 Open Loop Response of the system

This problem is continuation of Problem 3. Here, we give disturbance as a change in Load torque and remove it later (Positive load is given at $\theta = 50\tau_0$ and removed at $\theta = 1000\tau_0$)

The simulation timing and settings are adjusted accordingly. The simulations results are as follows 3 (The initial nominal values are added),

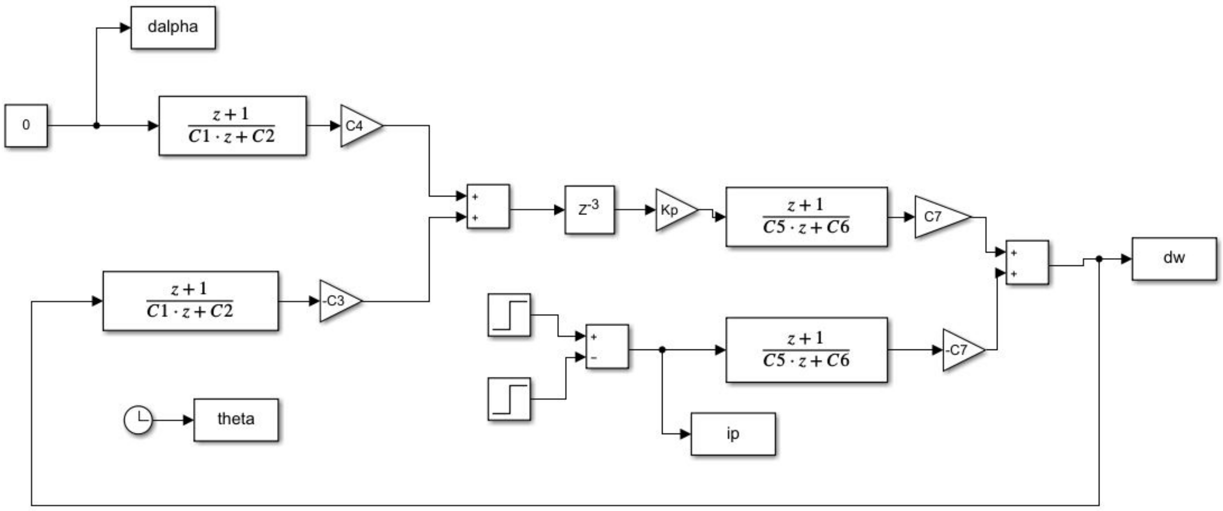

Figure 2: Simulink Block Diagram

From fig 3, as expected, the speed dropped when we load was applied. And when the load is removed, the speed is regained and reached the idea speed of 800 RPM.

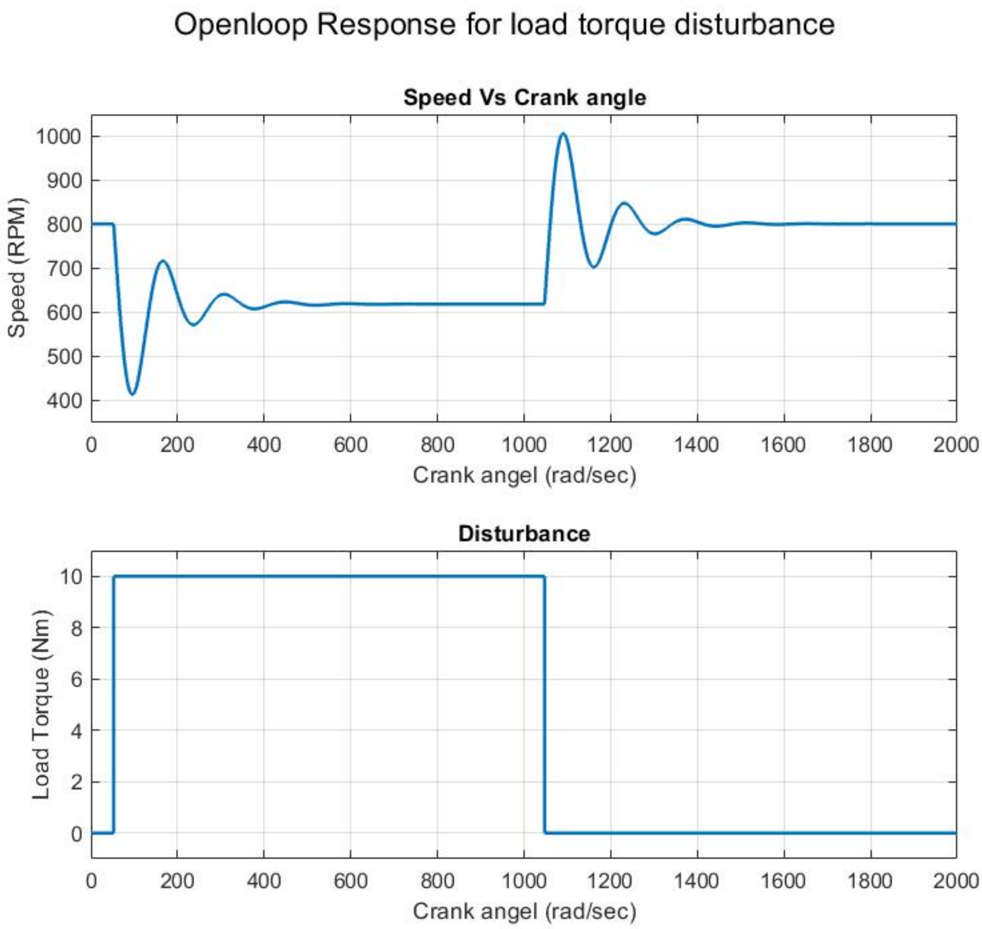

Figure 3: Open Loop Response for Load Torque disturbance