EE5605: Kernel Methods, Spring 2019 (56)

Indian Institute of Technology Hyderabad

HW 1, 60 points. Assigned: Wednesday 17.04.2019.

Due: Tuesday 23.04.2019 at 11:59 pm.

1. Show that product of kernels is a kernel. (5)

2. Use the following results about kernels to answer the questions that follow:

   - The sums of kernels are kernels.
   - The products of kernels are kernels.
   - If $T : \mathcal{X} \to \mathcal{X}'$ is a mapping from the set $\mathcal{X}$ to the set $\mathcal{X}'$ and k is a kernel defined on $\mathcal{X}'$, then $k(T(x), T(x'))$ is a kernel on $\mathcal{X}$.
   - Assume we can define a Taylor series $f(x) = \sum_{n=0}^{\infty} a_n x^n, |x| < r, x \in \mathbb{R}$ for $r \in (0, \infty]$, with $a_n \geq 0$ for all $n \geq 0$. Define $\mathcal{X}$ to be a $\sqrt{r}$-ball in $\mathbb{R}^d$. Then for $x, x' \in \mathbb{R}^d$ such that $\|x\| < \sqrt{r}$, $k(x, x') = f(\langle x, x' \rangle) = \sum_{n=0}^{\infty} a_n(\langle x, x' \rangle)^n$ is a kernel.

   Show that the following functions are valid kernels:

   (a) $k(x, y) := (\langle x, y \rangle + c)^n, n \geq 1, c \geq 0$, for all $x, y \in \mathbb{R}^d, d \geq 1$. (5)

   (b) $k(x, y) := \exp(\langle x, y \rangle), x, y \in \mathbb{R}^d, d \geq 1$. (5)

3. Moore-Aronszajn Theorem: Read Gretton2014.pdf and summarize how an RKHS can be constructed from a kernel. (5)

4. Implement the vanilla (linear) and kernel versions of a support vector machine (SVM). Use python. Do not use built-in functions for SVM. You can use open-source solvers for the convex optimization. (20)

5. Use *Xsvm.csv, ysvm.csv* to train the linear classifier. Test on the following points $x_t^1 = [1.9, 0.4], x_t^2 = [0.9, 0.9], x_t^3 = [1, 4, 1.5], x_t^4 = [0.01, 0.005]$. Report your predictions. (10)

6. Demonstrate the ability of the kernel SVM to learn the non-linear XOR function. Experiment with different kernels. Report the result of the kernel SVM and compare it with the vanilla version. (10)