Week - 3

1) Given,

$P(A) = 0.5$

$P(B) = 0.4$

To find :-

1) $P(A \cup B)$ 2) $P(A'B)$ 3) $P(AB')$ 4) $P(A'B')$

Soln :-

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ — ①

∴ if P(A) and P(B) are independent, then

$\Rightarrow P(A \cap B) = P(A) * P(B)$

$P(A \cap B) = P(0.5) * (0.4)$

$\boxed{P(A \cap B) = 0.2}$

① $\Rightarrow P(A \cup B) = 0.5 + 0.4 - 0.2$

$\boxed{P(A \cup B) = 0.7}$

2) $P(A'B) = P(A) - P(A \cap B)$

$= 0.5 - 0.2$

$\boxed{P(A'B) = 0.3}$

3) $P(AB') = P(B) - P(A \cap B)$

$= 0.4 - 0.2$

$\boxed{P(AB') = 0.2}$

4) $P(A'B') = 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - (0.5 + 0.4 - 0.2) = 1 - 0.7$

$= 0.3$

2) Sample Space :-

* It is a Set that Contains all possible Outcomes of an experiment or a random process. It is the Set of all possible results that a random Variable Can take.

Sample points :-

* The individual outcomes in the Sample Space is reffered as Sample points.

Eg - * Sample Space is {Heads, tails} only possible outcomes.

* Each outcome {Head}, {Tail} is a Sample point.

3) a) 3 attempt to hit the target,

⇒ Sample Space = {(Hit, miss, miss), (Miss, Hit, Miss), (Miss, Miss, Hit)}

b) Less than 6 attempt,

⇒ Sample Space = ~~{(Miss, Miss, Hit)}~~,

{(Hit), (Miss, Hit), (Miss, Miss, Hit), (miss, miss, miss, hit), (miss, miss, miss, miss, hit)}

c) Hits a target :-

⇒ Sample Space = {(Hit)}

d) Never hits the target :-

* If he never hits the target means, its empty set.

⇒ Sample Space = { }

4) ~~i) nCr~~ formula for nCr is

$$\therefore \boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

i) $nC_1 = \frac{n!}{1!\,(n-1)!}$

ii) $nC_0 = \frac{n!}{0!\,(n!)}$

iii) $nC_n = \frac{n!}{n!\,(n-n)!}$

5) Given :-

$nCr = nC(n-r)$

Soln:-

WKT,

$$\boxed{nCr = \frac{n!}{r!\,(n-r)!}} \quad \text{—— (1)}$$

$$\boxed{r = (n-r)}$$

$$nC_{(n-r)} = \frac{n!}{(n-r)!\,(n-(n-r))!} = \frac{n!}{(n-r)!\,(\cancel{n}-\cancel{n}+r)!}$$

$$\boxed{nC_{(n-r)} = \frac{n!}{(n-r)!\,r!}} \quad \text{—— (2)}$$

from ① & ②, we get

$$\boxed{LHS = RHS}$$

$$\boxed{{}^nC_r = {}^nC_{(n-r)}}$$

Hence Proved.

6) Permutation:-

* A permutation is an arrangement of objects in a specific order. The number of permutations of a set of n objects is given by n!

formula $\rightarrow$ $${}^nP_r = \frac{n!}{(n-r)!}$$

Combinations:-

* A Combination is a selection of objects without regard to the order in which they are arranged.

formula $\rightarrow$ $${}^nC_r = \frac{n!}{r!(n-r)!}$$

7) a) 2 red cards and 2 black cards :-

$n = 52$ (cards)

$\therefore$ 4 cards drawn, then

$$\boxed{r = 4}$$

Applying in formula

$\Rightarrow$ a) * Playing cards $= \left(\frac{26}{52}\right) \cdot \left(\frac{25}{51}\right) \cdot \left(\frac{26}{50}\right) \cdot \left(\frac{25}{49}\right)$

$= 0.053$ (or) 5.3%

b) <u>one Card of each Suit :-</u>

$\Rightarrow = \left(\frac{13}{52}\right) \cdot \left(\frac{12}{51}\right) \cdot \left(\frac{13}{50}\right) \cdot \left(\frac{12}{49}\right) = 0.00264$

$= 0.26\%$

c) <u>All cards of the Same Suit :-</u>

* 4 Cards out of 52 is $52C4$
* no. of ways to choose 4 cards of the Same Suit is $4C4$

$\Rightarrow \frac{4C4}{52C4} = \left(\frac{13}{52}\right) \cdot \left(\frac{12}{51}\right) \cdot \left(\frac{11}{50}\right) \cdot \left(\frac{10}{49}\right)$

$= 0.0072\%$ (or) 0.000072

d) <u>one king :-</u>

* There are 4 kings in a deck.

$\Rightarrow \frac{4}{52} = 0.0769$ (or) 7.69% //

$4C1 / 52C4 = \frac{4!}{1(4-1)!}$ $\frac{4}{3}$

1.25 $\quad$ 1.25 $\quad 3\overline{)4}$ $\quad$ 52!

8) Given :-

3 coins tossing = {(H, H, H), (H, H, T), (H, T, H), (T, H, H)
(T, T, T), (T, T, H), (T, H, T), (H, T, T)}

$n(S) = 8$

Soln :-

a) no. of 2 heads, n = 3 {(H, HT) (T, H, H) (H, T, H)}

$$\boxed{P(S) \; \frac{n(E)}{n(S)} = \frac{3}{8}} \; //$$

b) At most 2 heads :-

n = {TTT, HTT, THT, TTH, THH, HTH, HHT}

n = 7

$$\boxed{P(S) = \frac{n(E)}{n(S)} = 7/8} \; //$$

c) Atleast 2 heads :-

n = 4 {(H, H, T), (T H H), (H T H), (H H H)}

$$\boxed{P(S) = \cancel{4}/\cancel{8}_2 = 1/2} \; //$$

d) No head :-

n = 1 {TTT}

$$\boxed{P(S) = \frac{1}{8}} \; //$$

9) Given:-

no. of men's, $n = 4$, $r = 6$

no. of women's, $n = 7$, $r = 10$

| $n=4$, $r=6$
| $n=7$, $r=10$

WKT,

$$nCr = \frac{n!}{r!(n-r)!}$$

$$\cancel{\Rightarrow (C_{4,6})(C_{7,10}) = \left(\frac{4!}{(6!)(4-6)!}\right)\left(\frac{7!}{(10!)(7-10)!}\right)}$$

$$6C_4 = \frac{6!}{4!(6-4)!} = \frac{6!}{4!2!} = \frac{\cancel{6}^{3}\times5\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}\times\cancel{2}}$$

$$\Rightarrow \boxed{6C_4 = 15} \text{ --- } (1)$$

$$10C_7 = \frac{10!}{7!(10-7!)} = \frac{10!}{7!(3!)} = \frac{10\times\cancel{9}^{3}\times\cancel{8}^{4}\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}\times(\cancel{3}\times\cancel{2}\times1)}$$

$$\boxed{10C_7 = 120} \text{ --- } (2)$$

$\Rightarrow$ multiply (1) & (2)

$6C_4 \cdot 10C_7 = 15 \times 120 = \underline{1800 \text{ ways}}$

10) Given:-

gents = 3

ladies = 2

To find

* 2 ladies occupies extreme corner of the space.

Soln:-

no. of peoples = 5 (gents & ladies)

* There are 2 ladies are there, So one can sit at right Corner & another at left Corner.

$n = 5, \ r = 3$

so; $5C_3 = \dfrac{5!}{3!(5-3)!} = 10$

⇒ the probability of 2 ladies occuping extreme corners is $\dfrac{\cancel{2}}{\cancel{10}5} = 1/5$ //

11) Given:-

$P(A) = 0.8$

$P(B) = 0.7$

$P(A \cup B) = 0.3$

Soln:-

* False

* Because $P(A \cup B) = 0.3$ is less than $P(A)$ & $P(B)$.

* So, this is false.

12) Bag - 1 $\Rightarrow \{(4 - \text{maths}, 6 - \text{Stat})\}$, $n = 10$

Bag - 2 $\Rightarrow \{(2 - \text{math}, 4 - \text{Stat})\}$, $n = 6$

Bag - 3 $\Rightarrow \{(1 - \text{Math}, 5 - \text{Stat})\}$, $n = 6$

Soln:-

a) To find probability of bag 1 contains math book is selected :-

$\Rightarrow$ Using Baye's theorem :-

$$\Rightarrow P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

where, A = Bag 1 (P(1))

then,

$$P(1|B) = \frac{P(B|1) \cdot P(1)}{P(B)}$$

Where, P(B|1) is Prob. of finding math book that bag 1 is selected.

then, $P(1|B) = \frac{\cancel{4}\,2}{\cancel{10}\,5}$ $\frac{\text{(math book)}}{\text{(no. of books in bag 1)}}$

$$P(1) = \frac{1}{3}$$

$$P(B) = \left(\frac{4}{10} * \frac{1}{3}\right)$$

then, $P(1|B) = 6/35$ //

b) Selecting 2 books from bag II.

$$nCr = \frac{n!}{r!\,(n-r!)}$$

$n = 6,\ r = 2$

$$6C_2 = \frac{6!}{2!\,(4!)} = \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{2} \times 1 \times \cancel{4 \times 3 \times 2 \times 1}} = 15$$ //

The prob. of Selecting 2 books from bag II is,

$$P(II) = 6C_2 \cdot \left(\frac{2}{12}\right) \cdot \left(\frac{4}{11}\right)$$

$$= \overset{5}{\cancel{15}} \left(\frac{1}{\underset{3}{\cancel{6}}}\right) \left(\frac{\overset{2}{\cancel{4}}}{11}\right)$$

$$\boxed{P(II) = \frac{10}{11}}$$ //