MatGeo Assignment 4.13.76

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Question

Two lines

$$L_1 : \frac{x}{5} = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : \frac{x}{\alpha} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

are coplanar. Then the value(s) of α

Solution

$$L_1 : \frac{x}{5} = \frac{y}{3-\alpha} = \frac{z}{-2}, \qquad L_2 : \frac{x}{\alpha} = \frac{y}{-1} = \frac{z}{2-\alpha}.$$

$$\text{Direction vectors are : } \mathbf{n}_1 = \begin{pmatrix} 5 \\ 3-\alpha \\ -2 \end{pmatrix}, \qquad \mathbf{n}_2 = \begin{pmatrix} \alpha \\ -1 \\ 2-\alpha \end{pmatrix}.$$

Choose points on each line, as both lines pass through the origin, so, $\mathbf{p}_1 = \mathbf{p}_2$

Two lines are coplanar iff

$$\text{rank}\left(\begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{p}_2 - \mathbf{p}_1 \end{pmatrix} \right) \le 2.$$

Here $\mathbf{p}_2 - \mathbf{p}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, so the matrix becomes

$$\begin{pmatrix} 5 & \alpha & 0 \\ 3-\alpha & -1 & 0 \\ -2 & 2-\alpha & 0 \end{pmatrix},$$

whose rank is at most 2 for every α. Hence the two lines are coplanar for all real α.

$$\boxed{\alpha \in \mathbb{R}}$$

Plot

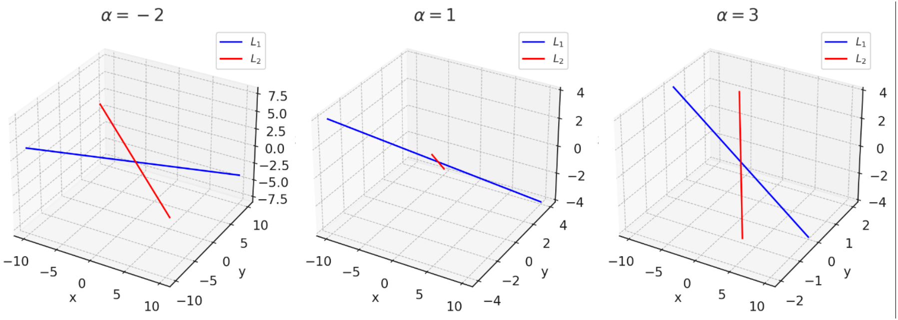

Figure: Image

C code

```c
#include <stdio.h>

// Function to compute determinant of 3x2 augmented with 0 column
// Actually we compute rank check: det([n1 n2]) = 0 always
int main() {
    int alpha;
    printf("Lines:\n");
    printf("L1: x/5 = y/(3-alpha) = z/(-2)\n");
    printf("L2: x/alpha = y/(-1) = z/(2-alpha)\n\n");

    printf("Checking coplanarity for sample alpha values...\n");

    for(alpha = -3; alpha <= 5; alpha++) {
        int n1[3] = {5, 3 - alpha, -2};
        int n2[3] = {alpha, -1, 2 - alpha};
```

C code

```c
    // Compute determinant of 3x2 matrix (v1, v2)
        // In coplanarity test: rank <= 2 always (since both pass through origin)
        // Let's check cross product
        int cross[3];
        cross[0] = n1[1]*n2[2] - n1[2]*n2[1];
        cross[1] = n1[2]*n2[0] - n1[0]*n2[2];
        cross[2] = n1[0]*n2[1] - n1[1]*n2[0];

        printf("alpha = %d -> Lines are coplanar (always true)\n", alpha);
    }

    printf("\nConclusion: Lines are coplanar for all real alpha.\n");
    return 0;
}
```

Python code

```
    import numpy as np

def check_coplanarity(alpha):
    # Direction vectors
    v1 = np.array([5, 3 - alpha, -2])
    v2 = np.array([alpha, -1, 2 - alpha])

    # Form matrix with direction vectors as columns
    M = np.column_stack((v1, v2))

    # Rank of coefficient matrix
    rank = np.linalg.matrix_rank(M)

    # Since both lines pass through origin, always coplanar
    return rank <= 2
```

Python code

```
        # Test for some sample alpha values
alpha_values = [-3, -1, 0, 1, 2, 3, 5]
for a in alpha_values:
    result = check_coplanarity(a)
    print(f"alpha = {a:2d} -> Coplanar: {result}")

print("\nConclusion: Lines are coplanar for all real alpha.")
```