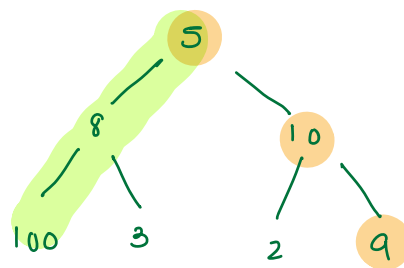


Agenda

- 1) Introduction
- 2) Fractional Knapsack
- 3) Activity Selection
- 4) Job scheduling

Introduction

Greedy algo: choosing the local best every time.



Q.1 Fractional Knapsack

We can consume K kg of food item. Find max protein we can get.

note: Eating any integral amount of an item is allowed.

| Food item | Eating complete item protein gained | $K = 70 \text{ kg}$ |
|-------------------------------------|--|---|
| | | $(PPK) \rightarrow \text{protein per kg}$ |
| Tomato $\rightarrow 20 \text{ kg}$ | 200 $\rightarrow 10u$ | |
| Apples $\rightarrow 15 \text{ kg}$ | 180 $\rightarrow 12u$ | $(50 \text{ kg} + 20 \text{ kg})$ |
| Onion $\rightarrow 50 \text{ kg}$ | 250 $\rightarrow 5u$ | $\times \text{max protein} = 250 + 200$ |
| chicken $\rightarrow 10 \text{ kg}$ | 150 $\rightarrow 15u$ | $= 450u$ |
| Potato $\rightarrow 25 \text{ kg}$ | 200 $\rightarrow 8u$ | |
| Mango $\rightarrow 12 \text{ kg}$ | 132 $\rightarrow 11u$ | |
| seafood $\rightarrow 5 \text{ kg}$ | 100 $\rightarrow 20u$ | |

$$K = 70 \text{ kg}$$

$$\text{ans} = 100 + 150 + 180 + 132 + 200 + 64$$

$$K = 70 \text{ (seafood)}$$

$$65 \text{ (chicken)}$$

$$55 \text{ (apples)}$$

$$40 \text{ (mango)}$$

$$28 \text{ (tomato)}$$

$$8 \text{ (potato : 8 kg)}$$

0

| Food item | Eating complete item protein gained |
|------------------------------|--|
| 0 Tomato → 20 kg | 200 → 10 u |
| 1 Apples → 15 kg | 180 → 12 u |
| 2 Onion → 50 kg | 250 → 5 u |
| 3 chicken → 10 kg | 150 → 15 u |
| 4 potato → 25 kg | 200 → 8 u |
| 5 Mango → 12 kg | 132 → 11 u |
| 6 seafood → 5 kg | 100 → 20 u |

class Pair {

int wt;

int protein;

double PPK;

3

```

int solve (int [] wt, int [] protein, int k) {
    int n = wt.length;
    Pair[] arr = new Pair[n];

    for (int i = 0; i < n; i++) {
        // create pair with the help of ith item
        Pair p = new Pair (wt[i], protein[i], protein[i]*1.0/wt[i]);
        arr[i] = p;
    }

    Arrays.sort (arr, — );
    // sort arr on the basis of PPK (protein per kg)
    // ascending order

    double ans = 0.0;

    for (int i = n-1; i >= 0; i--) {
        Pair ji = arr[i];
        if (ji.wt <= k) {
            // take ji completely
            ans += ji.protein;
            k = k - ji.wt;
        }
        else {
            // take k kg's of ji
            ans += k * ji.ppk;
            break;
        }
    }

    return ans;
}

```

TC: $O(n \log n)$

SC: $O(n)$

```
int solve (int [] wt, int [] protein, int k) {
```

```
    int n = wt.length;
```

```
    Pair[] arr = new Pair[n];
```

```
    for (int i = 0; i < n; i++) {
```

```
        // create pair with the help of ith item
```

```
        Pair p = new Pair (wt[i], protein[i], protein[i]*10/wt[i]);
```

```
        arr[i] = p;
```

```
    }
```

```
    Arrays.sort (arr, — );
```

```
    double ans = 0.0;
```

```
    for (int i = n-1; i >= 0; i--) {
```

```
        Pair ji = arr[i];
```

```
        if (ji.wt <= k) {
```

```
            // take ji completely
```

```
            ans += ji.protein;
```

```
            k = k - ji.wt;
```

```
        }
```

```
        else {
```

```
            // take k kg's of ji
```

```
            ans += k * ji.ppk;
```

```
            break;
```

```
        }
```

```
    }
```

```
    return ans;
```

```
}
```

k = 7

↓

| | 0 | 1 | 2 | 3 | 4 |
|----------|----|----|----|----|----|
| wt : | 5 | 2 | 1 | 3 | 4 |
| protein: | 25 | 20 | 15 | 18 | 20 |

| | | | | |
|----------|-----------|-----------|----------|----------|
| 5, 25, 5 | 2, 20, 10 | 1, 15, 15 | 3, 18, 6 | 4, 20, 5 |
|----------|-----------|-----------|----------|----------|

dry run

```
for (int i = n-1; i >= 0; i--) {
```

```
    Pair ji = arr[i];
```

```
    if (ji.wt <= k) {
```

```
        // take ji completely
```

```
        ans += ji.protein;
```

```
        k = k - ji.wt;
```

```
    }
```

```
    else {
```

```
        // take k kg's of ji
```

```
        ans += k * ji.ppk;
```

```
        break;
```

```
    }
```

```
}
```

after sorting

| | | | | |
|----------|----------|----------|-----------|-----------|
| 5, 25, 5 | 4, 20, 5 | 3, 18, 6 | 2, 20, 10 | 1, 15, 15 |
| 0 | 1 | 2 | 3 | 4 |

k = 7

ans = 0

6

15

4

35

2

53

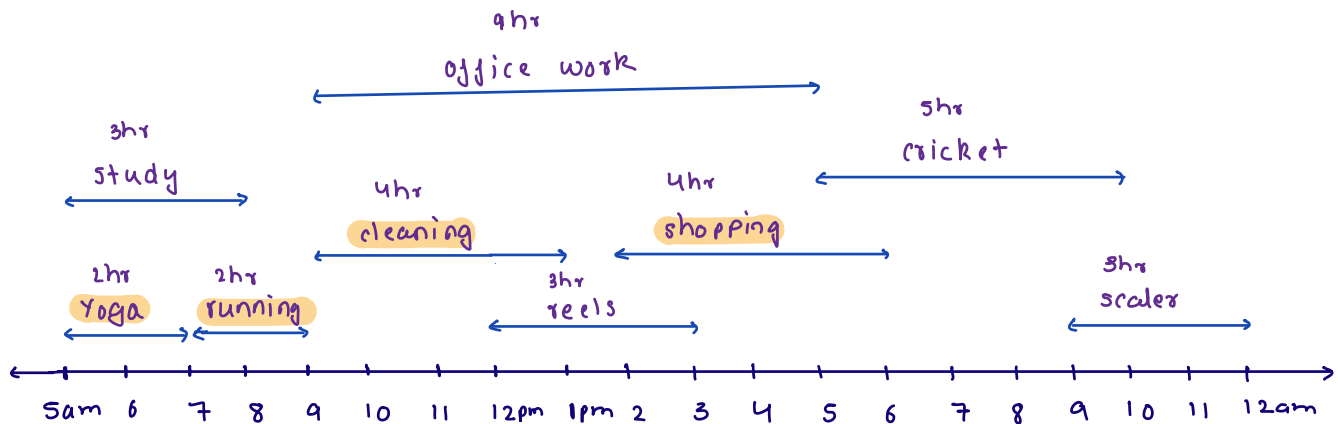
break

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Q.2 Activity Selection

Find max no. of task we can do.

Note : i) On starting a task, we need to complete it
ii) At any point of time do only a single task

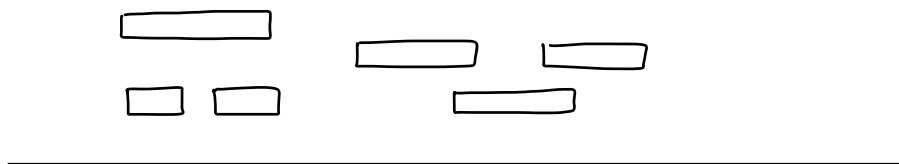


i) Short duration tasks (x)

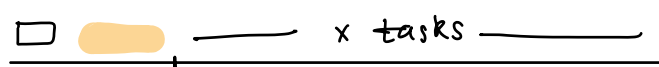
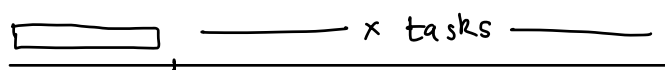
→ Yoga, running, reels, scaler ans = 4

ii) Ending early (✓)

→ yoga, running, cleaning, shopping, scaler ans = 5

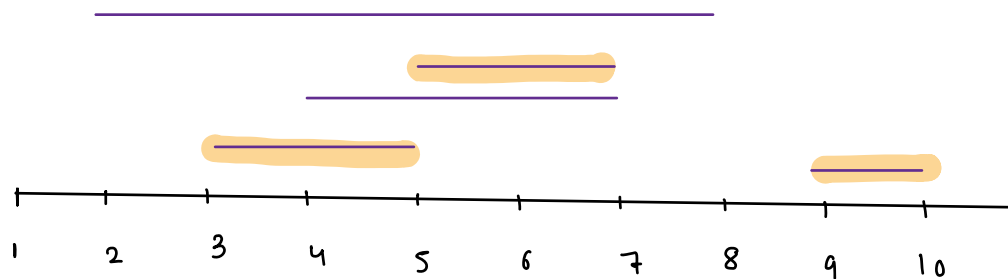


Correction



$\geq x$ tasks

| | | | | | |
|------|---|---|---|----|---|
| | 0 | 1 | 2 | 3 | 4 |
| st : | 4 | 5 | 3 | 9 | 2 |
| et : | 7 | 7 | 5 | 10 | 8 |



```
int solve ( int [] st, int [] et) {
```

```
    int n = st.length;
```

```
    Pair[] arr = new Pair[n];
```

```
    for (int i = 0; i < n; i++) {
```

```
        | Pair p = new Pair (st[i], et[i]);
```

```
        | arr[i] = p;
```

```
    }
```

```
    Arrays.sort (arr, —) ;
```



sort the arr based on ending

time of tasks [ascending order]

// pick non-overlapping tasks

```
    int ans = 1;
```

```
    int utet = arr[0].et; // last task's ending time
```

```
    for (int i = 1; i < n; i++) {
```

```
        | // can I do ith task
```

```
        | if (arr[i].st >= utet) {
```

```
            | ans++;
```

```
            | utet = arr[i].et;
```

```
        | }
```

```
    }
```

```
    return ans;
```

```
class Pair {
```

```
    int st;
```

```
    int et;
```

```
}
```

Tc: $O(n \log n)$

S(: $O(n)$

↓

| | 0 | 1 | 2 | 3 | 4 |
|------|---|---|---|----|---|
| st : | 4 | 5 | 3 | 9 | 2 |
| et : | 7 | 7 | 5 | 10 | 8 |

| | | | | |
|---|---|---|--|---|
| <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block;">4, 7</div> | <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block;">5, 7</div> | <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block;">3, 5</div> | <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block;">9, 10</div> | <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block;">2, 8</div> |
| 0 | 1 | 2 | 3 | 4 |

After sorting : (based on ending : ascending)

↓

| | | | | |
|--|--|---|---|---|
| <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block; background-color: #d4f1d4;">3, 5</div> | <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block; background-color: #d4f1d4;">5, 7</div> | <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block;">4, 7</div> | <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block;">2, 8</div> | <div style="border: 1px solid black; border-radius: 10px; padding: 2px 10px; display: inline-block; background-color: #d4f1d4;">9, 10</div> |
| 0 | 1 | 2 | 3 | 4 |

// pick non-overlapping tasks

int ans = 1;

int utet = arr[0].et; // last task's ending time

ans = ~~2~~ 3

utet = ~~8~~ 10

for (int i = 1; i < n; i++) {

// can i do ith task

if (arr[i].st >= utet) {

ans++;

utet = arr[i].et;

}

}

return ans;

Q.3 Job scheduling

Given N tasks to complete

- deadline assigned for each task, day on or before we can do task.
- Payment assigned to each task
- on any given day we can perform only 1 task and each task take 1 day to finish.
- find max payment we can get.

duration of → 1 day
each task

| | Job | deadline | Payment | |
|-----|-----|----------|---------|--|
| | a | 3 | 100 | |
| eg1 | b | 1 | 14 | |
| | c | 2 | 27 | |
| | d | 1 | 25 | |
| | e | 2 | 30 | |

Sort on the basis of deadline

du: 1 1 2 2 3

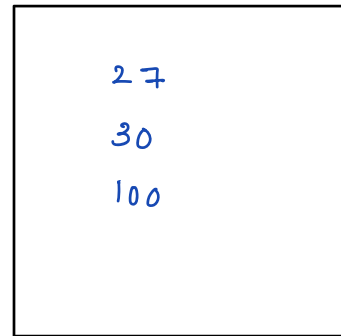
pay: 25 14 30 27 100

| | Job | deadline | Payment |
|-----|-----|----------|---------|
| | a | 3 | 5 |
| eg2 | b | 1 | 1 |
| | c | 3 | 6 |
| | d | 2 | 3 |
| | e | 3 | 9 |

The step after sorting pair array:

↓

| | 0 | 1 | 2 | 3 | 4 |
|-----|----|----|----|----|-----|
| du | 1 | 1 | 2 | 2 | 3 |
| pay | 25 | 14 | 30 | 27 | 100 |



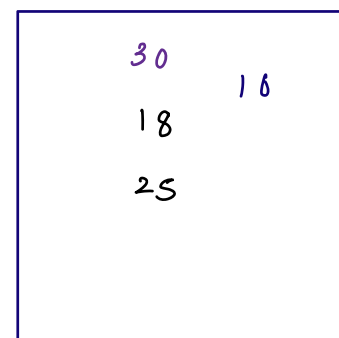
min PQ (Integer)

↓

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|----|----|----|----|----|----|----|----|---|----|
| du | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 | 4 |
| pay | 10 | 15 | 12 | 18 | 10 | 25 | 15 | 30 | 8 | 16 |

```

if (arr[i].du > pq.size()) {
    |   pq.add(arr[i].pay);
3
else {
    |   // replace if you want
    |   if (pq.peek() < arr[i].pay) {
    |       pq.remove();
    |       pq.add(arr[i].pay);
3
    |
3
  
```



min PQ

code: todo
(refer to ide)