MatGeo Assignment 5.5.24

AI25BTECH11007

September 28, 2025

Question

Using elementary row operations, find the inverse of the matrix $A = \left[\begin{matrix} 3 & -3 \\ 2 & -3 \\ 0 & -1 \end{matrix}\right.$

and hence solve the following system of equations:

$$\begin{cases} 3x - 3y + 4z = 21, \\ 2x - 3y + 4z = 20, \\ -y + z = 5. \end{cases}$$

Solution

$$\text{Let } \mathbf{A} = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Augment $\mathbf{A}$ with the identity: $$[\mathbf{A} \mid \mathbf{I}] = \left(\begin{array}{ccc|ccc} 3 & -3 & 4 & 1 & 0 & 0 \\ 2 & -3 & 4 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right)$$

Row transformation 1: $R_2 \to 3R_2 - 2R_1$

$$\left(\begin{array}{ccc|ccc} 3 & -3 & 4 & 1 & 0 & 0 \\ 0 & -3 & 4 & -2 & 3 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right)$$

Row transformation 2 : $R_3 \to 3R_3 - R_2$

$$\left(\begin{array}{ccc|ccc} 3 & -3 & 4 & 1 & 0 & 0 \\ 0 & -3 & 4 & -2 & 3 & 0 \\ 0 & 0 & -1 & 2 & -3 & 3 \end{array}\right)$$

Row transformation 3 : $R_3 \to -R_3$

$$\left(\begin{array}{ccc|ccc} 3 & -3 & 4 & 1 & 0 & 0 \\ 0 & -3 & 4 & -2 & 3 & 0 \\ 0 & 0 & 1 & -2 & 3 & -3 \end{array}\right)$$

Row transformation 4 and 5 : $R_2 \to R_2 - 4R_3, \quad R_1 \to R_1 - 4R_3$

$$\left(\begin{array}{ccc|ccc} 3 & -3 & 0 & 9 & -12 & 12 \\ 0 & -3 & 0 & 6 & -9 & 12 \\ 0 & 0 & 1 & -2 & 3 & -3 \end{array}\right)$$

Row transformation 6 : Scale rows to get leading 1's:

$$R_2 \to -\frac{1}{3}R_2, \qquad R_1 \to \frac{1}{3}R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 3 & -4 & 4 \\ 0 & 1 & 0 & -2 & 3 & -4 \\ 0 & 0 & 1 & -2 & 3 & -3 \end{array}\right)$$

Row transformation 7 : $\quad R_1 \to R_1 + R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -4 \\ 0 & 0 & 1 & -2 & 3 & -3 \end{array}\right)$$

$$\text{Thus } \boxed{\mathbf{A}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}}$$

Solving system of equations,

$$\text{We have } \mathbf{A}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}.$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \implies \boxed{x = 1,\ y = -2,\ z = 3}$$

C code

```c
        #include <stdio.h>

int main() {
    // Inverse of A
    double Ainv[3][3] = {
        {1, -1, 0},
        {-2, 3, -4},
        {-2, 3, -3}
    };

    // Vector b
    double b[3] = {21, 20, 5};

    // Result vector x = Ainv * b
    double x[3] = {0, 0, 0};
```

C code

```c
        // Matrix-vector multiplication
    for (int i = 0; i < 3; i++) {
        for (int j = 0; j < 3; j++) {
            x[i] += Ainv[i][j] * b[j];
        }
    }

    // Print the solution
    printf("Solution of the system:\n");
    printf("x = %.2lf\n", x[0]);
    printf("y = %.2lf\n", x[1]);
    printf("z = %.2lf\n", x[2]);

    return 0;
}
```

Python code

```python
        import numpy as np
# Inverse of A
A_inv = np.array([
    [1, -1, 0],
    [-2, 3, -4],
    [-2, 3, -3]
])

# Vector b
b = np.array([21, 20, 5])
# Solution x = A_inv * b
x = np.dot(A_inv, b)

print("Solution of the system:")
print(f"x = {x[0]}")
print(f"y = {x[1]}")
print(f"z = {x[2]}")
```