

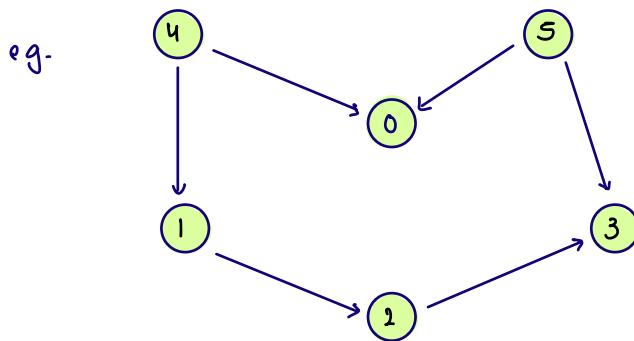
## Agenda

- 1) Topological sort
- 2) Cycle detection
- 3) Dijkstra (Single source shortest path Algo)

### Q.1 Topological sort

linear ordering of graph such that for every directed edge  $u \rightarrow v$ , vertex  $u$  comes before  $v$  in ordering. (applicable only of DAG)

↓  
Directed acyclic graph



0 1 2 5 4 3 (invalid)

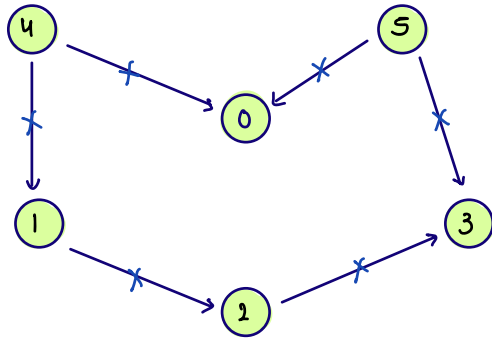
4 1 2 5 0 3 (valid)

4 5 1 2 0 3 (valid)

1 5 4 2 3 0 (invalid)

5 4 1 2 0 3 (valid)

## Kahn's Algo



indegree :

<del>2</del> 0	<del>1</del> 0	<del>1</del> 0	<del>2</del> 1 <sup>0</sup>	0	0
0	1	2	3	4	5

q :

<del>4</del>	<del>5</del>	<del>1</del>	<del>0</del>	<del>2</del>	<del>3</del>
--------------	--------------	--------------	--------------	--------------	--------------

4 5 1 0 2 3

0	1	2	3	4	5
	2	3		0 1	0 3

```
void topologicalSort (AL < AL < Integer >> graph) {
```

```
    int vtx = graph.size();
```

```
    int [] indegree = new int[vtx];
```

// fill indegree array

```
    for (int i=0; i<vtx; i++) {
```

```
        AL < Integer > list = graph.get(i);
```

```
        for (int nbr: list) {
```

```
            indegree[nbr]++;
```

```
        }
```

```
    }
```

// applying bfs (with correct starting pts)

```
Queue<Integer> q = new ArrayDeque<>();
```

```
for (int i=0; i<vtx; i++) {
```

```
    if (indegree[i] == 0) {
```

```
        q.add(i);
```

```
    }
```

```
}
```

```
while (q.size() > 0) {
```

```
    int rmv = q.remove();
```

```
    sop (rmv + " ");
```

```
    // add appropriate nbs
```

```
    AL<Integer> list = graph.get(rmv);
```

```
    for (int nbr: list) {
```

```
        indegree[nbr] --;
```

```
        if (indegree[nbr] == 0) {
```

```
            q.add(nbr);
```

```
        }
```

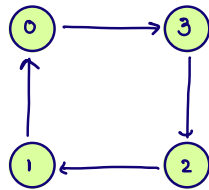
```
    }
```

```
}
```

}

## Cycle Detection in Directed graph

eg-1



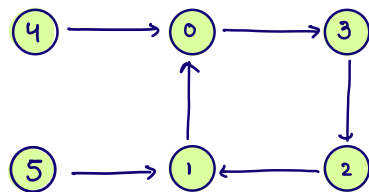
indegree : 

0	1	2	3
1	1	1	1

q: 

--

eg-2



indegree : 

0	1	2	3	4	5
2	2	1	1	0	0

q: 

4	5
---	---

4 5

If topological sort is incomplete (like in above two examples)

this indicates directed graph has cycle.



ques: How to detect cycle in directed graph

```
boolean cycleDetect (AL < AL < Integer >> graph) {
```

```
    int vtx = graph.size();
```

```
    int [] indegree = new int[vtx];
```

```
    // fill indegree array
```

```
    for (int i=0; i<vtx; i++) {
```

```
        AL < Integer > list = graph.get(i);
```

```
        for (int nbr: list) {
```

```
            indegree[nbr]++;
```

```
        }
```

```
    }
```

```
    // applying bfs (with correct starting pts)
```

```
    Queue < Integer > q = new ArrayDeque<>();
```

```
    for (int i=0; i<vtx; i++) {
```

```
        if (indegree[i] == 0) {
```

```
            q.add(i);
```

```
        }
```

```
    }
```

```
    int count = 0;
```

```
    while (q.size() > 0) {
```

```
        int rmv = q.remove();
```

```
        count++;
```

```
        // add appropriate nbs
```

```
        AL < Integer > list = graph.get(rmv);
```

```
        for (int nbr: list) {
```

```
            indegree[nbr]--;
```

```
            if (indegree[nbr] == 0) {
```

```
                q.add(nbr);
```

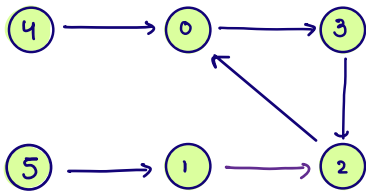
```
            }
```

```
        }
```

```
    }
```

```
    if (count < vtx) { return true; } (cycle is present)
```

```
    else { return false; }
```



indegree :

<del>1</del>	<del>0</del>	<del>1</del>	1	0	0
0	1	2	3	4	5

q :

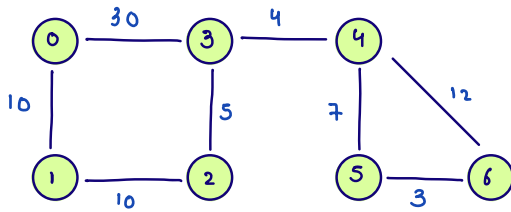
<del>1</del>	<del>2</del>	<del>3</del>
--------------	--------------	--------------

count = ~~1~~ 3

Shortest path (in terms edges)  $\rightarrow$  BFS

Dijkstra

$\hookrightarrow$  single source shortest path (wt wise)

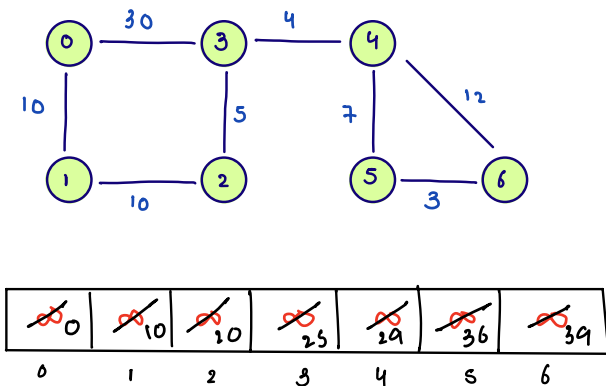


src = 0

Note: Instead of simple queue we will use PQ (minHeap)

0	10	20	25	29	36	39
0	1	2	3	4	5	6

src = 0

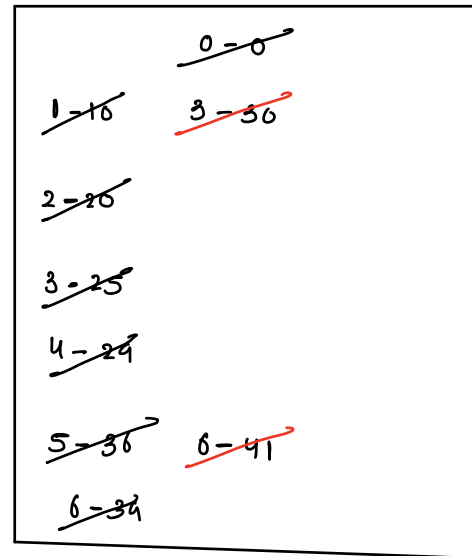


class Helper {

int v;

int wsg; // weight so far

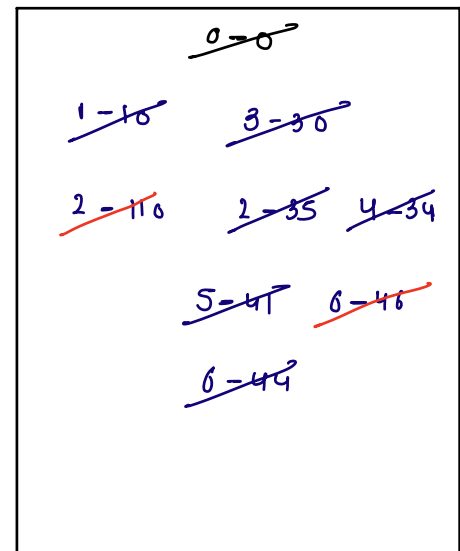
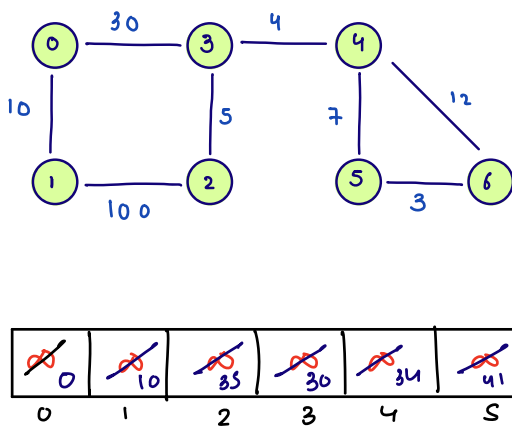
}



PQ (min Heap)

PQ < Helper > PQ;

src = 0



code in JDE

<https://www.interviewbit.com/snippet/26f8b2fde6bbefaa9309/>



Normal BFS (queue)  $\rightarrow$  TC:  $O(V+E)$

Dijkstra (Priority Queue)  $\rightarrow E \log E (V)$

$$[E \approx V^2]$$

$$E \log V^2$$

$$E (2 \log V)$$

$$\approx E \log V (VV)$$



PQ (size)

$\downarrow$   
at max:  $E$   
(single op  
 $\rightarrow \log E$ )

8285640010

$\rightarrow$  whatsapp no.

Slack

## Doubts

```
int vtx = 7;
int[][] edges = {
    {0,3,30},{0,1,10},{1,2,10},{2,3,5},{3,4,4},{4,5,7},{4,6,12},{5,6,3}
};

ArrayList<ArrayList<Pair>> graph = new ArrayList<>();
for(int i=0; i < vtx; i++) {
    graph.add(new ArrayList<>());
}

//fill graph with edges detail
for(int i=0; i < edges.length; i++) {
    int u = edges[i][0];
    int v = edges[i][1];
    int wt = edges[i][2];

    graph.get(u).add(new Pair(v,wt));
    graph.get(v).add(new Pair(u,wt));
}
```

vtx = 7

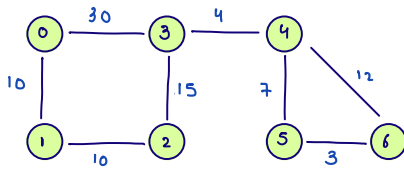
edges

0	3	30
0	1	10
2	3	15
1	2	10
3	4	4
4	5	7
5	6	3
4	6	12

graph:

0	1	2	3	4	5	6
3,30 1,10	0,10 2,10	3,15 1,10	0,30 2,15 4,4	3,4 5,7 6,12	4,7 6,3	5,3 4,12

src = 0



ans:

<del>0</del>	<del>16</del>	<del>20</del>	<del>30</del>	<del>34</del>	<del>41</del>	<del>44</del>
0	1	2	3	4	5	6

```

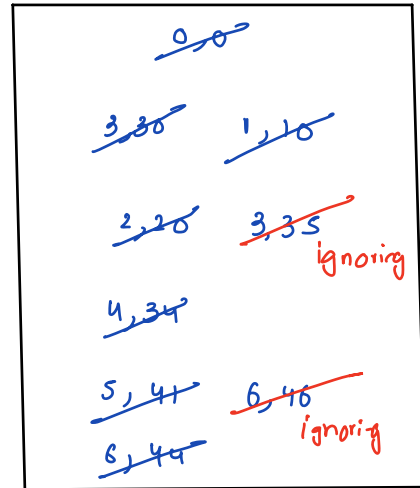
pq.add(new Helper(src,0));

while(pq.size() > 0) {
    Helper rmv = pq.remove();
    int v = rmv.v;
    int wsf = rmv.wsf;

    if(wsf < ans[v]) {
        //work
        ans[v] = wsf;

        //add unvisited nbr
        ArrayList<Pair> list = graph.get(v);
        for(Pair np : list) {
            if(ans[np.v] == Integer.MAX_VALUE) {
                pq.add(new Helper(np.v,wsf + np.wt));
            }
        }
    }
    else {
        //do nothing
    }
}

```



min PQ < Helper >

class Helper {

int v;

int wsf;

}

graph:

0	1	2	3	4	5	6
3,30 1,10	0,10 2,10	3,15 1,10	0,30 2,15 4,4	3,4 5,7 6,12	4,7 6,3	5,3 4,12