MatGeo Assignment 4.12.12

AI25BTECH11007

Question:
For what values of a and b the intercepts cut off on the coordinate axes by the line ax+by+8 = 0 are equal in length but opposite in signs to those cut off by the line 2x-3y = 0 on the axes.

Solution:

$$\text{Line}: ax + by + 8 = 0 \quad \iff \quad \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 8 = 0$$

$$\text{Intercept vector: } \begin{pmatrix} -\frac{8}{a} \\ -\frac{8}{b} \end{pmatrix}$$

$$\text{For } 2x - 3y = 0 \iff \frac{x}{3} + \frac{y}{-2} = 0, \quad \text{intercept vector: } \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\text{Condition: } \begin{pmatrix} -\frac{8}{a} \\ -\frac{8}{b} \end{pmatrix} = -\begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\Rightarrow \quad -\frac{8}{a} = -3, \quad -\frac{8}{b} = 2$$

$$\Rightarrow \quad a = \tfrac{8}{3}, \quad b = -4$$

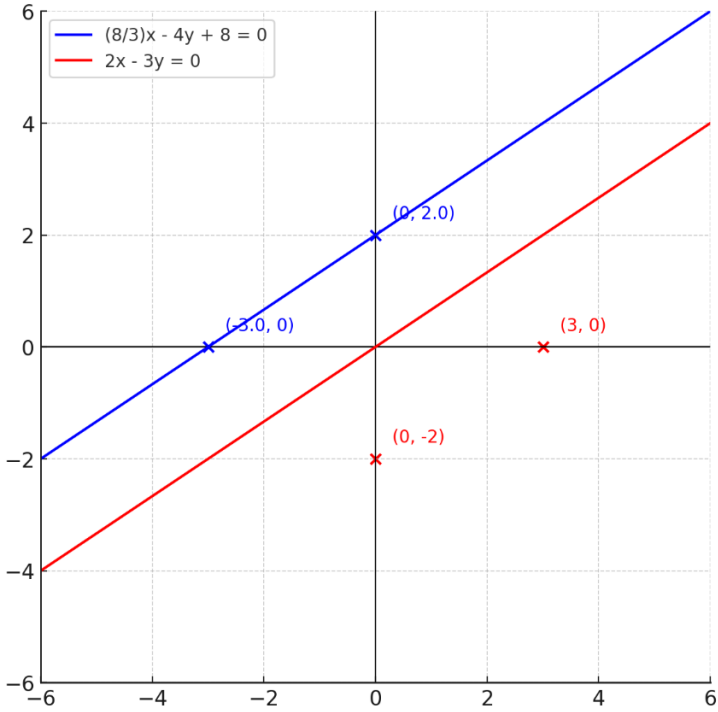

Fig. 0.1: Plot