

1) State pmf of Poisson Distribution.

**A. Probability mass function**

A discrete random variable  $X$  is said to have a Poisson distribution, with parameter  $\lambda > 0$ , if it has a probability mass function given by

$$f(k; \lambda) = \Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!},$$

where

- $k$  is the number of occurrences ( )
- $e$  is Euler's number ( )
- $!$  is the factorial function.

2) Let such that  $E(X)=4$  and  $SD(X) = \sqrt{3}$  find  $n$  and  $p$ .

A. mean= $np=4$ ,

Std.dev =  $(npq)^{1/2}$

$$2*(q)^{1/2} = (3)^{1/2}$$

Squaring on both sides

$$4(q)=3, \quad q=1-p$$

$$q=3/4, \text{ then } p=1-3/4=1/4$$

now, we know  $p$

$$np=4$$

$$n*1/4=4$$

$$n=16$$

3) State the additive property of Binomial Distribution

A. Additive property of binomial distribution.

Let  $X$  and  $Y$  be the two independent binomial variables.

$X$  is having the parameters  $n_1$  and  $p$

and

$Y$  is having the parameters  $n_2$  and  $p$ .

Then  $(X + Y)$  will also be a binomial variable with the parameters  $(n_1 + n_2)$  and  $p$

4) State pmf of multinomial Distribution.

A. The probability mass function of this multinomial distribution is:

$$f(x_1, \dots, x_k; n, p_1, \dots, p_k) = \Pr(X_1 = x_1 \text{ and } \dots \text{ and } X_k = x_k) \\ = \begin{cases} \frac{n!}{x_1! \dots x_k!} p_1^{x_1} \times \dots \times p_k^{x_k}, & \text{when } \sum_{i=1}^k x_i = n \\ 0 & \text{otherwise,} \end{cases}$$

for non-negative integers  $x_1, \dots, x_k$ .

The probability mass function can be expressed using the gamma function as:

$$f(x_1, \dots, x_k; p_1, \dots, p_k) = \frac{\Gamma(\sum_i x_i + 1)}{\prod_i \Gamma(x_i + 1)} \prod_{i=1}^k p_i^{x_i}.$$

6) A coin is tossed 12 times. What is the probability of getting exactly 7 heads?

A. pmf =  $nCk p^k q^{n-k}$  where  $n=12$ ,  $p=1/2$ ,  $q=1/2$  and  $k=7$

$$= {}^{12}C_7 (1/2)^7 (1/2)^5$$

$$= \frac{12!}{7!5!} (1/2)^{12}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5!} (1/2)^{12}$$

$$= 0.0483$$

8) The mean number of bacteria per millilitre of a liquid is known to be 6. Find the probability that in 1 ml of the liquid, there will be:

(a) 0,

A. pmf =  $f(k; \lambda) = \Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!},$

Where  $\lambda = 6$

$$P(k=0) = \frac{6^0 e^{-6}}{0!} = \frac{1}{e^6} = 0.002$$

(b) 1,

$$P(k=1) = \frac{6^1 e^{-6}}{1!} = \frac{6}{e^6} = 0.012$$

(c) 2,

$$P(k=2) = \frac{6^2 e^{-6}}{2!} = \frac{18}{e^6} = 0.036$$

(d) 3

A.  $P(k=3) = \frac{6^3 e^{-6}}{3!} = \frac{36}{e^6} = 0.072$

(e) less than 4,

A.  $P(k < 4) = P(k=0) + P(k=1) + P(k=2) + P(k=3)$

$$= 0.002 + 0.012 + 0.036 + 0.072$$

$$= 0.122$$

(f) 6 bacteria.

A.  $P(k=6) = 6^6 \cdot e^{-6} / 6! = 259.2 \cdot e^{-6} = 0.518$

10) State the pmf of negative binomial distribution.

A. The probability mass function of the negative binomial distribution is

where  $r$  is the number of successes,  $k$  is the number of failures, and  $p$  is the probability of success on each trial.

$$f(k; r, p) \equiv \Pr(X = k) = \binom{k+r-1}{k} (1-p)^k p^r$$