

Agenda

- 1) Max subsequence without adjacent elements.
- 2) count of paths in grid
- 3) count of paths in grid with blocked cells.
- 4) minimum sum path

Q.1 Given an $A[]$, find maximum subsequence sum such that no two consecutive elements are picked.

$$A = 9 \quad 14 \quad 3 \quad \text{ans} = 14$$

$$A = 13 \quad 14 \quad 2 \quad \text{ans} = 15$$

$$A = 4 \quad 3 \quad 2 \quad 1 \quad \text{ans} = 6$$

$$A = -4 \quad -3 \quad -2 \quad -1 \quad \text{ans} = 0$$

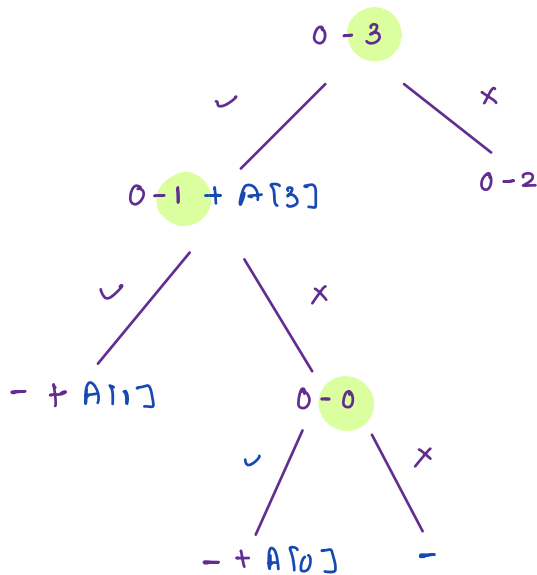
$$A = 9 \quad 4 \quad 13 \quad 24 \quad \text{ans} = 33$$

Alternative sum idea even-sum & odd-sum won't work.

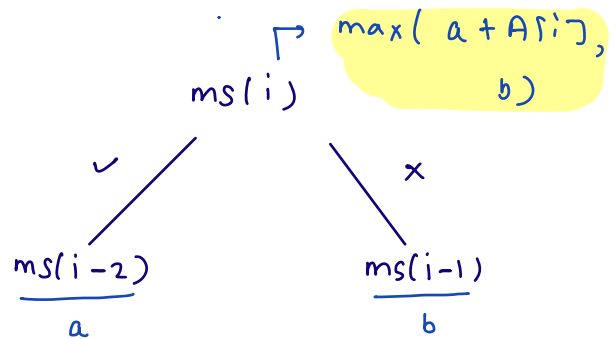
idea 1 : going on on all subseq, in base case check if subseq. is containing non-consecutive ele if yes then sum of this subseq. can be considered for updation of max ans till now.

TC: $O(2^n \times n)$

idea 2



0 1 2 3



```

int solve (int [] A) {
    int n = A.length;
    return maxSubseqSum(A, n-1);
}

```

3

```

int maxSubseqSum (int [] A, int i) {
    if (i < 0) {
        return 0;
    }
    int a = maxSubseqSum(A, i-2); // pick ith element
    int b = maxSubseqSum(A, i-1); // don't pick ith element
    int ans = Math.max(a+A[i], b);
    return ans;
}

```

3

```
int maxSubseqSum ( int [] A, int i) {
```

```
    if (i < 0) {
```

```
        return 0;
```

```
    }
```

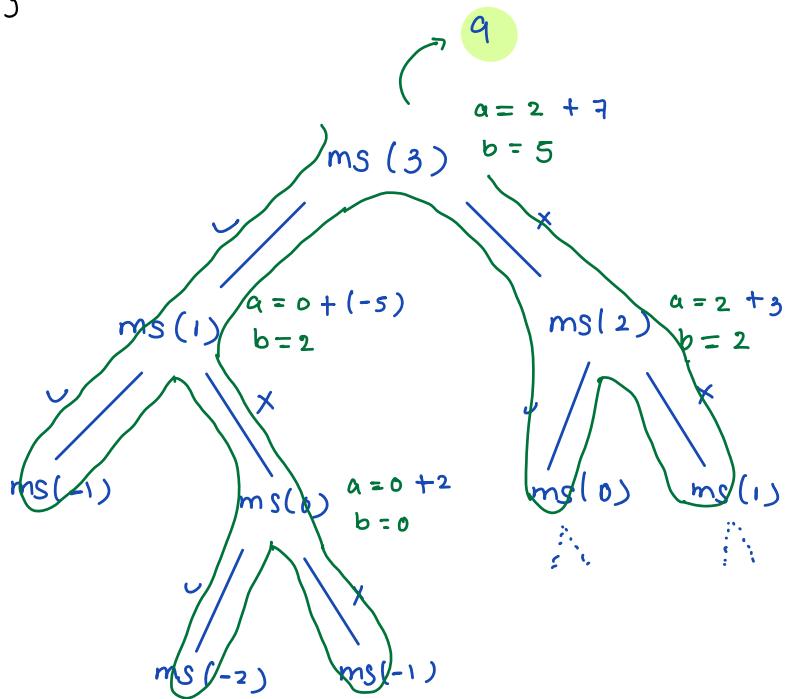
```
    int a = maxSubseqSum (A, i-2);
```

```
    int b = maxSubseqSum (A, i-1);
```

```
    int ans = Math.max (a+A[i], b);
```

```
    return ans;
```

```
}
```



$A = \begin{matrix} 2 & -5 & 3 & 7 \\ 0 & 1 & 2 & 3 \end{matrix}$

```
int ans = Math.max (a+A[i], b);
```



as repeated sub-problems are there, DP can be applied.

```
int [] dp = new int [n];
```

→ give it with -1

```
int maxSubseqSum ( int [] A, int i ) {
```

```
    if ( i < 0 ) {
```

```
        return 0;
```

```
    }
```

```
    if ( dp [i] != -1 ) {
```

```
        return dp [i];
```

```
    }
```

```
    int a = maxSubseqSum ( A, i-2 );
```

```
    int b = maxSubseqSum ( A, i-1 );
```

```
    int ans = Math.max ( a+A[i], b );
```

```
    dp [i] = ans;
```

```
    return ans;
```

```
}
```

TC : $O(n)$

SC : $O(n)$

Q.2 Count total no. of ways to go from $(0,0)$ to $(n-1, m-1)$

movement allowed $\rightarrow R$ or $\downarrow D$

A =

	0	1	2
0			
1			
2			

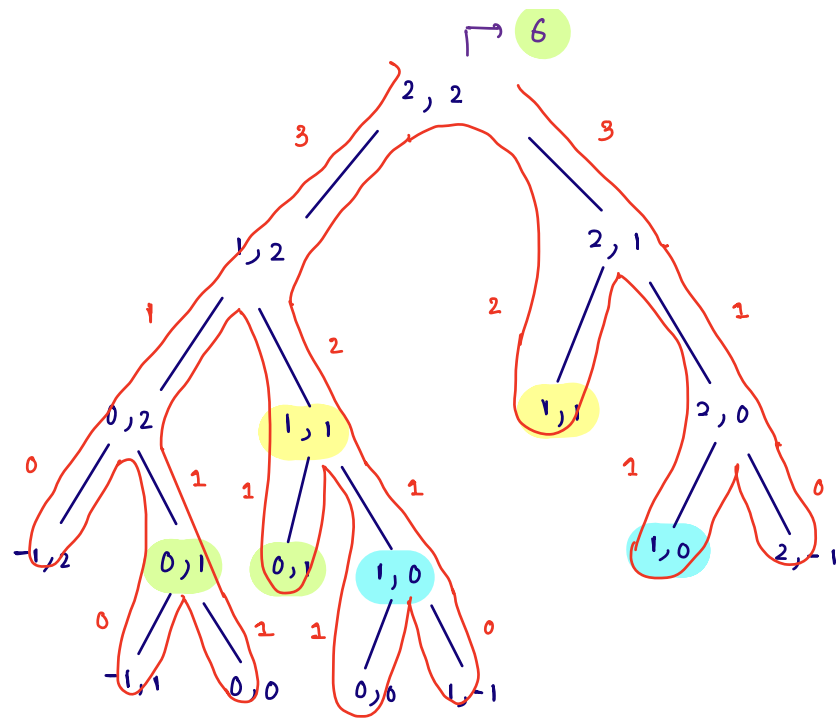
$RRDD$
 $RD RD$
 $RDDR$
 $DRRD$
 $DRDR$
 $DDRR$

$$\text{ways}(i, j) = \text{ways}(i-1, j) + \text{ways}(i, j-1)$$

no. of ways from $(0,0)$ to (i,j)	no. of ways from $(0,0)$ to $(i-1,j)$	+ no. of ways from $(0,0)$ to $(i,j-1)$
---	---	---

A =

	0	1	2
0			
1			
2			



$$\text{ways}(i, j) = \text{ways}(i-1, j) + \text{ways}(i, j-1)$$

```

int solve ( int n, int m ) {
    dp = new int [n] [m];
    // travel dp and fill it with -1
    return ways (n-1, m-1);
}

```

int dp[n][m]; (dimension: n x m and fill it with -1)

int ways (int i, int j) {

if (i < 0 || j < 0) { return 0; }

if (i == 0 && j == 0) { return 1; }

if (dp[i][j] != -1) {

return dp[i][j];

}

TC : $O(N \times m)$

int a = ways(i-1, j);

SC : $O(N \times m)$

int b = ways(i, j-1);

dp[i][j] = a + b;

return a + b;

}

Q.3 Count total no. of ways to go from $(0,0)$ to $(n-1, m-1)$

Note: if $mat[i][j] == 1$, it is a blocked cell, else it is unblocked.

→ R ↓ D

A =

	0	1	2	3
0	0	0	0	1
1	0	1	0	0
2	0	0	0	0

logic: last ques logic with base case condition & slight change.

```
int solve (int [][ ] A, int n, int m) {
```

```
    dp = new int [n][m];
```

```
    // travel dp and fill it with -1
```

```
    return ways (A, n-1, m-1);
```

```
int ways (int [][]A, int i, int j) {
```

```
    if (i < 0 || j < 0) { return 0; }
```

```
    if (A[i][j] == 1) { return 0; }
```

```
    if (i == 0 && j == 0) { return 1; }
```

```
    if (dp[i][j] != -1) {
```

```
        return dp[i][j];
```

```
    }
```

```
    int a = ways (A, i-1, j);
```

```
    int b = ways (A, i, j-1);
```

```
    dp[i][j] = a+b;
```

```
    return a+b;
```

}

Q.4 min path sum

Given a 2d matrix, filled with non-negative no., find the minimum cost path from $(0,0)$ to $(n-1, m-1)$.

→ R ↓ D

A =

	0	1	2
0	2	3	5
1	1	6	4
2	9	2	7

min cost = 18

A =

	0	1	2
0	2	2	100
1	10	100	200
2	1	1	3

min cost = 17

A =

	0	1	2
0	2	2	100
1	10	100	200
2	1	1	3

greedy soln won't work.

A =

	0	1	2
0	2	3	5
1	2	6	4
2	9	2	7

$a = \text{mincost}(i-1, j);$

$b = \text{mincost}(i, j-1);$

$\text{ans} = \min(a, b) + A[i][j];$

code on IDE

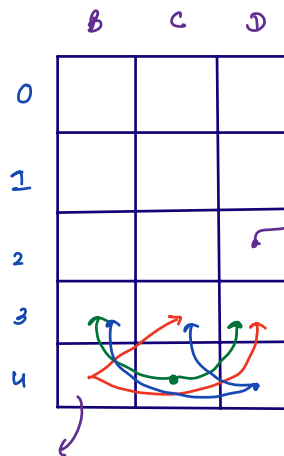
Doubts
=

$$A = \begin{bmatrix} 1 & 5 & -3 & 4 & -2 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$$B = 2$$

$$C = 1$$

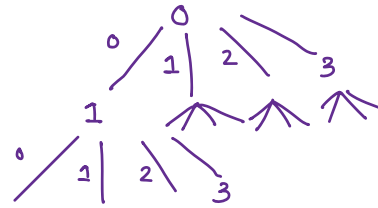
$$D = -1$$



ans till 2 such that 2nd indexed is involved with D.

ans till 4 such that 4 is involved with B

$$\begin{aligned} \max \begin{cases} \text{ans}(i, B) \Rightarrow \max (\text{ans}(i-1, C), \text{ans}(i-1, D)) + A[i][0] * B \\ \text{ans}(i, C) \Rightarrow \max (\text{ans}(i-1, B), \text{ans}(i-1, D)) + A[i][0] * C \\ \text{ans}(i, D) \Rightarrow \max (\text{ans}(i-1, B), \text{ans}(i-1, C)) + A[i][0] * D \end{cases} \end{aligned}$$



$$TC: O(N^N)$$

contest: syllabus is till backtracking.