Homework 6
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1) SISO - LQR

The Q and R weighting matrices are chosen as follows;

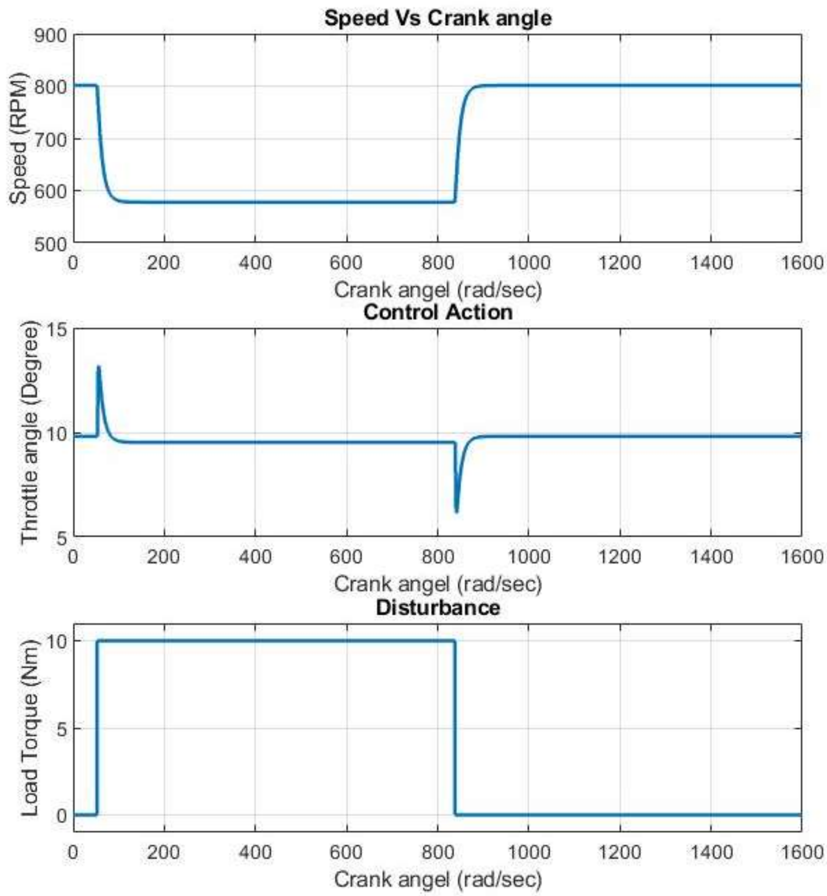
$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 999999 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad R = [99900]$$

The element Q(2,2) is chosen with high weightage because it is the state (Speed) that we are interested, while other states are given with low weightage. Similarly, the R matrix is chosen with significantly high value in order to maintain the throttle value in the desired range.

The gain is,

$$K = [-0.0080 \quad -1.8781 \quad 0.0007 \quad 0.0007 \quad 0.0007]$$

The response is as follows,



Closed loop Response for SISO

Inference:

The throttle angel, overshoot, settling time are within desired region.
The steady state error is non-zero (Because of NO integral action)

2) SISO Augmented - LQR

The Q and R weighting matrices are chosen as follows;

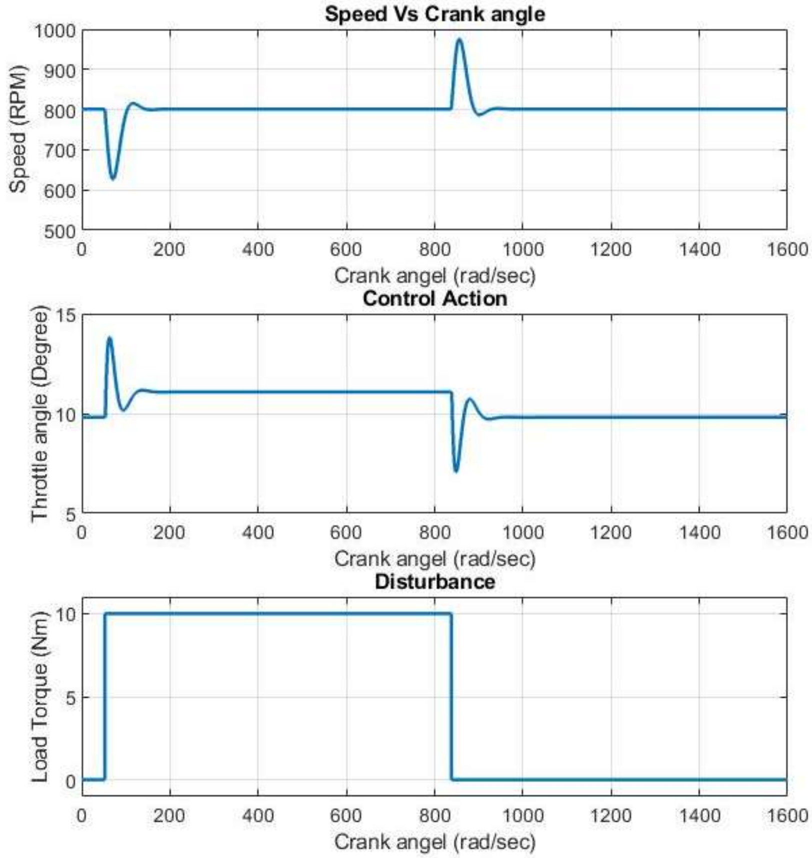
$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10000 \end{bmatrix} \qquad R = [9999999]$$

The element Q(2,2) and Q(6,6) is chosen with high weightage because it is the state (Speed and integral) that we are interested, while other states are given with low weightage. Similarly, the R matrix is chosen with significantly high value in order to maintain the throttle value in the desired range.

The gain is,

$$K = [-0.0021 \quad -0.6281 \quad 0.0002 \quad 0.0002 \quad 0.0002 \quad 0.0286]$$

The response is as follows,



Closed loop Response for SISO Augmented

Inference:

The throttle angel, overshoot, settling time are within desired region.
The steady state error is ZERO (Because of integral action)

3) MIMO - LQR

The Q and R weighting matrices are chosen as follows;

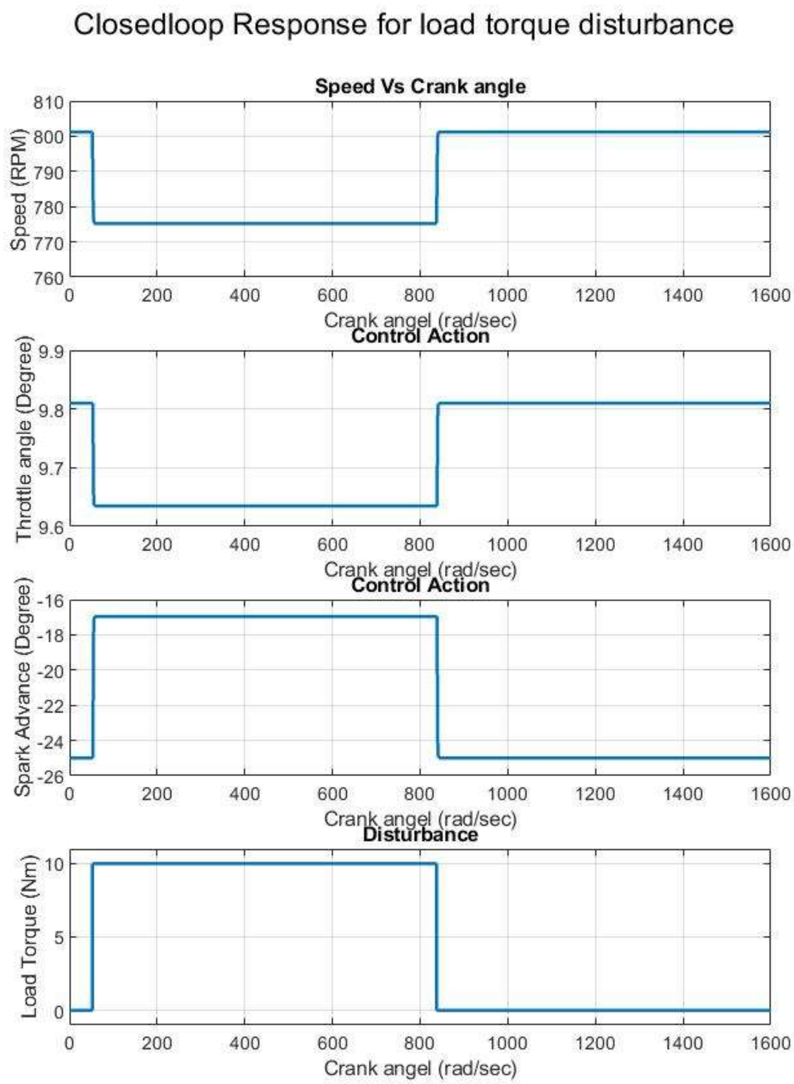
$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 99999999 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 9999 & 0 \\ 0 & 999999 \end{bmatrix}$$

The element Q(2,2) is chosen with high weightage because it is the state (Speed) that we are interested, while other states are given with low weightage. Similarly, the R(2,2) (Spark advance) is chosen with significantly high value than R(1,1) (Throttle angle) in order to use the throttle more than spark angle and also to maintain both the values in the desired range.

The gain is,

$$K = \begin{bmatrix} -0.0135 & 0.0063 & 0.0003 & 0.0001 & 0.0000 \\ 0.0000 & -4.2702 & 0.0001 & 0.0003 & 0.0015 \end{bmatrix}$$

The response is as follows,



Closed loop Response for MIMO

Inference:

The throttle angel, spark advance, overshoot, settling time are within desired region.
The steady state error is non-zero (Because of NO integral action)

4) MIMO Augmented - LQR

The Q and R weighting matrices are chosen as follows;

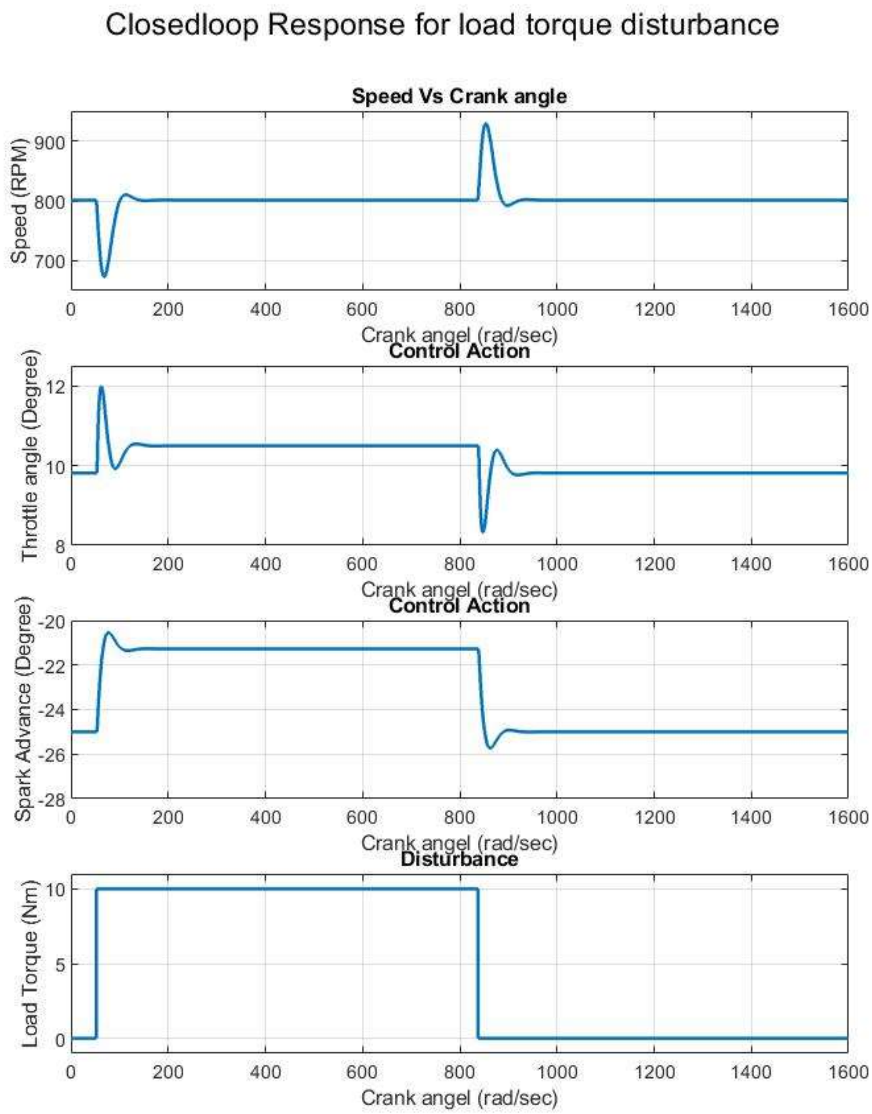
$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10000 \end{bmatrix} \qquad R = \begin{bmatrix} 9999990 & 0 \\ 0 & 19999999 \end{bmatrix}$$

The element Q(2,2) and Q(6,6) is chosen with high weightage because it is the state (Speed and integral) that we are interested, while other states are given with low weightage. Similarly, the R(2,2) (Spark advance) is chosen with significantly high value than R(1,1) (Throttle angle) in order to use the throttle more than spark angle and also to maintain both the values in the desired range.

The gain is,

$$K = \begin{bmatrix} -0.0016 & -0.3705 & 0.0001 & 0.0001 & 0.0001 & 0.0170 \\ -0.0004 & -0.2871 & 0.0001 & 0.0001 & 0.0001 & 0.0170 \end{bmatrix}$$

The response is as follows,



Closed loop Response for MIMO Augmented

Inference:

The throttle angel, Spark Advance, overshoot, settling time are within desired region.
The steady state error is ZERO (Because of integral action)

5) Non-linear system – LQR

The Q and R weighting matrices are chosen as follows;

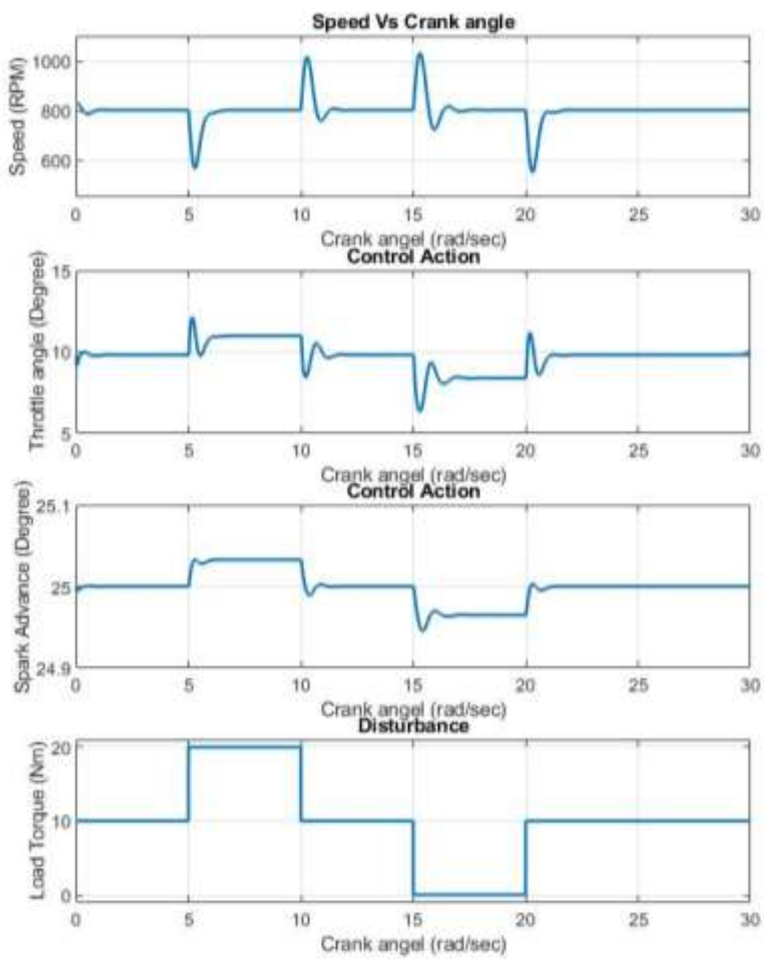
$$
Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10000 \end{bmatrix} \qquad R = \begin{bmatrix} 99999000 & 0 \\ 0 & 9999999999 \end{bmatrix}
$$

The element Q(2,2) and Q(6,6) is chosen with high weightage because it is the state (Speed and integral) that we are interested, while other states are given with low weightage. Similarly, the R(2,2) (Spark advance) is chosen with significantly high value than R(1,1) (Throttle angle) in order to use the throttle more than spark angle and also to maintain both the values in the desired range.

The gain is,

$$
K = \begin{bmatrix} -0.0011 & -0.2504 & 0.0001 & 0.0001 & 0.0001 & 0.0094 \\ -0.0000 & -0.0020 & 0.0000 & 0.0000 & 0.0000 & 0.0001 \end{bmatrix}
$$

The response is as follows,



Closed loop Response for NonLinear-MIMO

Inference:

The throttle angel, spark advance, overshoot, settling time are within desired region.
The steady state error is ZERO (Because of integral action)

6.1 Comparison of SISO

6.1.a)

The gain obtained using LQR for SISO Augmented (Prob 2);

$$K = [-0.0021 \quad -0.6281 \quad 0.0002 \quad 0.0002 \quad 0.0002 \quad 0.0286]$$

6.1.b)

The closed loop poles for enforced by LQR;

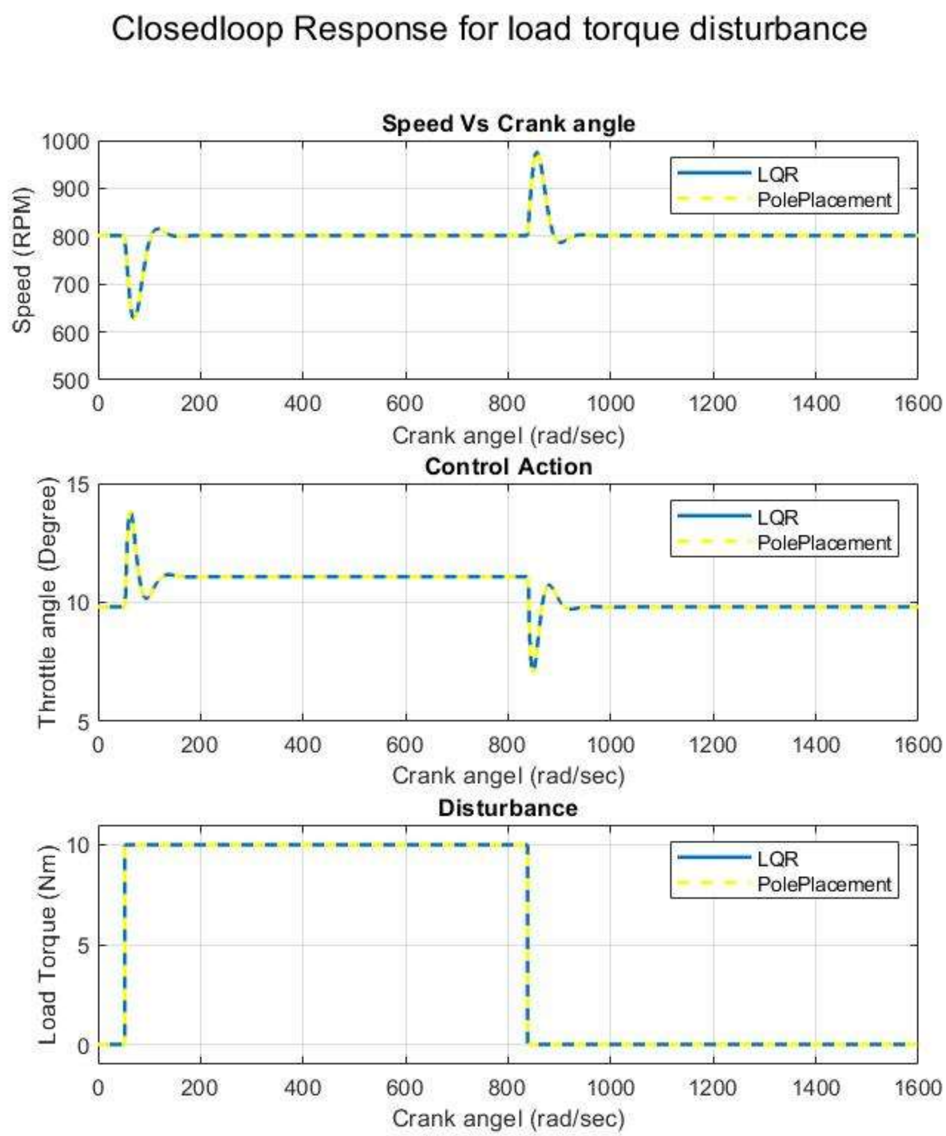
-0.0755 + 0.0000i
0.0354 + 0.0801i
0.0354 - 0.0801i
0.8906 + 0.0000i
0.9423 + 0.0747i
0.9423 - 0.0747i

6.1.c)

The gain obtained using Pole Placement (with LQR closed loop roots) for SISO Augmented;

$$K = [-0.0021 \quad -0.6281 \quad 0.0002 \quad 0.0002 \quad 0.0002 \quad 0.0286]$$

The response is as follows,



Inference:

Both the response for LQR and pole placement overlaps each other.

6.1.e)

Yes! Both the gain matrix obtained using LQR and pole placement are the same.

6.2 Comparison of MIMO

6.2.a)
The gain obtained using LQR for MIMO Augmented (Prob 4);

$$K = \begin{bmatrix} -0.0016 & -0.3705 & 0.0001 & 0.0001 & 0.0001 & 0.0170 \\ -0.0004 & -0.2871 & 0.0001 & 0.0001 & 0.0001 & 0.0170 \end{bmatrix}$$

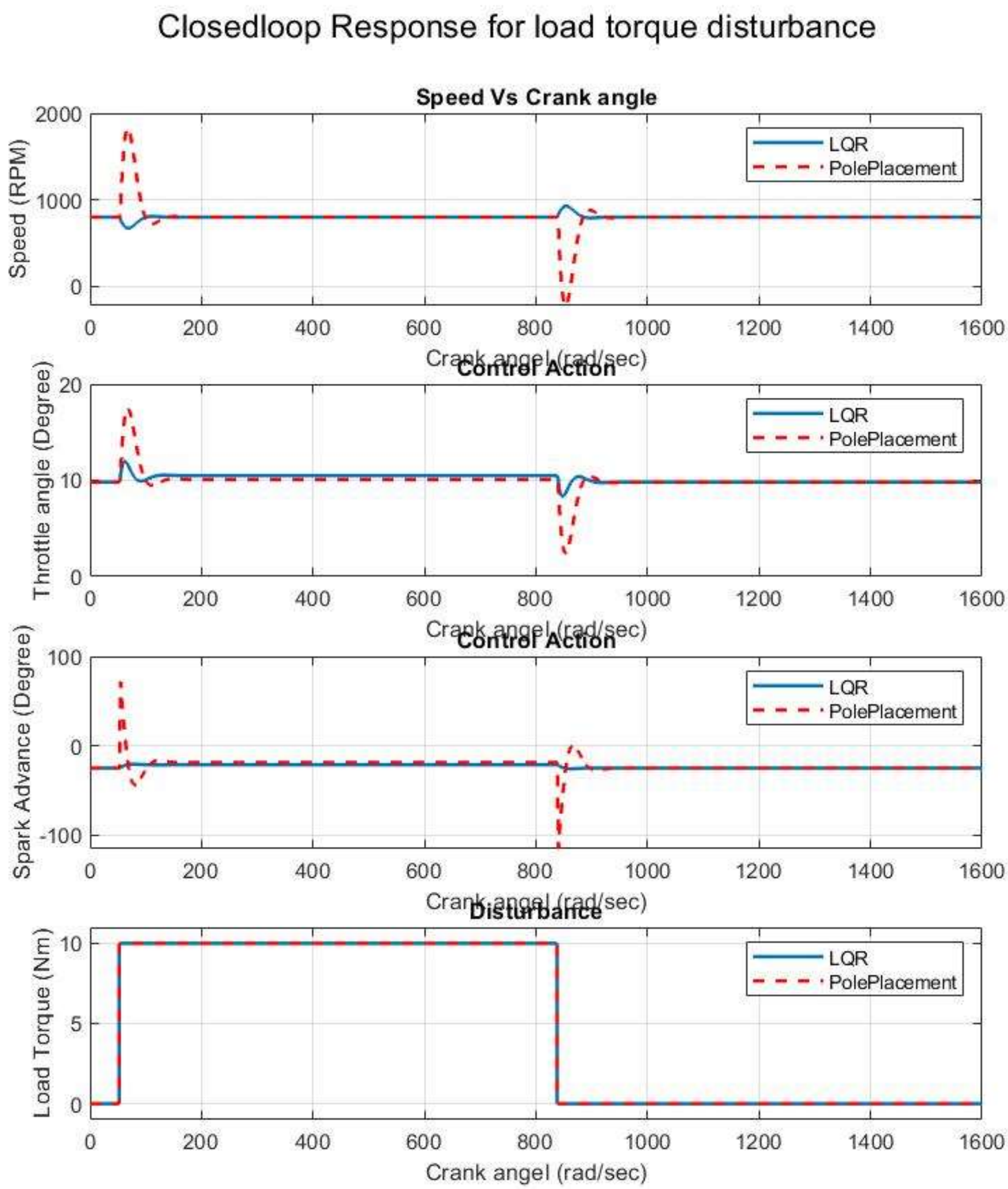
6.2.b)
The closed loop poles for enforced by LQR;

-0.0755 + 0.0000i
0.0354 + 0.0801i
0.0354 - 0.0801i
0.9383 + 0.0769i
0.9383 - 0.0769i
0.8908 + 0.0000i

6.2.c)
The gain obtained using Pole Placement (with LQR closed loop roots) for MIMO Augmented;

$$K = \begin{bmatrix} 0.1434 & 0.0286 & 0.1337 & -0.0013 & -0.0013 & 0.0033 \\ 19.2195 & -5.9542 & 17.8532 & -0.2264 & -0.1921 & 0.5032 \end{bmatrix}$$

The response is as follows,



Inference:

Both the response for LQR and pole placement DO NOT overlaps each other.

6.2.e)

No! the gain matrix obtained using LQR and pole placement are the NOT same.

To add on: Also tried for Non-Linear model and compared the result. The gain matrix were different and indeed, the response did not overlap. This is because, in pole placement we do not know which pole is assigned to while state (unless we diagonalize the system matrix)