

Assumption:

The following experiment uses the idle-speed engine model.

$$\begin{aligned}
 \dot{m}_{thr} &= \left[\frac{P_{amb}}{\sqrt{RT_{amb}}} \cdot \sqrt{\gamma} \cdot \sqrt{\left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \right] \cdot (a_2 g_{thr}^2 + a_1 g_{thr} + a_0) & 1 \\
 \frac{dp_{IM}}{dt} &= \frac{RT}{V_{IM}} (\dot{m}_{thr} - \dot{m}_{eng}) & 2 \\
 \dot{m}_{eng} &= \frac{(s_i p_{IM} - y_i) V_d N}{120 RT} & 3 \\
 T_{ind} &= \frac{V_d}{4\pi \cdot 10^{-5}} [a_1 MAP(t - t_d) + a_0 [b_2 g_{spark}^2 + b_1 g_{spark} + b_0]] & \\
 t_d &= \frac{120}{Z \cdot N} & 4 \\
 \frac{dN}{dt} &= \frac{30}{\pi J} [T_{ind} - T_{load} - T_{fr}(N)] & 5 \\
 T_{fr} &= c_2 N^2 + c_1 N + c_0 &
 \end{aligned}$$

Problem 1-a:

The steady state values of MAP, Indicated Torque and Engine speed, while changing External torque (load) in step of 5 Nm (0 Nm → 20 Nm) with a fixed throttle angle (10°) and spark advance (25° bTDC), are shown below.

TABLE 1: Steady State values of the engine parameters for the fixed Throttle angle and Spark advance

External Torque [Nm]	MAP [kPa]	Indicated Torque [Nm]	Engine Speed [RPM]
0	22.23	11.62	1070
5	24.32	16.48	934.7
10	26.43	21.40	828.9
15	28.55	26.34	744.2
20	30.68	31.30	674.9

For fixed throttle angle (10°) and spark advance (25° bTDC)

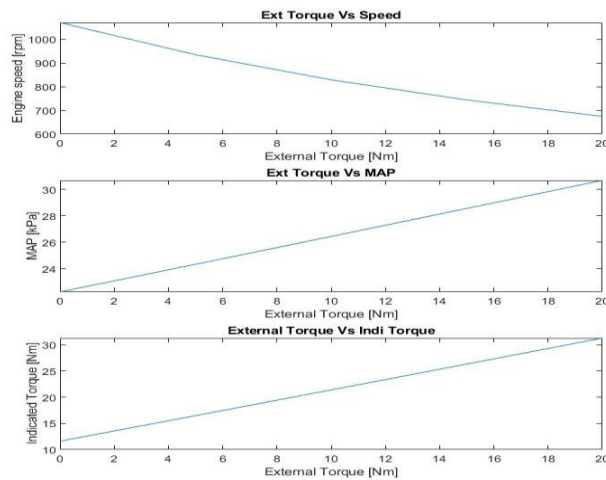


Fig 1: Steady State values for fixed Throttle angle and Spark advance

Inference:

From the above graph, it can be seen that, for a given throttle angle (10°) and spark advance (25° bTDC), the Indicated torque and MAP are directly proportional to external torque (load), while the speed is inversely related to external torque. In other words, using equations we can say that, as the external load torque increases the speed is reduced [equation 5] (considering the sign of the differential equation 5, or the slope), which in turn reduces the $M_{\dot{t}_{eng}}$ [equation 3] and MAP [equation 2] (considering the sign of differential). Indicated torque follows the pattern of MAP [equation 4] and reduces as we increase the external load torque. It can also be intuitively understood that, for the mentioned condition, as we increases the load torque the engine would not be able to deliver enough power, as a result, the speed reduces. This reduction in speed will reduces the air intake to the engine resulting in increased MAP and Indicated torque.

Problem 1-b:

The steady state values of MAP, Indicated Torque and Engine speed, while changing Throttle angle (from $7^\circ \rightarrow 16^\circ$) with the fixed external torque (10 Nm) and spark advance (25° bTDC), are shown below.

TABLE 2: Steady State values of engine parameters for the fixed External load torque and Spark advance

Throttle Angle [degree]	MAP [kPa]	Indicated Torque [Nm]	Engine Speed [RPM]
7	26.40	21.36	489 (Engine stalls)
8	26.37	21.26	576.6
10	26.43	21.40	828.9
12	26.57	21.73	1164
14	26.84	22.35	1573
16	27.28	23.37	2040 (exceeds boundary limit)

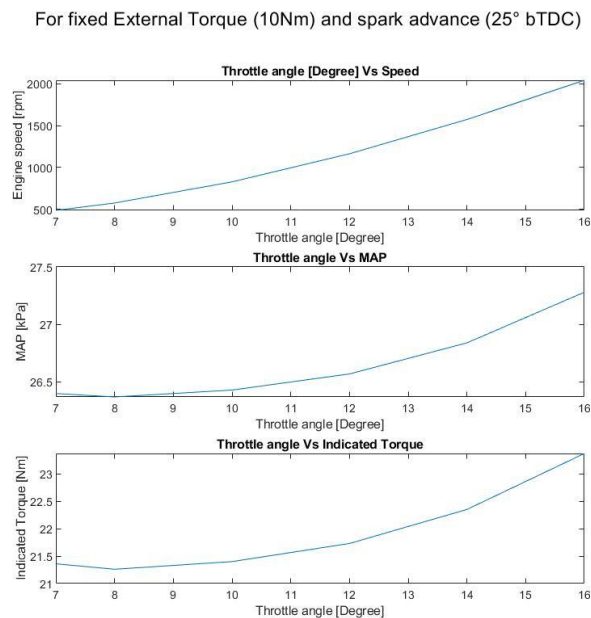


Fig 2: Steady State values for fixed External load torque and Spark advance

Inference:

From the graph, we can infer that, for the fixed external load torque (10Nm) and spark advance (25° bTDC), the steady state values (overall values) of Engine speed, MAP and Indicated torque are directly proportional to throttle angle. But if we look closely, in our first case for throttle angle 7, the resultant speed is undesirable (489 rpm). As a result of low speed (<500 rpm), the engine will eventually stall. Similarly, in the last case (throttle angle 16), the engine speed reaches 2040, which exceeds the boundary value (1600rpm). Therefore, we have to note that the actual values at the high speed might be different.

Boundary values for the model:
 $6 \text{ deg} < \Theta_{thr} < 20 \text{ deg}$
 $0 \text{ deg} < \Theta_{sp} < 30 \text{ deg}$
 $500 \text{ r/min} < N < 1600 \text{ r/min}$

The reason for the stall can be explained by [equation 2], in which, due to small throttle input the mass flow $M^o_{throttle}$ (supply) is lesser than M^o_{eng} (demand). i.e. the sign of equation will become negative and MAP will decrease gradually. This low air supply will eventually lead to stall of the engine. The remaining overall response can also be understood using other mathematical equations, as the throttle angle increases, mass flow rate $M^o_{throttle}$ increases [equation 1]. In turn, MAP is increased, as the sign of [equation 2] becomes positive at high throttle. Eventually, Indicated torque also increases by equation 5 along with MAP.

Problem 1-c:

The steady state values of MAP, Indicated Torque and Engine speed, while changing spark advance in step of 5° (from 5 → 30°) with fixed throttle angle (10°) and external load torque (10Nm), are shown below.

TABLE 3: Steady State values of engine parameters for the fixed Throttle angle and External load torque

Spark Advance [Degree]	MAP [kPa]	Indicated Torque [Nm]	Engine Speed [RPM]
5	23.53	21.53	981.8
10	23.48	21.53	984.7
15	23.81	21.51	964.6
20	24.64	21.47	917.1
25	26.43	21.40	828.9
30	30.94	21.30	667.5

For fixed throttle angle (10°) and Ext Torque (10Nm)

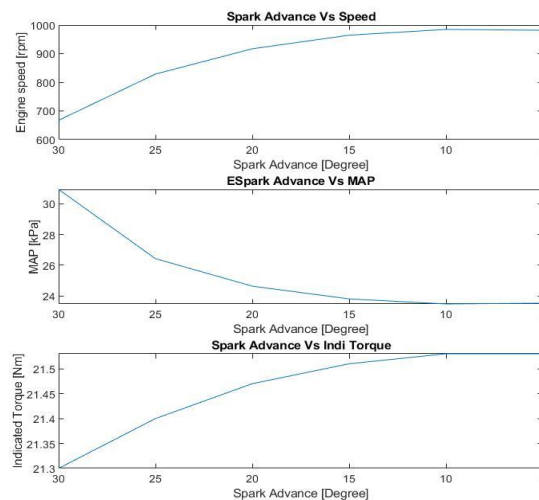


Fig 3: Steady State values for fixed Throttle angle and External load torque

Inference:

From fig 3, we can infer that as we reduce the spark advance, the speed initially increases and reaches a peak (at spark advance 10° bTDC). After the peak, the speed starts to reduce. The same can be intuitively understood that as we reduce the spark advance, we are allowing the piston to compress the fuel-air mixture before ignition. As the result of the increased compression, the generated force by explosion is comparatively higher. But, after reaching a peak value (say spark advance of 10°) the same concept is reversed i.e. after the peak value the speed starts to reduce. Thus, for the above mentioned settings, the spark advance of 10° is the best. The response of MAP can be inferred using [2 and 3], as the speed is increased, M_{eng}^o is increases, which eventually makes the equation 2 to be negative (Slope). In other words, as we reduces the spark advance, the Steady state value of MAP decreases. Similarly, we can infer that Indicated torque increases as we reduce the spark advance from [4].

For the above mentioned settings, the best spark advance value is 10° .

Problem 2:

The throttle angle needed to produce engine speed of 800 RPM with fixed spark advance of 25° bTDC is given by table 4.

TABLE 4: The desired needed throttle angle for various external torque

External load Torque [Nm]	Throttle angle [Degree]
0	8.3738
5	9.1319
10	9.8013
15	10.4072
20	10.9650

From the table, it can be seen that to delivery more load, we need to increase the throttle angle. Intuitively, we can say that to produce more torque and to maintain the speed, we need to increase Indicated Torque (Spark advance is fixed at 25° bTDC). For which, we need to increase the MAP (i.e. positive sign for equation 2). To have positive sign for equation 2, we have increase $M_{throttle}^o$ by increase the throttle angle (Since M_{eng}^o will fairly remain constant as the speed is 800rpm).

In short, to achieve the desire speed (800 rpm) for increasing load, we have to increase the throttle angle. The desired throttle angles for various loads are shown in table 4.

Problem 3:

For this problem, the values of throttle angle and spark advance are fixed at 9.8013° (from problem 2) and 25° bTDC respectively, and the external torque is varied using step functions. The response of the engine parameters are shown below,

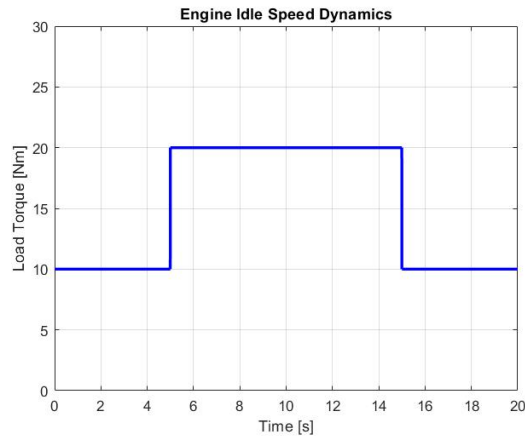


Fig4: Load Torque

The external load torque (input) is varied using step function as shown in fig 4. The responses of other variable are discussed below.

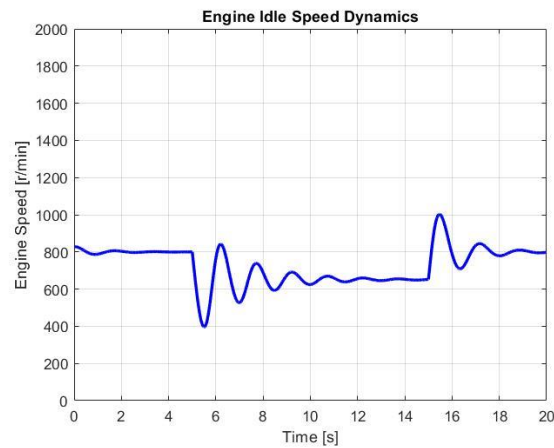


Fig 5: Response of engine speed

From previous problem, we know that as we increase the load, the engine speed reduce. In this specific condition as we turn on the A/C, the speed of the engine reduces to 640 rpm approx., for an additional load of 10Nm, as shown in figure 5. Converse results can be seen while turning off the A/C, the speed of the engine increases and returns to the desired value of 800rpm (Problem 2).

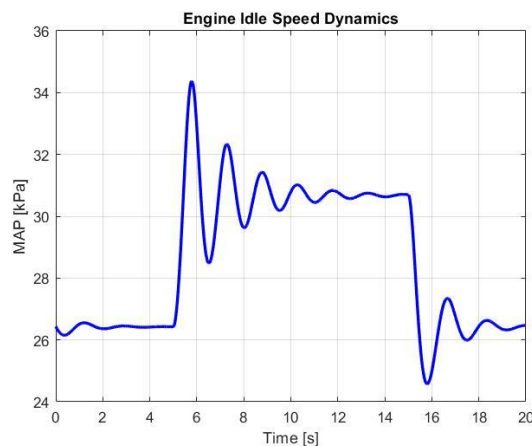


Fig 6: Response of MAP

We know that, when speed is reduced the mass flow rate M_{eng}^o reduces, as a result, the sign of equation 2 become more positive (slope increases) (i.e. MAP increases). In this case, the value of MAP is increased from 26 kPa to 31 kPa (approx.) for an additional load of 10Nm. The same can be inferred in figure 6. Similarly, when the load is removed, the speed is reduced, which eventually decreases MAP.

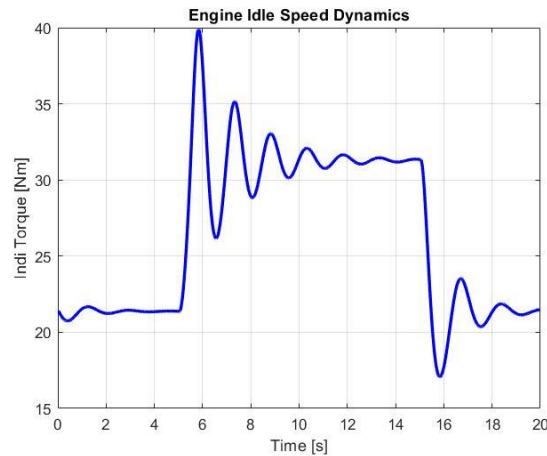


Fig 7: Response of Indicated Torque

We know that, for fixed spark advance and for nominal speed change (in this case, speed is fairly constant), the indicated torque is directly proportional to MAP [equation 4]. In other words, the Indicated torque is increased for additional load. The same is reflected in the figure 7. Thus, for additional load, the Indicated Torque is noticed to increase from 21 Nm to 31 Nm (approx.).

Problem 4:

In the same setup for problem 3, a feed forward controller is implemented using a look-up table (The same can be implemented using step functions). The lookup table is filled using the values in table 5 with an assumption that spark advance is 25° bTDC,

TABLE 5: Desired throttle values for various load condition

External Load torque [Nm]	Throttle Angle
10	9.8013
20	10.9650

The system response with feed forward controller (Throttle Angle) is shown below,

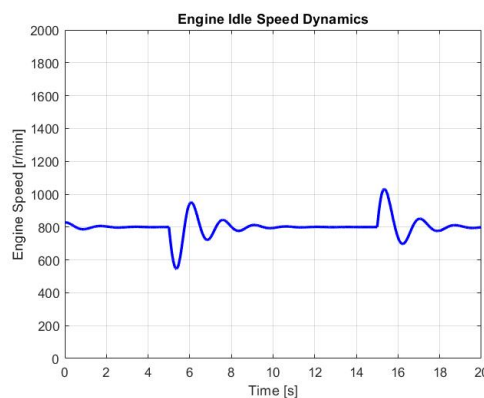


Fig 8: Response of Engine Speed (Control objective)

From fig 8, we can infer that the feed forward controller is working as expected. i.e. the desired speed of 800 rpm is maintained for the given load disturbance.

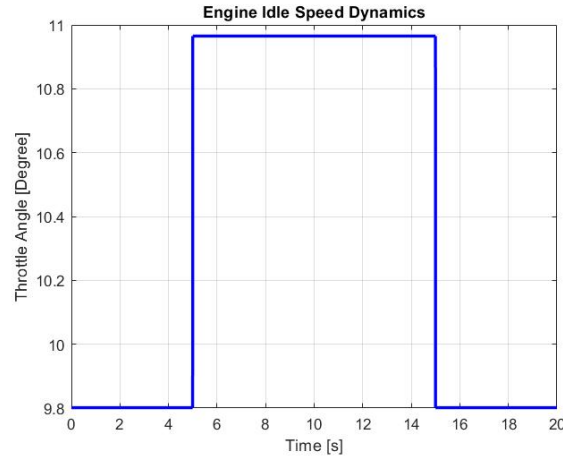


Fig 9: Throttle angle (Controller output / Manipulating Variable)

In order to maintain the desired speed, the throttle is varied as shown in fig 9. The throttle values for various load conditions are shown in Table 5. In general, the throttle value is increased for increasing load or vice-versa.

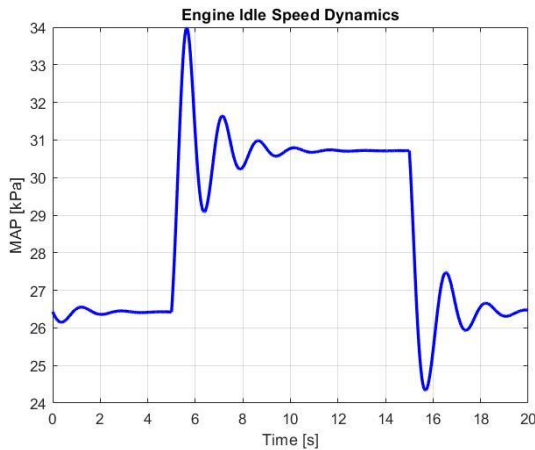


Fig 10: Response of MAP

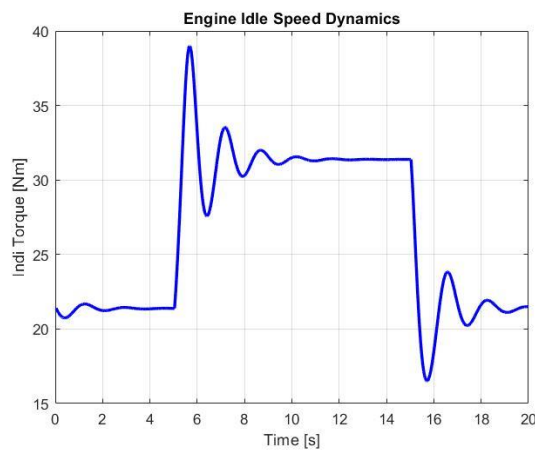


Fig 11: Response of Indicated Torque

As the throttle angle is increased more air is fed in, as a result, the pressure (MAP) increases. In other words, equation 1 increases and results in positive sign in equation 2 (i.e. MAP is increasing). The inverse is seen when the load is removed. The MAP value is approx. changing from 26.5 kPa to 31 kPa (approx.). Similarly, Indicated torque also increases for increasing load. We know, for given spark angle and nominal speed change, the indicated torque is directly proportional to MAP, which can be seen in fig 11.

In short, feedforward controller is implemented and desired speed for various load condition is achieved.

Problem 5:

This problem is similar to problem 4, except that we will be using spark advance to reach the desired response. The throttle angle is fixed to 9.8013° (from problem 2). The feedforward controller is designed to take load torque as input and to generate corresponding control signal (spark advance) using the values shown in table 6. The same can also be implemented using step functions.

TABLE 6: Desired Spark advance for various Load torque (for a given throttle angle)

External load torque [Nm]	Spark Advance [Degree]
10	25
20	11.54

The response of the system with feedforward controller (Spark Advance) is shown below,

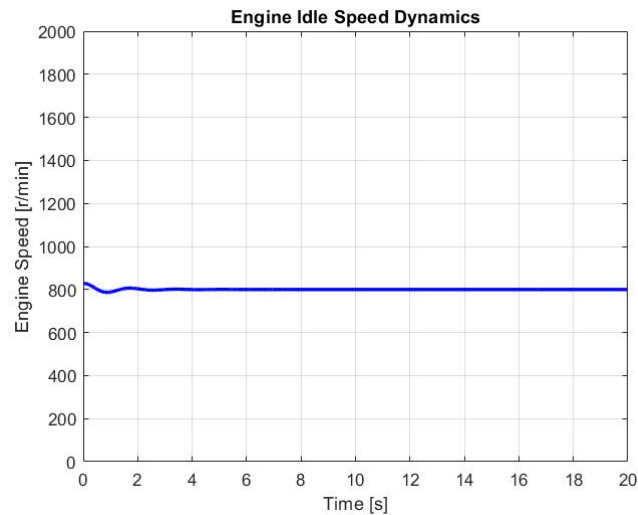


Fig 12: Response of Speed (Control Objective)

From fig 12, we can infer that the engine speed is maintained at the desired value of 800 rpm with the help of feed forward controller. If we compare the same with fig 8, we can notice that effect of load disturbance on speed is lesser if the feedforward controller use the spark advance as the manipulating variable. Therefore, if we want a steady output speed, controlling the spark advance would be a better choice than to adjust the throttle angle.

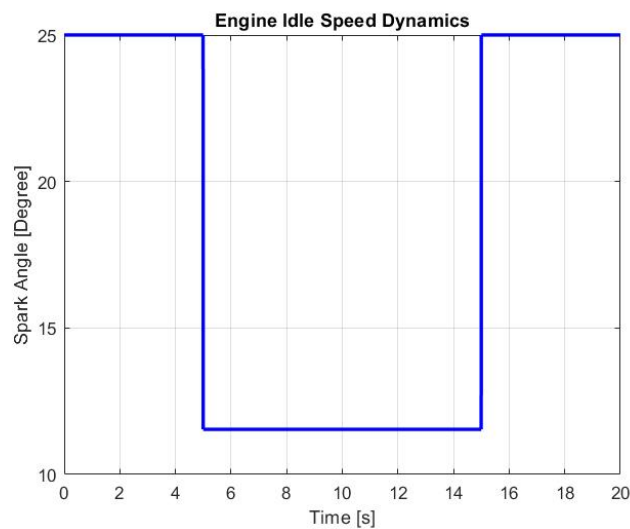


Fig 13: Spark Advance Angle (Controller output / Manipulating Variable)

Fig 13 shows the controller action (change of spark advance) in order to achieve the desire response. We know, from problem 1, one can achieve desire torque by adjusting the spark advance angle. The values of spark advance is changed as shown in table 6.

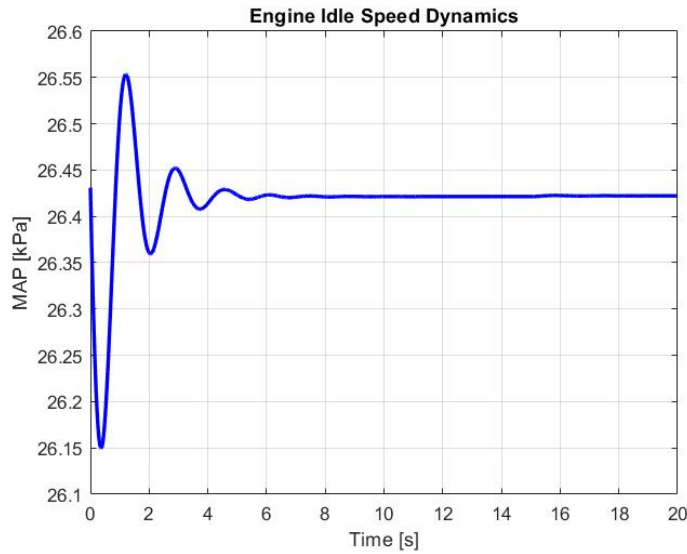


Fig 14: Response of MAP

We can infer from fig 14 that, although there is a huge fluctuation in the initial phase, the effect of disturbance is not much as compared to previous feedforward method (problem 4). The above effect can be inferred that, the needed speed to compensate the added load is gained using spark advance instead of feeding more air. As a result, the MAP fairly remains the same for the entire duration (once MAP is settled). The same can be also understood using the equation 2.

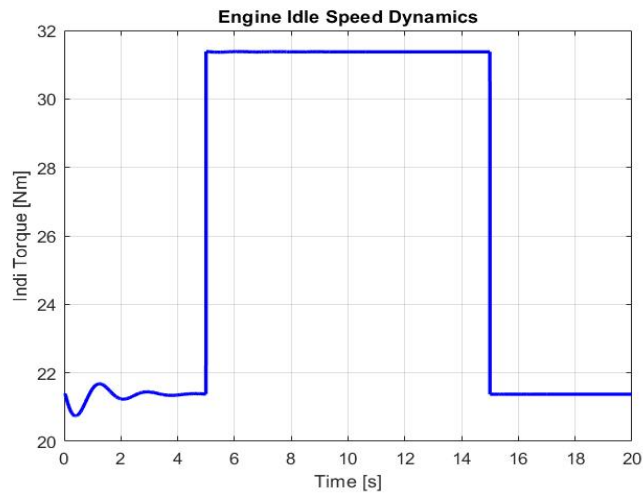


Fig 15: Response of Indicated Torque

From fig 15, we can infer that as the spark advance is reduced, the indicated torque is increased. That is the air-fuel mixture is compressed more before ignition, which results in higher output torque. The same can also be inferred using equation 4.

In short, the desire response is achieved by using feedforward controller using spark advance as the manipulating variable.

Problem 6:

In this problem, feedforward controller is used to control the throttle angle using the external load torque. The desired value for the feedforward controller is shown in the table 7.

TABLE 7: Desired Throttle angle for various load torque

External Load Torque [Nm]	Throttle Angle [degree]
0	8.3738
10	9.8013
20	10.9650

It is assumed that spark advance angle is 25° bTDC.

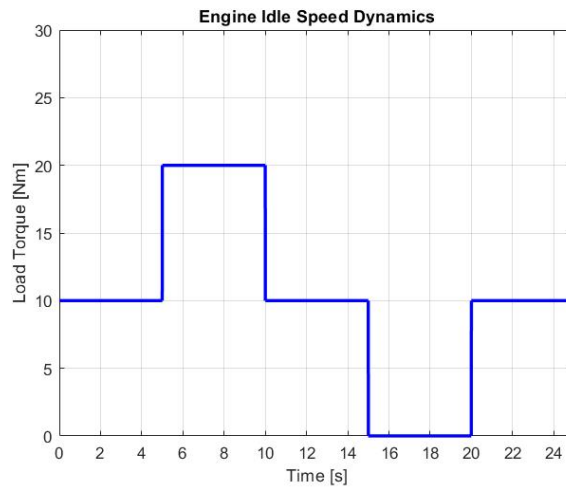


Fig 16: Load torque (Input)

The external load torque is varied as shown in the fig 16. The response with feedforward controller is shown below,

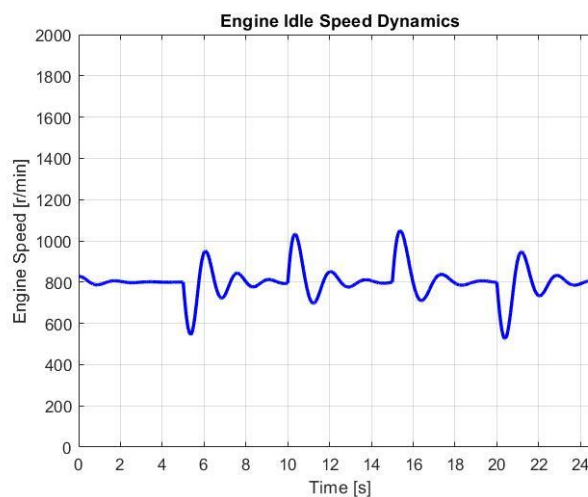


Fig 17: Response of Speed

From the fig 17, we can infer that for all the load torque condition, the speed is fairly in the desired range (approx. 800 rpm). The oscillation can be further reduced by controlling the spark advance instead of

throttle angle. It can be intuitively understood that, if we are controlling the throttle angel, the flow of signal is as follows [air → throttle → intake manifold → ...(engine)... → spark → explosion → torque → speed). As a result, if we control throttle angle, the systems take more time to reach steady state (Speed). In case if we use spark advance, then we can neglect the effect of initial components and directly (comparatevely faster) control the torque. As a result, the reponse of the speed for varing load will be steady, as we seen in problem 5.

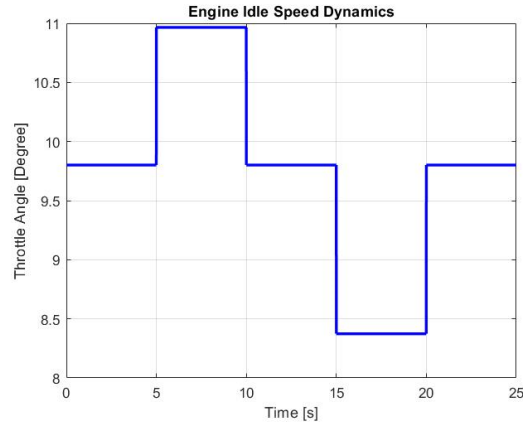


Fig 18: Throttle Angle (Controller output / Manipulating Variable)

The control action for varying load is shown in fig 18. As the load increases, the throttle angle is increased to maintain the desire speed. The values of throttle angle is shown in Table 7.

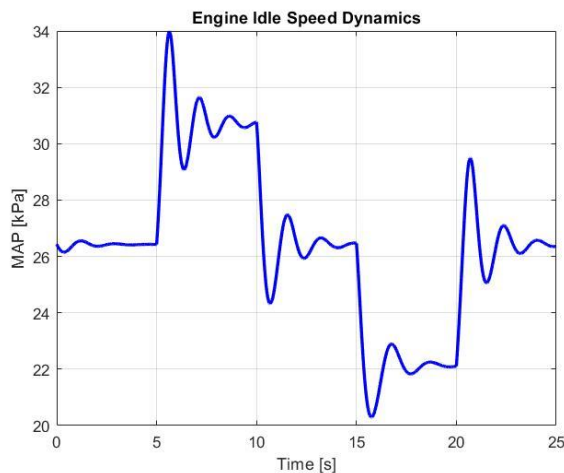


Fig 19: Response of MAP

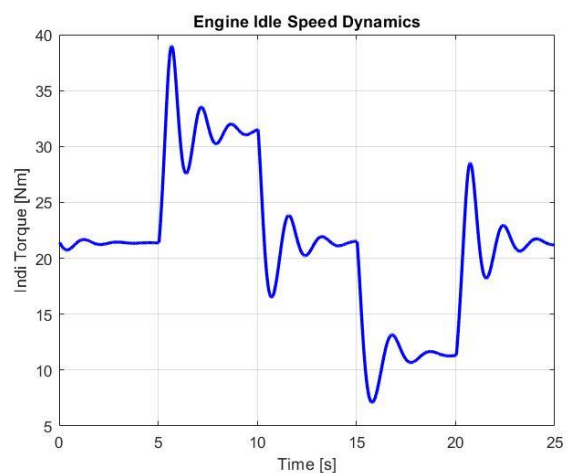


Fig 20: Response of Indicated Torque

From fig 19, we can infer that MAP follows the throttle angle. We know from [1 and 2] that MAP increases with increase in throttle angle. Since the speed and spark advance are also maintain constant, we can expect [from 4] that the indicated torque to follow the same patter as of MAP, which is evident from fig 20.

In short, a feedforward controller to maintain the desire speed of 800 rpm is implemented for varying external load torque by controlling the throttle angle with a fixed spark advance of 25° bTDC.

Recommendation: The response of feedforward controller can be intuitively improved by controlling the spark advance instead of throttle angle. Here, the taken control objective is to maintain the speed without oscillation.