Homework 5
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1) MIMO Model

a.

The State Space representation of 4-cylinder SI engine is given by,

$$x_1(k) = \frac{c_{T2}}{\sqrt{\tau_o}}\, p(k-1) + \frac{c_{T3}}{\sqrt{\tau_o}}\, w(k-1) - \frac{c_{T4}}{\sqrt{\tau_o}}\, \alpha(k-1)$$

$$x_2(k) = \frac{c_{T6}}{\sqrt{\tau_o}}\, w(k-1) - \frac{c_{T7}}{\sqrt{\tau_o}}\, T(k-1)$$

$$x_3(k) = \; p(k-1)$$

$$x_4(k) = x_3(k-1) = \; p(k-2)$$

$$x_5(k) = \; x_4(k-1) = p(k-3); \quad T(k) = \; K_p p(k-3) + K_V v(k)$$

State Space Matrices:

$$\begin{aligned} x(k+1) &= \quad \Phi_T x(k) + \Gamma_T\, \alpha(k) \\ y(k) &= \quad H_T x(k) + D_T\, \alpha(k) \end{aligned}$$

$$\Phi_T = \begin{bmatrix} (OM^{-1})_{11} & (OM^{-1})_{12} & 0 & 0 & Q_{12}K_p \\ (OM^{-1})_{21} & (OM^{-1})_{22} & 0 & 0 & Q_{22}K_p \\ (M^{-1})_{11} & (M^{-1})_{12} & 0 & 0 & -(M^{-1}N)_{12}K_p \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \Gamma_T = \begin{bmatrix} Q_{11} & Q_{12}K_v \\ Q_{21} & Q_{22}K_v \\ -(M^{-1}N)_{11} & -(M^{-1}N)_{12}K_v \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$H_T = \; [(M^{-1})_{21} \quad (M^{-1})_{22} \quad 0 \quad 0 \quad -(M^{-1}N)_{22}K_p] \qquad D_T = [-(M^{-1}N)_{21} \;\; -(M^{-1}N)_{22}K_v]$$

$$M = \begin{bmatrix} \frac{-c_{T_1}}{\sqrt{\tau_0}} & \frac{-c_{T_3}}{\sqrt{\tau_0}} \\ 0 & \frac{-c_{T_5}}{\sqrt{\tau_0}} \end{bmatrix} \qquad N = \begin{bmatrix} \frac{c_{T_4}}{\sqrt{\tau_0}} & 0 \\ 0 & \frac{c_{T_7}}{\sqrt{\tau_0}} \end{bmatrix}$$

$$\mathrm{O} = \begin{bmatrix} \frac{c_{T_2}}{\sqrt{\tau_0}} & \frac{c_{T_3}}{\sqrt{\tau_0}} \\ 0 & \frac{c_{T_6}}{\sqrt{\tau_0}} \end{bmatrix} \qquad P = \begin{bmatrix} \frac{-c_{T_4}}{\sqrt{\tau_0}} & 0 \\ 0 & \frac{-c_{T_7}}{\sqrt{\tau_0}} \end{bmatrix}$$

$$Q = \; -OM^{-1}N + P$$

$In\ general, X_{ij}\ stands\ for\ the\ element\ in\ i^{th} row\ and\ j^{th} column$

$$\Phi_T = \begin{bmatrix} 0.9724 & -6.1276 & 0 & 0 & 0.0011 \\ 0 & 0.9990 & 0 & 0 & -0.0004 \\ -1.0092 & 3.1353 & 0 & 0 & -0.0006 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \Gamma_T = \begin{bmatrix} -91.2330 & 0.5860 \\ 0 & -0.1912 \\ 46.6806 & -0.2998 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$H_T = [0 \quad -1.0228 \quad 0 \quad 0 \quad 0.0002] \qquad D_T = [0 \qquad 0.0978\,]$$

2. MIMO – Pole Placement

Design Specification:

- Settling time (radians) = 300 radian
- Overshoot = 250 rpm

Calculation for Desired location of the Dominant pole:

Finding the desire root location (region) for dominant poles,

Overshoot:

$$PO\ \% = 100 * e^{-\frac{\zeta\pi}{\sqrt{(1-\zeta^2)}}}$$

ζ = 0.34 (Approx.,)

Settling time:

$$T_s = \frac{4}{\zeta\omega_n} = 300\ rad$$

ω_n = 0.039 (Approx.,)

NOTE: The above parameter are in continuous time domain. One converting the above parameters we get the approximate pole location.

Region of interest:
The region of interest is approximately around (right side of unit circle) z = 0.93+0.1i (approx). The desired poles are chosen in the region of interest by trial and error method.

Desired poles:

The poles were chosen in the desired region, as follows;

0.939+0.1i,
0.939-0.1i,
0.089+0.65i,
0.089-0.65i,
-0.1

The K matrix is as follows,

$$K=\begin{bmatrix}0.0448 & 0.0302 & 0.0644 & -0.0289 & 0.0082\\ 0.0001 & -5.7489 & 0.0001 & 0.0000 & 0.0019\end{bmatrix}$$

The response is as follows,

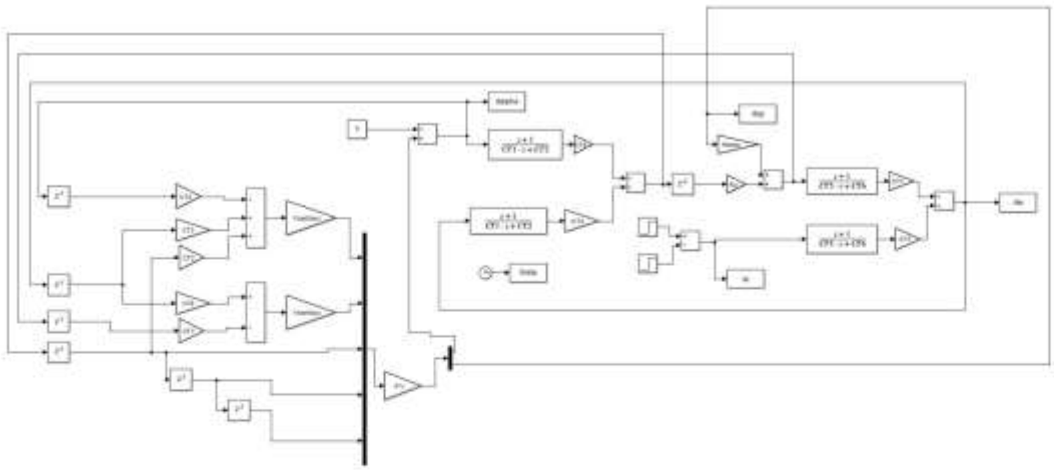


Fig 1: Simulink State Feedback (P alone)

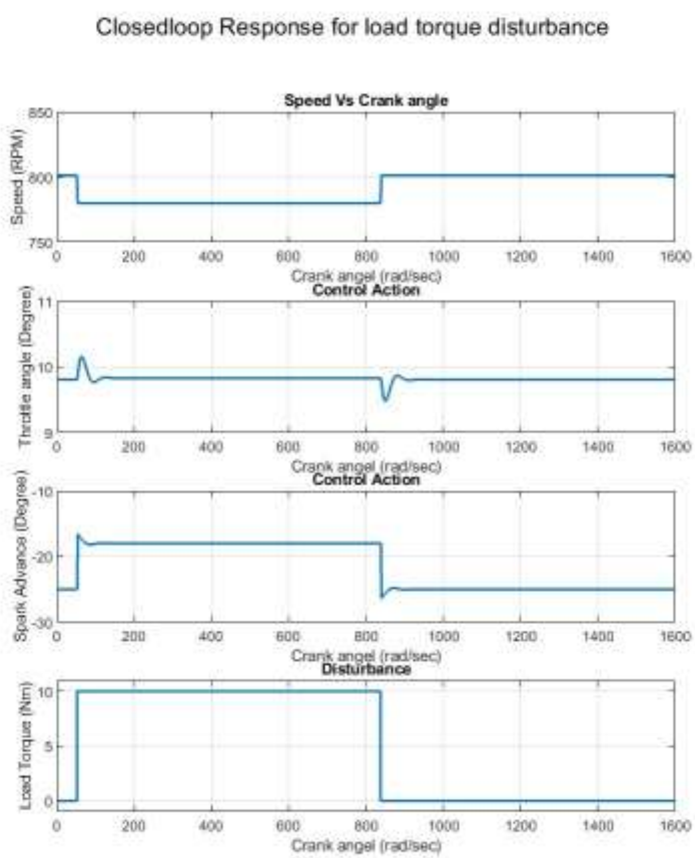


Fig 2: Closed Loop Response of State Feedback (P alone)

Inference,

The throttle angle, overshoot and settling time are within desired region.
The steady state error is non-zero (Because of P-alone control action)

3. MIMO Pole Placement – Augmented Systems;

The system is augmented to have integral action. The state space matrix is modifies as follows,

$$\Phi_T = \begin{bmatrix} (OM^{-1})_{11} & (OM^{-1})_{12} & 0 & 0 & Q_{12}K_p & 0 \\ (OM^{-1})_{21} & (OM^{-1})_{22} & 0 & 0 & Q_{22}K_p & 0 \\ (M^{-1})_{11} & (M^{-1})_{11} & 0 & 0 & -(M^{-1}N)_{12}K_p & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ (M^{-1})_{21} & (M^{-1})_{22} & 0 & 0 & -(M^{-1}N)_{22}K_p & 1 \end{bmatrix}$$

$$\Gamma_T = \begin{bmatrix} Q_{11} & Q_{12}K_v \\ Q_{21} & Q_{22}K_v \\ -(M^{-1}N)_{11} & -(M^{-1}N)_{12}K_v \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$H_T = \begin{bmatrix} (M^{-1})_{21} & (M^{-1})_{22} & 0 & 0 & -(M^{-1}N)_{22}K_p & 0 \end{bmatrix} \quad D_T = \begin{bmatrix} -(M^{-1}N)_{21} & -(M^{-1}N)_{22}K_v \end{bmatrix}$$

$$\Phi_T = \begin{bmatrix} 0.9724 & -6.1276 & 0 & 0 & 0.0011 & 0 \\ 0 & 0.9990 & 0 & 0 & -0.0004 & 0 \\ -1.0092 & 3.1353 & 0 & 0 & -0.0006 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1.0228 & 0 & 0 & 0.0002 & 1 \end{bmatrix} \qquad \Gamma_T = \begin{bmatrix} -91.2330 & 0.5860 \\ 0 & -0.1912 \\ 46.6806 & -0.2998 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$H_T = \begin{bmatrix} 0 & -1.0228 & 0 & 0 & 0.0002 & 0 \end{bmatrix} \qquad D_T = \begin{bmatrix} 0 & 0.0978 \end{bmatrix}$$

The desired poles for augmented are chosen in region of interest by trial and error method.
The chosen poles are as follows,

0.939+0.1i,
0.939-0.1i
0.089+0.65i
0.089-0.65i
0.73
-0.1

The gain matrix k is as follows

$$K=\begin{bmatrix} 0.0069 & 0.0562 & 0.0146 & -0.0072 & -0.0009 & 0.0013 \\ -0.2741 & -1.5735 & -0.3147 & -0.1534 & -0.0151 & 0.0968 \end{bmatrix}$$

The response of the systems for state feedback (PI) is as follows,

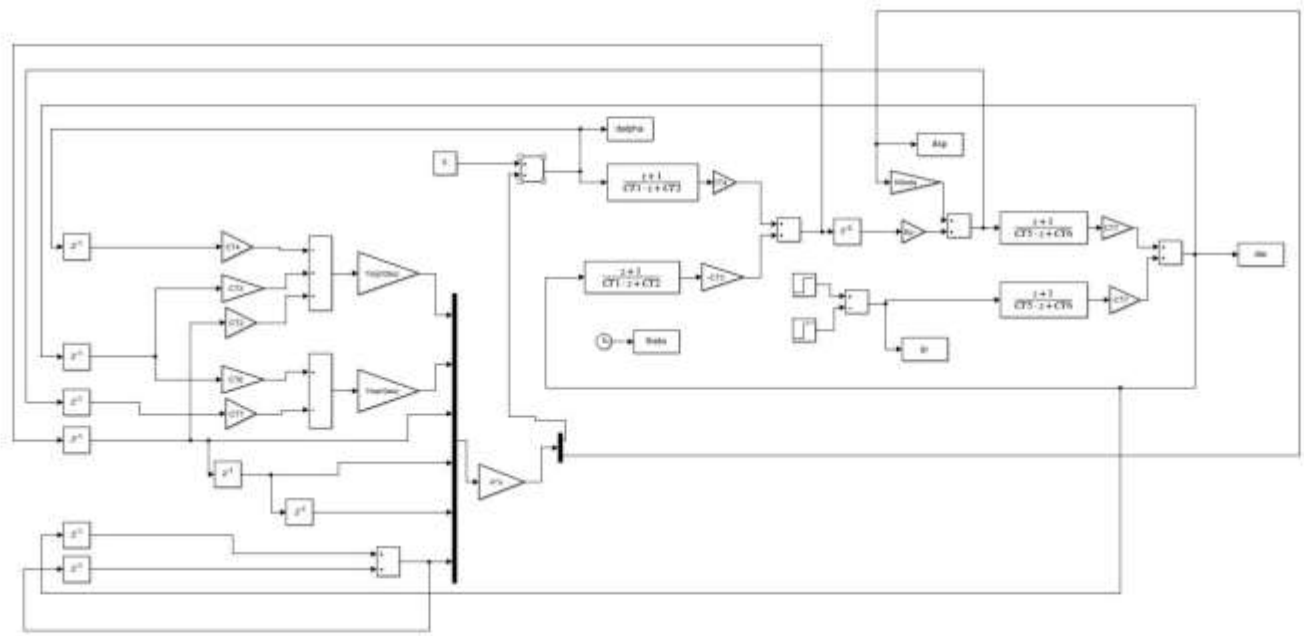


Fig 3: Simulink State Feedback (PI)

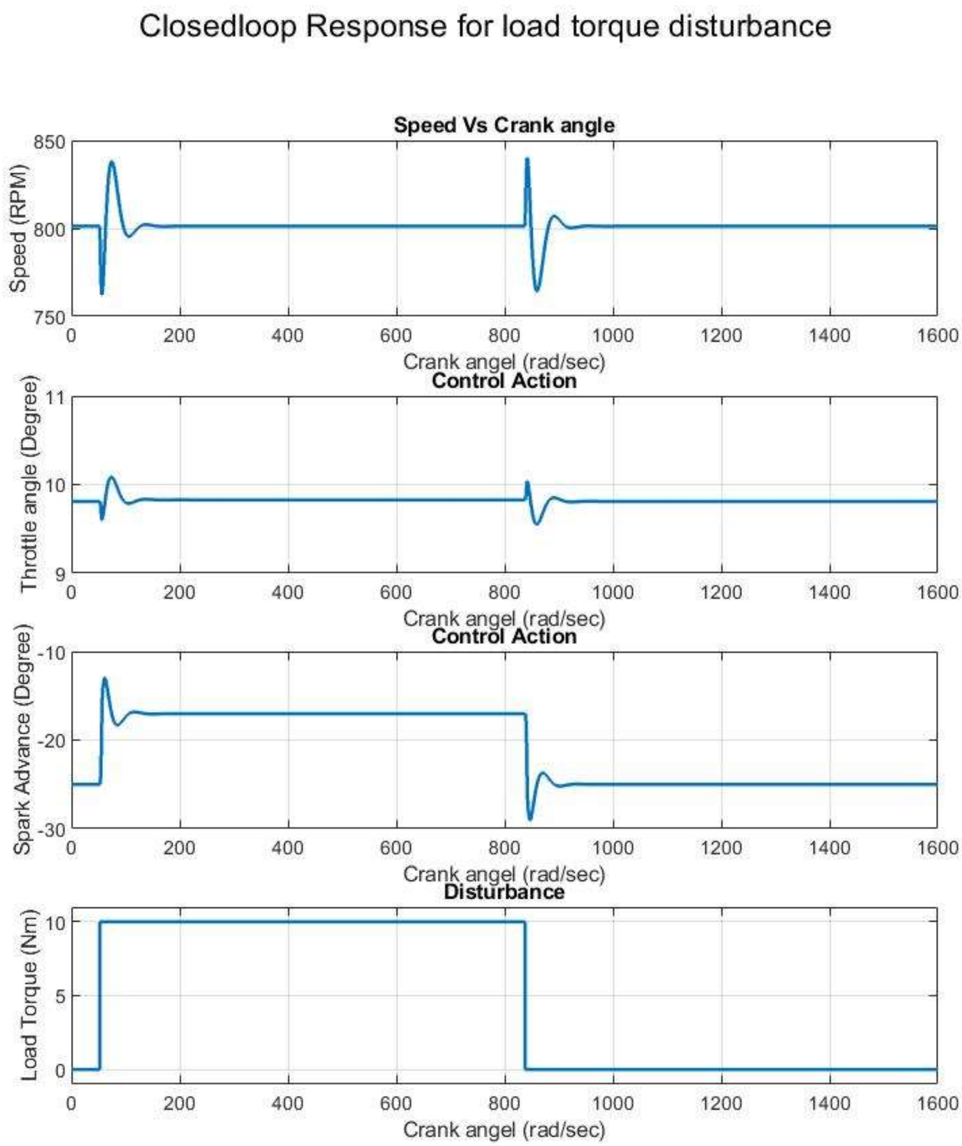
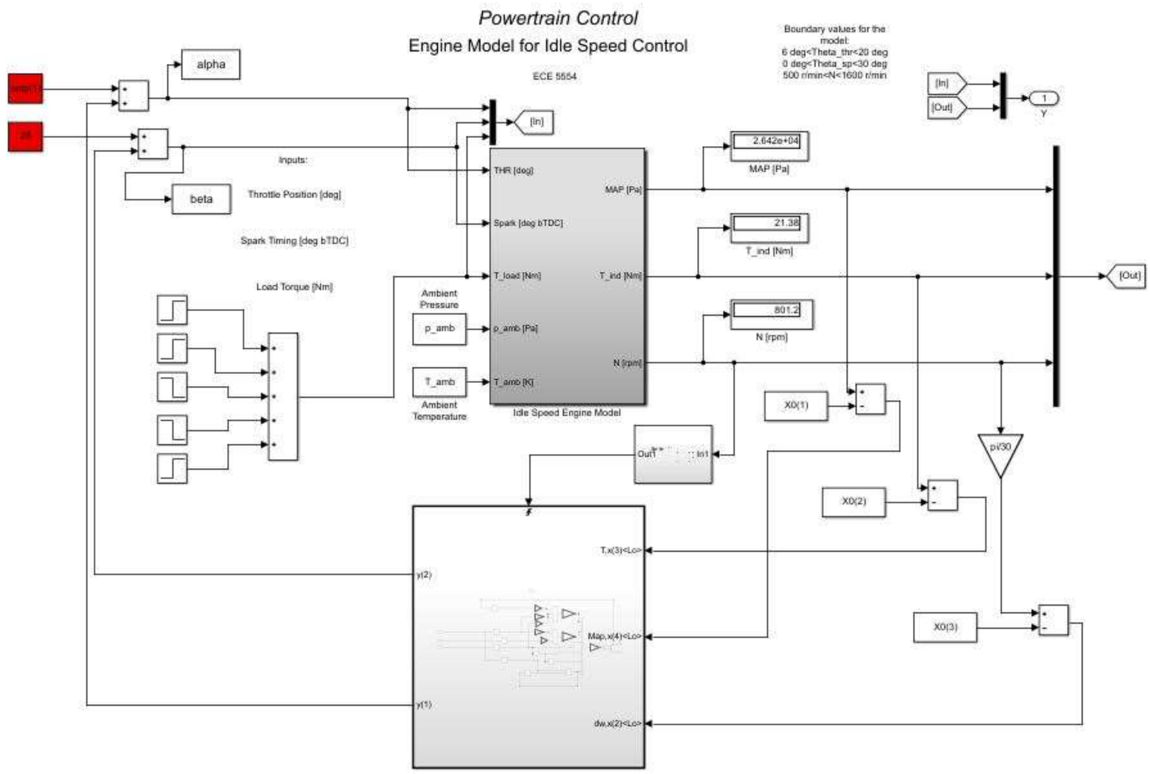


Fig 4: Closed Loop Response of State Feedback (PI)

Inference:

The throttle angel, overshoot, settling time are within desired region.
The steady state error is zero (Because of integral action)

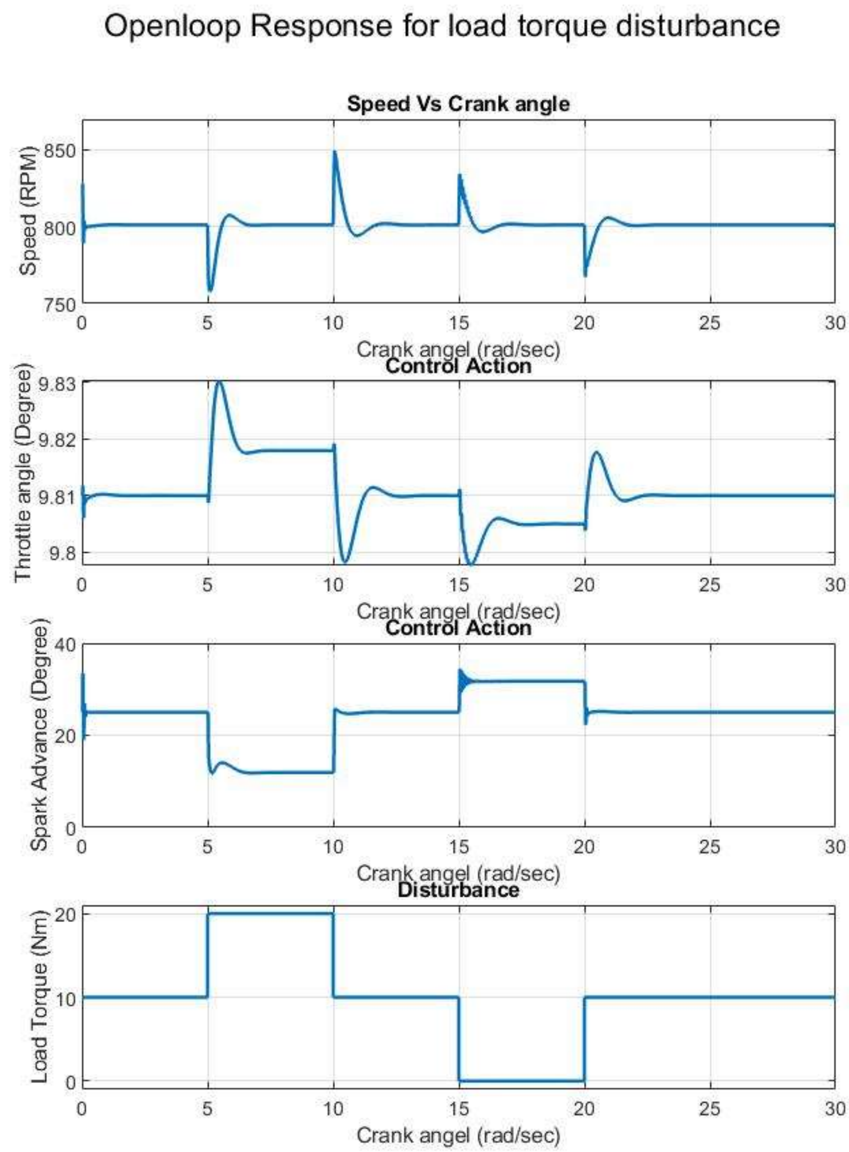
Problem 4:



By trial and error, (By adjusting poles, and state feedback gain directly and getting back the poles again), the final control state feedback is found to be of as follows;

$$K=\begin{bmatrix} 0.00009 & 0.003 & -0.0006 & -0.000012 & -0.000043 & 0.000087 \\ 0.0291 & 2.98 & 0.00173 & 0.10000545 & -0.00525 & -0.21 \end{bmatrix}$$

{-0.3409, -0.0022+0.0038i, -0.0022-0.0038i, 0.0044+0.0000i, 0.6885+0.2090i, 0.6885-0.2090i} (Using Trial and error)



Inference:

The above graph shows that output has reached the desire value. The throttle is in desire range, and spark advance is also in desire range (slightly greater at few instant). But Overall response is desirable and satisfactory.

Problems with state feedback:

Needs trial and error

While entering desired pole in "place" command, the order of poles matters and will result in different gain matrix for different ordering.

Place poles for SISO system is easy and it becomes difficult in case od MIMO

It has no factor to account to control authority. (No factor to give weight for different input in case of MIMO)