

MatGeo Assignment 5.12.2

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AI25BTECH11007

Question :

Use elementary column operation $C_2 \rightarrow C_2 + 2C_1$ in the following matrix equation

$$\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

Solution :

We have

$$A = \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

The column operation $C_2 \rightarrow C_2 + 2C_1$ is represented by the elementary matrix

$$E = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix},$$

since post-multiplication by E performs the same column operation.

Thus, by matrix theory,

$$AE = MNE,$$

where

$$AE = \begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix}, \quad NE = \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}.$$

Hence,

$$\boxed{\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}}.$$