MatGeo Assignment 4.8.2

AI25BTECH11007

Question:
Find the values of λ for which the distance of the point (2, 1, λ) from the plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.

Solution:

$$\text{Plane: } 3x + 5y + 4z = 11 \quad \Rightarrow \quad \mathbf{n} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}.$$

Let point be

$$\mathbf{p} = \begin{pmatrix} 2 \\ 1 \\ \lambda \end{pmatrix}.$$

The distance of a point $\mathbf{p}$ from plane $\mathbf{n}^T\mathbf{x} = 11$ is

$$d = \frac{|\mathbf{n}^T\mathbf{p} - 11|}{\|\mathbf{n}\|}. \tag{0.1}$$

Now,

$$\mathbf{n}^T\mathbf{p} = \begin{pmatrix} 3 & 5 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ \lambda \end{pmatrix} = 11 + 4\lambda, \tag{0.2}$$

and

$$\|\mathbf{n}\| = 5\sqrt{2}. \tag{0.3}$$

Hence,

$$d = \frac{|11 + 4\lambda - 11|}{5\sqrt{2}} = 2\sqrt{2}. \tag{0.4}$$

$$\therefore \quad \lambda = \pm 5. \tag{0.5}$$

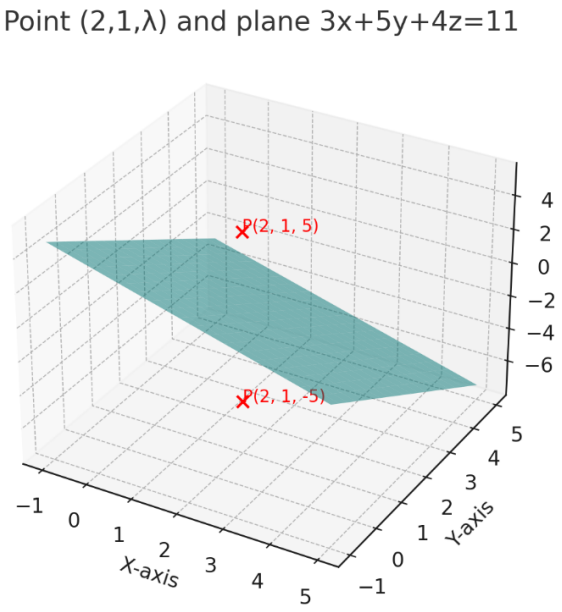

Fig. 0.1: Image