

Q.1 Given an Array. Create and return prefix sum Array.
where $ps[i] = A[0] + A[1] + A[2] + \dots + A[i]$.

$A = [2 \quad 4 \quad 5 \quad -3 \quad 17 \quad 8]$
 0 1 2 3 4 5

$ps = [2 \quad 6 \quad 11 \quad 8 \quad 25 \quad 33]$
 0 1 2 3 4 5

$A = [1, 2, 0, 4, -3, 5]$
 0 1 2 3 4 5

$ps = [1 \quad 3 \quad 3 \quad 7 \quad 4 \quad 9]$
 0 1 2 3 4 5

1st approach

→ for every index i , calculate the sum from 0 to i

T.C: $O(n^2)$

optimized version

$$A = \begin{matrix} [1, 2, 0, 4, -3, 5] \\ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \end{matrix}$$

$$PS = \begin{matrix} [1 \quad 3 \quad 3 \quad 7 \quad 4 \quad 9] \\ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \end{matrix}$$

$$PS[0] = A[0]$$

$$PS[1] = \underbrace{A[0] + A[1]}_{PS[0]} \Rightarrow PS[0] + A[1]$$

$$PS[2] = \underbrace{A[0] + A[1] + A[2]}_{PS[1]} \Rightarrow PS[1] + A[2]$$

$$PS[3] = \underbrace{A[0] + A[1] + A[2] + A[3]}_{PS[2]} \Rightarrow PS[2] + A[3]$$

$$PS[i] = PS[i-1] + A[i]$$

$$A = \begin{matrix} [1, 2, 0, 4, -3, 5] \\ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \end{matrix}$$

$$PS = \begin{matrix} [1 \quad 3 \quad 3 \quad 7 \quad 4 \quad 9] \\ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \end{matrix}$$

```
int[] prefixSum (int[] A) {
```

```
    int n = A.length;
```

```
    int[] ps = new int[n];
```

TC: $O(n)$

```
    ps[0] = A[0];
```

```
    for (int i = 1; i < n; i++) {
```

```
        ps[i] = ps[i-1] + A[i];
```

```
    }
```

```
    return ps;
```

}

Q.2 Range Sum queries

Given an array and Q queries. Find the answer for all queries in the given range.

$$1 \leq n \leq 10^5$$

$$1 \leq Q \leq 10^5$$

$A = [3 \quad 4 \quad -2 \quad 6 \quad 8 \quad 10 \quad 13 \quad 1]$
 $\quad \quad \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$

$Q = 4 \ (L \leq R)$

L	R	ans
1	3	8
2	6	35
5	5	10
0	3	11

void solve (int $A[]$, int $Q[][]$) {

for (int $i = 0$; $i < Q.length$; $i++$) {

int $l = Q[i][0]$;

int $r = Q[i][1]$;

// find sum of $A[]$ from l to r

int $sum = 0$;

for (int $k = l$; $k \leq r$; $k++$) {

sum += $A[k]$;

}

soP(sum);

}

$A = [2 \quad 4 \quad 4 \quad 5]$
 $\quad \quad \quad 0 \quad 1 \quad 2 \quad 3$

$Q = [[0, 2]$
 $\quad [1, 3]$
 $\quad]$

$i = 0, l = 0, r = 2, sum = 15$

$i = 1, l = 1, r = 3, sum = 18$

$O(Q * N) \rightarrow TLE$

improved idea

A = [3 4 -2 6 8 10 13 1]
 0 1 2 3 4 5 6 7

ps = [3 7 5 11 19 29 42 43]
 0 1 2 3 4 5 6 7

$$\begin{aligned} \text{sum}(2, 5) &= \text{sum}(0, 5) - \text{sum}(0, 1) \\ &= \text{ps}[5] - \text{ps}[1] = 29 - 7 = 22 \end{aligned}$$

$$\begin{aligned} \text{sum}(3, 6) &= \text{sum}(0, 6) - \text{sum}(0, 2) \\ &= \text{ps}[6] - \text{ps}[2] = 42 - 5 = 37 \end{aligned}$$

$$\text{sum}(0, 4) = \text{ps}[4]$$

==

$$\text{sum}(L \text{ to } R) = \text{ps}[R] - \text{ps}[L-1]$$

(L != 0)

go on query $\rightarrow l, r$

```
if (l == 0) {  
    sop (ps[r]);  
}  
else {  
    sop (ps[r] - ps[l-1]);  
}
```

```
void solve (int [] A, int [][] Q) {  
    int [] ps = prefixSum(A);
```

```
    for (int i = 0; i < Q.length; i++) {
```

```
        int l = Q[i][0];
```

```
        int r = Q[i][1];
```

```
        // find sum of A[] from l to r
```

```
        if (l == 0) {
```

```
            sop (ps[r]);
```

```
        }
```

```
        else {
```

```
            sop (ps[r] - ps[l-1]);
```

```
        }
```

```
    }
```

Tc: $O(N + Q)$

Q-3 Equilibrium Index

Amazon, Adobe

Given an Array, find the equilibrium index.

i is an equilibrium index when :

sum of all elements on the left side of i = sum of all elements on the right side of i

A: $[-7 \quad 5 \quad 1 \quad 2 \quad -4 \quad 3 \quad 0]$ ans = 3
 0 1 2 3 4 5 6

A: $[5 \quad 1 \quad 3 \quad 2 \quad 9]$ ans = 3
 0 1 2 3 4

A: $[1 \quad 2 \quad 3]$ ans = -1
 0 1 2

brute force \rightarrow hw

for every index i

\rightarrow find LS

\rightarrow find RS

and compare LS and RS

sum of all elements on the left side of i = sum of all elements on the right side of i

sum(0, i-1)

ps[i-1]

sum(i+1, n-1)

ps[n-1] - ps[i]

int equilibriumIndex (int[] A) {

int[] ps = prefixSum(A);

for (int i=0; i < A.length; i++) {

int ds = 0;

if (i > 0) {

ds = ps[i-1];

}

int rs = ps[n-1] - ps[i];

if (ds == rs) {

return i;

}

}

return -1;

}

TC: O(N)

SC: O(N)

A: [5 1 3 2 9]
 0 1 2 3 4

ps: [5 6 9 11 20]
 0 1 2 3 4

i	ds	rs
0	0	20 - 5 = 15
1	5	20 - 6 = 14
2	6	20 - 9 = 11
3	9	20 - 11 = 9

Q.4 Given an Array and Q queries, find the count of even numbers for every query.

A: [3 5 8 9 16 14 13 12]
0 1 2 3 4 5 6 7

Queries

L	R	ans
1	5	3
2	6	3
4	5	2
4	4	1
3	6	2

A: [3 5 8 9 16 14 13 12]
0 1 2 3 4 5 6 7

pc: [0 0 1 1 2 3 3 4]
0 1 2 3 4 5 6 7

pc[i]

↳ no. of even elements in A[]
from 0 to i.

A: [3 5 8 9 16 14 13 12]
 0 1 2 3 4 5 6 7

pc: [0 0 1 1 2 3 3 4]
 0 1 2 3 4 5 6 7

$$1, 5 \Rightarrow pc[5] - pc[0] = 3 - 0 = 3$$

$$3, 7 \Rightarrow pc[7] - pc[2] = 4 - 1 = 3$$

```
void solve (int[] A, int[][] Q) {
```

```
    int[] pc = prefixSumCount(A);
```

```
    for (int i = 0; i < Q.length; i++) {
```

```
        int L = Q[i][0];
```

```
        int R = Q[i][1];
```

Tc: $O(N+Q)$

```
        if (L == 0) {
```

```
            solve(pc[R]);
```

```
        }
```

```
        else {
```

```
            solve(pc[R] - pc[L-1]);
```

```
        }
```

```
    }
```

A: [3 5 8 9 16 14 13 12]
 0 1 2 3 4 5 6 7

pc: [0 0 1 1 2 3 3 4]
 0 1 2 3 4 5 6 7

```
int[] prefixEvenCount (int[] A) {
```

```
    int n = A.length;
```

```
    int[] pc = new int[n];
```

```
    if (A[0] % 2 == 0) {
```

```
        pc[0] = 1;
```

```
    }
```

```
    else {
```

```
        pc[0] = 0;
```

```
    }
```

```
    for (int i = 1; i < n; i++) {
```

```
        int temp = 0;
```

```
        if (A[i] % 2 == 0) {
```

```
            temp = 1;
```

```
        }
```

```
        pc[i] = pc[i-1] + temp;
```

```
    }
```

```
    return pc;
```

```
}
```

A = [2 4 4 3 16]
 0 1 2 3 4

pc = [1 1 2 2 3]
 0 1 2 3 4

Doubts
=

$2^N, 3^N \rightarrow$ exponentials

$x^5 + x^4 + 3x^2 \rightarrow$ polynomial

$x^2 + 3x + 2 \rightarrow$ quadratic (power 2)

buy and sell stock:

19 24 5 16 72

track of left-min

pick from both sides

$$B=4$$

$$A = [2 \quad 9 \quad 5 \quad 6 \quad 7 \quad -3 \quad 4]$$

possibilities:

pick from left

pick from right

4

0

3

1

2

2

1

3

0

4

$$A = \begin{matrix} i & & j \\ [2 & 9 & 5 & 1 & 7 & -3 & 4] \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix}$$

$$B=4$$

$$\text{sum} = 2 + 9 + 5 + 1, \text{ ans} = 17$$

$$i \rightarrow B-1$$

$$j \rightarrow n-1$$

i	j	Sum
3	6	$2 + 9 + 5 + \cancel{1} - \cancel{1} + 4$
2	5	$2 + 9 + \cancel{5} + 4 - \cancel{5} + (-3)$
1	4	$2 + \cancel{9} + 4 + (-3) - \cancel{9} + 7$
0	3	$\cancel{2} + 4 + (-3) + 7 - \cancel{2} + 1$

$$i \geq 0 \quad \left\{ \begin{array}{l} \text{sum} -= A[i] \\ \text{sum} += A[j] \\ i--; \\ j--; \end{array} \right.$$

```
for (int i=0; i < 2^n; i++) {
```

```
    int j=i;
```

```
    while (j>0) {
```

```
        j = j-1;
```

```
    }
```

```
}
```

i	j [i 1]	itr
0	0 to 1	0 +
1	1 to 1	1 +
2	2 to 1	2 +
3	3 to 1	3 +
⋮		⋮
2 ⁿ -1	2 ⁿ -1 to 1	2 ⁿ -1

$$0 + 1 + 2 + 3 + \dots + (2^{n-1} - 1)$$

sum of $2^{n-1} - 1$ natural no.

$$\text{sum of } N : \frac{N(N+1)}{2}$$

$$\Rightarrow \frac{(2^{n-1} - 1)(2^{n-1} - 1 + 1)}{2}$$

$$\Rightarrow 2^{2n-1} - 2^{n-1}$$

$$2^{2n} \Rightarrow (2^2)^n$$

$$2^{2n} \approx 4^n$$

```
for(int i=n; i>=1; i=i/2) {
```

```
    for(int j=1; j<=i; j++) {
```

```
        sop();
```

```
    }
```

```
}
```

i	j	iter
n	[1, n)	n-1 +
$\frac{n}{2}$	[1, $\frac{n}{2}$)	$\frac{n}{2}-1$ +
$\frac{n}{4}$	[1, $\frac{n}{4}$)	$\frac{n}{4}-1$ +
\vdots		\vdots
1	[1, 1)	0

$$n-1 + \frac{n}{2}-1 + \frac{n}{4}-1 + \dots + 0$$

$$n + \frac{n}{2} + \frac{n}{4} + \dots + 0 = (1 \times t)$$

~~~~~

$$a = n$$

$$r = \frac{1}{2}$$

$$t = \log_2 n$$

$$TC: O(n)$$

=

```

for(int i=1; i<=n; i=i*2) {
    for(int j=1; j<=n; j++) {
        SOP();
    }
}

```

$$N \times \log_2 N$$

| i | j     | itr    |
|---|-------|--------|
| 1 | 1 → n | n<br>+ |
| 2 | 1 → n | n<br>+ |
| 4 | 1 → n | n<br>+ |
| ⋮ |       | ⋮      |
| N | 1 → n | n<br>+ |

```

int i=1;
while (i<n) {
    int x=i;
    while (x-->0) {
    }
}

```

| i   | x       | itr    |
|-----|---------|--------|
| 1   | 1 → 1   | 1<br>+ |
| 2   | 2 → 1   | 2<br>+ |
| 3   | 3 → 1   | 3<br>+ |
| ⋮   |         | ⋮      |
| n-1 | n-1 → 1 | n-1    |

$$1 + 2 + 3 + \dots + n-1$$

$$\frac{N(N+1)}{2}$$

$$\Rightarrow \frac{N \rightarrow n-1}{(n-1)(n)} \Rightarrow \frac{(n-1)(n)}{2}$$

$$TC: O(n^2)$$