AI1103-Assignment 5

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Download all latex-tikz codes from

https://github.com/AravindShounik/AI1103/blob/main/Assignment−5/assignment−5.tex

Question GATE 2005 (ME), Q.30

Consider a single server queuing model with Poisson arrivals ($\lambda = 4/hour$) and exponential service ($\mu = 4/hour$). The number in the system is restricted to a maximum of 10. The probability that a person who comes leaves without joining the queue is

Solution

Let P_n represent the probability that there are n people in the queue
The queue is in the notation $(M/M/1):(10/FCFS)$ in Kendall's notation.
Here, we can say that

$$P_{n+1} = \rho P_n \tag{0.0.1}$$

$$\rho = \frac{\lambda}{\mu} \tag{0.0.2}$$

where, ρ is the traffic intensity. which implies

$$P_n = \rho^n P_0 \tag{0.0.3}$$

Here, given $\lambda = 4/hour$ and $\mu = 4/hour$. i.e, traffic intensity is given by

$$\rho = \frac{\lambda}{\mu} = \frac{4}{4} = 1 \tag{0.0.4}$$

Here, the maximum number of people that can stand in the line is 10. So, we can say that

$$\sum_{n=0}^{n=10} P_n = 1 \tag{0.0.5}$$

$$P_0 + P_1 + P_2.... + P_{10} = 1 \tag{0.0.6}$$

$$P_0 + \rho P_0 + \rho^2 P_0... + \rho^1 0 P_0 = 1 \tag{0.0.7}$$

$$P_0(1 + \rho + \rho^2.. + \rho^{10}) = 1 \tag{0.0.8}$$

$$P_0(1 + 1 + ..1) = 1 \tag{0.0.9}$$

$$P_0(11) = 1 \tag{0.0.10}$$

$$P_0 = \frac{1}{11} \tag{0.0.11}$$

Here, the person doesn't stand in queue if he comes after the 10 people i.e, if he is the 11th person

$$P_{11} = \rho^{11} P_0 \tag{0.0.12}$$

$$P_{11} = 1^{11} P_0 \tag{0.0.13}$$

$$P_{11} = \frac{1}{11} \tag{0.0.14}$$

So, the probability that a person who comes and doesn't join the queue is $\frac{1}{11}$