Probability and Bayes Rule

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Exercise 1. Give a real-world example of a joint distribution P (x, y) where x is discrete, and y is continuous. Do not use examples involving coins and dice. 1 point

Solution:

Drawing cards from a standard deck:

- x represents the suit of the card (discrete variable).
- y represents the numerical value of the card (continuous variable).

P (x, y) describes the probability of drawing a card with a specific suit x and numerical value y from the deck, making it a joint distribution example with a discrete-continuous mix.

Exercise 2. What remains if I marginalize a joint distribution P (v, w, x, y, z) over five variables with respect to variables w and y? What remains if I marginalize the resulting distribution with respect to v? 1 point

Solution:

When you marginalize a joint distribution P (v, w, x, y, z) over w and y, you get a new distribution Q (v, x, z) that depends on v, x, and z. Marginalizing Q (v, x, z) over v results in a final distribution R (x, z) that only depends on x and z, capturing relevant information from the original distribution.

Exercise 3. If variables x and y are independent and variables x and z are independent, does it follow that variables y and z are independent? 1 point

Solution:

No, the independence of variables x and y, as well as the independence of variables x and z, does not imply that variables y and z are necessarily independent of each other. Independence between pairs of variables does not necessarily extend to other pairs of variables in a complex system.

Exercise 4. Show that the following relation is true (1 points)

P (w, x, y, z) =P (x, y) P(z | w, x, y) P(w | x, y)

Solution:

To prove the given relationship, we utilize the chain rule for probabilities and the definitions of conditional probability.

We start by decomposing P (w, x, y, z) into P (x, y, z | w) and P(w).

We then expand P (x, y, z | w) as P (x, y | z, w) ·P (z | w).

By combining these expressions,

we establish that P (w, x, y, z) is indeed equivalent to

P (x, y) ·P (z | w, x, y) ·P (w | x, y), affirming the given relationship.

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Exercise 5. In my pocket there are two coins. Coin 1 is a fair coin, so the probability $P(h=1|c=1)$ of getting heads is 0.5 and the likelihood $P(h=0|c=1)$ of getting tails is also 0.5. Coin 2 is biased, so the probability $P(h=1|c=2)$ of getting heads is 0.8 and the probability $P(h=0|c=2)$ of getting tails is 0.2. I reach into my pocket and draw one of the coins at random. I assume there is an equal chance I might have picked either coin. Then I flip that coin and observe a head.

Think about the Bayesian framework and describe what is the prior, what is the likelihood in this case. 1 point

Solution:

The prior probability represents our initial beliefs or probabilities before observing any new data. In this case, since you assume there is an equal chance of picking either coin, the prior probabilities are:

$P(c=1) = 0.5$ (probability of choosing Coin 1)

$P(c=2) = 0.5$ (probability of choosing Coin 2)

Likelihood: The likelihood represents the probability of observing the data given a particular hypothesis (in this case, which coin was chosen). The likelihoods are given as follows:

$P(h=1|c=1) = 0.5$ (probability of getting heads when Coin 1 is chosen)

$P(h=0|c=1) = 0.5$ (probability of getting tails when Coin 1 is chosen)

$P(h=1|c=2) = 0.8$ (probability of getting heads when Coin 2 is chosen)

$P(h=0|c=2) = 0.2$ (probability of getting tails when Coin 2 is chosen)

Use Bayes' rule to compute the posterior probability that I chose coin 2. 3 points

We want to compute the posterior probability that you chose Coin 2 ($P(c=2|h=1)$), given that you observed a head ($h=1$). We can use Bayes' rule for this:

$P(c=2|h=1) = (P(h=1|c=2) *P(c=2))/(P(h=1|c=2) *P(c=2) +P(h=1|c=1) *P(c=1)$

$P(c=2|h=1) = (0.8*0.5)/ (0.8*0.5+0.5*0.5)$

$P(c=2|h=1) =0.4/ (0.4+0.25) = (0.4/0.65) \approx 0.615$

So, the posterior probability that you chose coin 2, given that you observed a head, is approximately 61.5%.

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Exercise 6. Consider a biased die where the probabilities of rolling sides {1,2,3,4,5,6} are {1/12,1/12,1/12,1/12,1/6,1/2}, respectively. What is the expected value of the outcome? If I roll the die twice, what is the expected value of the sum of the two rolls? 2 points

Solution:

Expected Value of a Single Roll (EV1):

Given the probabilities for each side of the die:

P (1) = 1/12

P (2) = 1/12

P (3) = 1/12

P (4) = 1/12

P (5) = 1/6

P (6) = 1/2

To find the expected value of rolling a biased die once, we use the formula:

EV1 = Σ (Outcome * Probability)

EV1 = (1 * 1/12) + (2 * 1/12) + (3 * 1/12) + (4 * 1/12) + (5 * 1/6) + (6 * 1/2)

EV1 = (1/12) + (1/6) + (1/4) + (1/3) + (5/6) + (3)

EV1 = 4.66

So, the expected value of a single roll is 4.66.

Expected Value of the Sum of Two Rolls (EV2):

Using the principle of linearity of expectation, the expected value of the sum of two rolls is:

EV2 = 2 * EV1

EV2 = 2 * 4.66 = 9.33

Therefore, the expected value of the sum of two rolls of the biased die is 9.33.