

1. Normal or Gaussian distribution is a continuous probability distribution that has a bell-shaped probability density function (Gaussian function), or informally a bell curve. In a normal distribution the mean is 0 and variance is 1.

$$f(x) = \left[\frac{1}{\sigma \sqrt{2\pi}} \right] \exp \left[-\frac{(x-u)^2}{2\sigma^2} \right]$$

Mean: μ , Variance: σ^2

2. Find the area under the standard normal curve between $z = 0$ and $z = 1.53$?

Using a standard normal distribution table, we can find the area under the curve between $z = 0$ and $z = 1.53$ by looking up the values for $z = 0$ and $z = 1.53$ in the table and subtracting the value for $z = 0$ from the value for $z = 1.53$. The table gives us the area to the left of the z -score, so we need to subtract the area to the left of $z = 0$ from the area to the left of $z = 1.53$.

Looking up $z = 0$ in the table gives us an area of 0.5000, since the standard normal distribution is symmetric around the mean of 0. Looking up $z = 1.53$ in the table gives us an area of 0.9382. Subtracting the area to the left of $z = 0$ from the area to the left of $z = 1.53$

gives us:

$$0.9382 - 0.5000 = 0.4382$$

Therefore, the area under the standard normal curve between $z = 0$ and $z = 1.53$ is approximately 0.4382.

3. Why normal distribution called symmetric?

A normal distribution comes with a perfectly symmetrical shape. This means that the distribution curve can be divided in the middle to produce two equal halves. The symmetric shape occurs when one-half of the observations fall on each side of the curve.

4. $\mu = 1000$, $\sigma = 200$, $N = 2000$, $x = 700$?

$$Z = \frac{(x - \mu)}{\sigma} = \frac{(700 - 1000)}{200} = -1.5$$

$$P(X < 700) = P(Z < -1.5) = 1 - P(Z < 1.5) = 1 - 0.9332 = 0.0668$$

The probability that a bulb will fail in the first 700 burning hours is 6.7% (app.)

5. $\mu = 10$, $x = 7$, $m = 1/10 = 0.1$?

$$P(X > 7) = 1 - P(X \leq 7) = 1 - e^{-mx} = e^{-mx}$$

$$= e^{-(0.1)(7)} = 0.4966$$

The probability that a computer will last more than 7 years is 50% (app.)

6. What is the pdf of Log- Normal distribution?

In probability theory, a log-normal (or lognormal) distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Thus, if the random variable X is log-normally distributed, then $Y = \ln(X)$ has a normal distribution.

7. Suppose that the reaction time in seconds of a person can be modeled by a lognormal distribution with parameter values, mean = -0.35 and sd = 0.2.?

$$\text{mean} = -0.35 \text{ and } \text{sd} = 0.2 \quad \ln X \sim N(-0.35, 0.2^2)$$

a)

$$P(X < 0.6) = P(\ln X < \ln(0.6))$$

$$= P(Z < \ln(0.6) - (-0.35) / 0.2) = P(Z < -0.8)$$

$$= 1 - 0.7881 = 0.2119$$

The probability that the reaction time is less than 0.6 seconds is 21.2 % (app.)

b)

$$P(X < x) = 0.05$$

$$P(\ln X < \ln x) = 0.05$$

$$P(Z < \ln(x) - (-0.35) / 0.2) = 0.05$$

$$(\ln(x) - (-0.35) / 0.2) = Z_{0.05}$$

$$\ln(x) = -1.645 * 0.2 - 0.35$$

$$x = 0.507 \text{ seconds}$$

The reaction time that is exceeded by 95% of the population is 0.507 seconds (app.)

8. $\beta = 1.5$ and a scale parameter $\eta = 100.0$ hours, $x = 25$ hours?

$$P(X < 25) = 1 - e^{-(25/100)^{1.5}}$$

$$= 1 - e^{-0.125} = 0.118$$

the probability that the item fails before achieving a life of 25 hours is 11.8 % (app.)