MatGeo Assignment 4.12.12

AI25BTECH11007

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For what values of a and b the intercepts cut off on the coordinate axes by the line ax+by+8 = 0 are equal in length but opposite in signs to those cut off by the line 2x-3y = 0 on the axes.

Solution

$$\text{Line}: ax + by + 8 = 0 \quad \iff \quad \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 8 = 0$$

$$\text{Intercept vector: } \begin{pmatrix} -\frac{8}{a} \\ -\frac{8}{b} \end{pmatrix}$$

$$\text{For } 2x - 3y = 0 \quad \iff \quad \frac{x}{3} + \frac{y}{-2} = 0, \quad \text{intercept vector: } \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\text{Condition: } \begin{pmatrix} -\frac{8}{a} \\ -\frac{8}{b} \end{pmatrix} = - \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\Rightarrow \quad -\frac{8}{a} = -3, \quad -\frac{8}{b} = 2$$

$$\Rightarrow \quad a = \frac{8}{3}, \quad b = -4$$
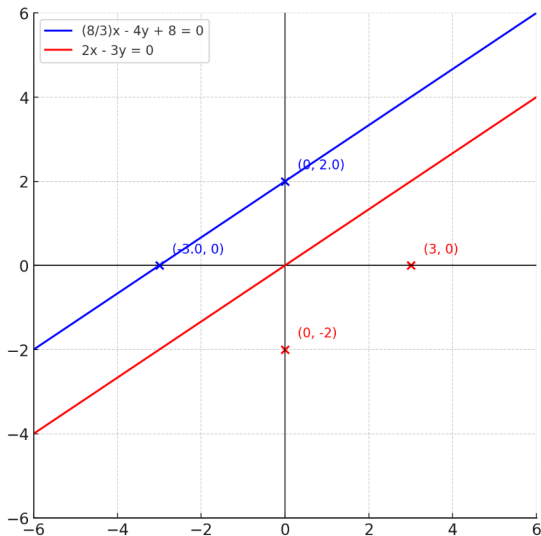
Figure: Plot

C code

```c
#include <stdio.h>
int main() {
    // Line 1: (8/3)x - 4y + 8 = 0
    float a = 8.0/3.0;
    float b = -4;
    // Intercepts of line 1
    float x1 = -8 / a; // x-intercept
    float y1 = -8 / b; // y-intercept
    // Line 2: 2x - 3y = 0
    // Intercepts of line 2
    float x2 = 3;
    float y2 = -2;
    printf("Line 1: (8/3)x - 4y + 8 = 0\n");
    printf("Intercepts: (%.1f, 0) and (0, %.1f)\n\n", x1, y1);
    printf("Line 2: 2x - 3y = 0\n");
    printf("Intercepts: (%.1f, 0) and (0, %.1f)\n", x2, y2);
    return 0;
}
```

Python code

```
        import matplotlib.pyplot as plt
import numpy as np
# Define the lines
x = np.linspace(-10, 10, 400)
# Line 1: (8/3)x - 4y + 8 = 0 -> y = (2/3)x + 2
y1 = (2/3)*x + 2
# Line 2: 2x - 3y = 0 -> y = (2/3)x
y2 = (2/3)*x
# Axes
fig, ax = plt.subplots(figsize=(6,6))
ax.axhline(0, color='black', linewidth=0.8)
ax.axvline(0, color='black', linewidth=0.8)
```

Python code

```python
# Plot the lines
ax.plot(x, y1, 'r', label=r'$\tfrac{8}{3}x - 4y + 8 = 0$')
ax.plot(x, y2, 'b', label=r'$2x - 3y = 0$')

# Mark intercepts for line 1
x_int1 = -8/(8/3) # = -3
y_int1 = -8/(-4) # = 2
ax.scatter([x_int1, 0], [0, y_int1], color='red')
ax.text(x_int1, 0.5, f"(-3,0)", color="red", fontsize=9)
ax.text(0.3, y_int1, f"(0,2)", color="red", fontsize=9)

# Mark intercepts for line 2
x_int2 = 3
y_int2 = -2
ax.scatter([x_int2, 0], [0, y_int2], color='blue')
ax.text(x_int2, -0.7, f"(3,0)", color="blue", fontsize=9)
ax.text(0.3, y_int2, f"(0,-2)", color="blue", fontsize=9)
```

Python code

```python
# Limits and grid
ax.set_xlim(-6, 6)
ax.set_ylim(-6, 6)
ax.grid(True, linestyle="--", alpha=0.6)
ax.legend()
plt.show()
```