MatGeo Assignment 8.2.43

AI25BTECH11007

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Question

Find the equation of the conic, that satisfies the given conditions.
Focus at (-1, 2), directrix $x - 2y + 3 = 0$.

Solution

Let :

$$\mathbf{F} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{1}$$

$$\text{directrix equation is : } \quad x - 2y + 3 = 0 \quad \implies \quad \begin{pmatrix} 1 \\ -2 \end{pmatrix}^T \mathbf{x} = -3 \tag{2}$$

The equation of a conic with directrix $\mathbf{n}^T\mathbf{x} = c$, eccentricity e and focus $\mathbf{F}$ is given by:

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{3}$$

where :

$$\begin{aligned} \mathbf{V} &= \|\mathbf{n}\|^2\mathbf{I} - e^2\mathbf{n}\mathbf{n}^T, \\ \mathbf{u} &= ce^2\mathbf{n} - \|\mathbf{n}\|^2\mathbf{F}, \\ f &= \|\mathbf{n}\|^2\|\mathbf{F}\|^2 - c^2e^2 \end{aligned}$$

From the question we can say that the conic is a parabola so $e = 1$.
Calculating $\mathbf{V}$, $\mathbf{u}$ and f by using the above equations we get :

$$\mathbf{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \qquad \|\mathbf{n}\|^2 = 5, \qquad c = -3, \qquad \|\mathbf{F}\|^2 = 5$$

$$\mathbf{V} = 5\mathbf{I} - \mathbf{n}\mathbf{n}^T = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \tag{4}$$

$$\mathbf{u} = c\mathbf{n} - 5\mathbf{F} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \tag{5}$$

$$f = 5 \cdot 5 - (-3)^2 = 16 \tag{6}$$

Finding eigen values of $\mathbf{V}$:

$$\det|\mathbf{V} - \lambda\mathbf{I}| = 0 \tag{7}$$

$$\det \begin{pmatrix} 4 - \lambda & 2 \\ 2 & 1 - \lambda \end{pmatrix} = 0 \tag{8}$$

(9)

$$\lambda = 5 \quad \text{and} \quad 0 \tag{10}$$

Eigen vectors $\mathbf{v}$ for any square matrix $\mathbf{A}$ are defined by:

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v} \tag{11}$$

For $\lambda = 0$ and $\lambda = 5$ we get

$$\lambda = 0 \quad \Rightarrow \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \tag{12}$$

$$\lambda = 5 \quad \Rightarrow \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \tag{13}$$

Substituting in the equation (0.3) we get the equation of the conic to be :

$$\mathbf{x}^T \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 2 & -4 \end{pmatrix} \mathbf{x} + 16 = 0 \tag{14}$$

Plot

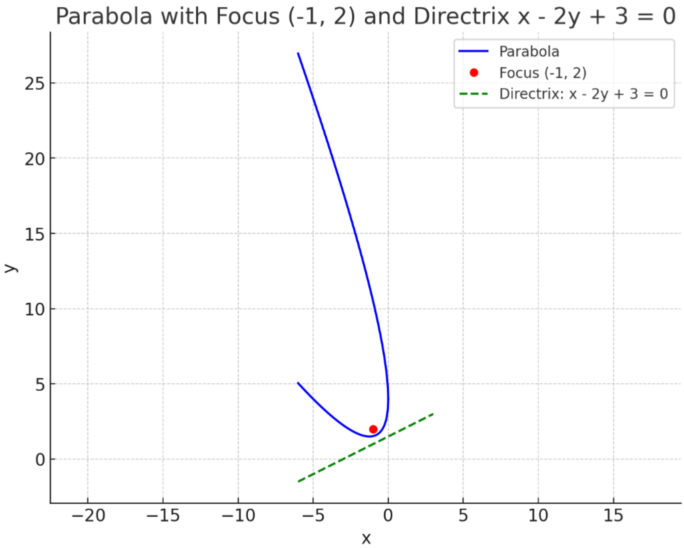

Figure: Image

C code

```c
        #include <stdio.h>
#include <math.h>

int main() {
    // Define vectors and constants
    double F[2] = {-1, 2}; // Focus
    double n[2] = {1, -2}; // Normal vector to directrix
    double c = -3; // Constant term from directrix n^T x = c
    double e = 1; // Parabola => e = 1
    // Compute ||n||^2
    double norm_n2 = n[0]*n[0] + n[1]*n[1];
    // Identity matrix I
    double I[2][2] = {{1, 0}, {0, 1}};
    // Compute n*n^T
    double nnT[2][2];
    for(int i=0; i<2; i++)
        for(int j=0; j<2; j++)
            nnT[i][j] = n[i]*n[j];
```

C code

```c
    // Compute V = ||n||^2 * I - e^2 * (n*n^T)
double V[2][2];
for(int i=0; i<2; i++)
    for(int j=0; j<2; j++)
        V[i][j] = norm_n2*I[i][j] - e*e*nnT[i][j];

// Compute u = c*e^2*n - ||n||^2 * F
double u[2];
for(int i=0; i<2; i++)
    u[i] = c*e*e*n[i] - norm_n2*F[i];

// Compute f = ||n||^2 * ||F||^2 - c^2 * e^2
double f = norm_n2 * (F[0]*F[0] + F[1]*F[1]) - (c*c*e*e);

// Display results
printf("Matrix Form of Parabola:\n\n");

printf("V = [ [%.2f, %.2f], [%.2f, %.2f] ]\n",
       V[0][0], V[0][1], V[1][0], V[1][1]);
```

C code

```
        printf("u = [ %.2f, %.2f ]^T\n", u[0], u[1]);
    printf("f = %.2f\n\n", f);

    printf("Conic Equation: x^T V x + 2u^T x + f = 0\n");
    printf("=> [x y] [%.2f %.2f; %.2f %.2f] [x; y] + 2[%.2f %.2f ][x; y] + %.2f = 0\n",
           V[0][0], V[0][1], V[1][0], V[1][1], u[0], u[1], f);

    return 0;
}
```

Python code

```
import numpy as np
import matplotlib.pyplot as plt

# Parabola equation from matrix form:
# 4xÂš + 4xy + yÂš + 4x - 8y + 16 = 0
# We'll use this to plot the curve.

# Define grid
x = np.linspace(-8, 6, 400)
y = np.linspace(-4, 8, 400)
X, Y = np.meshgrid(x, y)

# Define the equation of the parabola
F = 4*X**2 + 4*X*Y + Y**2 + 4*X - 8*Y + 16

# Plot the parabola (F=0)
plt.contour(X, Y, F, levels=[0], colors='b', linewidths=2, label=
    'Parabola')
```

C code

```
        # Plot directrix: x - 2y + 3 = 0 âĘŠ y = (x + 3)/2
x_dir = np.linspace(-8, 6, 200)
y_dir = (x_dir + 3) / 2
plt.plot(x_dir, y_dir, 'g--', label='Directrix')

# Focus point (-1, 2)
plt.plot(-1, 2, 'ro', label='Focus (-1, 2)')

# Graph settings
plt.axis('equal')
plt.grid(True, linestyle='--', alpha=0.6)
plt.title('Parabola with Focus (-1, 2) and Directrix x - 2y + 3 = 0')
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.legend()
plt.show()
```