

Agenda

- i) Linear search to find K in $A[]$
- ii) Binary search to find K in $A[]$
- iii) First occurrence of K in array
- iv) Floor of K in array
- v) Local minima

Q-1 Given an $A[]$, find if K is present or not.

$A =$ 2 4 8 17 42 1
 0 1 2 3 4 5

$K = 13 \Rightarrow -1$

$K = 17 \Rightarrow 3$

→ apply Linear search

```
int search (int [] A, int K) {
```

```
    int n = A.length;
```

```
    for (int i = 0; i < n; i++) {
```

```
        if (A[i] == K) {
```

```
            return i;
```

```
        }
```

```
    }
```

```
    return -1;
```

```
}
```

T.C: $O(n)$

Searching becomes easy when data is organised.
(Dictionary ...)

Q.2 Given an sorted array, find if x is present or not.

$A =$

2	9	13	15	19	24	31	48	52
0	1	2	3	4	5	6	7	8

 $K = 15 \Rightarrow 3$
 $K = 48 \Rightarrow 7$
 $K = 20 \Rightarrow -1$

binary search

$K = 13$

2	9	13	15	19	24	31	48	52
0	1	2	3	4	5	6	7	8

 do hi

do	hi	mid
0	8	4
0	3	1
2	3	2

$A[mid]$

Binary
search

$A[mid] == K \Rightarrow$ got the ans

$A[mid] < K \Rightarrow$ discard left side

$do = mid + 1$

$A[mid] > K \Rightarrow$ discard right side

$hi = mid - 1$

```
int search (int [] A, int k) {
```

$k = 19$

```
    int n = A.length;
```

```
    int lo = 0, hi = n-1;
```

```
    while (lo <= hi) {
```

```
        int mid = (lo+hi) / 2;
```

```
        if (A[mid] == k) {
```

```
            return mid;
```

```
        }
```

```
        else if (A[mid] < k) {
```

// discard left side

```
            lo = mid+1;
```

```
        }
```

```
        else if (A[mid] > k) {
```

// discard right side

```
            hi = mid-1;
```

```
        }
```

```
    }
```

```
    return -1;
```

```
}
```

A =	2	4	6	7	10	15	19	21
	0	1	2	3	4	5	6	7

lo hi
m

\Rightarrow ans: 6

$k = 21$

A =	2	4	6	7	10	15	19	21
	0	1	2	3	4	5	6	7

hi
lo
m

\Rightarrow ans: 7

$k = 5$

A =	2	4	6	7	10	15	19	21
	0	1	2	3	4	5	6	7

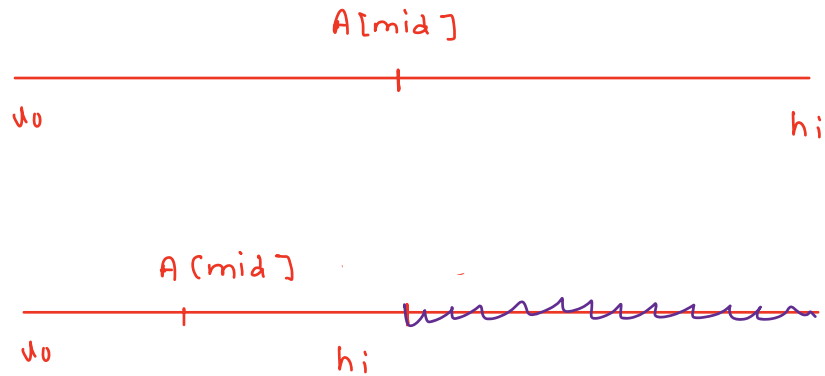
hi

lo

m

get out of loop

ans \Rightarrow -1



$\Rightarrow n$
 \downarrow
 $\frac{n}{2}$
 \downarrow
 $\frac{n}{4}$
 \vdots
 1

$$Tc: O(\log_2 n)$$

$$10^5 \approx 2^{16}$$

$$\log_2(10^5) = \log_2(2^{16}) = 16$$

*** In an array of 10^5 elements we can find K in just 16-17 ops.

Q.3 Given a sorted `A[]`, find first occurrence of `K`.

A = 2 2 3 4 5 5 5 7 7 8 9 12 14 20
 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Slight modification in Binary search:

$\Rightarrow A[mid] == K$

keep on searching in left side

$K=3$, ans $\Rightarrow 2$

$K=5$, ans $\Rightarrow 4$

$K=10$, ans $\Rightarrow -1$

```
int search (int [] A, int K) {
```

```
    int n = A.length;
```

```
    int lo = 0, hi = n-1, ans = -1;
```

```
    while (lo <= hi) {
```

```
        int mid = (lo+hi) / 2;
```

```
        if (A[mid] == K) {
```

```
            ans = mid;  
            hi = mid - 1;
```

```
        }
```

```
        else if (A[mid] < K) {
```

```
            // discard left side
```

```
            lo = mid + 1;
```

```
        }
```

```
        else if (A[mid] > K) {
```

```
            // discard right side
```

```
            hi = mid - 1;
```

```
        }
```

```
    }
```

```
    return ans;
```

```
}
```

$K=5$

A = 2 2 5 5 5 7 9 12 12
 0 1 2 3 4 5 6 7 8
 hi lo

ans = ~~-1~~ 2

$K=5$

A = 2 5 5 5 5 5 5 5 10
 0 1 2 3 4 5 6 7 8
 do
 hi
 m

ans = ~~-1~~ 1

Q-4 Given a sorted array, find floor of k in the array.

floor(k) \Rightarrow max of all the values which are $\leq k$

A = 12 14 21 25 28 32 35 38 42 51
 0 1 2 3 4 5 6 7 8 9

k	floor
25	25
26	25
24	21
37	35

floor(k) = { k if k is present in array
 just smaller than k if k is not present in array

$k = 24$

A = 12 14 21 25 28 32 35 38 42 51
 0 1 2 3 4 5 6 7 8 9

hi

do
m

ans = ~~21~~ 21

if ($A[mid] \leq k$) {

else {

ans = $A[mid]$;

$hi = mid - 1$;

do = $mid + 1$;

}

3

```
int floor ( int [] A, int k ) {
```

```
    int n = A.length;
```

```
    int lo = 0, hi = n-1, ans = -1;
```

```
    while ( lo <= hi ) {
```

```
        int mid = (lo+hi) / 2;
```

```
        if ( A[mid] <= k ) {
```

```
            ans = A[mid];
```

```
            lo = mid + 1;
```

```
        }
```

```
        else {
```

```
            hi = mid - 1;
```

```
        }
```

```
    }
```

```
    return ans;
```

```
}
```

k = 19

A =	15	18	21	25	31	38	46	48
	0	1	2	3	4	5	6	7

hi

lo

m

ans = ~~18~~

k = 50

A =	15	18	21	25	31	38	46	48
	0	1	2	3	4	5	6	7

hi

lo

m

ans = ~~36~~

~~38~~ ~~46~~ 48

Q.5 Local minima

Given an array, find any local minima. Local minima is the element smaller than ^{or equals to} both of its neighbours. Corner elements will have only one neighbour. { Array might have duplicates }

A = 12 > 10 < 15 20 ans: 10

A = 12 10 9 > 7 < 15 ans: 7

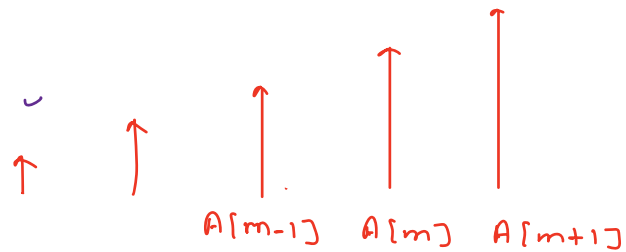
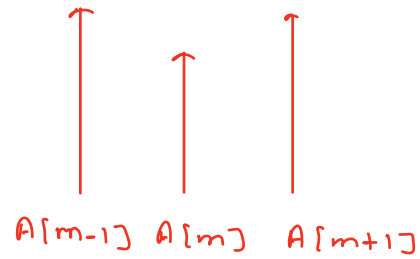
A = 12 15 17 14 8 20 ans: 8 or 12

any one of them.

If array contains distinct values, local minima will definitely exist.



Find local minima using binary search :



20	18	22	25	28	16	5
0	1	2	3	4	5	6
lo	m	hi				

5	5	5	5	5	3	2
lo						hi

ans = 5
(considering equality also)

18	20	22	25	28	16	19
----	----	----	----	----	----	----

corner cases

```

int local-minima(int[] A) {
    int n = A.length;
    if (A[0] <= A[1]) {
        return A[0];
    }
    else if (A[n-1] <= A[n-2]) {
        return A[n-1];
    }

    int lo = 1, hi = n-2;
    while (lo <= hi) {
        int m = (lo + hi) / 2;
        if (A[m] <= A[m-1] && A[m] <= A[m+1]) {
            return A[m];
        }
        else if (A[m-1] < A[m]) {
            hi = m-1;
        }
        else {
            lo = m+1;
        }
    }
    return -1;
}

```

}

```
while (do <= hi) {
```

```
    int m = (do + hi) / 2;
```

```
    if (A[m] <= A[m-1] && A[m] <= A[m+1]) {
```

```
        return A[m];
```

```
    }
```

```
    else if (A[m-1] < A[m]) {
```

```
        hi = m-1;
```

```
    }
```

```
    else {
```

```
        do = m+1;
```

```
    }
```

```
}
```

A =

9	7	5	4	3	2	8
0	1	2	3	4	5	6

hi
do
m

A =

11	5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

do
hi
m