MatGeo Assignment 4.13.76

AI25BTECH11007

Question:
Two lines

$$L_1 : \frac{x}{5} = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : \frac{x}{\alpha} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

are coplanar. Then the value(s) of α

Solution:

$$L_1 : \frac{x}{5} = \frac{y}{3-\alpha} = \frac{z}{-2}, \qquad L_2 : \frac{x}{\alpha} = \frac{y}{-1} = \frac{z}{2-\alpha}.$$

$$\text{Direction vectors are : } \mathbf{n}_1 = \begin{pmatrix} 5 \\ 3-\alpha \\ -2 \end{pmatrix}, \qquad \mathbf{n}_2 = \begin{pmatrix} \alpha \\ -1 \\ 2-\alpha \end{pmatrix}.$$

Choose points on each line, as both lines pass through the origin, so, $\mathbf{p}_1 = \mathbf{p}_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Two lines are coplanar iff

$$\operatorname{rank}\left(\begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{p}_2 - \mathbf{p}_1 \end{pmatrix} \right) \le 2.$$

Here $\mathbf{p}_2 - \mathbf{p}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, so the matrix becomes

$$\begin{pmatrix} 5 & \alpha & 0 \\ 3-\alpha & -1 & 0 \\ -2 & 2-\alpha & 0 \end{pmatrix},$$

whose rank is at most 2 for every α. Hence the two lines are coplanar for all real α.

$$\boxed{\alpha \in \mathbb{R}}$$

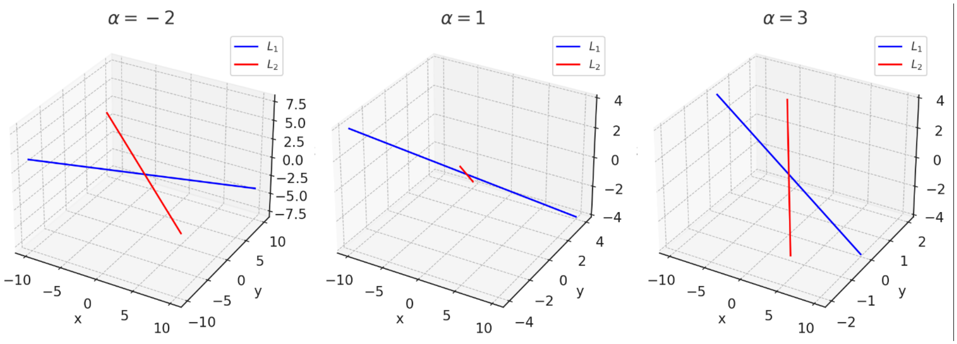

Fig. 0.1: Image