ECE 5554 - PowerTrain Control
Home Work 2

Aravind Chandradoss

1. Solution

1.1 Given

Following equations represent the linerized crank-angle domain equation for the idle speed control:

$$\frac{\mathrm{d}\delta p(\theta)}{\mathrm{d}\theta} = -K_3\delta p - \frac{K_2}{\omega_0^2}\delta w + \frac{K_1}{\omega_0}\delta\alpha \tag{1.1}$$

$$\frac{\mathrm{d}\delta\omega}{\mathrm{d}\theta} = -\frac{K_f}{J\omega_0^2}\delta\omega + \frac{1}{J\omega_0}\delta T - \frac{1}{J\omega_0}\delta T_l \tag{1.2}$$

$$\delta T = K_p\delta p(\theta-\theta_d)+K_v\delta_v \tag{1.3}$$

where:

- δin the equations represents the perturbation from the nominal values: (δ_0, v_0, T_l) and (p_o, ω_0, T_0);
- θ is the crank angle in radians;
- θ_d is the delay in radians;
- p is the manifold pressure in Pa;
- ω is the engine speed in rad/sec;
- α is the throttle angle in degrees;
- v is the spark angle advance in degrees;
- T is the engine indicated torque in Nm;
- T_l is the load torque (disturbance) in Nm;
- the values of K_* is given in table below.

Expression	**Value**
$K_1 = K_{p1}K_{thr}(2a_2\alpha_0 + a_1)$	$7584Pa/s^2/deg$
$K_2 = K_{p1}K_{thr}(a_2\alpha_0^2 + a_1\alpha_0 + a_0)$	$41790Pa/s$
$K_3 = K_{p2}$	$0.02671/deg$
$K_p = K_T b_1(c_2v_0^2 + c_1v_0 + c_0)$	$0.00233Nm/Pa$
$K_f = 2K_{fr1}\omega_0^2 + Kfr2\omega_0$	$0.553Nm/s$
$K_v = K_T(b_1p_0 - b_0)(2c_2v_0 + c_1)$	$1.2373Nm/deg$

1.2 Problem 1

Discretization of linearized crank angle domain equations (1.1), (1.2), (1.3) using bilinear transformation,

Bilinear Transformation: For Bilinear Transformation, we have to substitute (1.4) and (1.5) in (1.1), (1.2), and (1.3) and then, $Z-transform$ the resultant equations.

$$\frac{dx}{d\theta} = \frac{x(K\tau_0+\tau_0)-x(K\tau_0)}{\tau_0} = \frac{x(K+1)-x(K)}{\tau_0} \tag{1.4}$$

$$x = \frac{x(K\tau_0+\tau_0)+x(K)}{2} = \frac{x(K+1)+x(K)}{2} \tag{1.5}$$

Substituting (1.4) and (1.5) in (1.1),

$$\frac{p(k+1)-p(k)}{\tau_0} = -K_3\frac{p(k+1)+p(k)}{2} - \frac{K_2}{\omega_0^2}\frac{\omega(k+1)+\omega(k)}{2} + \frac{K_1}{\omega_0}\frac{\alpha(k+1)+\alpha(k)}{2} \tag{1.6}$$

Rearranging,

$$\left(\frac{1}{\tau_0}+\frac{K_3}{2}\right)p(k+1)+\left(\frac{-1}{\tau_0}+\frac{K_3}{2}\right)p(k) = -\frac{K_2}{2\omega_0^2}(\omega(k+1)+\omega(k)) + \frac{K_1}{2\omega_0}(\alpha(k+1)+\alpha(k)) \tag{1.7}$$

Similarly for (1.2)

$$\frac{\omega(k+1)-\omega(k)}{\tau_0} = -\frac{K_f}{J\omega_0^2}\frac{\omega(k+1)+\omega(k)}{2} + \frac{1}{J\omega_0}\frac{T(k+1)+T(k)}{2} - \frac{1}{J\omega_0}\frac{T_l(k+1)+T_l(k)}{2} \tag{1.8}$$

Rearranging,

$$\left(\frac{1}{\tau_0}+\frac{K_f}{2J\omega_0^2}\right)\omega(k+1)+\left(\frac{-1}{\tau_0}+\frac{K_f}{2J\omega_0^2}\right)\omega(k) = \frac{1}{2J\omega_0}(T(k+1)-T(k)) - \frac{1}{2J\omega_0}(T_l(k+1)+T_l(k)) \tag{1.9}$$

Similarly for (1.3), after rearranging,

$$T(k) = K_p p(k-n) + K_v v(k) \tag{1.10}$$

We know that (1.7) and (1.9) will be in form of,

$$C_{T1}p(k+1)+C_{T2}p(k) = -C_{T3}[\omega(k+1)+\omega(k)] + C_{T4}[\alpha(k+1)+\alpha(k)] \tag{1.11}$$

$$C_{T5}\omega(k+1)+C_{T6}\omega(k) = C_{T7}[T(k+1)+T(k)] - C_{T7}[T_l(k+1)+T_l(k)] \tag{1.12}$$

On Comparing, (1.7) and (1.9) with (1.11) and (1.12),

$$C_{T1} = \left(\frac{1}{\tau_0} + \frac{K_3}{2}\right) \qquad C_{T2} = \left(\frac{-1}{\tau_0} + \frac{K_3}{2}\right) \qquad C_{T3} = \frac{K_2}{2\omega_0^2}$$
$$C_{T4} = \frac{K_1}{2\omega_0} \qquad C_{T5} = \left(\frac{1}{\tau_0} + \frac{K_f}{2J\omega_0^2}\right) \qquad C_{T6} = \left(\frac{-1}{\tau_0} + \frac{K_f}{2J\omega_0^2}\right)$$
$$C_{T7} = \frac{1}{2J\omega_0}$$

The above C_i represents the needed coefficients.

2. Problem 2

2.1 Transfer Function

A.Applying $Z-Transform$ on equations (1.10), (1.11) and (1.12),

$$(C_{T1}Z + C_{T2})p(z) = -C_{T3}(Z+1)\omega(z) + C_{T4}(Z+1)\alpha(z) \tag{2.13}$$
$$(C_{T5}Z + C_{T6})\omega(z) = C_{T7}(Z+1)T(z) - C_{T7}(Z+1)T_l(z) \tag{2.14}$$
$$T(z) = K_p Z^{-n} p(z) + K_v v(z) \tag{2.15}$$

Since, we assumed the system is linear, we can remove T_l and v from equation, (2.14), (2.15) will become

$$(C_{T5}Z + C_{T6})\omega(z) = C_{T7}(Z+1)T(z) \tag{2.16}$$
$$T(z) = K_p Z^{-n} p(z) \tag{2.17}$$

Substituting (2.17) in (2.16)

$$(C_{T5}Z + C_{T6})\omega(z) = C_{T7}K_p(Z+1)Z^{-n}\ p(z) \tag{2.18}$$

Substituting $p(z)$ from (2.18) in (2.13),

$$(C_{T1}Z + C_{T2})\frac{(C_{T5}Z + C_{T6})\omega(z)}{C_{T7}K_p(Z+1)Z^{-n}} = -C_{T3}(Z+1)\omega(z) + C_{T4}(Z+1)\alpha(z) \tag{2.19}$$

Rearranging,

$$\frac{\omega(z)}{\alpha(z)} = \frac{C_{T4}C_{T7}K_p(Z^2+2Z+1)}{C_{T5}C_{T1}Z^{n+2} + (C_{T5}C_{T2} + C_{T1}C_{T6})Z^{n+1} + C_{T6}C_{T2}Z^n + C_{T3}C_{T7}K_p(Z^2+2Z+1)} \tag{2.20}$$

Taking n=3, as $\tau_0 = 60degree$ and torque delay $\theta_d = 180degree$, $n = \frac{\tau_0}{\theta_0} = 3$
Thus, we get,

$$\frac{\omega(z)}{\alpha(z)} = \frac{C_{T4}C_{T7}K_p(Z^2+2Z+1)}{C_{T5}C_{T1}Z^5 + (C_{T5}C_{T2}+C_{T1}C_{T6})Z^4 + C_{T6}C_{T2}Z^3 + C_{T3}C_{T7}K_p(Z^2+2Z+1)} \quad (2.21)$$

The above equation represents the transfer function from throttle angle α to speed ω.

2.2 B. Numerical Values

On substituting the values of C_is in MatLab we get,

$$\frac{\omega(z)}{\alpha(z)} = \frac{0.007955Z^2 + 0.01591Z + 0.007955}{0.9251Z^5 - 1.824Z^4 + 0.8987Z^3 + 0.0005224Z^2 + 0.001045Z + 0.0005224} \quad (2.22)$$

2.3 Z-plane

From matlab function **pzmap,**the poles were found to be inside the unit circle. The locations of poles are as follows,

$$\mathbf{2-Zeros}: (-1+0i), (-1+0i) \quad (2.23)$$

$$\mathbf{5-Poles}: (-0.0754+0i), (0.0354 \pm 0.08i), (0.988 \pm 0.046i) \quad (2.24)$$

The plot representing the poles and zeros is shown below 1.

3. Problem 3

3.1 Open Loop System Block Diagram

Using the following equations, the transfer function blocks are created in simulink. The system is treated as **SISO** system, that is, effect of spark is not considered. Once, the equations are rearragned, we get the needed transfer function as follows,

$$(C_{T1}Z + C_{T2})p(z) = -C_{T3}(Z+1)\omega(z) + C_{T4}(Z+1)\alpha(z) \quad (3.25)$$

$$(C_{T5}Z + C_{T6})\omega(z) = C_{T7}(Z+1)T(z) - C_{T7}(Z+1)T_l(z) \quad (3.26)$$

$$T(z) = K_p Z^{-n} p(z) \quad (3.27)$$

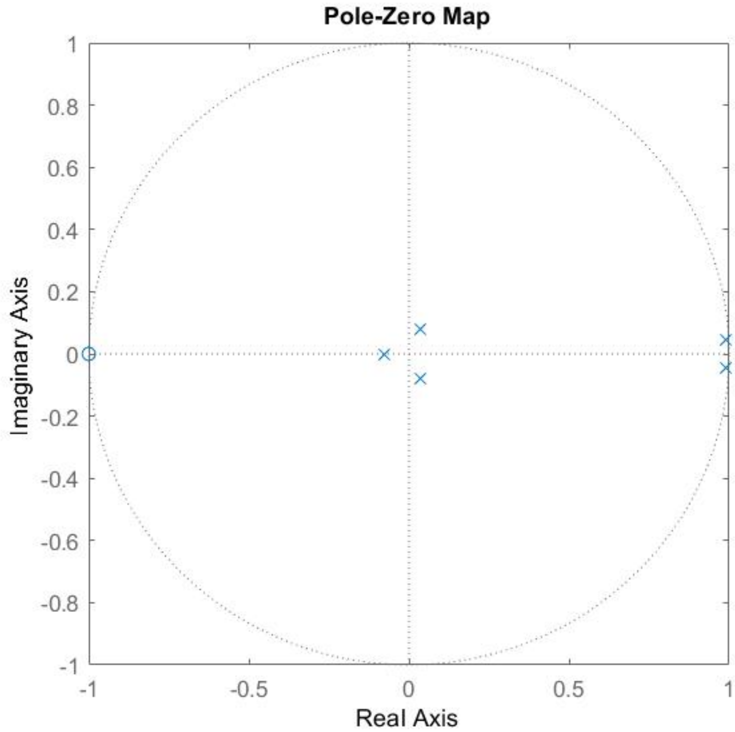

Figure 1: Zeros and Poles in Z-plane

The transfer function (Asked) blocks are listed below,

$$TF1 = TF2 = \frac{Z+1}{C_{T1}Z + C_{T2}}$$
$$TF3 = TF4 = \frac{Z+1}{C_{T5}Z + C_{T6}}$$

The completed simulink block diagram is shown below 2,

4. Problem 4

4.1 Open Loop Response of the system

This problem is continuation of Problem 3. Here, we give disturbance as a change in Load torque and remove it later (Positive load is given at $\theta = 50\tau_0$ and removed at $\theta = 1000\tau_0$)

The simulation timing and settings are adjusted accordingly. The simulations results are as follows 3 (The initial nominal values are added),

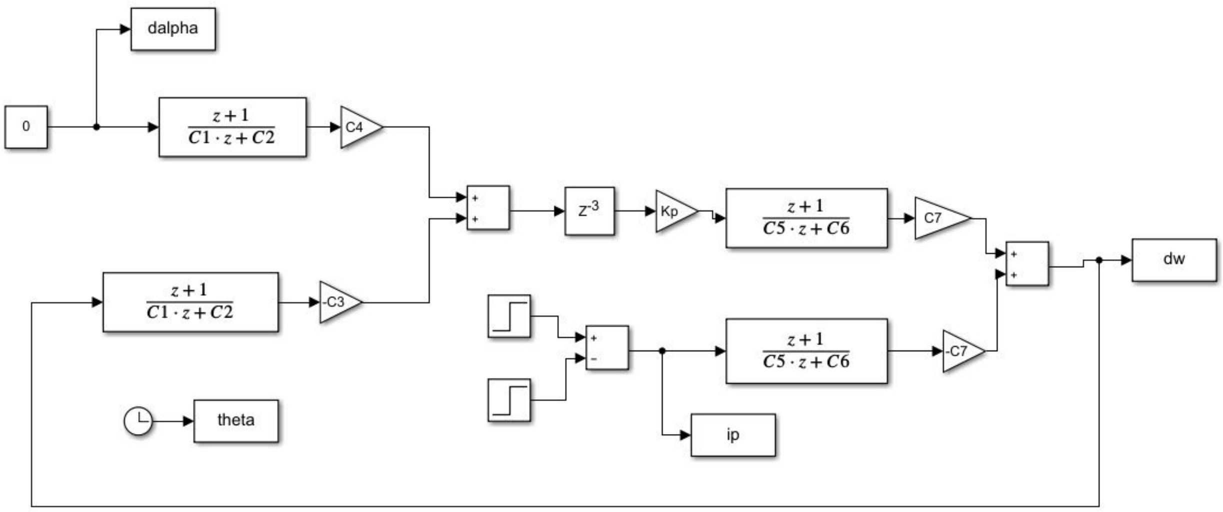

Figure 2: Simulink Block Diagram

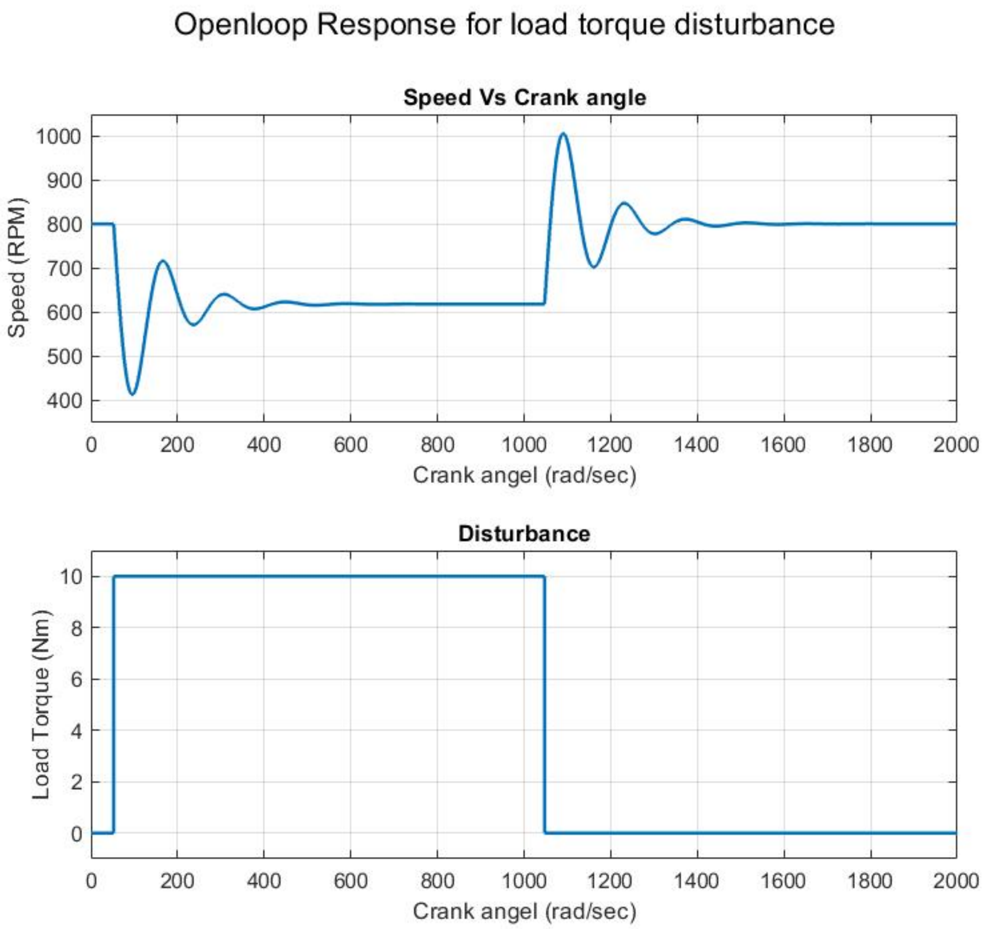

Figure 3: Open Loop Response for Load Torque disturbance