MatGeo Assignment 9.2.41

AI25BTECH11007

Question :
Find the area of the region bounded by the curves y^2 = 9x, y = 3x.

Solution :

$$\text{Given curves: } y^2 = 9x, \quad y = 3x \tag{0.1}$$

The conic can be written as

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.2}$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -\frac{9}{2} \\ 0 \end{pmatrix}, \quad f = 0 \tag{0.3}$$

The equation of the line is

$$\mathbf{x} = \mathbf{h} + \kappa\mathbf{m} \tag{0.4}$$

where

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{0.5}$$

Substituting in $g(\mathbf{x}) = 0$:

$$(\mathbf{h} + \kappa\mathbf{m})^T\mathbf{V}(\mathbf{h} + \kappa\mathbf{m}) + 2\mathbf{u}^T(\mathbf{h} + \kappa\mathbf{m}) + f = 0 \tag{0.6}$$

Expanding,

$$\kappa^2(\mathbf{m}^T\mathbf{V}\mathbf{m}) + 2\kappa(\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u})) + (\mathbf{h}^T\mathbf{V}\mathbf{h} + 2\mathbf{u}^T\mathbf{h} + f) = 0 \tag{0.7}$$

Substituting the known values,

$$9\kappa^2 - 9\kappa = 0 \tag{0.8}$$

$$\kappa_1 = 0, \quad \kappa_2 = 1$$

Finally, intersecting points are

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.9}$$

$$\mathbf{x}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{0.10}$$

Thus, the points of intersection are

$$A = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Hence, the required area is

$$A = \int_0^1 (3\sqrt{x} - 3x)\, dx \tag{0.11}$$

$$\tag{0.12}$$

$$\boxed{A = \frac{1}{2} \text{ square units}}$$

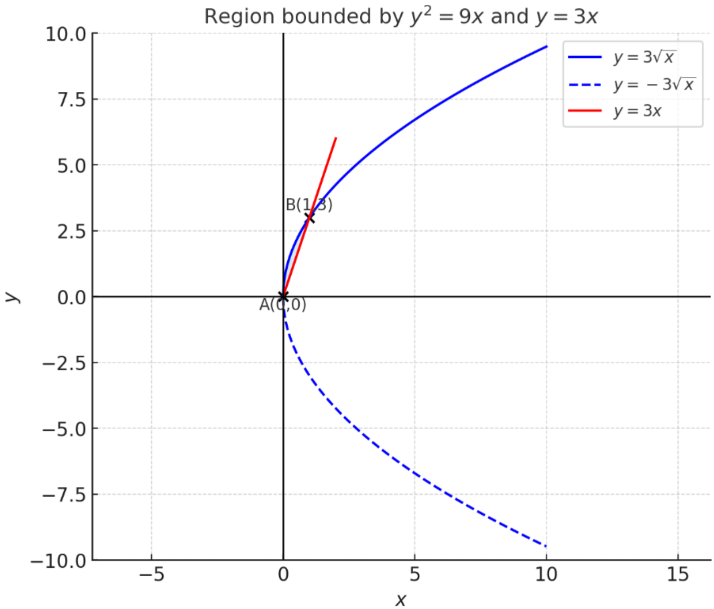

Fig. 0.1: Image