Prelab report 6a

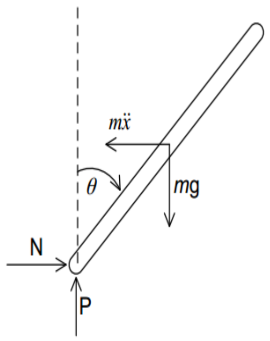
Yujian An

3.2 Equations of motion of the Mechanical system

Under the small-angle approximation $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, derive the equations of motion (1) and (2) of the inverted pendulum-cart system. In (1), F_a is the force exerted on the cart by the motor.

$$(M+m)\ddot{x} + mL_p\ddot{\theta} = F_a \tag{1}$$

$$mL_p\ddot{x} + \frac{4mL_p^2}{3}\ddot{\theta} - mgL_p\theta = 0 \tag{2}$$



From Newton's second law. We have F= am

In the question

Assume acceleration=a and angular acceleration=α

Fa= (M+m)*a+m*α*Lp

And we have a $=\ddot{x}$ α$=\ddot{\theta}$

Then we have Fa=(m+M)*$\ddot{x}$+m*$\ddot{\theta}$*Lp, which is (1)

Then calculate the moment of the stick, from the left figure, we have Lp*(m$\ddot{x}$cosθ-mgsinθ)+I*α=0, where α is the angular acceleration and I is the moment of inertia.

I$=\int_0^L r^2 dr=\frac{1}{3}$ mL^2$=\frac{4}{3}$ mLp^2. So we have Lp*(m$\ddot{x}$cosθ-mgsinθ)+ $\frac{4}{3}$ mLp^2.$\ddot{\theta}$=0

Use linearization cosθ=1 and sinθ=θ, we have Lp*(m$\ddot{x}$-mgθ)+ $\frac{4}{3}$ mLp^2.$\ddot{\theta}$=0, which is (2)

3.2 Full System Dynamics of Linearized System

Fa=(m+M)*$\ddot{x}$+m*$\ddot{\theta}$*Lp (1)

Lp*(m$\ddot{x}$-mgθ)+ $\frac{4}{3}$ mLp^2.$\ddot{\theta}$=0 (2)

$$Fa=\frac{Kg*Kt}{Rm*r}V-\frac{Km*Kg^2*Kt}{Rm*r^2}\dot{x}-\frac{Kg^2*Jm}{r^2}\ddot{x} \qquad (3)$$

From (1) and (2), we have $(M+m+\frac{Kg^2*Jm}{r^2}-\frac{3m}{4})\ddot{x}+mLp\ddot{\theta}=\frac{Kg*Kt}{Rm*r}V-\frac{Km*Kg^2*Kt}{Rm*r^2}\dot{x}$ (4)

From (3) we have $\ddot{\theta}=\frac{3g}{4Lp}\theta-\frac{3}{4Lp}\ddot{x}$ (5)

From (4) and (5) we have

$$\ddot{x}=-\frac{3g}{4\,(M+m+\frac{Kg^2*Jm}{r^2}-\frac{3m}{4})}\theta-\frac{Km*Kg^2*Kt}{Rm*r^2(M+m+\frac{Kg^2*Jm}{r^2}-\frac{3m}{4})}\dot{x}+\frac{Kg*Kt}{Rm*r\,(M+m+\frac{Kg^2*Jm}{r^2}-\frac{3m}{4})}V$$

Then $\ddot{\theta}=(\frac{3g}{4Lp}+\frac{9}{16Lp\,(M+m+\frac{Kg^2*Jm}{r^2}-\frac{3m}{4})})\theta+\frac{3Km*Kg^2*Kt}{4LpRm*r^2(M+m+\frac{Kg^2*Jm}{r^2}-\frac{3m}{4})}\dot{x}$ -

$$\frac{3Kg*Kt}{4LpRm*r\left(M+m+\frac{Kg^2*Jm}{r^2}-\frac{3m}{4}\right)}V$$

$$A = \begin{bmatrix} 0 & a_{12} & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 \\ 0 & 0 & 0 & a_{34} \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b4 \end{bmatrix}$$

So for matrix A,

We have a12=1, a22= $-\frac{4Km*Kg^2*Kt}{Rm*(4Mr^2+4mr^2+4Kg^2*Jm)}$, a23= $-\frac{3mr^2g}{4Mr^2+mr^2+4Kg^2*Jm}$

b2= $\frac{4r*Kg*Kt}{Rm*(4Mr^2+mr^2+4Kg^2*Jm)}$, a34 =1, a42= $\frac{3Km*Kg^2*Kt}{LpRm\,(4Mr^2+mr^2+4Kg^2*Jm)}$,

a43 = $\frac{3g[Kg^2*Jm+r^2(M+m)]}{Lp(4Mr^2+mr^2+4Kg^2*Jm)}$, b4 = $-\frac{3r*Kg*Kt}{LpRm\,(4Mr^2+m\,r^2+Kg^2*Jm)}$

A System Parameters

Parameter	Value	Description
	439.6 counts/cm	Resolution of the cart position encoder
	651.9 counts/rad	Resolution of the angle encoder
M	0.94 kg	Mass of cart and motor
m	0.230 kg	Mass of pendulum
L_p	0.3302 m	Pendulum distance from pivot to center of mass
I_c	$m\,L_p^2/3$	Moment of inertia of pendulum about its center
I_e	$4m\,L_p^2/3$	Moment of inertia of pendulum about its end
K_t	$7.67 \cdot 10^{-3}$ Nm/A	Motor torque constant
K_m	$7.67 \cdot 10^{-3}$ Vs/rad	Motor back EMF constant
K_g	3.71	Motor gearbox ratio
R_m	2.6 Ω	Motor winding resistance
r	$6.36 \cdot 10^{-3}$ m	Radius of motor gear
J_m	$3.9 \cdot 10^{-7}$ kg m^2	Motor moment of inertia

Table 1: Parameters of the inverted pendulum setup

Put the values in ,we get A=[0 1 0 0; 0 -6.81 -1.50 0; 0 0 0 1;0 15.47 25.66 0]

B=[0 ; 1.52 ; 0 ; -3.46]

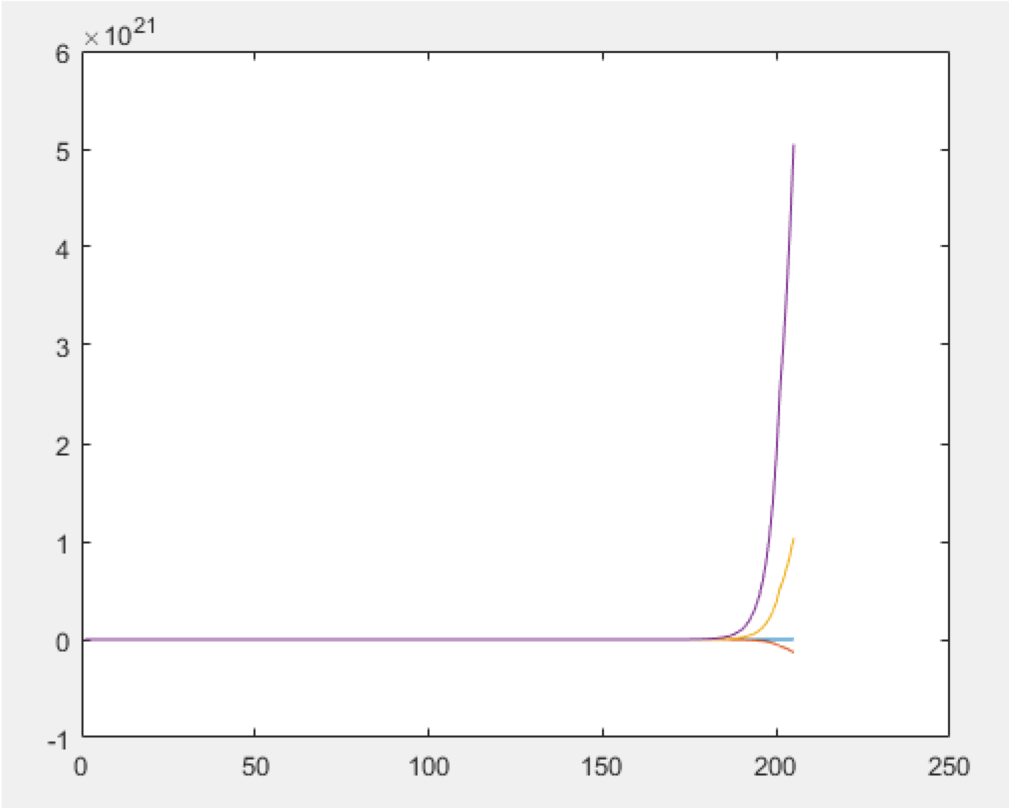
3.3 Analysis and Controller Design

1. Use MATLAB, we get 4 eigenvalues 0 -9.0525 -3.1232 5.3634, we have one pole on the RHP, so it's not internally stable, it's not internally stable, so it can't be BIBO stable.

2. simulation

```
F = @(t,x) [x(1);
            1.52 - 1.50*x(3) - 6.81*x(2);
            x(4);
            15.47*x(2) + 25.66*x(3) - 3.46];
tSpan = [0 10];
initialCondition = [pi/2;0;pi/2;0];
[tSol,xSol] = ode45(F, tSpan, initialCondition);
plot(xSol)
```

plot:



That means in the simulation , the stick will keep rotating as the car goes forward, but actually in the real world's physical system, we expect the stick become stable at a specific angle. I think that's because we have things like friction in real world's physical system, so the stick will lose energy in the process of rotating and finally becomes stable like a tail of the running car, but we don't have interference like friction in simulation, so the stick can't be stable and will keep rotating.

3. (a) Ak=[0 1 0 0 ;

-1.52k1 -6.81-1.52k2 -1.50-1.52k3 -1.52k4 ;

0 0 0 1 ;

-3.46k1 15.47-3.46k2 25.66-3.46k3 -3.46k4]

(b) use MATLAB, I get P(k,s)= det(sI-A) to be s^4+(6.81-3.46k4+1.52k2)s^3+(1.52k1-3.46k3-25.66)s^2+(-155-33.93k2)s-33.93k1

(c) Pdes(s)= (s-s1)(s-s2)(s-s3)(s-s4)=s^4+7s^3+120.02s^2+347.7s+440.34

(d) use the P(k,s)=Pdes(s), I get k1=-12.98 k2=-14.72 k3=-47.85 k4=-6.54

Use the command "place", k1=-12.9795 k2=-14.7230 k3=-47.8456 k4=-6.5363

4. $\dot{x}$=Akx+BKr, use Laplace transform we get sx(s)=AkX(s)+BKR(s)

The input of system is x, so we only use the first line, so C is [1 0 0 0]

Code:

```
 AK  =  A  -  B*K;
BK  =  B*K;
C  =  [1  0  0  0];
D  =  zeros(1,4);
[num,den]  =  ss2tf(AK,BK,C,D,1);
SYS  =  tf(num,den);
figure(2)
bode(SYS)
```