

## Prelab 6b report

### 3 Pre-Lab

#### 3.1 Controllability and Observability

$$\dot{x} = Ax + B u \quad y = Cx$$

$$x = \begin{bmatrix} x \\ \theta \\ \dot{\theta} \end{bmatrix}^T$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -6.81 & -1.50 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 15.47 & 25.66 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1.52 \\ 0 \\ -3.46 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Use MATLAB, I got the controllability matrix of this system,  
1.0e+03 \*

$$\begin{bmatrix} 0 & 0.0015 & -0.0104 & 0.0758 \\ 0.0015 & -0.0104 & 0.0758 & -0.5518 \\ 0 & -0.0035 & 0.0236 & -0.2492 \\ -0.0035 & 0.0236 & -0.2492 & 1.7778 \end{bmatrix}$$

Its rank is 4, so it's controllable.

The observability matrix is

$$\begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0 & -6.8123 & -1.4957 & 0 \\ 0 & 15.4731 & 25.6566 & 0 \\ 0 & 46.4072 & 10.1894 & -1.4957 \\ 0 & -105.4069 & -23.1437 & 25.6566 \end{bmatrix}$$

Its rank is 4, it's full rank. so it's observable.

#### 3.2 Observer Design

1. A is a 4x4 matrix and C is a 2x4 matrix, to make A-LC has the same size as A, L should be a 4x2 matrix, we assume L is  $\begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \\ l_{31} & l_{32} \\ l_{41} & l_{42} \end{bmatrix}$

2. A-LC is

$$\begin{bmatrix} -1.0 \cdot l_{11} & 1.0 & -1.0 \cdot l_{12} & 0 \\ -1.0 \cdot l_{21} & -6.8123 & -1.0 \cdot l_{22} - 1.4957 & 0 \\ -1.0 \cdot l_{31} & 0 & -1.0 \cdot l_{32} & 1.0 \\ -1.0 \cdot l_{41} & 15.473 & 25.657 - 1.0 \cdot l_{42} & 0 \end{bmatrix}$$

we can just use the place command in MATLAB, the eigenvalues of (A-LC) is the same as  $(A^T - C^T L^T)$ ,

$$A^T = A^T;$$

$$C^T = C^T;$$

$$L^T = L^T;$$

$$P = [-10+15i; -10-15i; -12+17i; -12-17i];$$

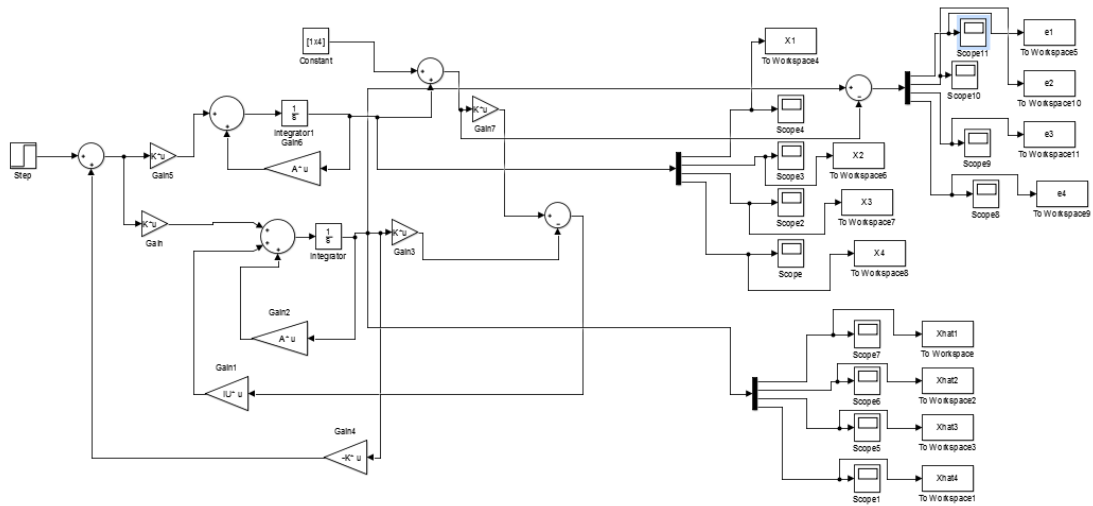
$L T = \text{place}(A T, C T, P)$

Get L=

16.1595	-2.3767
254.9780	-5.4818
15.7136	21.0282
180.7734	378.1934

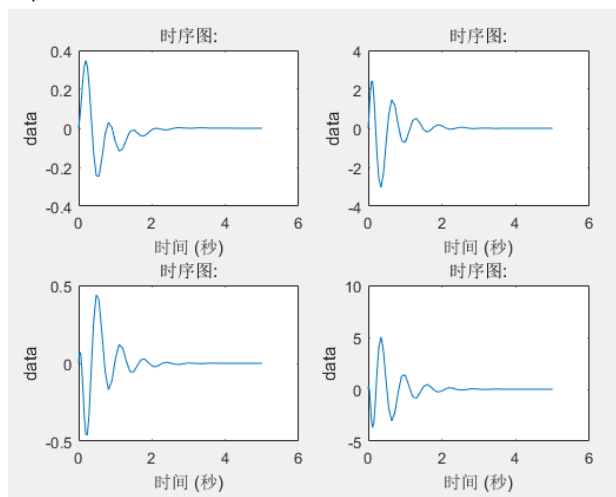
### 3.3 Simulation

#### 1.The Simulation block

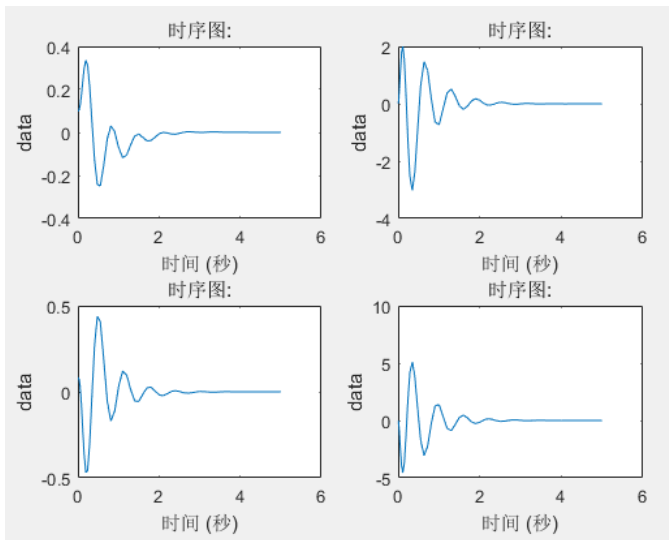


#### 2.4 plots of $x^{\wedge}$ and 4 plots of $x$

4 plots for  $X^{\wedge}$ :



4 plots for  $X$



3.

4 plots for error

