MatGeo Assignment 5.3.1

AI25BTECH11007

Question:
For what value of k, the system of linear equations x + y + z = 2
2x + y - z = 3
3x + 2y + kz = 4 has a unique solution?

Solution:

$$\text{System: } \begin{cases} x + y + z = 2 \\ 2x + y - z = 3 \\ 3x + 2y + kz = 4 \end{cases}$$

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ k \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

$$\text{Augmented matrix(M) : } \begin{pmatrix} A & B & C & D \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 3 \\ 3 & 2 & k & 4 \end{pmatrix}$$

by row reducing,

$$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - 3R_1, \quad R_3 \to R_3 - R_2$$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 \\ 0 & 0 & k & -1 \end{pmatrix}$$

- **If** $k \neq 0$**:** the augmented matrix has three non-zero rows , so rank$(M) = 3$.
  hence, Unique Solution for system
- **If** $k = 0$**:** the row-echelon form becomes

$$\left[\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 0 \end{pmatrix} \middle| \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}\right].$$

  Here rank$(M) = 2$
  so, system is Inconsistent; No Solution.

Therefore, the system has a unique solution precisely when $k \neq 0$.

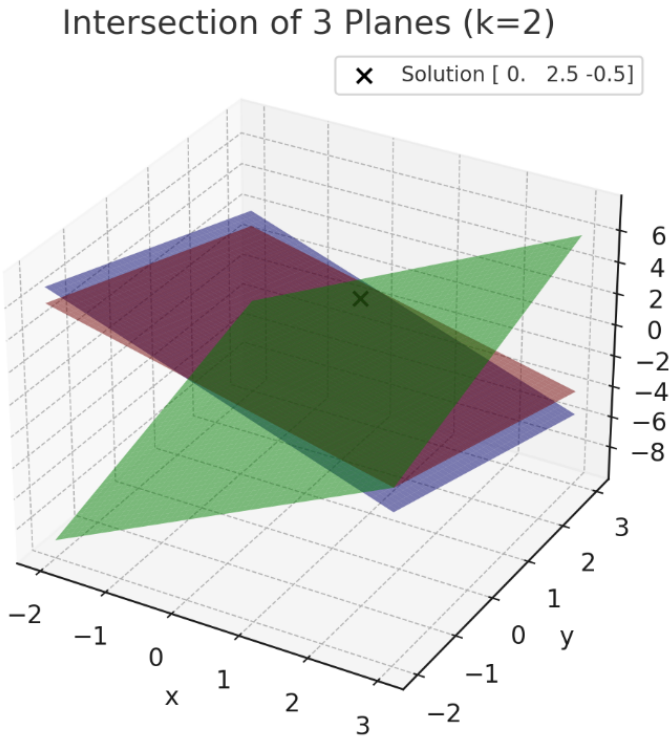
Intersection of 3 Planes (k=2)
Solution [0. 2.5 -0.5]
6
4
2
0
−2
−4
−6
−8
−2
−1
0
1
2
3
x
3
2
1
0
−1
−2
y

Fig. 0.1: Image