

→ Comparing 2 Algos

i) using execution time

ii) using iterations and graphs

→ Why Big O needed

i) Why lower order terms are neglected

ii) Why constant coefficients are neglected

iii) Issues with Big O

iv) Worst case scenario

→ Space complexity

→ TLE (Time Limit exceeded)

Comparison using execution time

→ both algo's are running on same input size

Algo1 (P1)

↓
15 sec (Windows XP)

↓
Macbook

↓
8 sec (C++)

↓
8 sec

Algo2 (P2)

↓
10 sec (Macbook Pro)

↓
10 sec (Python)

↓
C++
↓
8 sec

Conclusion : Comparing two algo's on the basis of execution time is not correct, because execution time depends on a lot of external factors (processor, language, temp etc.)

Comparing using iterations

```
for (int i=1; i<=n; i++) {  
    SOP(i);  
}
```

} n itr

itr

Algo1 (P1)

$100 \log_2 N$

Algo2 (P2)

$\frac{N}{10}$

$N < 3550$

$100 \log_2 N > \frac{N}{10}$

Algo 2 is better

$N \geq 3550$

$100 \log_2 N < \frac{N}{10}$

Algo 1 is better

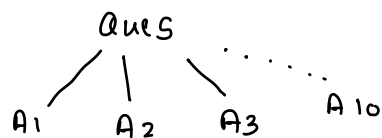
Google search \rightarrow millions of results

Hotstar \rightarrow Millions

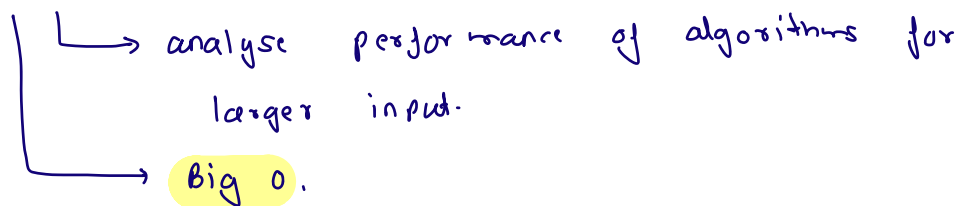
} data is always inc.

\rightarrow Pick Algo that is better performer for larger value of N

\Rightarrow Algo1



Asymptotic analysis of algo's



How to find Big O of a code?

- calculate total no. of iterations.
- neglect all lower order terms (keep highest order term)
- remove constant coefficient.

Why neglect lower order terms:

$$\text{itr} : N^2 + 10N$$

N	total itr	% of lower order term contribution in total itr
10	$100 + 100$	$\frac{100}{200} \times 100 = 50\%$
100	$10^4 + 10^3$	$\frac{10^3}{10^4 + 10^3} \times 100 \approx 10\%$
1000	$10^6 + 10^4$	$\frac{10^4}{10^6 + 10^4} \times 100 \approx 1\%$

Conclusion:

for larger value of N , contribution of lower order terms in total itr is very less.

that's why we can ignore lower order terms while calculating Big O.

Why to remove constant coefficient:

	Algo 1	Algo 2	(better)
i) itr \rightarrow	$10 \log_2 N$	$\frac{N}{100}$	Algo 1
ii) itr \rightarrow	$3N^2$	$15N$	Algo 2

for larger values of N , const. coeff. don't play a very big role in itr.

Issues with Big O

1)	Algo 1	Algo 2
its :	$10N$	N^2
	$O(N)$	$O(N^2)$

Claim: Algo 1 is always better than Algo 2 X

N	its in Algo 1	its in Algo 2	better
5	50	25	Algo 2
8	80	64	Algo 2
10	100	100	Same

$N > 10$ Algo 1 is always better

Algo 1 is not always better, it is better than Algo 2 after a specific value (threshold value).

2)

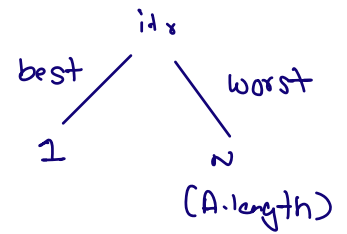
	Algo 1	Algo 2
its:	$2N^2 + 7N$	$5N^2$
BigO:	$O(N^2)$	$O(N^2)$

When Algo's have same Big O, then we should do the comparison based on its.

```

boolean search (int[] A, int k) {
    for (int i = 0; i < A.length; i++) {
        if (A[i] == k) {
            return true;
        }
    }
    return false;
}

```



always consider its.

in worst case scenario

its: N

TC: $O(N)$

Space complexity

```
void fun (int n) {
```

```
    int x = 10;
```

```
    long y = 300;
```

```
    int z = 99;
```

```
}
```

int \rightarrow 4 bytes

long \rightarrow 8 bytes

$$\text{total space} = 3 \times 4 + 8$$

$$= 20 \text{ bytes}$$

```
void fun (int n) {
```

```
    int x = 10;
```

```
    int[] A = new int [N];
```

```
    int[][] B = new int [5] [N];
```

```
}
```

total space

$$= 8 + 4N + 4 \times 5N$$

$$= 8 + 24N$$

How to calculate space complexity

input \rightarrow { Algo } \rightarrow return ans

Note: when we calculate space complexity of Algo we don't consider input space, consider space taken by Algo.


```
int max (int [] A) {
```

```
    int max = A[0];
```

```
    for (int i=0; i<A.length; i++) {
```

```
        if (A[i] > max) {
```

```
            max = A[i];
```

```
        }
```

```
    }
```

```
    return max;
```

```
}
```

Time	Space
itr: N	Space: 8 Bytes (max, i)
TC $\rightarrow O(N)$	SC $\rightarrow O(1)$

```
int solve (int [] A, int k) {
```

```
    int n = A.length;
```

```
    int [] B = new int [n];
```

```
    for (.....) {
```

```
        ==  
        ==  
        ==
```

```
    }
```

```
    return B[k];
```

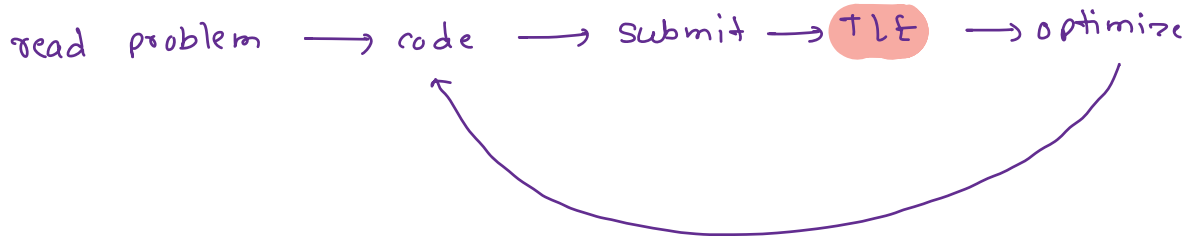
```
}
```

TC: $O(N)$

SC: $O(N)$

Time limit exceeded

Amazon contest \rightarrow 20, 1 hour



how to check that approach will give TLE
without writing code.

online contest | \longrightarrow Server \longrightarrow processing power
online I/O $\qquad\qquad\qquad \approx 10^8$ it/s

\hookrightarrow to avoid TLE our code should
within a second.

Your code should have at max :

** Safe side

10^7 to 10^8 it/s

Ques
=

constraints $1 \leq N \leq 10^5$

rough code

(nested loop) \rightarrow itr = N^2 , itr = 10^{10}
if ($N = 10^5$)



don't write code



improved idea

Count factors

Beginner

$1 \leq N \leq 10^6$

N itr

if ($N = 10^6$)

\rightarrow itr : 10^6

DSA module

$1 \leq N \leq 10^{12}$

N itr

if ($N = 10^{12}$)

\rightarrow itr : 10^{12}

TLE

int a = 0, i = N;

while (i > 0) {

 a += i;

 i /= 2;

}

loop breaks at i = 0

itr	i value after
1	$\frac{N}{2}$
2	$\frac{N}{4}$
3	$\frac{N}{8}$

// assume the loop k times

$$i = \frac{N}{2^k}$$

no. of itr till i = 1

$$\frac{N}{2^k} = 1$$

$$k = \log_2 N$$

$$\text{total itr} = \log_2 N + 1$$

$$\text{Big O} \rightarrow O(\log_2 N)$$

```

for (int i=1; i<=n; i=i*2) {
    for (int j=1; j<=n; j++) {
        sop();
    }
}

```

$0 \rightarrow O(n \times \log_2 n)$

i	j	iter
1	[1 n]	n
2	[1 n]	n
4	[1 n]	n
8	[1 n]	n
...		+
n	[1 n]	n

$1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow \dots \rightarrow n$
 $\underbrace{\hspace{10em}}_{\text{approx. } \log_2 n \text{ times}}$

```

for (int i=1; i<=100; i=i*2) {
    for (int j=1; j<=n; j++) {
        sop();
    }
}

```

Big O $\rightarrow O(n)$

i	j	iter
1	[1 n]	n
2	[1 n]	n
4	[1 n]	n
8	[1 n]	n
16	[1 n]	n
32	[1 n]	n
64	[1 n]	n
128		<u>7n</u>