

Lab 6b: Luenberger Observer Design for Inverted Pendulum

“The observer, when he seems to himself to be observing a stone, is really, if physics is to be believed, observing the effects of the stone upon himself.” – Bertrand Russell

1 Objectives

The objective of this lab is to design a full-state observer to estimate the state of an inverted pendulum system given just the position of the cart and the pendulum. We will utilize this estimate for full state feedback control of the system.

2 Theory

Pole placement design is performed under the assumption that measurements of all states of the system are available. However, in many physical systems not all states may be easily measurable and thus states need to be estimated based on the limited sensing available. In this case the state feedback becomes $u = -K\hat{x}$, where \hat{x} is the estimated state. We cannot use the controller $u = -Kx$, because the only measurements we have available are y .

Recall from class the dynamics of a Luenberger observer:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \quad (1)$$

where $y = Cx$ and $\hat{y} = C\hat{x}$. The first two terms in the above equation, $A\hat{x} + Bu$, can be called the predictor part and is a replica of the plant dynamics. However, because of uncertainties or errors in the plant model, the estimate of the state using only the predictor (“open-loop”) will generally not match the actual state of the system. The corrective term $L(y - \hat{y})$ is thus needed. Together, these form the Luenberger observer.

The $L(y - \hat{y})$ term corrects future estimates of the state based on the present error in estimation. The gain matrix L can be considered a parameter which weighs the relative importance between the predictor and the corrector in state estimation. Intuitively, a “low” value for L is chosen when our confidence in the model (i.e. the predictor) is high and/or confidence in measurement y is low (i.e. when the measurements are noisy) and vice-versa for a “high” value of L .

The objective of this lab is to design the observer gain matrix L and use the state estimator for feedback control of the inverted-pendulum system instead of our previous derivative-based approximation.

3 Pre-Lab

3.1 Controllability and Observability

Consider the linearized open-loop system from last week’s lab, in state-space form. Check whether the system is controllable and / or observable. You can use the Matlab commands `ctrb`, `obsv` and `rank`.

3.2 Observer Design

Recall our state-feedback control from Lab 6a. Although we only measure the position x and angle θ , we assumed that we have access to the full state, and estimated \dot{x} and $\dot{\theta}$ simply by using derivative blocks in Simulink. As we observed during the last lab, this yields a poor-quality estimate of \dot{x} and $\dot{\theta}$ due to the amplification of noise. As a result, the controller output (actuation of the motor) was of poor quality, and this manifested in particular in a loud grinding noise at high frequencies.

In this lab we will solve these problems by implementing a Luenberger observer, which will provide a state estimate \hat{x} . We use this estimate for state feedback, i.e. $u(t) = K(r(t) - \hat{x}(t))$.

Controller gain The model for inverted-pendulum system and the desired closed-loop poles $s_{1,2} = -1.9 \pm 10j$ and $s_{3,4} = -1.6 \pm 1.3j$ are the same as in the previous lab.

Observer gain The gain L is chosen such that the matrix $A - LC$ has eigenvalues in the left half-plane. Further, the exact position of the eigenvalues of $A - LC$ govern the rate at which the state estimate \hat{x} converges to the actual state x of the system. It is desirable that the observer estimate of the state converges to the actual state much faster than the system dynamics. This helps the controller in obtaining a “good” estimate of the actual state of the system in relatively short time and thus it can take appropriate control action. A general rule of thumb is that the error dynamics should be at least an order of magnitude faster than the dynamics of the controlled system.

1. Given that the size of $A - LC$ must be the same as A , what are the dimensions of L ?
2. For this lab, we want to place the eigenvalues of the observer at $-10 \pm 15j$ and $-12 \pm 17j$. Note that they have been chosen to be relatively far “away” from the desired closed-loop poles. Using MATLAB, find the matrix L such that this is achieved. How would you use the `place` command to do this? *Hint:* For any real square matrix M , the eigenvalues of M are the eigenvalues of its transpose M^T .

3.3 Simulation

1. Implement the designed observer in MATLAB. As usual, there should be *no* derivative blocks used. Remember that the observer is placed in feedback around the actual system. Use the estimate \hat{x} of the state for state feedback. You can use the feedback gain matrix K designed in the previous lab, since the desired locations of the closed-loop poles have not changed.
2. Simulate the system with a 10cm position perturbation and 5 degrees angle perturbation of the plant. You can achieve this by using an initial condition x_0 for the plant. Plot the observer estimate \hat{x} of the state and the actual state of the plant x , which may be obtained from the **plant model in Simulink** (again, do *not* use derivative blocks), on 4 separate plots, one for each state variable. Note that in practice, this cannot be done with the physical plant, as we have no measurement of the actual state x (that’s the whole point of the observer).
3. Plot the estimation error $e = \hat{x} - x$ and discuss how it varies with time.

4 Lab

1. Implement the state feedback controller operating on the state estimate \hat{x} provided by the Luenberger observer on the hardware. For a zero reference signal, observe and record the output \hat{y} of the observer and the actual measurement y when manually applying small perturbations. That is, plot both the estimated and actual signals on the same graph for the position of the cart and the pendulum. The difference between these two signals indicates how well the observer estimates the state of the system.
2. We will now compare the controllers from Lab 6a and Lab 6b. Remember that the closed loop poles of both systems are the same.
For each of the following reference signals, *qualitatively* describe any noticeable differences in performance, and plot the cart position and the angular position of the rod for both controllers on top of each other and compare their tracking abilities.
 - zero reference
 - zero reference with small perturbations (try to be consistent in how you apply the perturbations)
 - sinusoidal reference position with amplitude 5 cm and frequency 1 rad/s, i.e. $r_1(t) = .05 \sin(t)$. The reference velocity, angle, and angular velocity should be set to 0.
3. Now we will look at the differences in performances a little more closely. Compare the estimates of the cart and pendulum velocities from this lab with the measurements obtained by taking the derivatives of the position and angle signals from the previous lab. How do these two schemes differ when a noise is present in the actual measurement of the positions?
4. Which scheme do you think gives the “better” performance, and more importantly, *why*? There is no definite answer here. Just form your own opinion and defend it.