BTP: Search Algorithms for E Commerce

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MOTIVATION

To survive in today's market E commerce companies need to cut down their delivery costs while satisfying customer's requirements. Thus finding optimal delivery routes is a major and interesting problem to solve.

Problem Statement

→To identify an efficient routing between a set of storage depots to various customer locations satisfying their respective demands and time constraints.

→The goal is to minimize the total distance from all the depots respecting those constraints.

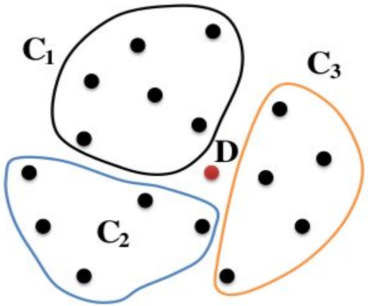
Proposed Algorithm: Beam Search based Approximate Algorithm

Why Beam Search?

- In the real world scenario the set of customers can be very large.
- Optimal Algorithms like best first search have space issues since exploring all nodes is not efficient.
- Beam search solves the issue and finds an approximate optimal solution.
- We also use local search algorithms on top of the solution found by beam search to further improve the solution.

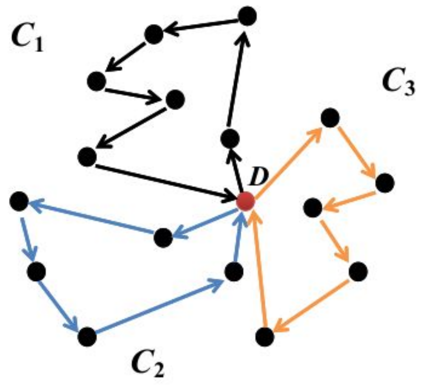
Algorithm Phase 1: K means Clustering

- Divide the set of customers into disjunctive clusters using K means clustering.
- Each cluster will be assigned one delivery vehicle respecting its capacity constraints.



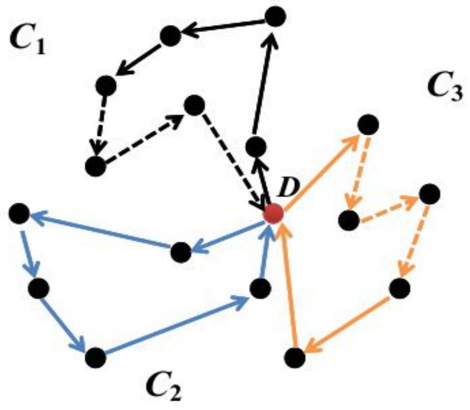
Algorithm Phase 2: Beam Search

- Determine a feasible route for the vehicle in each cluster using Beam search algorithm.
- The time windows of the customers must be satisfied.



Algorithm Phase 3: Local Search

- Apply local search algorithms to further improve the solutions found using beam search. (covered in detail later)



Error: Relaxed time windows

- Solution not found for many cases for strict time windows.
- Introduced Error as a measure of time window violation, which depends on customer priority.
- The algorithm now finds a solution even when the time windows are not satisfied, but with some error.

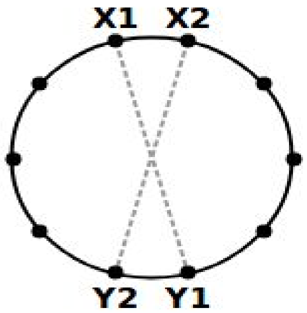
Extension to multiple Depot locations

- Sorted customers based on 3 different heuristics:
  - Earliest time and Latest time and Priority
  - Distance
- Assigned sorted customers to their nearest depot making sure that the total customer demand does not exceed the depot total capacity.

Local Search Analysis

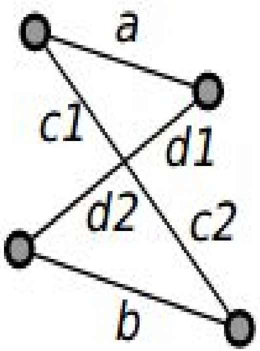
2-OPT Algorithm

- Given any route and 2 different pairs of consecutive nodes in the route, it can be shortened by exchanging 2 links (hence 2-opt)
- If the condition, $d(X1, Y1) + d(X2, Y2) < d(X1, X2) + d(Y1, Y2)$ holds and the time windows are satisfied, then we remove links X1-X2 and Y1-Y2 and replace with X1-Y1 and X2-Y2.



Observation

- Since our distance metric is Euclidean, the tour without intersections would be shorter than the tour with intersections



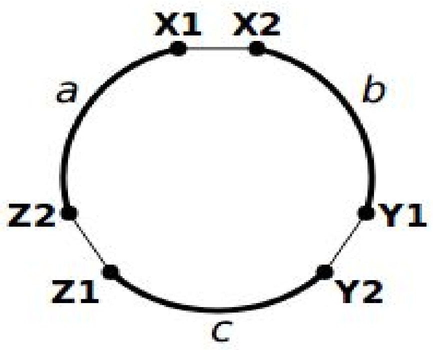
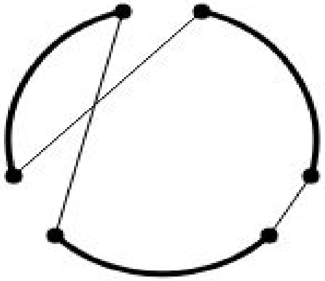
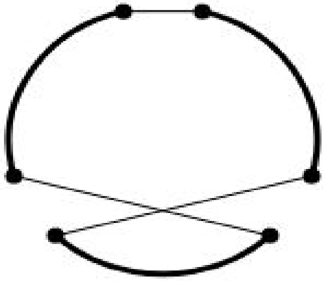
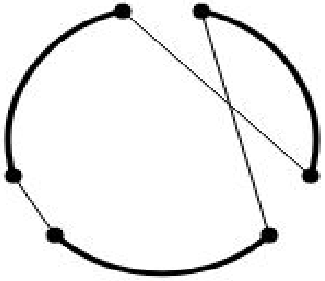
$$a < c_1 + d_1$$

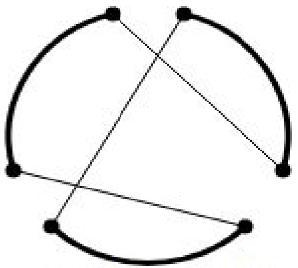
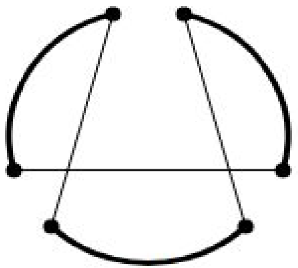
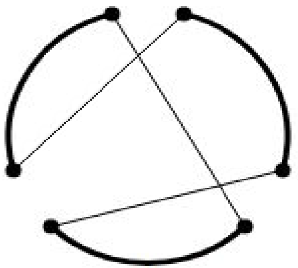
$$b < c_2 + d_2$$

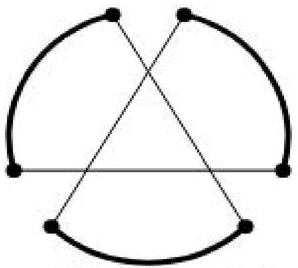
$$a + b < (c_1 + c_2) + (d_1 + d_2)$$

Multiple swaps: 3-OPT

- In 2-OPT move we remove 2 links from cyclic tour, thus obtaining 2 open segments, which we manipulate and combine them to get a new tour.
- In 3-OPT we remove 3 links, obtaining 3 segments to manipulate.
- This gives us 8 combinations including the tour identical with the initial one.

equals original tour		
case = 0		
		
equivalent to a single 2-opt move		
case=1	case=2	case=3
		
a'bc (=ac'b')	*abc'*	*ab'c*

equivalent to two subsequent 2-opt moves		
case=4	case=5	case=6
 ab'c' (=a'cb)	 a'b'c	 a'bc'

equivalent to three subsequent 2-opt moves
case=7
 a'b'c' (=acb)

Why 3-OPT?

- Each 3-OPT move is either identical with 2-OPT move of is equal to a sequence of two or three 2-OPT moves
- It is possible that there exists a sequence of 2-OPT moves that improves the tour but it begins with 2-OPT move that increases the length of the tour.
- This sequence is not achieved when we use 2-OPT only because of that initial “bad” move.

Efficiency Issues

- Any K-OPT(performing K swaps) is better than (K-1)-OPT(performing K-1 swaps)
- But Complexity for K-OPT for finding one single improvement is $O(n^k)$.
- So, can we improve the efficiency of K-OPT.

First Improvement

- If for a given customer X1, we previously did not find any improving move and it still has the same neighbours, then chances that we will find an improvement now is small.
- We use special flag for each of the customers. Initially all the flags are turned off, which means we allow searching.
- If the search for improvement from customer X fails then the bit for that customer is turned on. If a move is performed which involves customer X, then the flag is turned off.
- Now, when we are searching for candidates for customer Y, we skip all customers with their flag turned on.

Second Improvement

- Suppose that for some X1, X2, Y1, Y2 occurs:

$$\begin{cases} d(X2, Y2) \geq d(X1, X2) \\ d(X1, Y1) \geq d(Y1, Y2) \end{cases}$$

- But, for 2-OPT to hold

$$d(X1, Y1) + d(X2, Y2) < d(X1, X2) + d(Y1, Y2)$$

- Therefore, one of the below 2 conditions must hold

$$d(X2, Y2) < d(X1, X2)$$

$$d(X1, Y1) < d(Y1, Y2)$$

Second Improvement

- Let us analyze the first condition

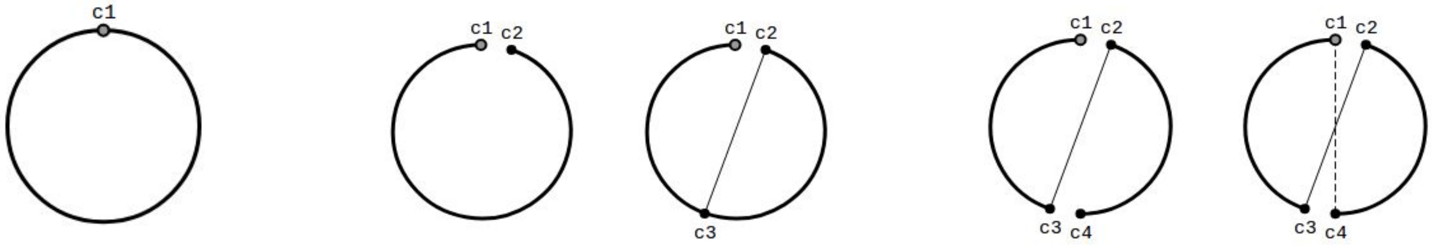
$$d(X2, Y2) < d(X1, X2)$$

- So In 2-opt, for a given vertex X1, considering it's neighbour X2, we can search around X2 for vertices Y2 which are closer than d(X1,X2) (Basically in a fixed radius neighbourhood) and accept the first improving 2-exchange move.

These are small improvements and vary on a case to case basis. Is there a generic improvement which works on any K-OPT algorithm?

Sequential Moves

- Consider a sequence of 2-OPT moves
- Start from certain customer C1 on tour.
- Remove link between C1 and one of its tour neighbours, C2
- Add link between C2 and some other customer C3
- Remove link between C3 and its neighbor C4.
- Add link between C4 and the first customer C1, to close the tour.



Sequential Moves

- Note that steps 1 and 2 have the same scheme: remove a link between a city and it's tour neighbor and then add link between this neighbor and some other city. Each step exchanges two links.
- This can be done similarly for 3-opt as well.
- Basically this is to show that every k-opt swap can be expressed as a sequence of moves where each move is equivalent to removing a link between a city and its tour neighbour and then add a link between this neighbour and some other city.
- So in K-opt should I consider all possible K sequential moves(there are totally $O(n^k)$ moves) or is there a better solution ?

Improving Move Condition

- A move is improving when it is valid and it improves a tour. Any k-opt move that improves a tour must fulfill the condition: Sum of lengths of links removed from tour must be greater than sum of links added to tour.
- In other words sum of links removed from tour must be greater than sum of links added to tour.

Improving Move Condition

Let us take a sequential move of a K-swap solution. Define

g1 = distance(c1, c2) - distance(c2, c3) # gain from step 1
g2 = distance(c3, c4) - distance(c4, c5) # gain from step 2
g3 = distance(c5, c6) - distance(c6, c7) # gain from step 3

And so on $$G_k = \sum_{i=1}^{k} g_i$$

- For solution to be improving $G_k > 0$
- Although some of g1, g2, g3… may be negative, when this *sum* of numbers is positive then the move is an improving move

Observation

- We should note that sequential moves are *cyclic*, we can start from any step of move and apply them one by one, until we make them all.
- So is there a specific order in which we can process these sequential moves instead of checking all possible orders for a given set of sequential moves?
- YES!

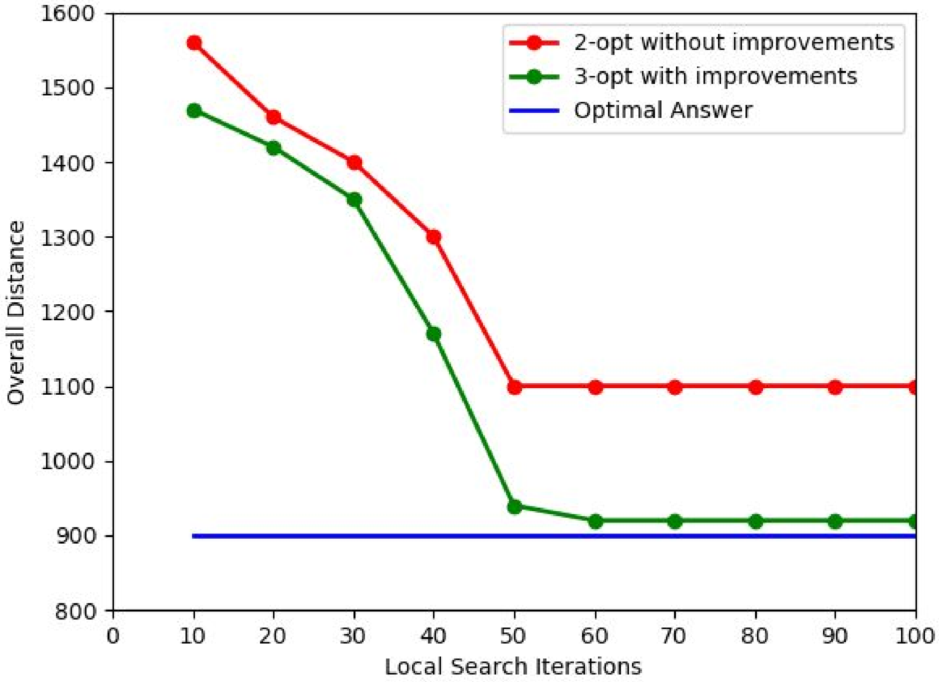
Observation

- Theorem: If a sequence of numbers has a positive sum, there is a cyclic permutation of these numbers such that *every partial sum* is positive.
- In particular, then, since we are looking for sequences of gains g_i's that have positive sum, *we need only consider sequences of gains whose partial sum is always positive*. This gain criterion enables us to reduce enormously the number of sequences we need to examine!
- Thus we found a huge reduction in the number of moves we have to check for a K-opt solution
- Therefore during process of building a sequential move we check partial sum of gains. If this sum remains positive before the last, closing step, then a sequence of exchanges is *promising* and we can continue, even if it is not valid move now.

Results

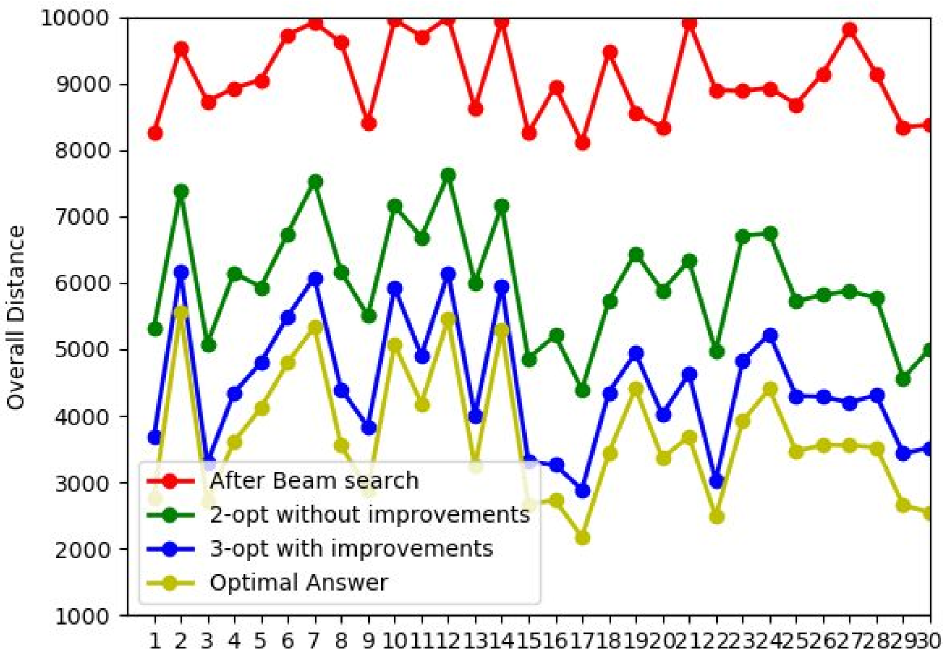
Local Search Improvement

- This is for one depot and 200 customers
- As the iterations increase the local search solution improves but after a point it gets stuck at local minimum
- It's clear that 3-OPT with improvements is better than 2-OPT.



Local Search Improvement

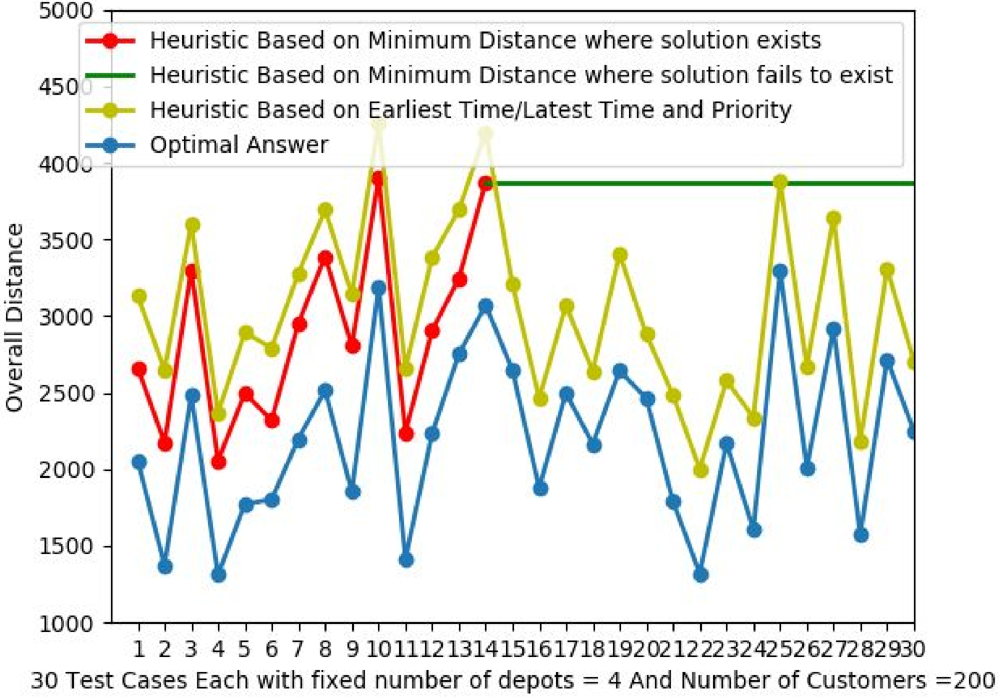
- The gap between beam search solution and 2-OPT solution is much more than the gap between the 3-OPT and 2-OPT solution



This is for a fixed Number of Depots = 1. The number of customers = 200

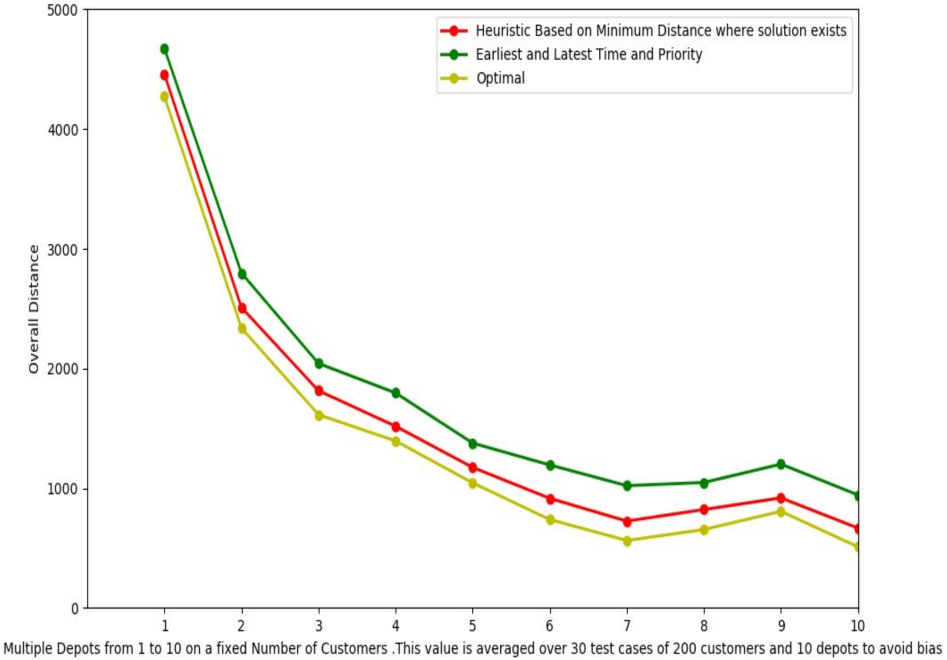
Heuristics Performance Comparison

- In the heuristic criteria of minimum distance we just assigned based on minimum distance.
- Even though it performs better than other heuristics, it fails to find a solution on certain cases highlighted by green



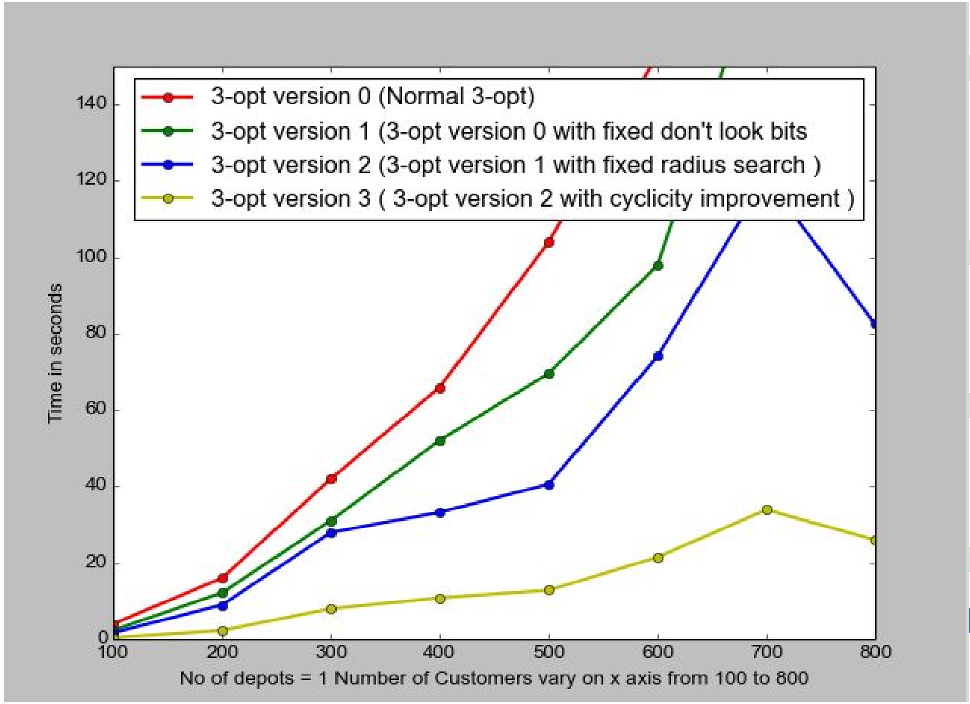
Heuristics Performance Comparison

- On an average the distance reduces by a factor of the number of depots



Local Search Time Performance Comparison

- Since 3-opt is $O(n^3)$ the general observation is that as number of customers double the time factor increase proportionately.
- Also notice that the cyclicity observation decreased the time by a larger amount since for every cyclic permutation we reduced by a factor of 3 if there existed a solution
- Other improvements were minor and did not much effect on the time



Thank You