

* Important observations around Prefix Sum

↳ carrying the term

$$A = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ [0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0] \end{matrix}$$

$$PS = \begin{matrix} 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{matrix}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ [0 & 0 & 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0] \end{matrix}$$

$$PS = \begin{matrix} 0 & 0 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ [0 & 0 & 2 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0] \end{matrix}$$

$$PS = \begin{matrix} 0 & 0 & 2 & 2 & 2+4 & 2+4 & 2+4 & 2+4 & 2+4 & 2+4 & 2+4 \end{matrix}$$

$$A = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ [0 & 0 & 2 & 0 & 4 & 0 & -2 & 0 & 0 & 0 & -4] \end{matrix}$$

$$PS = \begin{matrix} 0 & 0 & 2 & 2 & 2+4 & 2+4 & 4 & 4 & 4 & 4 & 0 \end{matrix}$$

Q.1 Continuous sum query

(Frequently asked in Google)

Given array with all elements = 0 and Q queries. For every query {start, end, val} do +val in the range start to end. Return the final answer array.

A =

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0

Queries

S	e	val
---	---	-----

3	6	1
---	---	---

2	7	3
---	---	---

4	6	5
---	---	---

1	5	-4
---	---	----

			+1	+1	+1	+1			
			+3	+3	+3	+3	+3	+3	
						+5	+5	+5	
		-4	-4	-4	-4	-4			

0	-4	-1	0	5	5	9	3	0	0
---	----	----	---	---	---	---	---	---	---

i) Brute idea : $Tc \rightarrow O(Q \cdot N)$

A =

			1				-1		
0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9

PS =

0	0	0	1	1	1	1	0	0	0
---	---	---	---	---	---	---	---	---	---

S	e	val
---	---	-----

3	6	1
---	---	---

$A =$

		-4	3	1	5		4	-5	-3	
	0	0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7	8	9

 $PS =$

0	-4	-1	0	5	5	9	3	0	0
---	----	----	---	---	---	---	---	---	---

Queries

s	e	val	
3	6	1	$\rightarrow A[3] += 1, A[7] += -1$
2	7	3	$\rightarrow A[2] += 3, A[8] += -3$
4	6	5	$\rightarrow A[4] += 5, A[7] += -5$
1	5	-4	$\rightarrow A[1] += -4, A[6] += 4$

$A =$

	0	1	2	3	4	5
	0	0	0	0	0	0
		+3	+3	+3	+3	+3
		+2	+2	+2		
	-1	-1	-1	-1		

Queries
=

1	5	3	
1	3	2	
0	3	-1	

-1	4	4	4	3	3
----	---	---	---	---	---

		2			+1	
	-1	3			-2	
A =	0	0	0	0	0	0
	0	1	2	3	4	5
PS:	-1	4	4	4	3	3

Queries
=

1 5 3 , A[0] += 3

1 3 2 , A[1] += 2 A[4] += -2

0 3 -1 , A[0] += -1 A[4] += 1

```
int[] solve ( int[] A , int[][] Q ) {
```

```
    for (int i = 0; i < Q.length; i++) {
```

```
        int s = Q[i][0];
```

```
        int e = Q[i][1];
```

```
        int val = Q[i][2];
```

```
        // give impact of val in s to e
```

```
        A[s] += val;
```

```
        if (e+1 < A.length) {
```

```
            A[e+1] += -val;
```

```
        }
```

```
    }
```

// converting A[] into its prefix sum array

```
    for (int i = 1; i < A.length; i++) {
```

```
        A[i] = A[i-1] + A[i];
```

```
    }
```

```
    return A;
```

```
}
```

A =

	1	5		-1	
	0	0	0	0	0
	0	1	2	3	4
	1	6	6	0	0

Q: 1 2 5
0 2 1

Tc: $O(Q + N)$

Sc: $O(1)$

Q. 2 Create prefixMax and suffixMax array.

$A = \begin{bmatrix} 2 & 4 & 3 & 1 & 12 & 5 & 6 & 8 \end{bmatrix}$
0 1 2 3 4 5 6 7

prefixMax[i] \Rightarrow max of (0, i)

suffixMax[i] \Rightarrow max of (i, n-1)

	0	1	2	3	4	5	6	7
$A =$	2	4	3	1	12	5	6	8
pfmax	2	4	4	4	12	12	12	12
sfmax	12	12	12	12	12	8	8	8

$pfmax[i] = \max(pfmax[i-1], A[i]);$

$sfmax[i] = \max(sfmax[i+1], A[i]);$

```
int [] prefixMax ( int [] A ) {
```

```
    int n = A.length;
```

```
    int [] pJmax = new int [n];
```

```
    pJmax [0] = A[0];
```

```
    for (int i=1; i<n; i++) {
```

```
        pJmax[i] = Math.max ( pJmax [i-1], A[i] );
```

```
    }
```

```
    return pJmax;
```

```
}
```

```
int [] suffixMax ( int [] A ) {
```

```
    int n = A.length;
```

```
    int [] sJmax = new int [n];
```

```
    sJmax [n-1] = A[n-1];
```

```
    for (int i=n-2; i>=0; i--) {
```

```
        sJmax[i] = Math.max ( sJmax [i+1], A[i] );
```

```
    }
```

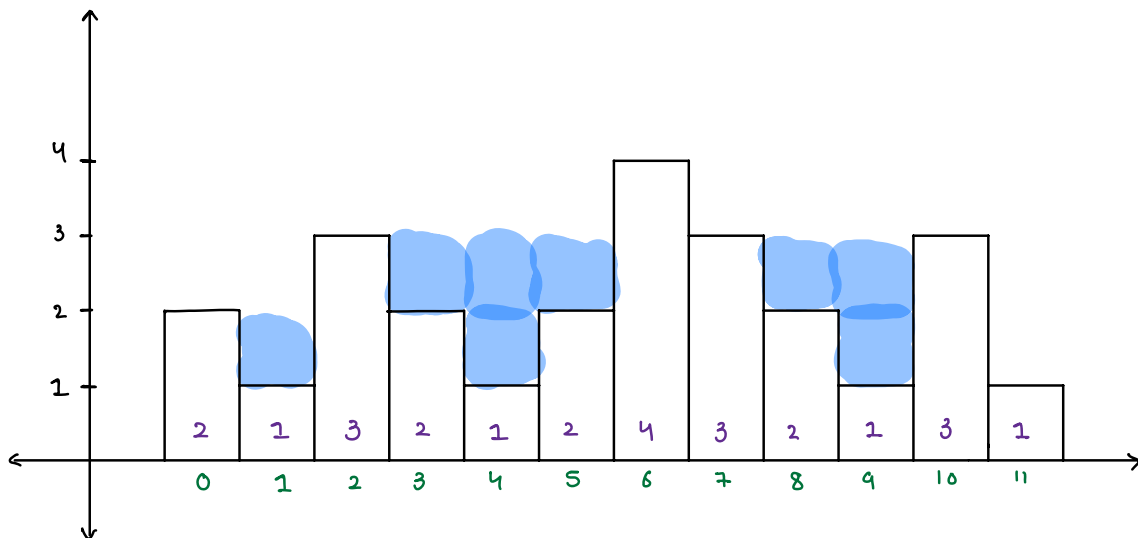
```
    return sJmax;
```

```
}
```

Q-3 Rainwater trapping

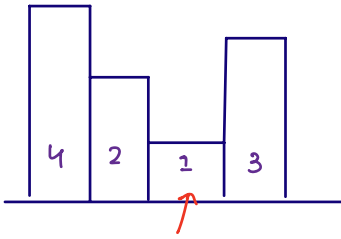
Given an array $A[]$, where $A[i]$ denotes height of i^{th} building.
Return amount of water trapped on all buildings.

0 1 2 3 4 5 6 7 8 9 10 11
 $A = [2 \quad 1 \quad 3 \quad 2 \quad 1 \quad 2 \quad 4 \quad 3 \quad 2 \quad 1 \quad 3 \quad 1]$



Ans = 8

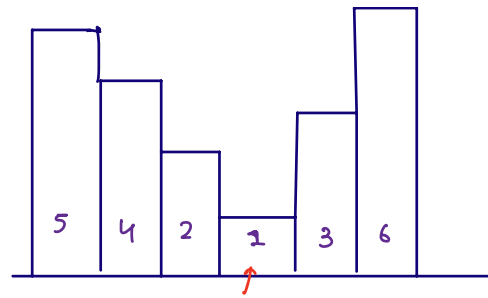
total water trapped = sum of water units trapped
on each building's rooftop.



$$ub = 4$$

$$lb = 3$$

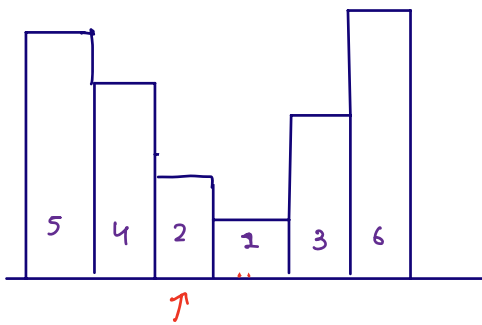
$$amt = 3 - 1 = 2$$



$$ub = 5$$

$$lb = 6$$

$$amt = 5 - 1 = 4$$



$$ub = 5$$

$$lb = 6$$

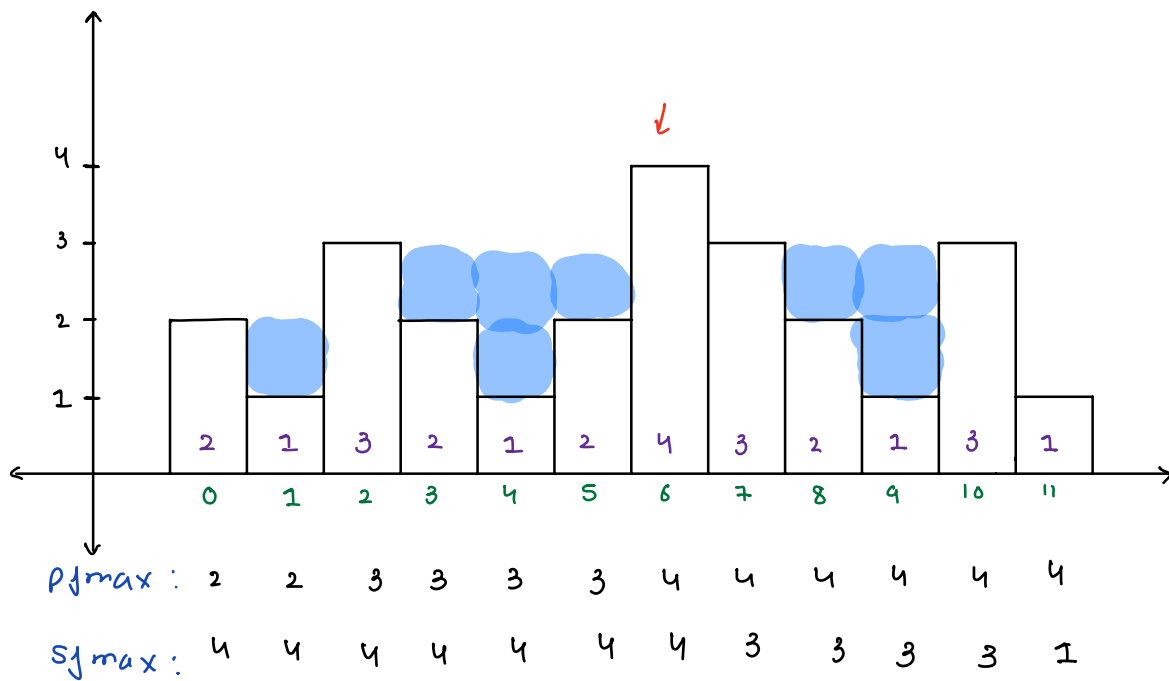
$$amt = 5 - 2 = 3$$

$$ub \rightarrow \max(0, i-1)$$

$$= Pjmax[i-1]$$

$$lb \rightarrow \max(i+1, n-1)$$

$$= Sjmax[i+1]$$



```
int rainwater (int [] A) {
```

```
    int [] L = prefixMax (A);
```

```
    int [] R = suffixMax (A);
```

```
    int ans = 0;
```

```
    for (int i = 1; i < A.length - 1; i++) {
```

```
        int L = L[i - 1];
```

```
        int R = R[i + 1];
```

```
        int amt = Math.min(L, R) - A[i];
```

```
        if (amt > 0) {
```

```
            ans += amt;
```

```
        }
```

```
    }
```

```
    return ans;
```

```
}
```

i	L	R	amt
1	2	4	2-2=0
2	2	4	2-3=-1
3	3	4	3-2=1
4	3	4	3-1=2
5	3	4	3-2=1
...

Q-4 Maximum Sum Subarray (Kadane's Algo)

Given an array, find max sum subarray.

↳ continuous part of an array

$$A = [\underset{0}{3} \quad \underset{1}{2} \quad \underset{2}{-6} \quad \underset{3}{8} \quad \underset{4}{2} \quad \underset{5}{9} \quad \underset{6}{4}] \quad \text{ans} = 23$$

$$A = [\underset{0}{-3} \quad \underset{1}{2} \quad \underset{2}{4} \quad \underset{3}{-1} \quad \underset{4}{3} \quad \underset{5}{-4} \quad \underset{6}{3}] \quad \text{ans} = 8$$

$$A = [\underset{0}{3} \quad \underset{1}{4} \quad \underset{2}{2} \quad \underset{3}{-14} \quad \underset{4}{16} \quad \underset{5}{-20} \quad \underset{6}{5}] \quad \text{ans} = 16$$

i) idea 1 Tc: $O(N^2)$

create prefix sum array, now find sum of every subarray and keep a track of max ans.

ii) idea 2 Tc: $O(N)$

↙

	0	1	2	3	4	5	6
A	[3	4	2	-14	16	-20	5]
Sum = 0	3	7	9	-5	16	-4	5
ans = -∞	3	7	9	9	16	16	16

<div>3</div> <div>3 4</div> <div>3 4 2</div> <div>3 4 2 -14</div> <div>3 4 2 -14 16</div> <div>3 4 2 -14 16 -20</div> <div>3 4 2 -14 16 -20 5</div>	<div>4</div> <div>4 2</div> <div>4 2 -14</div> <div>4 2 -14 16</div> <div>4 2 -14 16 -20</div> <div>4 2 -14 16 -20 5</div>	<div>2</div> <div>2 -14</div> <div>2 -14 16</div> <div>2 -14 16 -20</div> <div>2 -14 16 -20 5</div>	
<div>-14</div> <div>-14 16</div> <div>-14 16 -20</div> <div>-14 16 -20 5</div>	<div>16</div> <div>16 -20</div> <div>16 -20 5</div>	<div>-20</div> <div>-20 5</div>	5

↙

A =	[-3	2	4	-1	3	-4	3]
	0	1	2	3	4	5	6
sum = 0	-3	2	6	5	8	4	7
ans = -∞	-3	2	6	6	8	8	8

```
int maxSumSubarray (int [] A) {
```

```
    int sum = 0;
```

```
    int ans = Integer.MIN_VALUE;
```

```
    for (int i = 0; i < A.length; i++) {
```

```
        if (sum > 0) {
```

```
            sum += A[i];
```

```
        }
```

```
        else {
```

```
            sum = A[i];
```

```
        }
```

```
        if (sum > ans) {
```

```
            ans = sum;
```

```
        }
```

```
    }
```

```
    return ans;
```

```
}
```

A = { 2 3 -7 4 1 -2 5 -1 }

0 1 2 3 4 5 6 7

sum = 0 2 5 -2 4 5 3 8 7

ans = -∞ 2 5 5 5 5 5 8 8