MatGeo Assignment 4.4.3

AI25BTECH11007

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Question

Find the values of λ for which the distance of the point $(2, 1, \lambda)$ from the plane
$3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.

Solution

$$\text{Plane: } 3x + 5y + 4z = 11 \quad \Rightarrow \quad \mathbf{n} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}.$$

Let point be

$$\mathbf{p} = \begin{pmatrix} 2 \\ 1 \\ \lambda \end{pmatrix}.$$

The distance of a point $\mathbf{p}$ from plane $\mathbf{n}^T\mathbf{x} = 11$ is

$$d = \frac{|\mathbf{n}^T\mathbf{p} - 11|}{\|\mathbf{n}\|}. \tag{1}$$

Now,

$$\mathbf{n}^T\mathbf{p} = \begin{pmatrix} 3 & 5 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ \lambda \end{pmatrix} = 11 + 4\lambda, \tag{2}$$

and

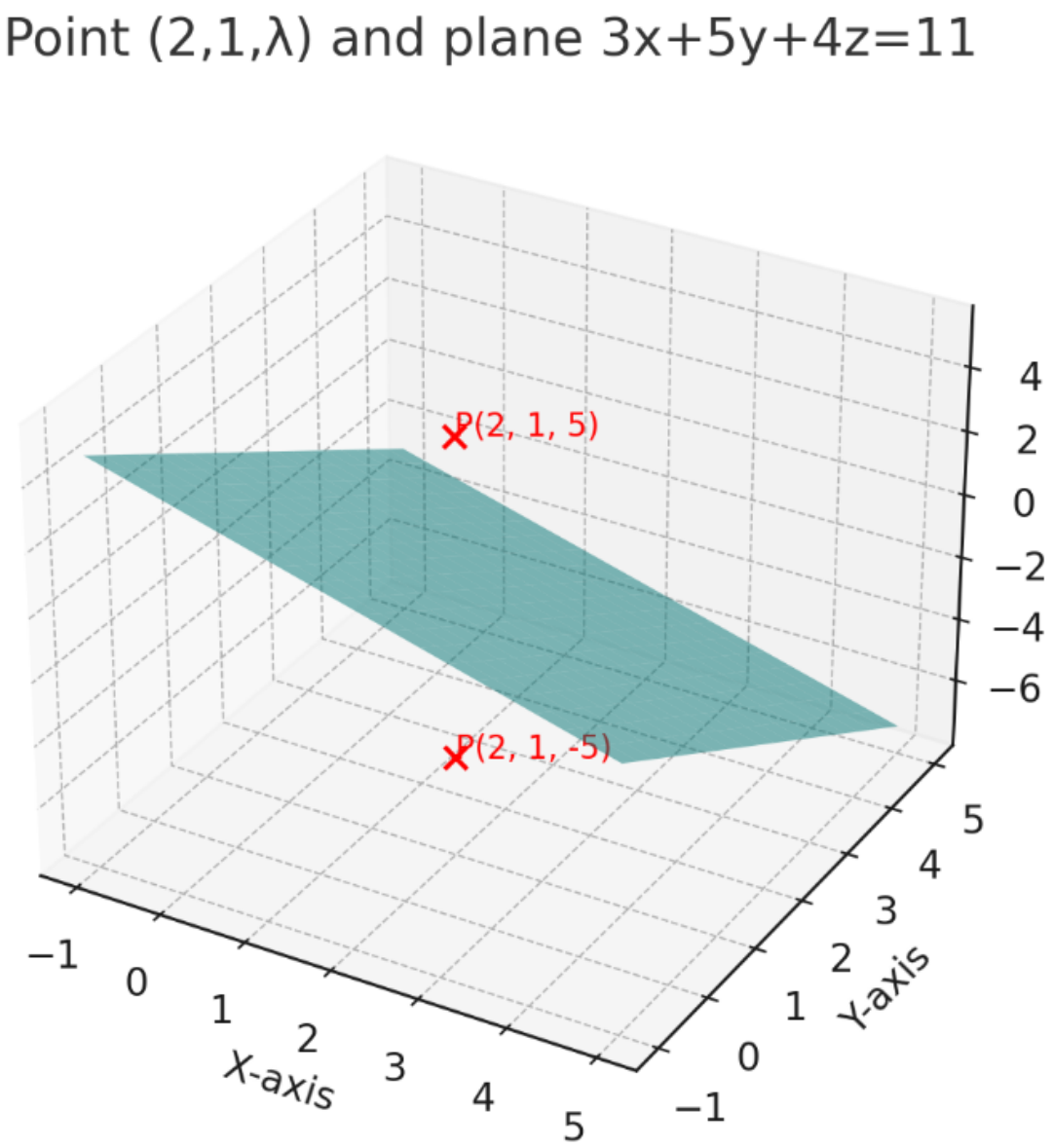
$$\|\mathbf{n}\| = 5\sqrt{2}. \tag{3}$$

Hence,

$$d = \frac{|11 + 4\lambda - 11|}{5\sqrt{2}} = 2\sqrt{2}. \tag{4}$$

$$\therefore \quad \lambda = \pm 5. \tag{5}$$

Plot


Point (2,1,λ) and plane 3x+5y+4z=11
P(2, 1, 5)
P(2, 1, -5)
4
2
0
−2
−4
−6
−1
0
1
2
3
4
5
X-axis
−1
0
1
2
3
4
5
Y-axis

C code

```c
#include <stdio.h>
#include <math.h>
#include <stdlib.h> // For fabs()

int main() {
    // Plane: 3x + 5y + 4z = 11
    double n[] = {3, 5, 4}; // Normal vector components
    double d_plane = 11; // Plane constant
    double distance = 2 * sqrt(2); // Given distance

    // Point: (2, 1, lambda)
    double x = 2, y = 1;
    double lambda;

    // Distance formula: d = |n . p - d_plane| / ||n||
    // ||n|| = sqrt(3^2 + 5^2 + 4^2) = sqrt(50) = 5*sqrt(2)
    double norm_n = sqrt(n[0]*n[0] + n[1]*n[1] + n[2]*n[2]);
```

C code

```
    // Solve for lambda: |n . p - d_plane| / ||n|| = distance
    // n . p = 3*2 + 5*1 + 4*lambda = 11 + 4*lambda
    // |11 + 4*lambda - 11| / (5*sqrt(2)) = 2*sqrt(2)
    // => |4*lambda| / (5*sqrt(2)) = 2*sqrt(2)
    // => |lambda| = 5
    lambda = 5;
    printf("Lambda = %.2f or %.2f\n", lambda, -lambda);

    return 0;
}
```

Python code

```
import math

# Plane: 3x + 5y + 4z = 11
n = [3, 5, 4] # Normal vector
d_plane = 11 # Plane constant
distance = 2 * math.sqrt(2) # Given distance

# Point: (2, 1, lambda)
x, y = 2, 1

# Norm of the normal vector
norm_n = math.sqrt(n[0]**2 + n[1]**2 + n[2]**2) # ||n||

lambda_val = 5
print(f"Lambda = {lambda_val} or {-lambda_val}")
```