
Practice Modelling and Simulation
Assignment Report



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Table of Content

1. State Space Equations.....	03
1.1 State Space Equations of motor and state space from simulation	
1.2 State Space Equations of door lift and state space from simulation	
1.3 State Space Equations of water level and state space from simulation	
2. Transfer Functions.....	04
2.1 Transfer Functions of motor	
2.2 Transfer Functions of door lift	
2.3 Transfer Functions of water level	
3. Model Verification.....	04
3.1. Model Verification with Step Input	
3.2. Model Verification with own Input	
4. Model Validation.....	05
4.1. Model Validation using given test data	
4.1.1 Model Validation using given test data and actual output	
4.1.2 Model Validation using given test data and input from data	
4.2. Model Validation using sensitivity analysis.....	06
4.2.1. Varying the mass of the gate	
4.2.2. Varying the torque constant	
4.2.3. Varying the area of the reservoir	
5. Conclusion	08
6. Bonus Question's answer.....	08

1. State Space Equations

1.1 State Space of Motor:

The Differential equations for dc motor are as follows

Mechanical equation is $\frac{di}{dt} = \frac{1}{L} (V_S - i \cdot R - K_b \cdot \omega)$ and electrical equation is $\frac{d\omega}{dt} = \frac{1}{J} (K_t \cdot i - D \cdot \omega)$

considering V_{in} as the input we derive the state space equation to calculate Tm of the motor, the state space is

$$\frac{d}{dt} \begin{bmatrix} \theta \\ i \end{bmatrix} = \begin{bmatrix} -D/J & K_t/J \\ -K_b/L & -R/L \end{bmatrix} \begin{bmatrix} \theta \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} [vin]$$

$$y(Tm) = [0 \quad K_t] \begin{bmatrix} \theta \\ i \end{bmatrix} + [0][vin]$$

state space obtained from simulation results for dc motor with provided data is

A =	B =	C =	D =
x1 x2	u1	x1 x2	u1
x1 -2530 4940	x1 0	y1 0 2.47	y1 0
x2 -1435 -108.7	x2 4348		

1.2 State Space of door lift:

The differential equation of door lift is

$$\frac{d^2 y_{lift}}{dt^2} = \frac{F_{rope}}{m} - g \quad == \quad \frac{d^2 y_{lift}}{dt^2} = \frac{T_m \cdot N_2}{N_1 \cdot r_p \cdot m} - g \text{ let } dy'' = x'2, x = x1 \text{ thus } x' = x'1$$

This is system has multiple input (2 inputs, Tm and g) and single output y (lift) and its state space is obtained as follows

$$\frac{d}{dt} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ p & -1 \end{bmatrix} \begin{bmatrix} Tm \\ g \end{bmatrix} \quad \text{where, } P = ((N2)/(N1 * r_p * M));$$

$$y(lift) = [1 \quad 0] \begin{bmatrix} x1 \\ x2 \end{bmatrix} + [0] \begin{bmatrix} Tm \\ g \end{bmatrix}$$

state space obtained from simulation results for door lift with provided data is

A =	B =	C =	D =
x1 x2	u1 u2	x1 x2	u1 u2
x1 0 1	x1 0 0	y1 1 0	y1 0 0
x2 0 0	x2 0.007018 -1		

1.3 State Space for water level in reservoir:

The differential equation for water level is

$$A_r \cdot \frac{dL_{water}}{dt} = A_{opening} \cdot v_{water} \quad == \quad \frac{dL_{water}}{dt} = \frac{Y_{lift} \cdot L_m}{L_r \cdot W_r} \cdot v_{water}$$

considering Y_{lift} of the gate as the input we derive the state space equation to calculate $y_{water level}$ of the reservoir.

$$\frac{d}{dt} [x1] = [0][x1] + [Q][ylift] \quad \text{where, } Q = (lm * vw) / (lr * wr);$$

$$y(waterlevel) = [1][x1] + [0][ylift]$$

state space obtained from simulation results for water level with provided data is

A =	B =	C =	D =
x1	u1	x1	u1
x1 0	x1 0.015	y1 1	y1 0

2. Transfer Function

The transfer function of each sub system is obtained by using MATLAB to convert the state space into the transfer function using **ss2tf** and the **tf (num, den)** commands. It is not possible to obtain a single equivalent transfer function of the system since this is MIMO system.

2.1 Transfer Function of motor:

The transfer function obtained from simulation results for motor is

$$\text{Motor_tf} = \frac{1.074e04 s + 2.717e07}{s^2 + 2639 s + 7.363e06}$$

2.2 Transfer Function of door lift:

The transfer functions obtained from simulation results for door lift are

$$\text{Door_tf1} = \frac{0.007018}{s^2} \quad \text{for the first input (Tm) and} \quad \text{Door_tf2} = \frac{-1}{s^2} \quad \text{for the second input (g)}$$

Since this sub system has two inputs two transfer functions are obtained

2.3 Transfer Function of water level:

The transfer function obtained from simulation results for water level is

$$\text{Water_level_tf} = \frac{0.015}{s}$$

3. Model Verification

3.1 Model Verification with given Input

The system has been divided into three subsystems motor, gate lift and water level, and it is modelled and it has been translated from differential equation into a Simulink diagram with other modelling techniques, State space and transfer functions. And it is seen that outputs of differential equations, state space and transfer function are found to be superimposed on each other (Fig 1), thus the system is modelled correctly using different modelling techniques. And it is verified that upon running the system for different scenarios with provided parameters the results are consistent thus model is verified.

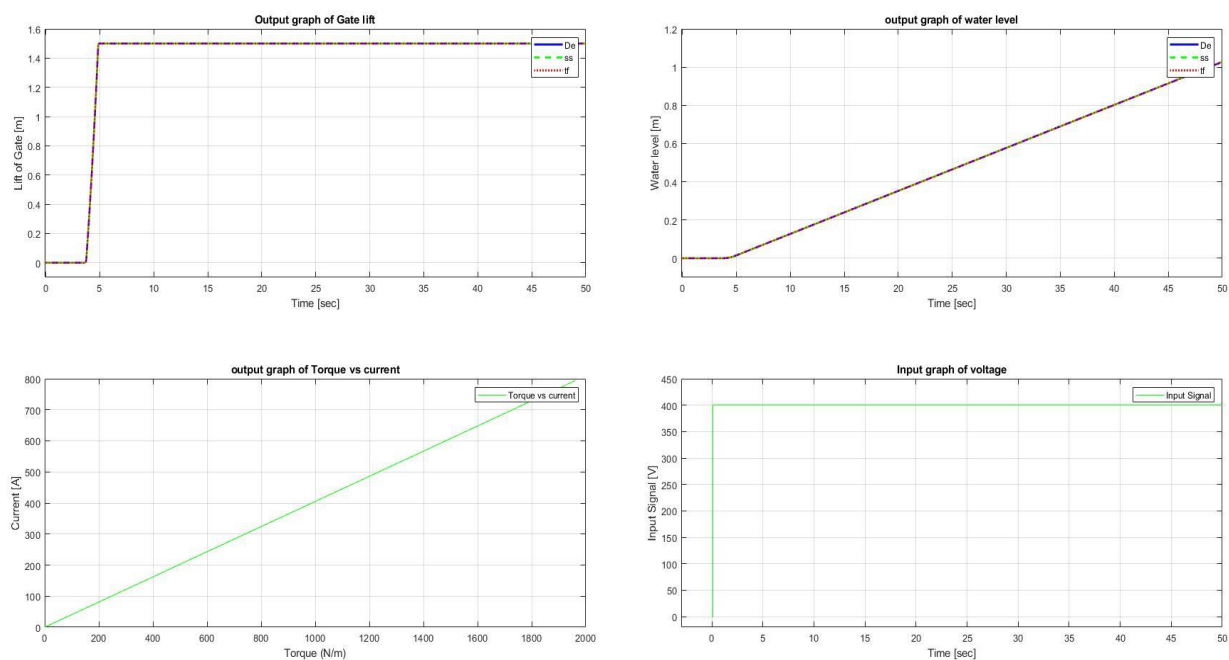


Fig 1 Out graph for the model

3.1 Model Verification with own Input

The same model is given a ramp input with slope of 250 and found that the output produced sensible results. There is decrease in torque hence the lift is delayed and which in turn delays the water level increase. But when the system is tested with higher slopes the torque increases and system response similar to the step input. Thus, the model is verified with different inputs. The torque vs current is commented, motor doesn't generate necessary power to lift in required time which lift the gate after long time (Fig 2).

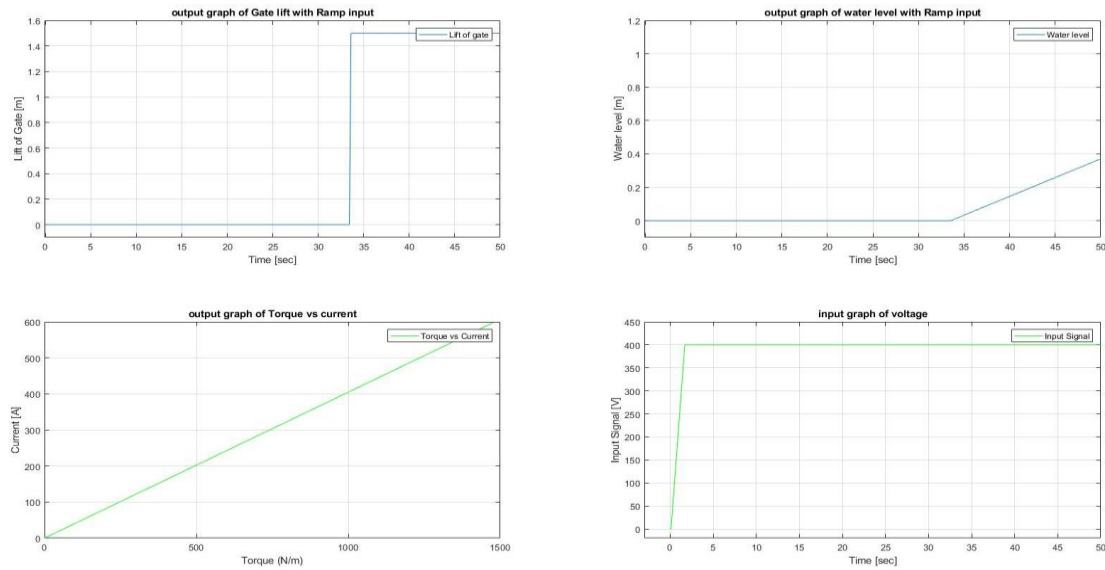


Fig 2 Out graph with Ramp input

4. Model Validation

4.1 Model Validation using given test data

4.1.1 Model Validation using given test data and actual input

The model output is plotted and matched with given test results data for validation and it is found with some deviations which are due to the different step time and sim time from the test results data (Fig 3).

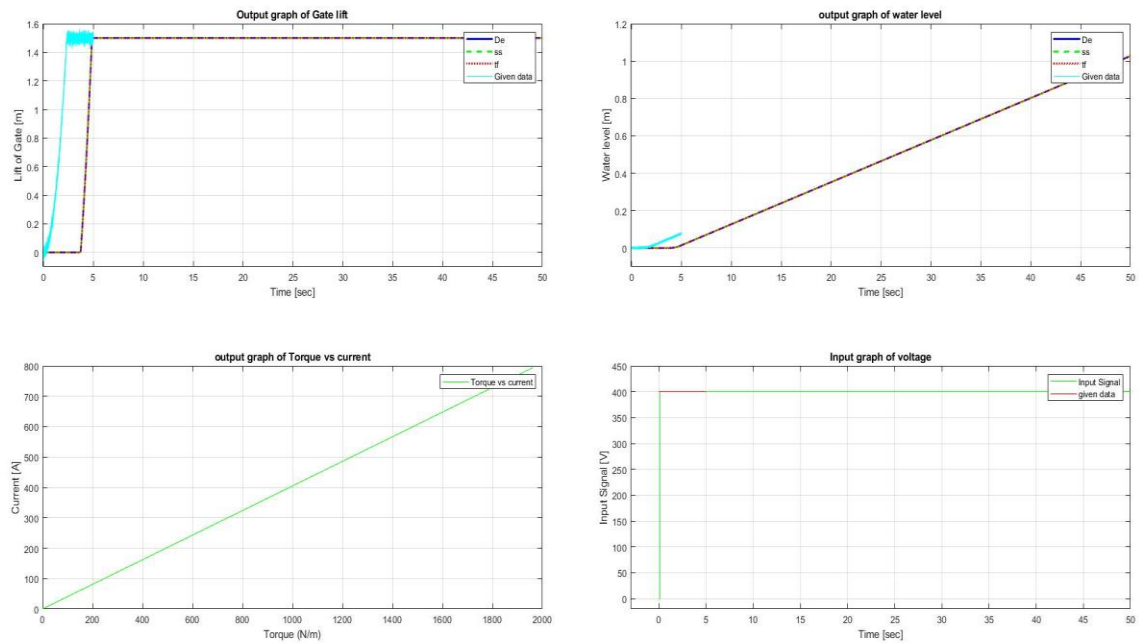


Fig 3 Output graph with Data validation using actual input

4.1.2 Model Validation using given test data and input from data

When the input data from the imported data is used to run the simulation and compared with the model results it is found to be superimposed on each other matching the actual model, hence the model is validated with given test results (Fig 4).

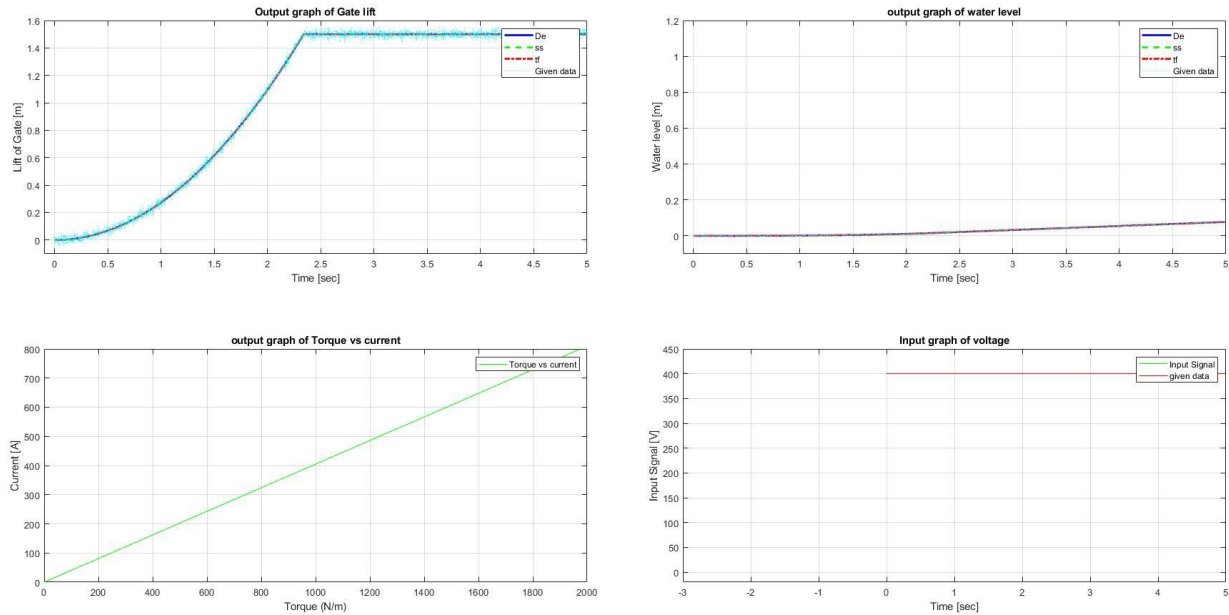


Fig 4 Output graph with Data validation using input from data

4.2 Model Validation using sensitivity analysis

sensitivity analysis is done by 10% increase and decrease of three parameters, i.e., mass, torque constant and area of reservoir on the outputs of the system by varying one parameter at a time and found that the results are consistent with the developed model.

4.2.1 Varying the mass of the gate

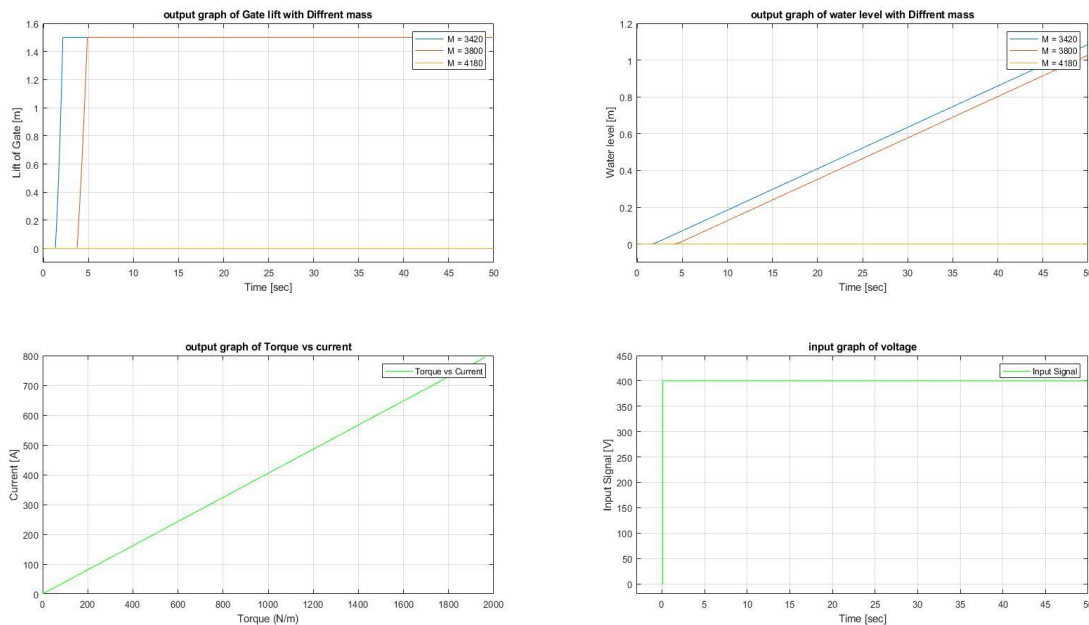


Fig 5 Sensitive analysis by varying mass

When the model is validated with varying mass it is found that the system response is consistent ,for less mass ($m = 3420$) the lift time is less as compared with actual mass ($m = 3800$) which also influences the water level to rise in shorter time and the system is unable to lift the mass ($m = 4180$) hence no rise in water level. comment has been made on output of gate lift since it creates variation in outputs (Fig5)

4.2.2 Varying the torque constant of motor

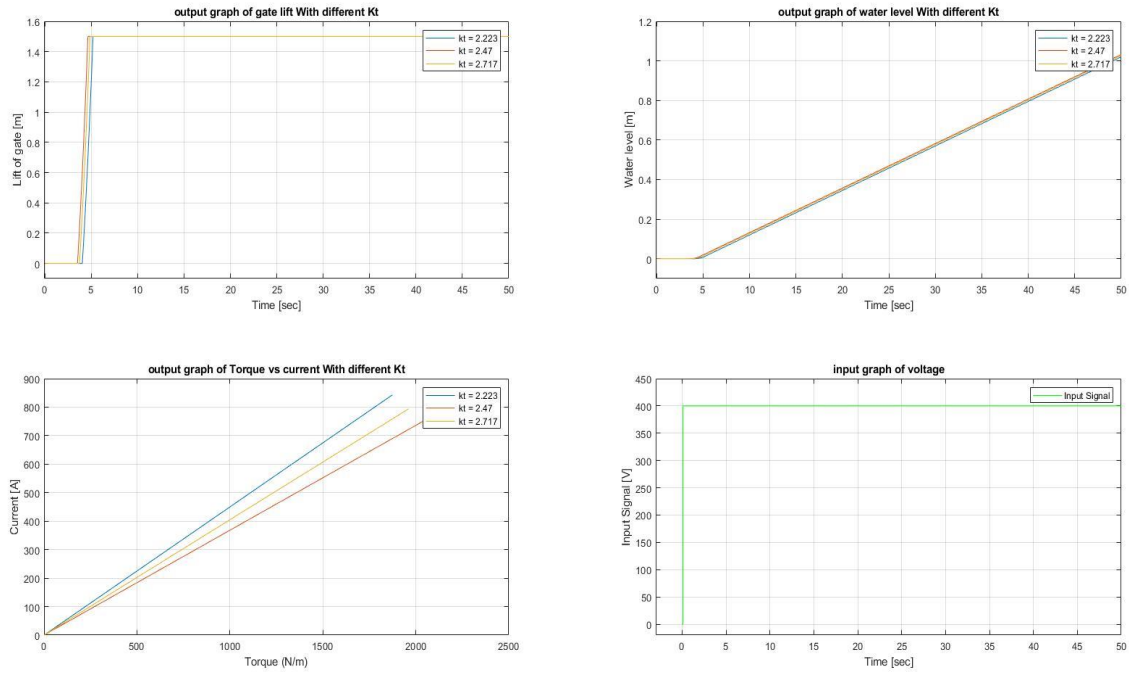


Fig 6 Sensitive analysis by varying Torque constant of motor

When the model is validated by varying torque constant of motor it is found that the system response is consistent and it is inferred that the torque vs current graph varies for varying torque constants, which influences the lift of gate time and water level. comment has been made on torque vs current graph since it creates variation in outputs (Fig 6).

4.2.3 Varying the Area of reservoir

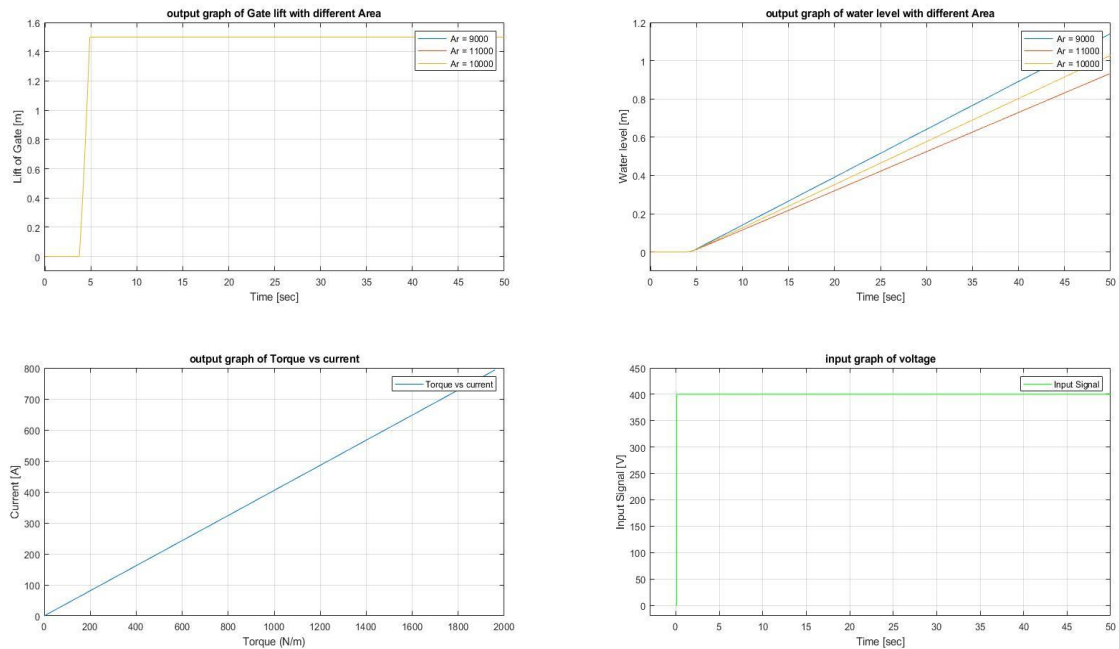


Fig 7 Sensitive analysis by varying Area of reservoir

When the model is validated by varying area of reservoir it is found that the system response is consistent and it is inferred that the water level graph varies for varying area of reservoir, which doesn't influence the lift of gate time and water level. comment has been made on output of water level since it creates variation in outputs (Fig 6).

7. Conclusion

A detailed study has been made on the given problem statement by modeling the system using modeling techniques like differential equations, state space and transfer function in MATLAB and Simulink, from the obtained results after verification and validation of created model, it is concluded that the given DC motor with the given supplied input voltage will be able to lift the dam gate in case of extreme situations.

8. Bonus Question

It is possible to obtain the similar outputs of same system with smaller motor. One is by changing the Gear set for lower torque motor and other way is to increase the output torque with same gear set. Here motor with higher torque is obtained by varying the parameters and tested with the same system.

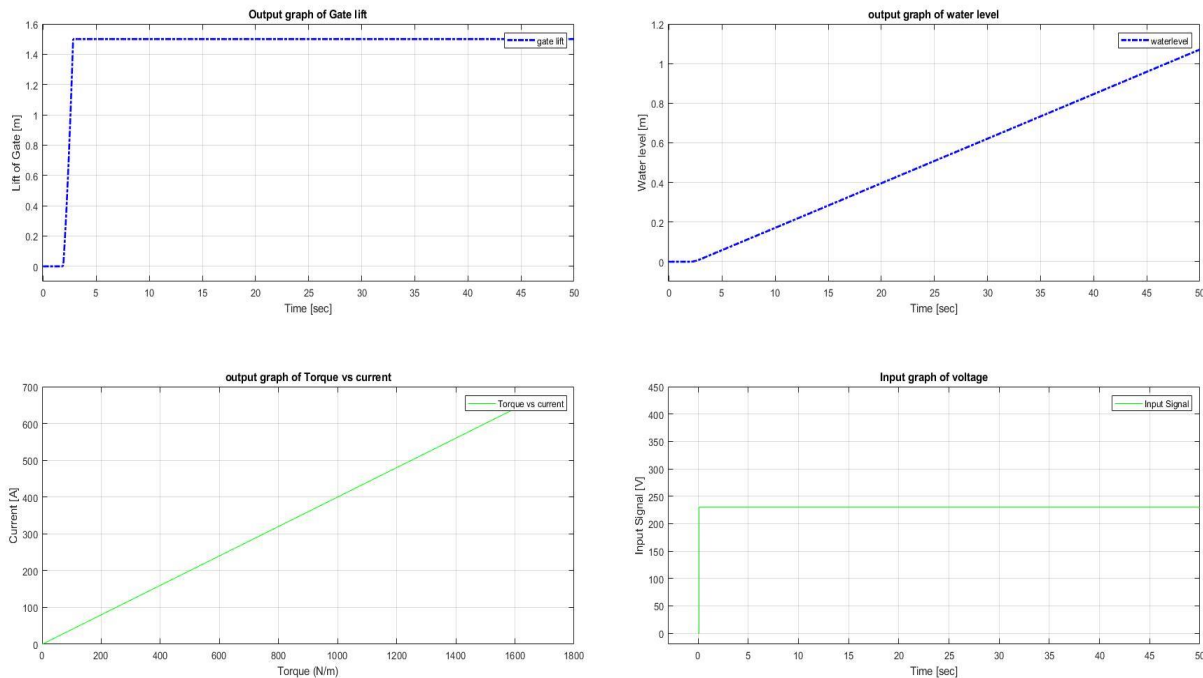


Fig 8 Bonus Question graph

The output graph (Fig 8) show that system produces similar results with lower voltage of 230V, it also states that motor is stable and able to lift the gate by 3 seconds and system is compatible with provided motor voltage.

The proposed model is feasible to make since the model is based only on varying the motor parameters as stated below and in real-time it is feasible to obtain motor with higher torques with low inputs or create one with parameters stated below.

Electrical constants $R = 0.00025$; % Resistance in [N/m] $L = 0.0023$; % Inductance in [H] $K_b = 0.23$; % Back emf constant	Mechanical constants $J = 0.005$; % Inertia $D = 1.565$; % damping coefficient [Ns/m] $K_t = 2.50$; % torque Constant
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The proposed model is able to lift the gate in case of emergencies and it produces results with values near to the actual input of system. From the graphical data's (fig 1 and fig 8) it is observed that the actual model reaches 1.5-meter lift in 5 secs and the proposed model take 3 secs. It is possible to obtain the similar result by varying the motor parameters

Conclusion: Thus, it is possible to downgrade the motors with needed characteristics the system will remain same and a low volt high torque motor will be replaced with the current motor.