# Powertrain Control Final
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## 1) Observability

The State Space representation of 4-cylinder SI engine is given by,

$$
\begin{aligned}
x_1(k) &= \frac{c_{T2}}{\sqrt{\tau_o}}\, p(k-1) + \frac{c_{T3}}{\sqrt{\tau_o}}\, w(k-1) - \frac{c_{T4}}{\sqrt{\tau_o}}\, \alpha(k-1) \\
x_2(k) &= \frac{c_{T6}}{\sqrt{\tau_o}}\, w(k-1) - \frac{c_{T7}}{\sqrt{\tau_o}}\, T(k-1) \\
x_3(k) &= p(k-1) \\
x_4(k) &= x_3(k-1) = p(k-2) \\
x_5(k) &= x_4(k-1) = p(k-3)
\end{aligned}
$$

**State Space Matrices:**

$$
\begin{aligned}
x(k+1) &= \Phi_T x(k) + \Gamma_T\, \alpha(k) \\
y(k) &= H_T x(k) + D_T\, \alpha(k)
\end{aligned}
$$

$$
\Phi_T = \begin{bmatrix} (OM^{-1})_{11} & (OM^{-1})_{12} & 0 & 0 & Q_{12}K_p \\ (OM^{-1})_{21} & (OM^{-1})_{22} & 0 & 0 & Q_{22}K_p \\ (M^{-1})_{11} & (M^{-1})_{12} & 0 & 0 & -(M^{-1}N)_{12}K_p \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \Gamma_T = \begin{bmatrix} Q_{11} \\ Q_{21} \\ -(M^{-1}N)_{11} \\ 0 \\ 0 \end{bmatrix}
$$

$$
H_T = [(M^{-1})_{21} \quad (M^{-1})_{22} \quad 0 \quad 0 \quad -(M^{-1}N)_{22}K_p] \qquad D_T = [-(M^{-1}N)_{21}\ ]
$$

$$
M = \begin{bmatrix} \frac{-c_{T_1}}{\sqrt{\tau_0}} & \frac{-c_{T_3}}{\sqrt{\tau_0}} \\ 0 & \frac{-c_{T_5}}{\sqrt{\tau_0}} \end{bmatrix} \qquad N = \begin{bmatrix} \frac{c_{T_4}}{\sqrt{\tau_0}} & 0 \\ 0 & \frac{c_{T_7}}{\sqrt{\tau_0}} \end{bmatrix}
$$

$$
\mathrm{O} = \begin{bmatrix} \frac{c_{T_2}}{\sqrt{\tau_0}} & \frac{c_{T_3}}{\sqrt{\tau_0}} \\ 0 & \frac{c_{T_6}}{\sqrt{\tau_0}} \end{bmatrix} \qquad P = \begin{bmatrix} \frac{-c_{T_4}}{\sqrt{\tau_0}} & 0 \\ 0 & \frac{-c_{T_7}}{\sqrt{\tau_0}} \end{bmatrix}
$$

$$
Q = -OM^{-1}N + P
$$

$In\ general, X_{ij}\ stands\ for\ the\ element\ in\ i^{th} row\ and\ j^{th} column$

$$\Phi_T = \begin{bmatrix} 0.9724 & -6.1276 & 0 & 0 & 0.0011 \\ 0 & 0.9990 & 0 & 0 & -0.0004 \\ -1.0092 & 3.1353 & 0 & 0 & -0.0006 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \Gamma_T = \begin{bmatrix} -91.2330 \\ 0 \\ 46.6806 \\ 0 \\ 0 \end{bmatrix}$$

$$H_T = [0 \quad -1.0228 \quad 0 \quad 0 \quad 0.0002] \qquad D_T = [0]$$

**Observability:**

**Rank of observability matrix is 5** (=n). Thus, the system is **observable.**

$$Rank\ (W_o) = Rank(\begin{bmatrix}\mathrm{H}_t & \mathrm{H}_t\phi_t & \mathrm{H}_t\phi_t^2 & \mathrm{H}_t\phi_t^3 & \mathrm{H}_t\phi_t^4\end{bmatrix}^T) = 5 = n\,,$$

**2. Observer design**

The observer is designed such that,

$$u(k) = -K\,x_a(k)$$

Where, $x_a(k) = [x(k) \quad x_I(k)]$. Once the state are estimated, the estimated states will be used for state feedback.

$$u(k) = -K\,x\hat{}_a(k)$$

Where, $x\hat{}_a(k) = [x\hat{}(k) \quad x_I(k)]$.

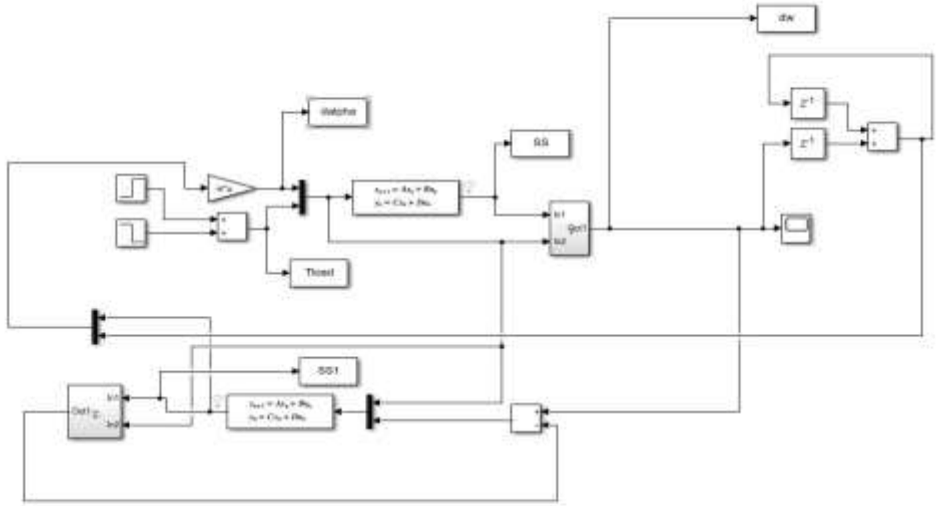
The state feedback gain for **Luenberger gain is found using pole placement**. The **poles were places more towards the origin** to have **fast response**. The chosen poles were as follows,

**0.2190 - 0.1000i**
**0.2190 + 0.1000i**
**0.3590 - 0.1000i**
**0.3590 + 0.1000i**
**0.1000 + 0.0000i**

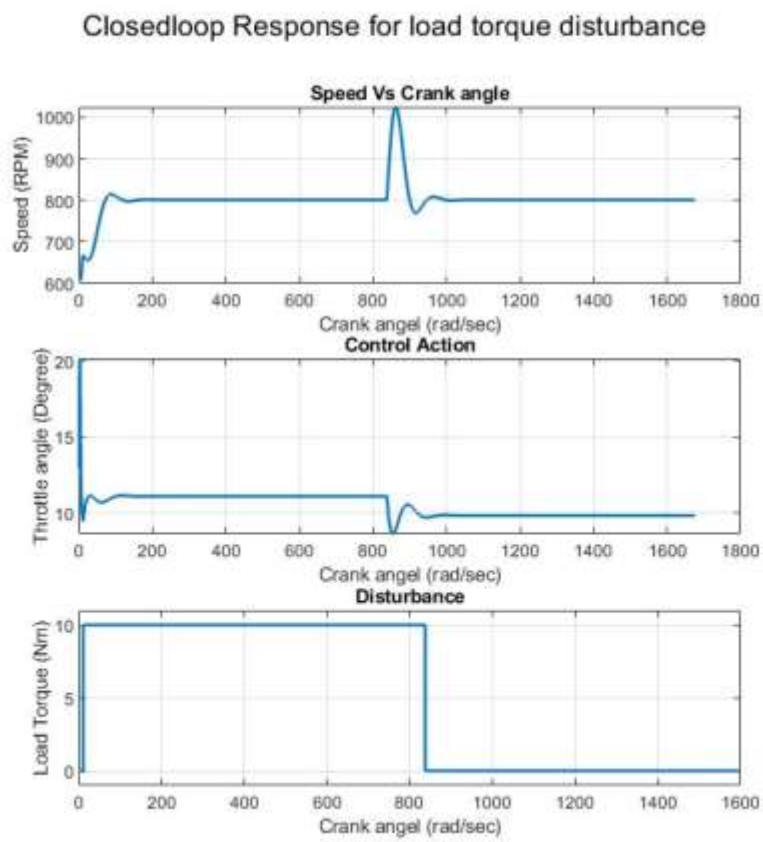
The feedback gain obtained using pole placement is

**K=[-524.8;**
**-0.4397;**
**552.3;**
**464.5;**
**1441]**

As expected the gain were high to have fast convergence.



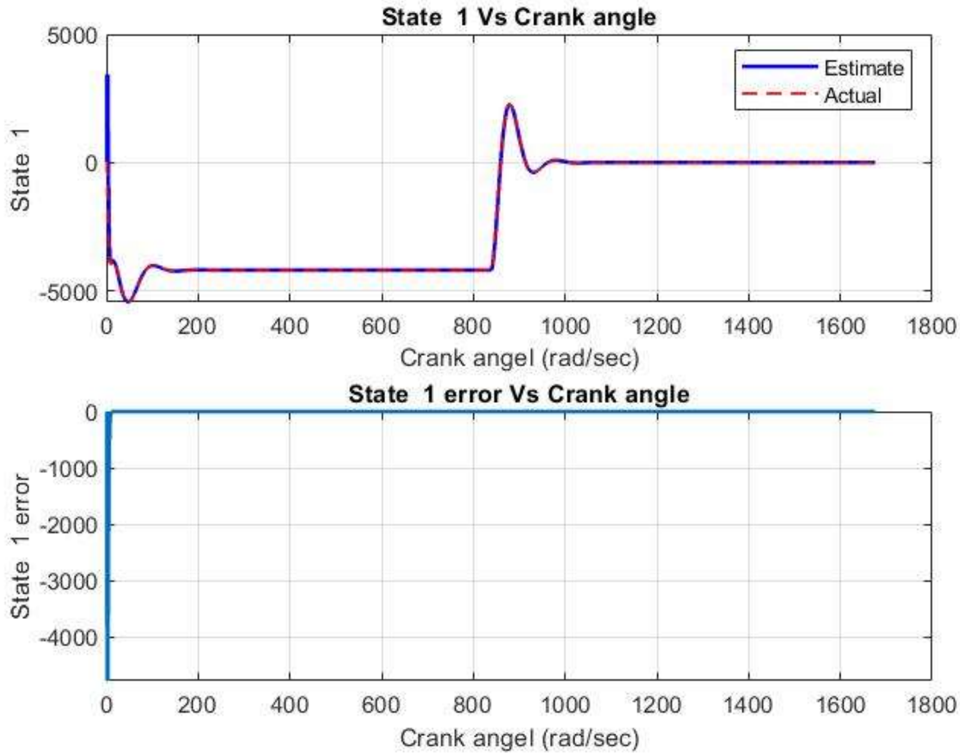
The response for state feedback using estimated state is as follows,

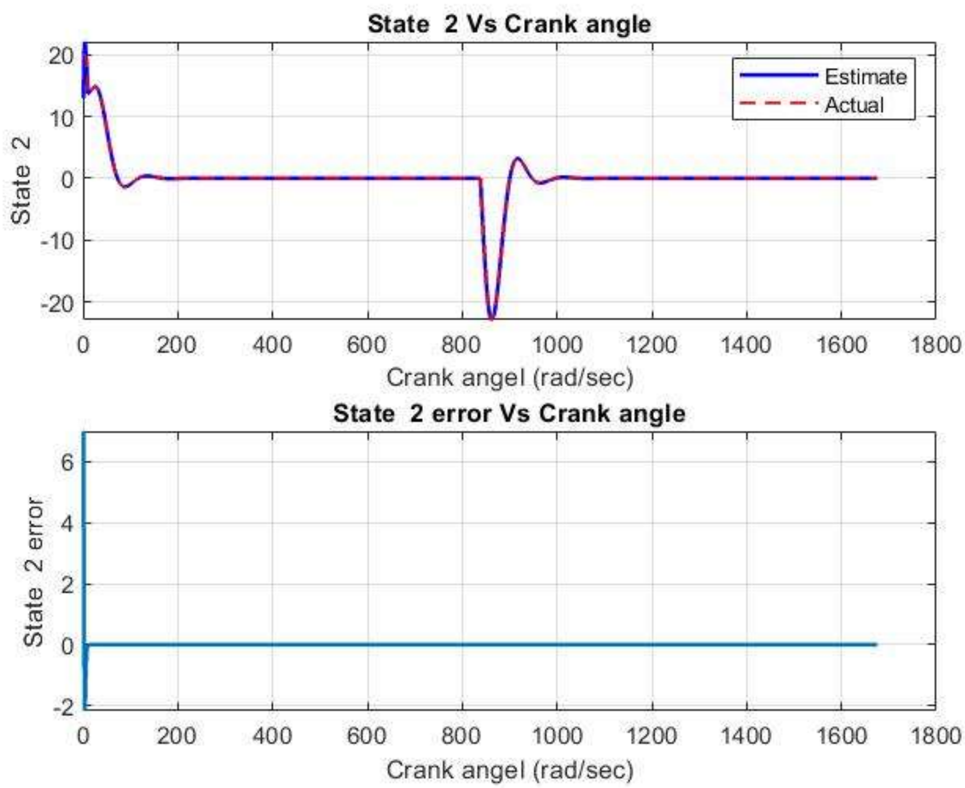


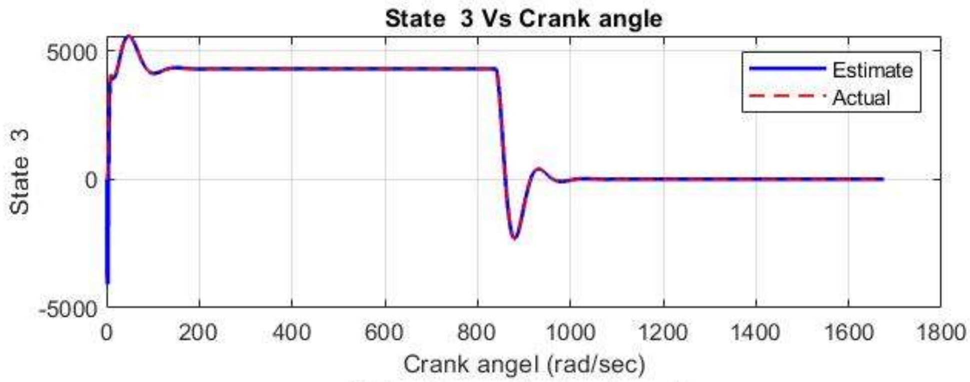

**Closed loop Response using estimated state**

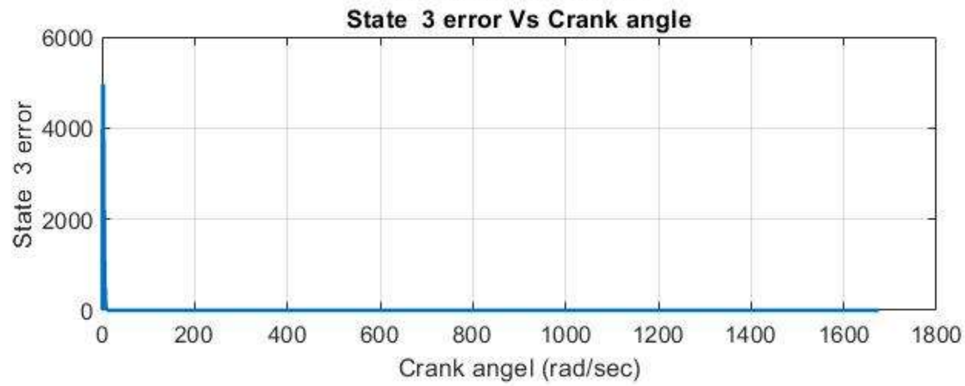
**NOTE:** The speed in rpm starts at approx. 650rpm because the initial states is chosen non-zero for the plant also (to check regulatory response) and for the observer (to check convergence of the estimates) (both the initial conditions are not equal)

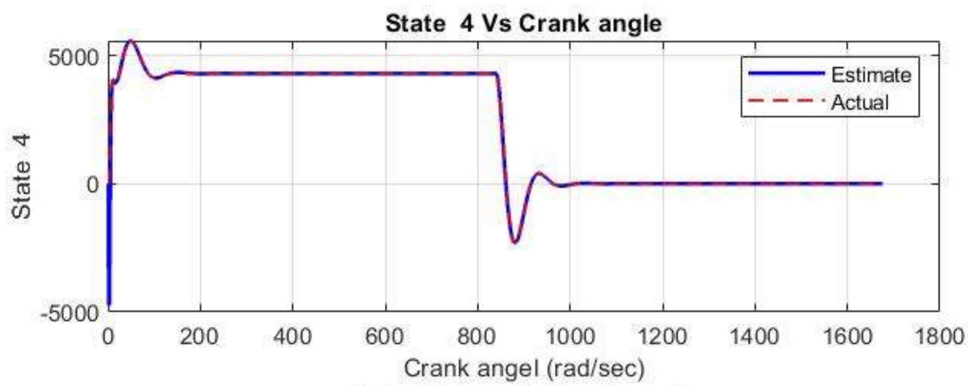
**The estimates and error of each states is as follows;**

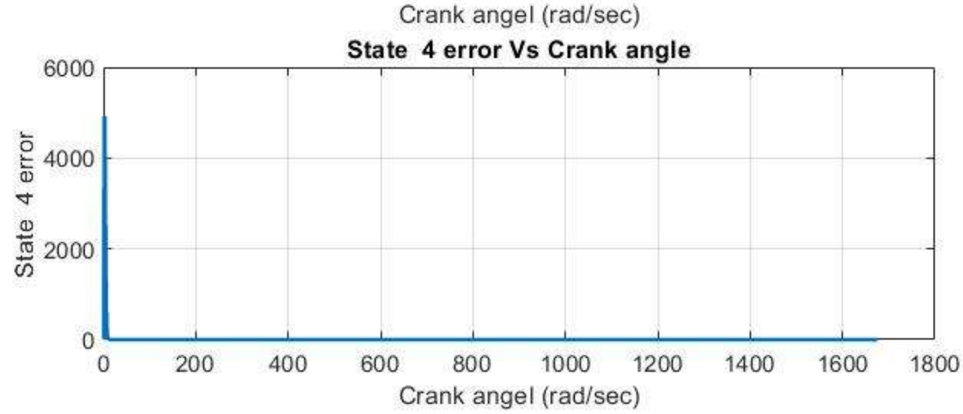


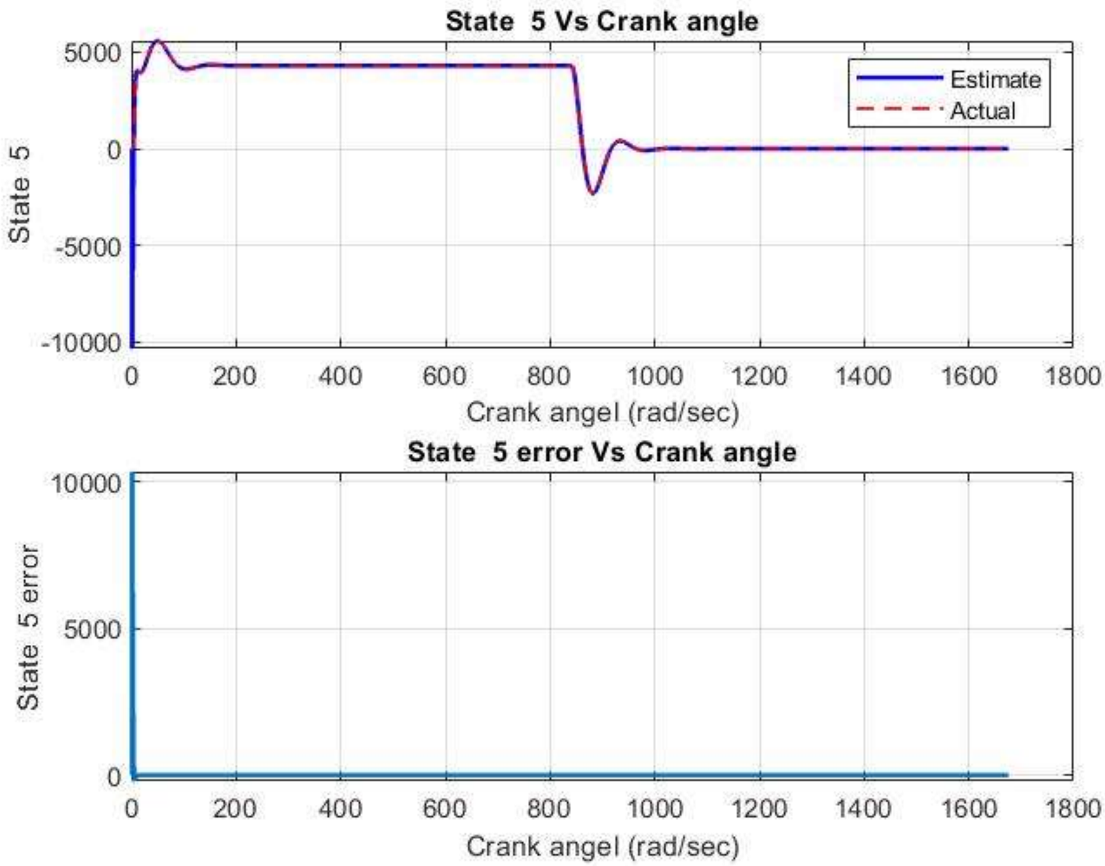




State 3 Vs Crank angle
State 3
Estimate
Actual
5000
0
-5000
0
200
400
600
800
1000
1200
1400
1600
1800
Crank angel (rad/sec)


State 3 error Vs Crank angle
State 3 error
6000
4000
2000
0
0
200
400
600
800
1000
1200
1400
1600
1800
Crank angel (rad/sec)


State 4 Vs Crank angle
State 4
Estimate
Actual
5000
0
-5000
0
200
400
600
800
1000
1200
1400
1600
1800
Crank angel (rad/sec)


State 4 error Vs Crank angle
State 4 error
6000
4000
2000
0
0
200
400
600
800
1000
1200
1400
1600
1800
Crank angel (rad/sec)

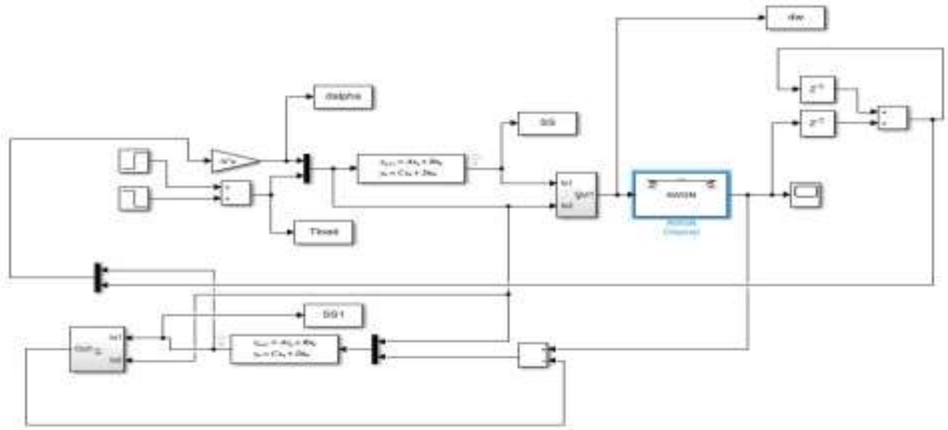



**Inference:**

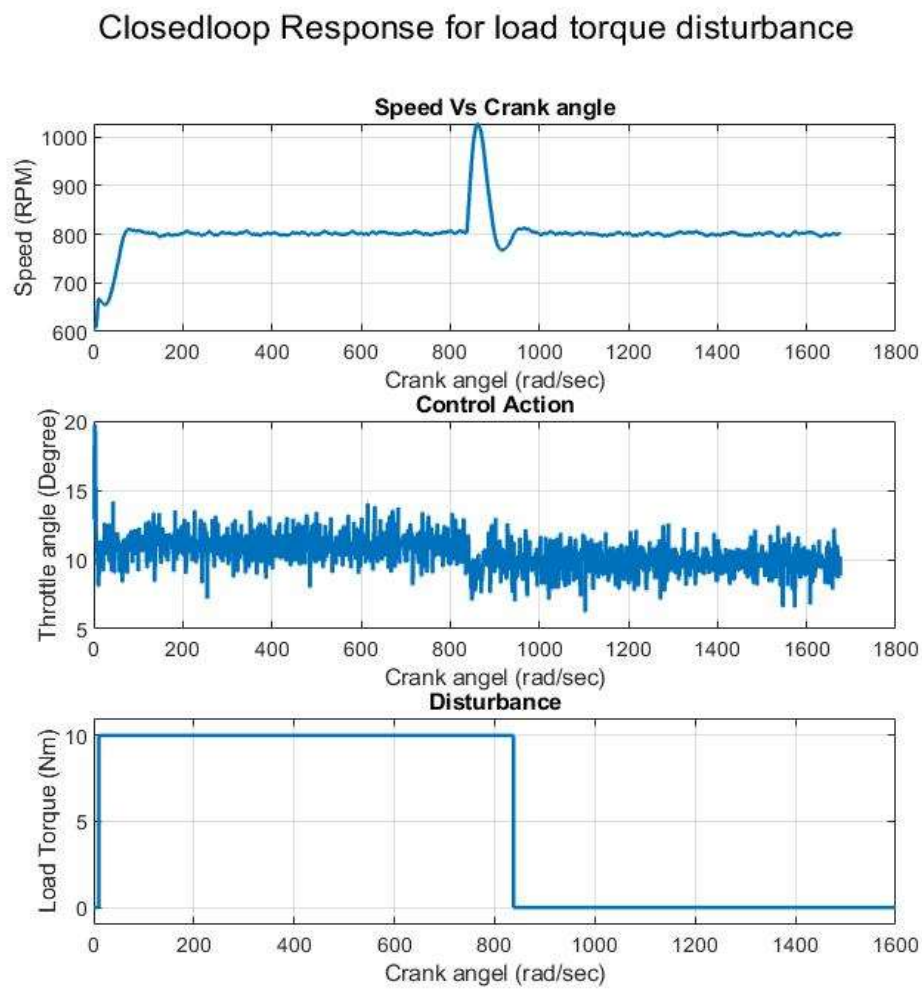
All the state are tracked properly (Error between estimate and actual states tends to zero)
The throttle angle, overshoot, settling time are within desired region.
The steady state error is zero (Because of integral action)

**3. Gaussian Noise**

Adding Gaussian noise (Variance=0.8) to speed measurement and using the same control as in previous problem.



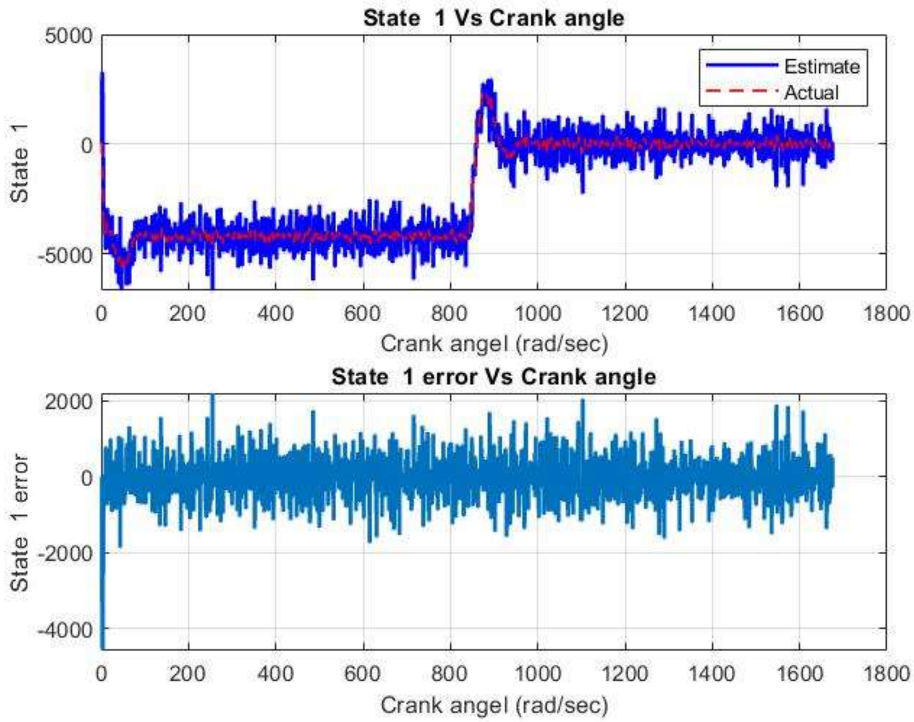
We get the following response.

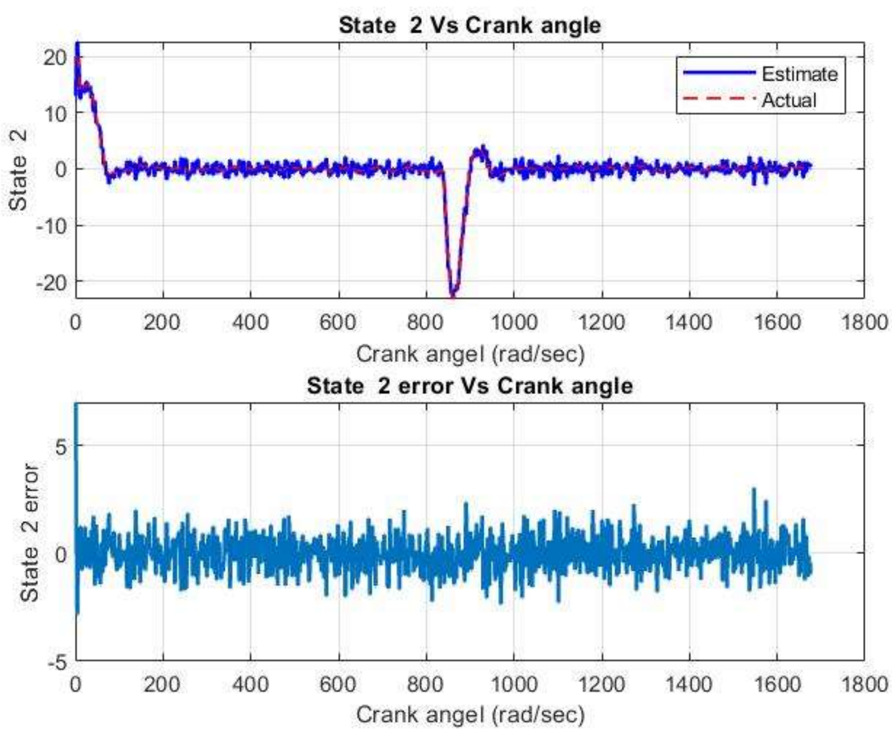


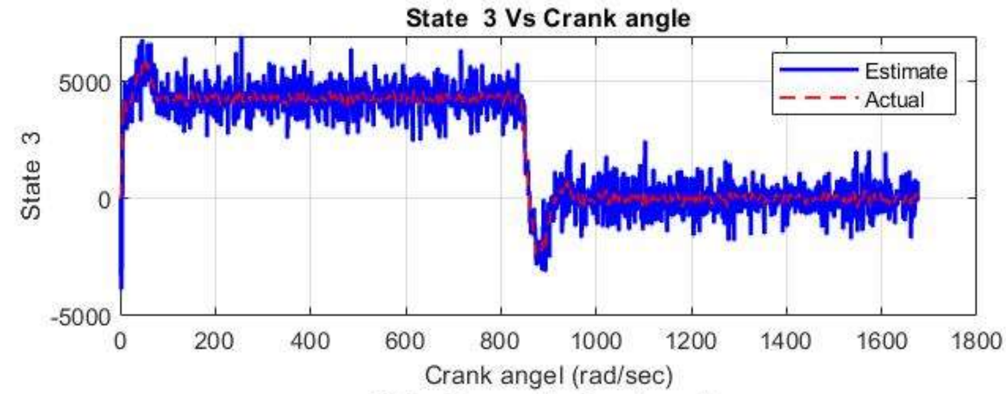

**Closed Loop Response with Gaussian noise**

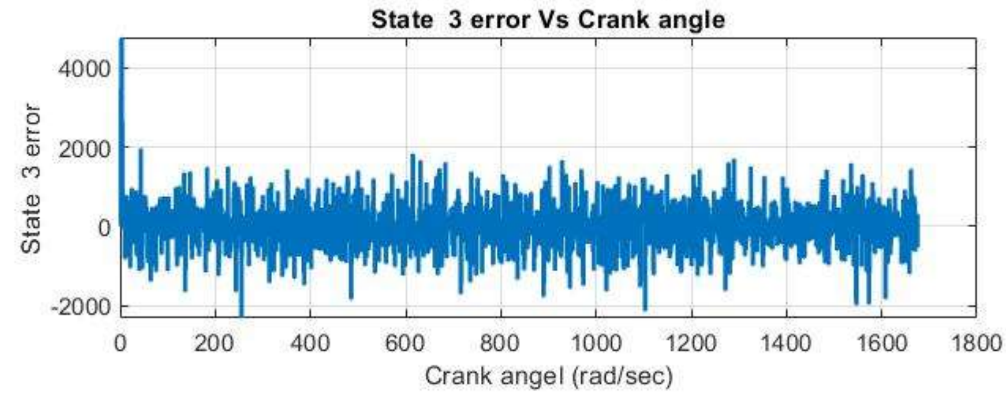
**NOTE:** The speed in rpm starts at approx. 650rpm because the initial states is chosen non-zero for the plant also (to check regulatory response) and for the observer (to check convergence of the estimates) (both the initial conditions are not equal)

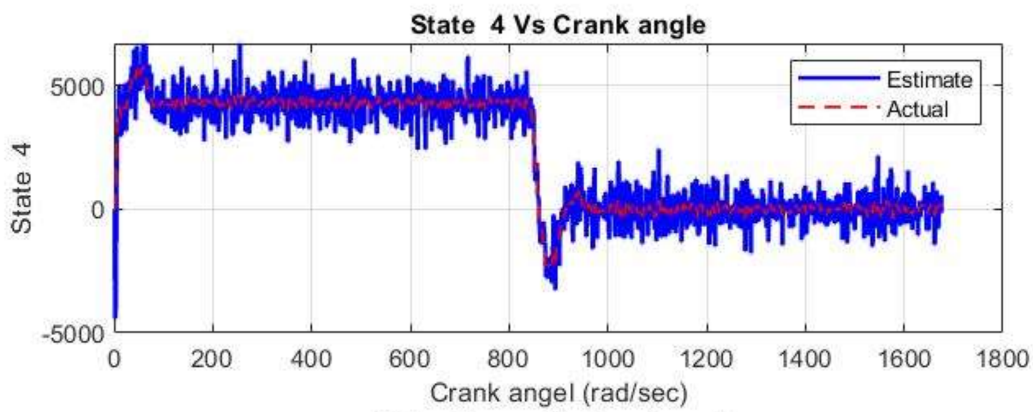
**The state estimate and errors are,**

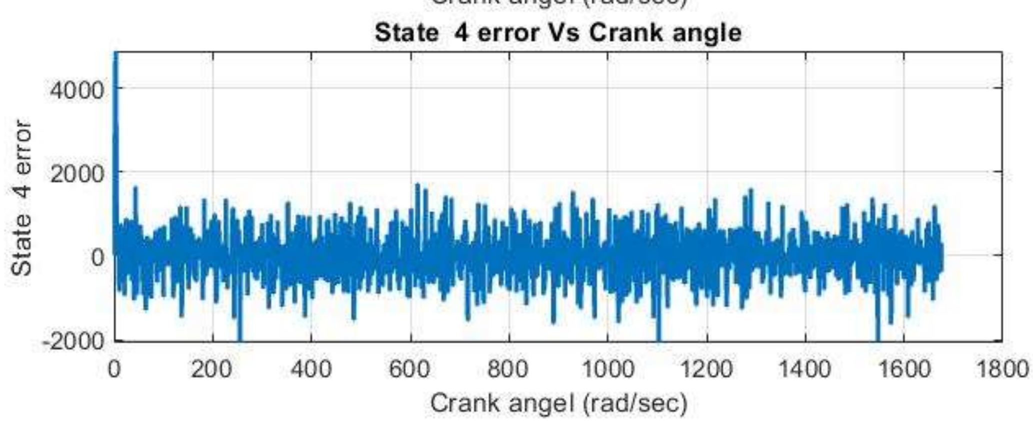


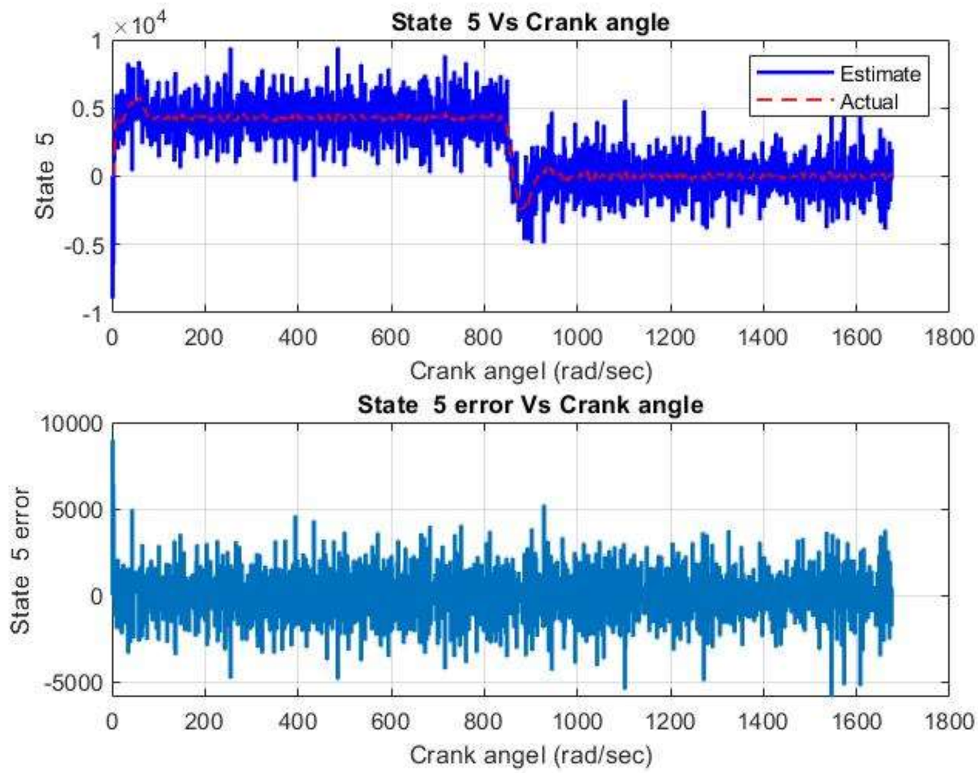




State 3 Vs Crank angle
Estimate
Actual
State 3
5000
0
-5000
0
200
400
600
800
1000
1200
1400
1600
1800
Crank angel (rad/sec)


State 3 error Vs Crank angle
State 3 error
4000
2000
0
-2000
0
200
400
600
800
1000
1200
1400
1600
1800
Crank angel (rad/sec)


State 4 Vs Crank angle
Estimate
Actual
State 4
5000
0
-5000
0
200
400
600
800
1000
1200
1400
1600
1800
Crank angel (rad/sec)


State 4 error Vs Crank angle
State 4 error
4000
2000
0
-2000
0
200
400
600
800
1000
1200
1400
1600
1800
Crank angel (rad/sec)




**Inference:**

All the state were tracked significantly (In particular, the states 1 and 2 are tracked with comparably lesser error than the states 3, 4 and 5. This is due to the added Gaussian noise and inherent delay in the system)

The throttle angle, overshoot, settling time are within desired region (But oscillates).

The steady state error is non-zero (But chatters around zero value)

**4. Kalman Filter design**

To the same system (with Gaussian noise), Kalman filter is implemented as follows;

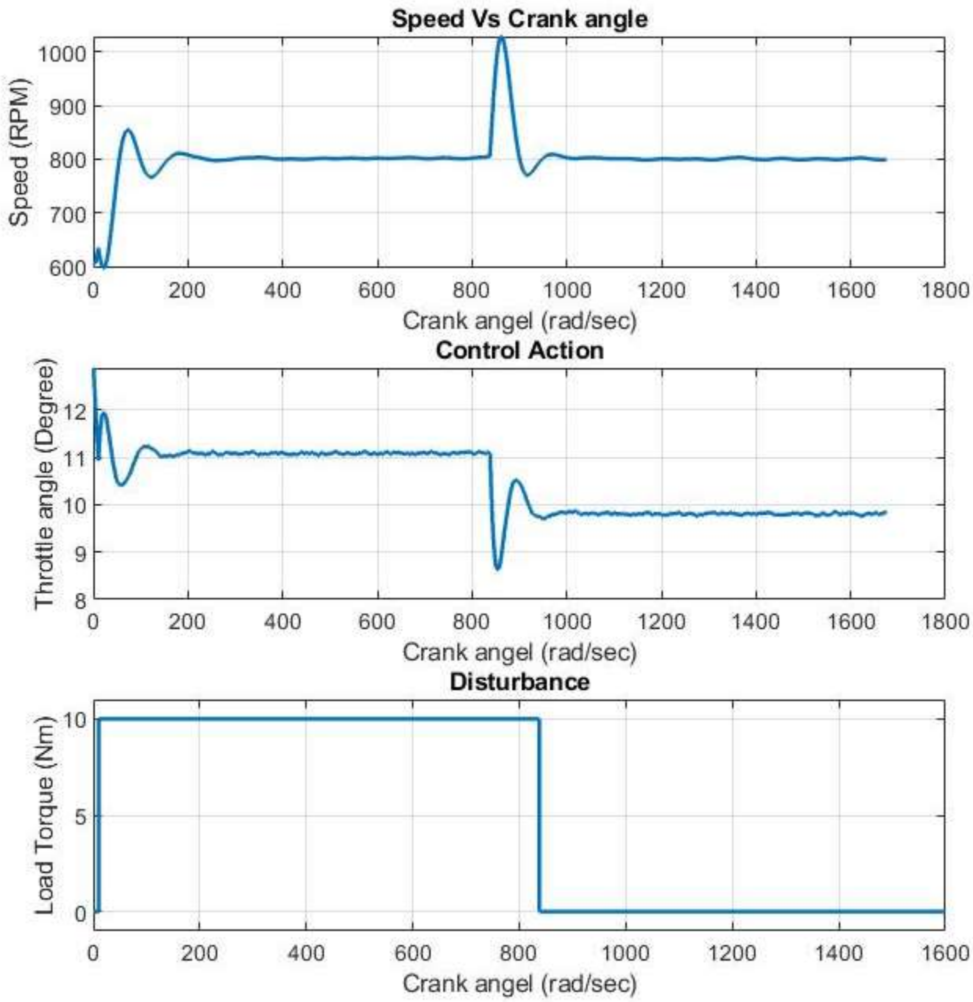
The Process noise (Q) and measurement noise (R) is assumed to be 0.01 and 0.8 respectively. The filter gain is determined using state-state kalman filter gain,

**[KEST,K_bar,P]=kalman(Plant,Q_noise,R_noise)**

The K_bar is used as filter gain. (With same LQR controller as in previous problem) The system response is as follows, The kalman filter gain (K_bar) is,

$$\begin{matrix} 0.4340 \\ -0.0076 \\ -0.4259 \\ -0.3892 \\ -0.3524 \end{matrix}$$

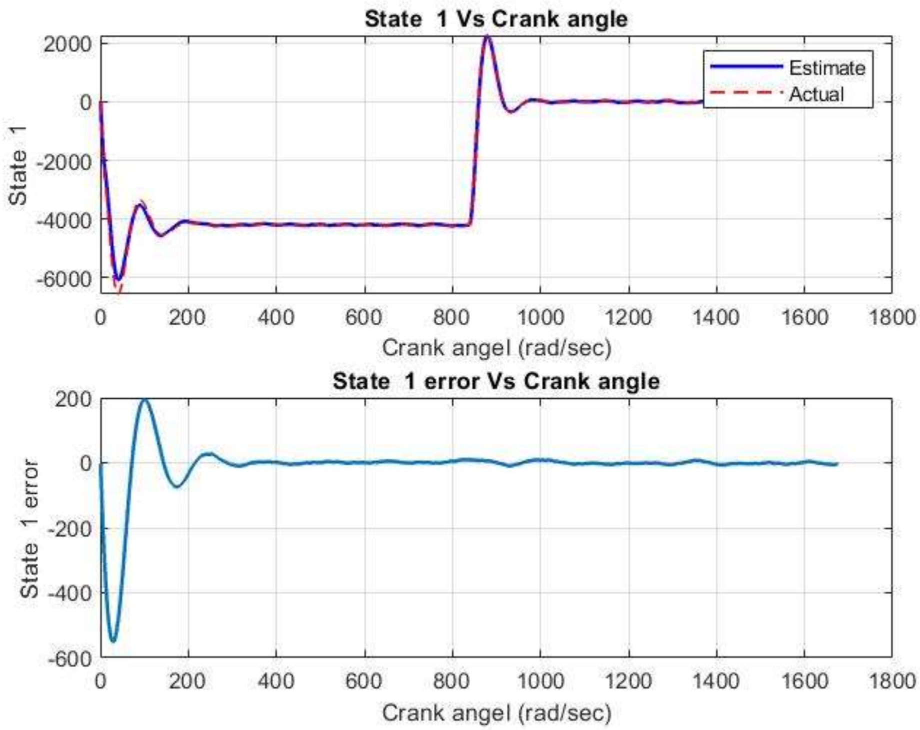
The overall response of the system with kalman filter is,

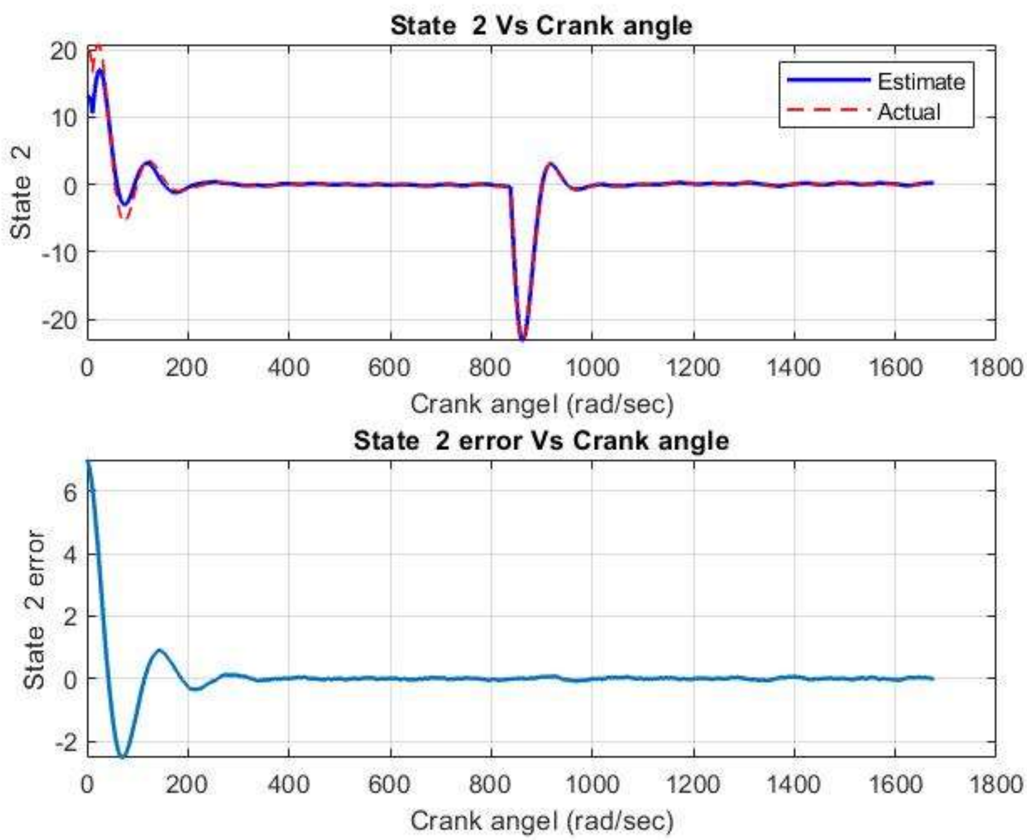


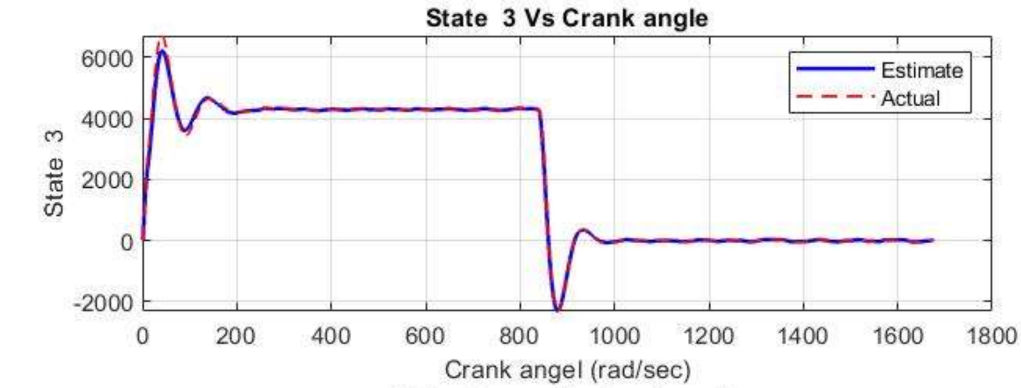

**Closed Loop Response with Kalman Filter**

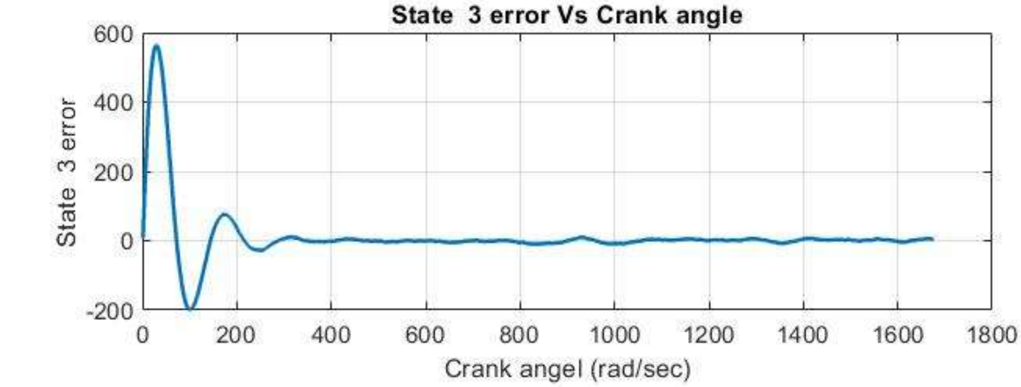
**NOTE:** The speed in rpm starts at approx. 650rpm because the initial states is chosen non-zero for the plant also (to check regulatory response) and for the observer (to check convergence of the estimates) (both the initial conditions are not equal)

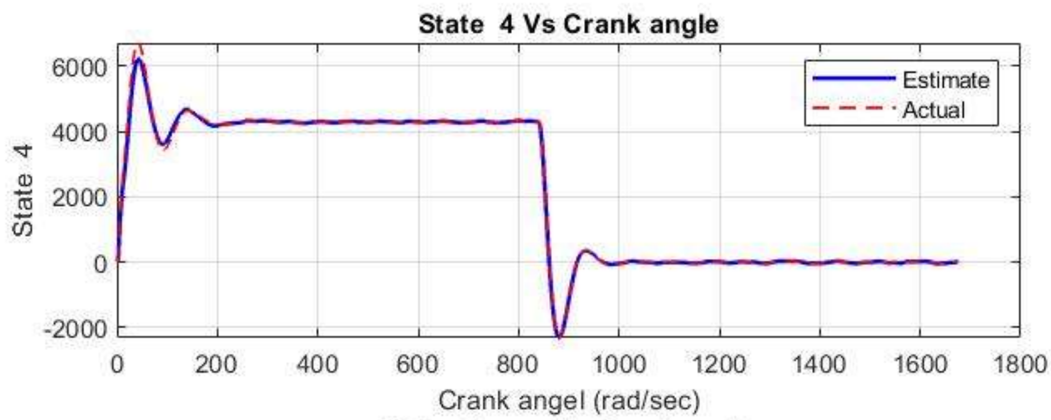
**The State Estimates and Error:**

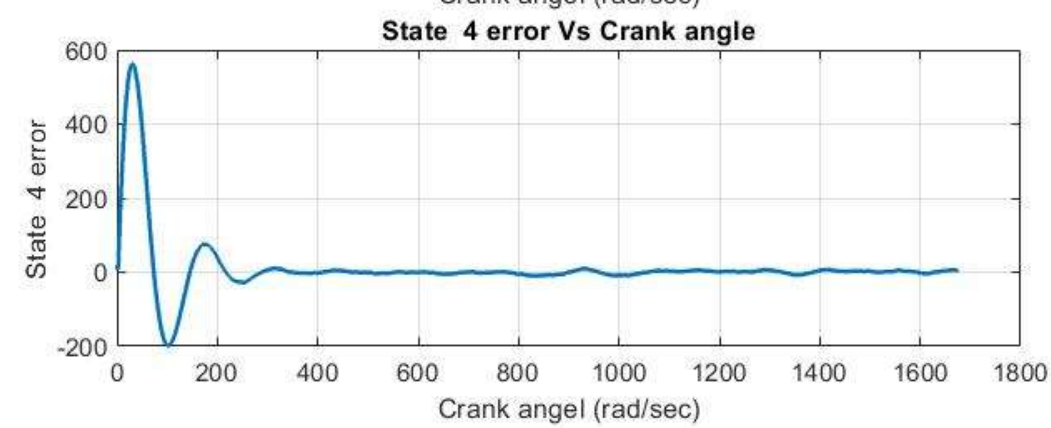


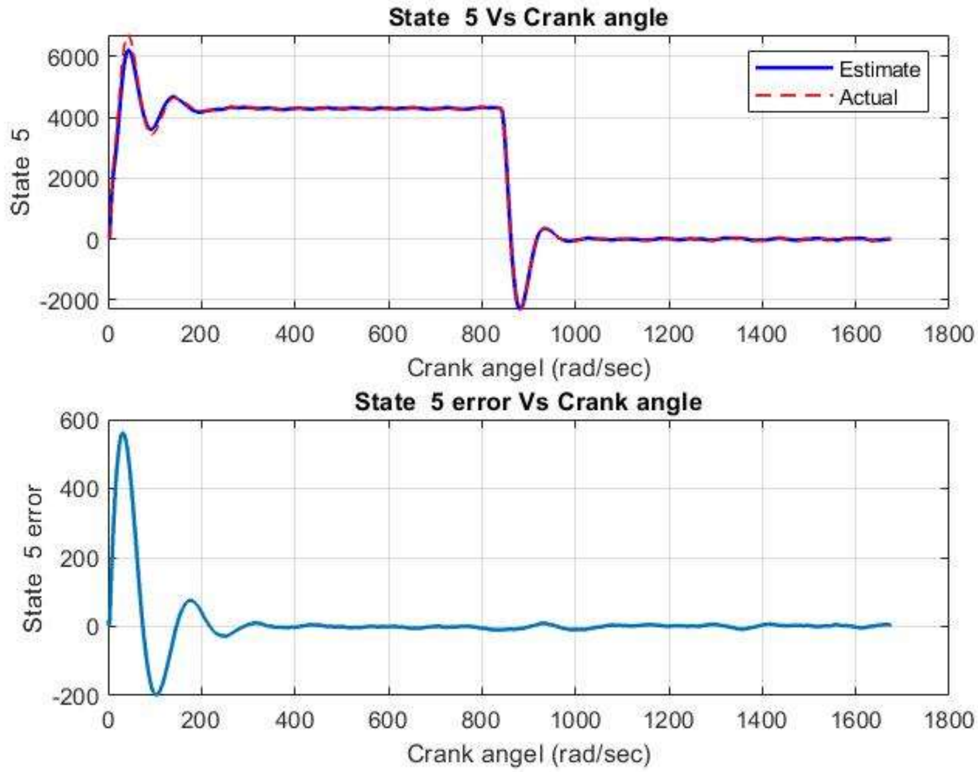




State 3 Vs Crank angle
Estimate
Actual
State 3
Crank angel (rad/sec)


State 3 error Vs Crank angle
State 3 error
Crank angel (rad/sec)


State 4 Vs Crank angle
Estimate
Actual
State 4
Crank angel (rad/sec)


State 4 error Vs Crank angle
State 4 error
Crank angel (rad/sec)




**Inference:**

All the state were tracked significantly (response is satisfactory)

The throttle angle, overshoot, settling time are within desired region.

The steady state error is fairly zero (But chatters around zero value with small variation)