

## 1. Decimal No. System

base : 10

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

0	10
1	11
2	12
3	13
4	14
5	15
6	16
7	17
8	18
9	19

## 2. Binary No. System

base : 2

digit  $\rightarrow$  0, 1

00  $\rightarrow$  0

10  $\rightarrow$  2

100  $\rightarrow$  4

01  $\rightarrow$  1

11  $\rightarrow$  3

101  $\rightarrow$  5

## Conversions

### 1. Decimal to binary

$$(27)_{10} = (11011)_2$$

2	27	
2	13	1
2	6	1
2	3	0
2	1	1
	0	1

$$(28)_{10} = (11100)_2$$

2	28	
2	14	0
2	7	0
2	3	1
2	1	1
	0	1

2. binary to decimal

$$(1101)_2 = (13)_{10}$$

$$\begin{array}{cccc} 1 & 1 & 0 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \end{array}$$

$$8 + 4 + 0 + 1 = 13$$

$$\begin{array}{c} 384 \\ \downarrow \\ 3 \times 10^2 + 8 \times 10^1 + 4 \times 10^0 \\ 300 + 80 + 4 = 384 \end{array}$$

$$\begin{array}{cccc} & 1 & 1 & 0 & 1 \\ & \downarrow & & & \downarrow \\ \text{MSD} & & & & \text{LSD} \end{array}$$

$$(1010)_2 = (10)_{10}$$

$$\begin{array}{cccc} 1 & 0 & 1 & 0 \\ \downarrow & & \downarrow & \\ 2^3 & & 2^1 & \end{array}$$

$$= 8 + 2 = 10$$

quiz

Binary of 25

$$(25)_{10} \rightarrow (11001)_2$$

2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

$$(10110)_2 \rightarrow (22)_{10}$$

1	0	1	1	0
↓		↓	↓	
16		4	2	

## Addition of two binary numbers

$$\begin{array}{r}
 1 \ 1 \ 1 \\
 3 \ 8 \ 4 \ 9 \\
 + \ 1 \ 7 \ 6 \\
 \hline
 4 \ 0 \ 2 \ 5
 \end{array}$$

$$15$$

$$\begin{array}{r}
 1 \ 1 \ 1 \ 1 \\
 1 \ 0 \ 1 \ 1 \\
 + \ 0 \ 1 \ 1 \ 1 \\
 \hline
 1 \ 0 \ 0 \ 1 \ 0
 \end{array}$$

$$\begin{array}{r}
 11 \\
 + 7 \\
 \hline
 18
 \end{array}$$

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

$$\begin{array}{r}
 1 \ 1 \ 1 \\
 1 \ 0 \ 1 \ 1 \ 0 \\
 + \ 1 \ 0 \ 1 \ 1 \\
 \hline
 1 \ 0 \ 0 \ 0 \ 0 \ 1
 \end{array}$$

## Bitwise operators

$\&$  , bitwise AND  
 $|$  , bitwise OR  
 $\wedge$  , XOR  
 $!$  , NOT  
 left shift, right shift

$\&$   $\rightarrow$  0 is dominating

$|$   $\rightarrow$  1 is dominating

$\wedge$   $\rightarrow$  same same  
puppy shame

a	b	$a \& b$	$a   b$	$a \wedge b$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

$$1) \quad 10 \& 12$$

$$\begin{array}{r} 1010 \\ \& 1100 \\ \hline 1000 \end{array} \rightarrow 8$$

$$\text{SOPAN}(10 \& 12); \text{ // } 8$$

$$2) \quad 10 | 12$$

$$\begin{array}{r} 1010 \\ | \quad 1100 \\ \text{(OR)} \quad \hline 1110 \end{array} \rightarrow 14$$

$$\text{SOPAN}(10 | 12); \text{ // } 14$$

$$3) \quad 10 \wedge 12$$

$$\begin{array}{r} 1010 \\ \wedge 1100 \\ \hline 0110 \end{array} \rightarrow 6$$

$$\text{SOPAN}(10 \wedge 12); \text{ // } 6$$

## Properties of bitwise operators

$a$  and  $b$  are  
two no.

1) Commutative

$$a \& b = b \& a$$

$$a | b = b | a$$

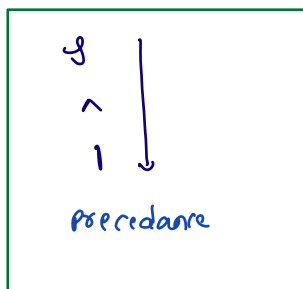
$$a \wedge b = b \wedge a$$

2) Associative

$$a \& b \& c = (a \& b) \& c = a \& (b \& c)$$

$$a | b | c = (a | b) | c = a | (b | c)$$

$$a \wedge b \wedge c = (a \wedge b) \wedge c = a \wedge (b \wedge c)$$



$$n \& 0 = 0$$

$$n \& n = n$$

$$n | 0 = n$$

$$n | n = n$$

$$n \wedge 0 = n$$

$$n \wedge n = 0$$



Q-1 Given a int no., check whether it is even or odd.

Note: we can't use  $+$ ,  $-$ ,  $*$ ,  $!$ ,  $/$ .

10  $\rightarrow$  1010

13  $\rightarrow$  1101

if (LSB == 0) {

even

}

else {

odd

}

$n = 13$

1101  
8 0001  
-----  
0001

o/p: odd

$n = 18$

10010  
8 00001  
-----  
00000

o/p: even

```
void solve(int n) {  
    if ((n & 1) == 1) {  
        solve("odd");  
    }  
    else {  
        solve("Even");  
    }  
}
```

## Q.2 Single element

Given an  $A[]$ , every value is coming except for a no. that is coming once. Find the single no.

T.C:  $O(n)$

S.C:  $O(1)$

$A = 5 \quad 2 \quad 3 \quad 2 \quad 8 \quad 5 \quad 3$

$$\text{ans} = 5^2 \wedge 3^2 \wedge 8^1 \wedge 5^2 \wedge 3^2$$

$$= \underbrace{5^5}_0 \wedge \underbrace{2^4}_0 \wedge \underbrace{3^4}_0 \wedge 8^1 = 8$$

$A = 3 \quad 5 \quad 3 \quad 8 \quad 5$

```
int singleElement(int [] A) {
    int ans = 0;
    for (int i = 0; i < A.length; i++) {
        ans = ans ^ A[i];
    }
    return ans;
}
```

$$\begin{array}{r}
 000 \\
 \wedge 011 \\
 \hline
 011 \\
 \wedge 101 \\
 \hline
 110 \\
 \wedge 011 \\
 \hline
 0101 \\
 \wedge 1000 \\
 \hline
 1101 \\
 \wedge 0101 \\
 \hline
 1000
 \end{array}$$

Left shift

<<

with every left shift  
no. is getting  
twiced.

$$a = 13 \quad 1 \ 1 \ 0 \ 1 \rightarrow 13$$

$$a << 1 \quad 1 \ 1 \ 0 \ 1 \ 0 \rightarrow 26$$

$$a << 2 \quad 1 \ 1 \ 0 \ 1 \ 0 \ 0 \rightarrow 52$$

$$a << 1 \rightarrow 2a$$

$$a << 2 \rightarrow 4a$$

$$a << 3 \rightarrow 8a$$

⋮

$$a << N \rightarrow 2^N \times a$$

$$1 << N = 2^N$$

↙  
 $O(1)$

## Right shift

>>

with every rightshift  
no. is getting  
halved.

$a = 13$                       1 1 0 1                       $\rightarrow 13$

$a >> 1$                       1 1 0                       $\rightarrow 6$

$a >> 2$                       1 1                       $\rightarrow 3$

$$a >> 1 = a / 2$$

$$a >> 2 = a / 4$$

$$a >> 3 = a / 8$$

⋮

$$a >> N = a / 2^N$$

Doubts  
=

$$\text{ans} \% (10^9 + 7)$$

```
int m = (int) (Math.pow(10, 9) + 7);
```

// a, b, c → int

```
ans = a + b + c;  
return ans % m;
```

i) long variables

ii) apply mod in correct manner

$$(a + b) \% m = (a \% m + b \% m) \% m$$

$$(a - b) \% m = (a \% m - b \% m + m) \% m$$

$$a = 5 \quad b = 2$$

$$(5 \% m + 2 \% m) \% m$$

$$(5 + 2) \% m = 7$$

$$(5 \% m - 2 \% m + m) \% m$$

$$(5 - 2 + m) \% m$$

$$(3 + m) \% m = 3$$

```
int j = m-1;
```

```
for (i → 0 to n-1)
```

```
while (A[i][j] == 1 && j >= 0)
```

```
    j--;
```

```
    ans = i;
```

```
}
```

j

0	1	1	1	1
0	0	0	1	1
1	1	1	1	1
0	1	1	1	1

i

ans = ~~0~~ 2

j

	1	1	1	1	1
	1	1	1	1	1
i	1	1	1	1	1

ans = 0