

Agenda

- 1) Bubble sort
 - 2) Insertion sort
 - 3) Merge 2 sorted arrays (ques)
 - 4) Merge sort
- } Algorithms

What is sorting: Arranging the data

2 9 15 20 31 → increasing order

15 12 10 5 -8 → decreasing order

	7	6	8	12		→ inc. order based on
# of	2	4	4	6		count of factors

How to sort?

```
int [] A = { 7, 9, 5, 1, 33 };
```

```
Arrays.sort(A);
```

↳ Array variable name

1) A ⇒ 1 3 5 7 9

note : $T_c = \text{Arrays.sort}(A) \Rightarrow O(n \log n)$

Bubble Sort

A = 3 8 6 2 4
 0 1 2 3 4

itr1: 3 6 2 4 8

itr2: 3 2 4 6 8

itr3: 3 2 4 6 8

itr4: 2 3 4 6 8

A = 3 8 6 2 4
 0 1 2 3 4

itr1: 2 4
 6 ~~8~~ ~~8~~ 8
 3 ~~8~~ ~~6~~ 2 4

itr2: 2 4
 3 ~~8~~ ~~2~~ 6 4 8

itr3: 2 3
 ~~8~~ ~~2~~ 4 6 8

itr4: 2 3 4 6 8

2 3 4 6 8

if ($A[j] > A[j+1]$) {
 swap

}

$A = \quad 4 \quad 3 \quad 1$
 $\text{itr1:} \quad \begin{array}{ccc} & 1 & \\ 3 & 4 & 4 \\ 4 & 8 & 1 \end{array}$
 $\text{itr2:} \quad \begin{array}{cc} 1 & 3 \\ 3 & 2 \end{array} \quad 4$
 $\quad \quad 1 \quad 3 \quad 4$

$A = \quad 3 \quad 8 \quad 6 \quad 2 \quad 4$
 $\quad \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

$n-1 \text{ itr}$

$\text{itr1:} \quad \begin{array}{ccccc} & 2 & 4 & & \\ & 6 & 8 & 8 & \\ 3 & 8 & 8 & 2 & 4 \end{array}$
 $\text{itr2:} \quad \begin{array}{ccc} 2 & 4 & \\ 3 & 8 & 2 \end{array} \quad 6 \quad 8$
 $\text{itr3:} \quad \begin{array}{cc} 2 & 3 \\ 3 & 2 \end{array} \quad 4 \quad 6 \quad 8$
 $\text{itr4:} \quad 2 \quad 3 \quad 4 \quad 6 \quad 8$
 $\quad \quad 2 \quad 3 \quad 4 \quad 6 \quad 8$

i	j ^{0 to n-i-1}
0	0 to 3
1	0 to 2
2	0 to 1
3	0 to 0

```
void bubble-sort (int [ ] A) {
```

```
    int n = A.length;
```

```
    for (int i = 0; i < n-1; i++) {
```

```
        for (int j = 0; j < n-i-1; j++) {
```

```
            if (A[j] > A[j+1]) {
```

```
                int temp = A[j];
```

```
                A[j] = A[j+1];
```

```
                A[j+1] = temp;
```

```
            }
```

```
        }
```

```
    }
```

```
}
```

TC: $O(n^2)$

SC: $O(1)$

dry run

```
void bubble-sort (int [ ] A) {
```

```
    int n = A.length;
```

```
    for (int i = 0; i < n-1; i++) {
```

```
        for (int j = 0; j < n-i-1; j++) {
```

```
            if (A[j] > A[j+1]) {
```

```
                int temp = A[j];
```

```
                A[j] = A[j+1];
```

```
                A[j+1] = temp;
```

```
            }
```

```
        }
```

```
    }
```

```
}
```

A = 8 5 4 1

i	j	steps
0	0, 1, 2	5 8 4 8 1 8 8 5 4 1
1	0, 1	4 5 1 5 5 4 1 8
2	0	1 4 4 1 5 8
		=> 1 4 5 8

Insertion sort

A = 3 8 6 2 4

3] 8 6 2 4
↖

3 8 6 2 4
↖

3 6 8 2 4
↖

2 3 6 8 4
↖

2 3 4 6 8

A = 3 8 6 2 4
 0 1 2 3 4

1 to n-1	
i	j \rightarrow i-1 to 0
1	0 to 0
2	1 to 0
3	2 to 0
4	3 to 0

itr 1 : 3] 8 6 2 4
 0 1 2 3 4

itr 2 : 3 8] 6 2 4
 0 1 2 3 4

itr 3 : 2 3 6 8] 4
 0 1 2 3 4

itr 4 : 2 3 4 6 8] 4
 0 1 2 3 4

2 3 4 6 8

```
void insertion-sort (int[] A) {
```

```
    int n = A.length;
```

TC: $O(n^2)$

```
    for (int i = 1; i < n; i++) {
```

SC: $O(1)$

```
        for (int j = i - 1; j >= 0; j--) {
```

```
            if (A[j] > A[j + 1]) {
```

```
                int temp = A[j];
```

```
                A[j] = A[j + 1];
```

```
                A[j + 1] = temp;
```

```
            }
```

```
            else {
```

```
                break;
```

```
            }
```

```
        }
```

```
    }
```

```
}
```

```
void insertion-sort (int[] A) {
```

```
    int n = A.length;
```

```
    for (int i = 1; i < n; i++) {
```

```
        for (int j = i - 1; j >= 0; j--) {
```

```
            if (A[j] > A[j + 1]) {
```

```
                int temp = A[j];
```

```
                A[j] = A[j + 1];
```

```
                A[j + 1] = temp;
```

```
            }
```

```
            else {
```

```
                break;
```

```
            }
```

```
        }
```

```
    }
```

```
}
```

A = 8 9 4 6

i	j	
1	0 to 0	$\begin{array}{cccc} & & \downarrow & \\ 8 & 9 & 4 & 6 \\ \hline & 8 & & \\ 4 & 8 & 9 & 6 \\ \hline \end{array}$
2	1 to 0	$\begin{array}{cccc} & & 8 & \\ 4 & 8 & 9 & 6 \\ \hline 8 & 9 & 4 & 6 \\ \hline \end{array}$
3	2 to 0	$\begin{array}{cccc} & & 8 & \\ & 6 & 9 & \\ 4 & 8 & 9 & 6 \\ \hline 4 & 6 & 8 & 9 \\ \hline \end{array}$
		⇒ 4 6 8 9

**

Q-1 merge two sorted arrays.

Tc: $O(n+m)$

A \Rightarrow 2 5 9 12 15

B \Rightarrow 3 6 8 10 16 18

ans =

2	3	5	6	8	9	10	12	15	16	18
0	1	2	3	4	5	6	7	8	9	10

A \Rightarrow 2 5 9 12 15

B \Rightarrow 3 6 8 10 16 18

ans =

2	3	5	6	8	9	10	12	15	16	18
0	1	2	3	4	5	6	7	8	9	10

A = 3 5 9 12

B = 10 15 18

3	5	9	10	12	15	18
0	1	2	3	4	5	6

```

while(i < n && j < m) {
    if(A[i] < B[j]) {
        ans[k] = A[i];
        i++;
        k++;
    }
    else {
        ans[k] = B[j];
        j++;
        k++;
    }
}

```

//if first array values are pending

```

while(i < n) {
    ans[k] = A[i];
    i++;
    k++;
}

```

//if second array values are pending

```

while(j < m) {
    ans[k] = B[j];
    j++;
    k++;
}

```

A = 3 9 12 15
0 1 2 3

B = 2 3 8
0 1 2

ans =

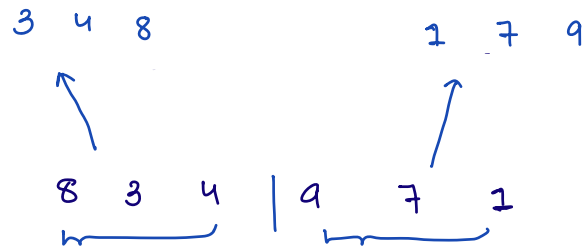
2	3	3	8	9	12	15
0	1	2	3	4	5	6

k

TC: $O(n+m)$

merge sort

Tc: $O(n \log n)$



```
int[] mergeSort (int[] arr, int lo, int hi) {
```

8 3 4 9 7 1
lo mid hi

```
int mid = (lo+hi)/2;
```

```
int[] A = mergeSort(arr, lo, mid);
```

```
int[] B = mergeSort(arr, mid+1, hi);
```

```
int[] ans = merge(A, B);
```

```
return ans;
```

3

```
int[] mergesort (int[] arr, int lo, int hi) {
```

```
int mid = (lo+hi) / 2;
```

```
int [] A = mergeSort(arr, lo, mid);
```

```
int[] B = mergeSort(arr, mid+1, hi);
```

```
int [] ans = merge(A, B);
```

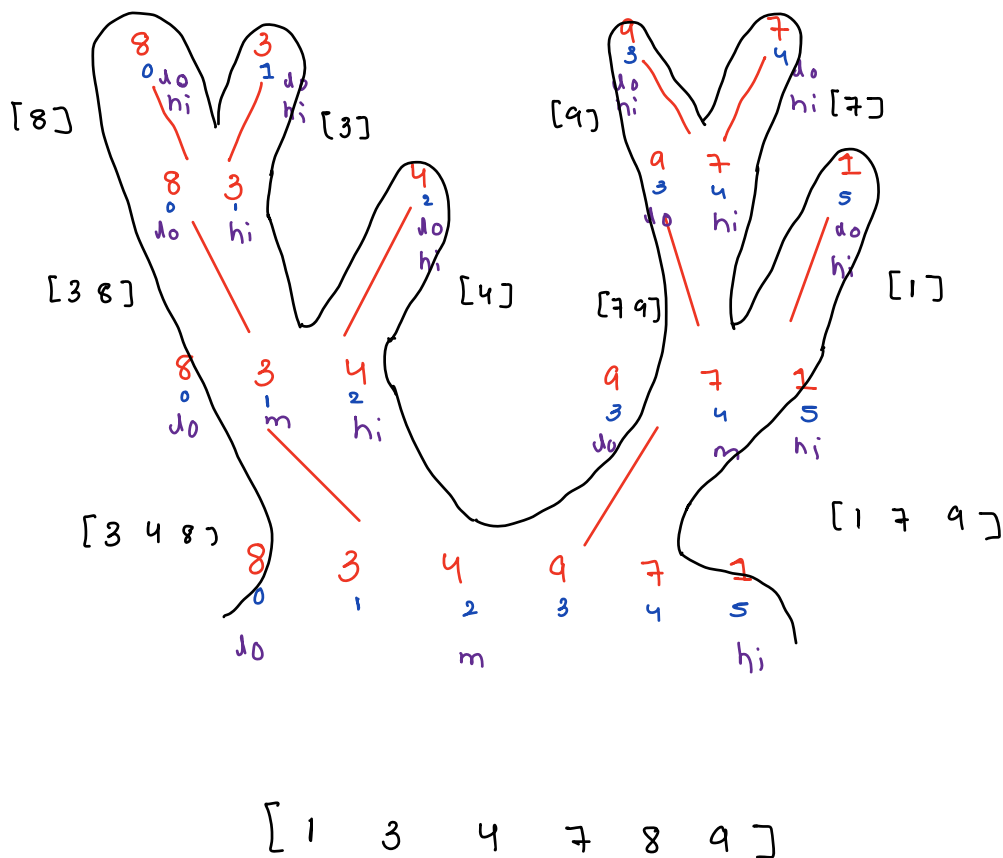
```
return ans;
```

$$ij(a_0 = h_i) \quad ?$$
$$in + [] sa = new in + [1] ;$$
$$S_{\alpha}[0] = \alpha x x[10];$$

```
return sa;
```

3

3



todo:

merge sort \Rightarrow

TC: $O(n \log n)$

SC: $O(n)$

Doubts
=

```
int[] mergeSort (int[] arr, int lo, int hi) {
```

```
    int mid = (lo + hi) / 2;
```

```
    int[] A = mergeSort (arr, lo, mid);
```

```
    int[] B = mergeSort (arr, mid + 1, hi);
```

```
    int[] ans = merge (A, B);
```

```
    return ans;
```

if (lo == hi) {

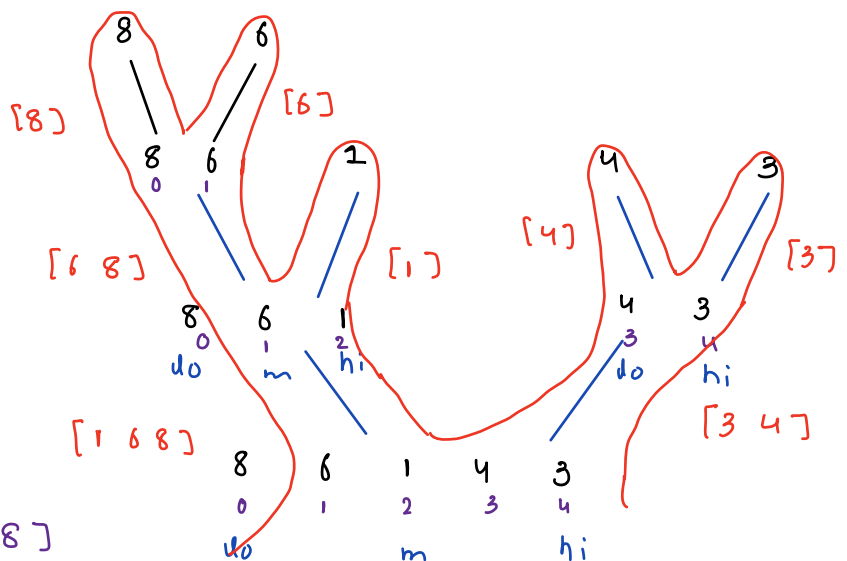
int[] sa = new int[1];

sa[0] = arr[lo];

return sa;

}

3



Subarray with sum k

$$k = 9$$

	-1	0	1	2	3	4	5	6	7	8
	↑	2	4	7	1	-6	4	1	3	6
sum :	0	2	6	13	14	8	12	13	16	22

$$\text{sum} - k$$

$$22 - 9 = 13$$

$$\text{map.get}(13) = 2$$

0 → -1	8 → 4
2 → 0	12 → 5
6 → 1	16 → 7
13 → 2	
14 → 3	

map contains key
(sum - k)

$$\begin{cases} s = \text{map.get}(\text{sum} - k) + 1 \\ e = i \end{cases}$$

Sum vs first
index