ПРАКТИЧЕСКОЕ ЗАНЯТИЕ № 1

Тема: Непосредственное интегрирование. Замена переменной в неопределенном интеграле.

1.1. Найти $\int\left(2x^3-2\sqrt[3]{x^2}+3\right)dx$.

► $\int\left(2x^3-2\sqrt[3]{x^2}+3\right)dx=2\int x^3dx-2\int x^{2/3}dx+3\int dx=\frac{2x^4}{4}-\frac{2x^{5/3}}{5/3}+3x+c=$

$=\frac{x^4}{2}-\frac{6}{5}\sqrt[3]{x^5}+3x+c$. (См. ф. 1).

1.2. Найти $\int\left(\frac{5}{x}+\frac{2}{x^5}-\frac{3}{\sqrt[4]{x}}\right)dx$.

► $\int\left(\frac{5}{x}+\frac{2}{x^5}-\frac{3}{\sqrt[4]{x}}\right)dx=5\int\frac{dx}{x}+2\int x^{-5}dx-3\int x^{-1/4}dx=5\ln|x|+2\frac{x^{-4}}{-4}-3\frac{x^{3/4}}{3/4}+c=$

$=5\ln|x|-\frac{1}{2x^4}-\frac{1}{4}\sqrt[4]{x^3}+c$. (См. ф. 13; 1).

1.3. Найти $\int\frac{x^2-2x+3}{x\sqrt{x}}dx$.

► $\int\frac{x^2-2x+3}{x\sqrt{x}}dx=\int\frac{x^2dx}{x\sqrt{x}}-2\int\frac{x\,dx}{x\sqrt{x}}+3\int\frac{dx}{x\sqrt{x}}=\int x^{1/2}dx-2\int\frac{dx}{\sqrt{x}}+3\int x^{-3/2}dx=$

$=\frac{2x^{3/2}}{3}-4\sqrt{x}+3\frac{x^{-1/2}}{-1/2}+c=\frac{2}{3}\sqrt{x^3}-4\sqrt{x}-\frac{6}{\sqrt{x}}+c$. (См. ф. 1; 16).

1.4. Найти $\int 2^x\cdot 7^x dx$.

► $\int 2^x\cdot 7^x dx=\int 14^x dx=\frac{14^x}{\ln 14}+c$. (См. ф. 3).

1.5. Найти $\int\frac{\sqrt{x^2+3}-\sqrt{9-x^2}}{\sqrt{x^2+3}\cdot\sqrt{9-x^2}}dx$.

► $\int\frac{\sqrt{x^2+3}-\sqrt{9-x^2}}{\sqrt{x^2+3}\cdot\sqrt{9-x^2}}dx=\int\left(\frac{\sqrt{x^2+3}}{\sqrt{x^2+3}\cdot\sqrt{9-x^2}}-\frac{\sqrt{9-x^2}}{\sqrt{x^2+3}\cdot\sqrt{9-x^2}}\right)dx=$

$\int\frac{dx}{\sqrt{9-x^2}}-\int\frac{dx}{\sqrt{x^2+3}}=\arcsin\frac{x}{3}-\ln\left|x+\sqrt{x^2+3}\right|+c$. (См. ф. 17; 18).

1.6. Найти $\int \cos x\left(2-\frac{3}{\cos^3 x}\right)dx$.

► $\int \cos x\left(2-\frac{3}{\cos^3 x}\right)dx = 2\int \cos x\,dx - 3\int \frac{dx}{\cos^2 x} = 2\sin x - 3\operatorname{tg} x + c$. (См. ф. 6; 10).

1.7. Найти $\int \frac{\ln^4 x\,dx}{x}$.

► $\int \frac{\ln^4 x\,dx}{x} = \left\{\begin{matrix} u=\ln x \\ u'=\frac{1}{x}\end{matrix}\right\} = \int u^4\cdot u'dx = \frac{u^5}{5}+c = \frac{\ln^5 x}{5}+c$. (См. ф. 1).

1.8. Найти $\int \frac{dx}{\left(1+x^2\right)\cdot\sqrt[3]{\operatorname{arctg} x}}$.

► $\int \frac{dx}{\left(1+x^2\right)\cdot\sqrt[3]{\operatorname{arctg} x}} = \left\{\begin{matrix} u=\operatorname{arctg} x \\ u'=\frac{1}{1+x^2}\end{matrix}\right\} = \int \frac{u'dx}{\sqrt[3]{u}} = \int u^{-1/3}u'dx = \frac{\operatorname{arctg}^{2/3} x}{2/3}+c =$

$= \frac{3}{2}\sqrt[3]{\operatorname{arctg}^2 x}+c$. (См. ф. 1).

1.9. Найти $\int \frac{x+2}{\sqrt{x^2+4x+5}}dx$.

► $\int \frac{x+2}{\sqrt{x^2+4x+5}}dx = \left\{\begin{matrix} u=x^2+4x+5 \\ u'=2x+4=2(x+2)\end{matrix}\right\} = \frac{1}{2}\int \frac{u'dx}{\sqrt{u}} = \sqrt{x^2+4x+5}+c$.

(См. ф. 16).

1.10. Найти $\int \frac{\cos 2x\,dx}{\sqrt{\sin^2 2x+3}}$.

► $\int \frac{\cos 2x\,dx}{\sqrt{\sin^2 2x+3}} = \left\{\begin{matrix} u=\sin 2x \\ u'=\cos 2x\cdot 2\end{matrix}\right\} = \int \frac{u'dx}{2\sqrt{u^2+3}} =$

$= \frac{1}{2}\ln\left|\sin 2x+\sqrt{\sin^2 2x+3}\right|+c$. (См. ф. 18).

1.11. Найти $\int \frac{2-3x}{x^2+2}dx$.

► $\int \frac{2-3x}{x^2+2}dx = \int\left(\frac{2}{x^2+2}-\frac{3x}{x^2+2}\right)dx = 2\int \frac{dx}{x^2+2} - 3\int \frac{x\,dx}{x^2+2} =$

$= \frac{2}{\sqrt{2}}\operatorname{arctg}\frac{x}{\sqrt{2}} - \frac{3}{2}\ln\left|x^2+2\right|+c$. (См. ф. 14; 13).

1.12. Найти $\int \frac{x\,dx}{x^4-25}$.

► $\int \frac{x\,dx}{x^4-25} = \left\{\begin{matrix} u = x^2 \\ u' = 2x \end{matrix}\right\} = \int \frac{u'dx}{2\left(u^2-25\right)} = \frac{1}{2}\cdot\frac{1}{10}\ln\left|\frac{x^2-5}{x^2+5}\right| + c =$

$= \frac{1}{20}\ln\left|\frac{x^2-5}{x^2+5}\right| + c$. (См. ф. 15).

1.13. Найти $\int e^{\cos 5x}\cdot\sin 5x\,dx$.

► $\int e^{\cos 5x}\cdot\sin 5x\,dx = \left\{\begin{matrix} u = \cos 5x \\ u' = -5\sin 5x \end{matrix}\right\} = \int -e^u\cdot\frac{u'}{5}dx = -\frac{1}{5}e^{\cos 5x} + c$. (См. ф. 4).

1.14. Найти $\int \sin\left(3x-1\right)dx$.

► $\int \sin\left(3x-1\right)dx = \left\{\begin{matrix} u = 3x-1 \\ u' = 3 \end{matrix}\right\} = \frac{1}{3}\int \sin u\cdot u'dx = -\frac{1}{3}\cos\left(3x-1\right) + c$. (См. ф. 5).

1.15. Найти $\int \frac{dx}{\cos^2\frac{x}{3}}$.

► $\int \frac{dx}{\cos^2\frac{x}{3}} = \left\{\begin{matrix} u = \frac{x}{3} \\ u' = \frac{1}{3} \end{matrix}\right\} = 3\int \frac{u'dx}{\cos^2 u} = 3\,\mathrm{tg}\,\frac{x}{3} + c$. (См. ф. 10).

1.16. Найти $\int e^{-2x}\left(1+\frac{e^{2x}}{\sin x}\right)dx$.

► $\int e^{-2x}\left(1+\frac{e^{2x}}{\sin x}\right)dx = \int\left(e^{-2x}+\frac{1}{\sin x}\right)dx = \int e^{-2x}dx + \int\frac{dx}{\sin x} =$

$= -\frac{1}{2}e^{-2x} + \ln\left|\,\mathrm{tg}\,\frac{x}{2}\right| + c$. (См. ф. 4; 11).

САМОСТОЯТЕЛЬНАЯ РАБОТА

Найти неопределенные интегралы.

1.17. $\int \frac{2x^3+3\sqrt{x}-1}{2x}dx$. Ответ: $\frac{x^2}{2}+3\sqrt{x}-\frac{1}{2}\ln|x| + c$.

1.18. $\int\left(5x^7-\frac{3}{\sqrt[5]{x^3}}+\frac{3}{x^4}\right)dx$. Ответ: $\frac{5}{8}x^8-\frac{15}{2}\sqrt[5]{x^2}-\frac{1}{x^3}+c$.

1.19. $\int\frac{x\,dx}{3x^2+4}$. Ответ: $\frac{1}{6}\ln\left|3x^2+4\right|+c$.

1.20. $\int\sqrt[7]{(5x+3)^3}\,dx$. Ответ: $\frac{7}{50}\sqrt[7]{(5x+3)^{10}}+c$.

1.21. $\int\cos^4 2x\cdot\sin 2x\,dx$. Ответ: $-\frac{1}{10}\cos^5 2x+c$.

1.22. $\int\left(e^{-3x}+3^{2x}\right)dx$. Ответ: $-\frac{1}{3}e^{-3x}+\frac{1}{2\ln 3}\cdot 3^{2x}+c$.

1.23. $\int\left(\sin 7x+\frac{1}{\cos^2 4x}-e^{3-2x}\right)dx$. Ответ: $-\frac{1}{7}\cos 7x+\frac{1}{4}\operatorname{tg}4x+\frac{1}{2}e^{3-2x}+c$.

1.24. $\int\frac{x^2dx}{x^6+5}$. Ответ: $\frac{1}{3\sqrt{5}}\operatorname{arctg}\frac{x^3}{\sqrt{5}}+c$.

1.25. $\int\frac{dx}{\sqrt{6-5x^2}}$. Ответ: $\frac{1}{\sqrt{5}}\arcsin\frac{x\sqrt{5}}{\sqrt{6}}+c$.

1.26. $\int\frac{dx}{\left(1+4x^2\right)\operatorname{arctg}2x}$. Ответ: $\frac{1}{2}\ln\left|\operatorname{arctg}2x\right|+c$.

1.27. $\int\left(\operatorname{ctg}(3x+1)-\operatorname{tg}4x\right)dx$. Ответ: $\frac{1}{3}\ln\left|\sin(3x+1)\right|-\frac{1}{4}\ln\left|\cos 4x\right|+c$.

1.28. $\int\frac{\sin 2x\,dx}{5+\cos^2 x}$. Ответ: $-\ln\left|5+\cos^2 x\right|+c$.

1.29. $\int\frac{dx}{x\sqrt{2+\ln^2 x}}$. Ответ: $\ln\left|\ln x+\sqrt{2+\ln^2 x}\right|+c$.

1.30. $\int\frac{e^x dx}{e^{2x}-5}$. Ответ: $\frac{1}{2\sqrt{5}}\ln\left|\frac{e^x-\sqrt{5}}{e^x+\sqrt{5}}\right|+c$.

1.31. $\int\frac{\sin 2x\,dx}{\cos^2 2x+9}$. Ответ: $-\frac{1}{6}\operatorname{arctg}\frac{\cos 2x}{3}+c$.

1.32. $\int\left(\frac{x^3}{\sqrt{1+2x^4}}-\frac{e^x}{\sin^2\left(3e^x\right)}\right)dx$. Ответ: $\frac{1}{4}\sqrt{1+2x^4}+\frac{1}{3}\operatorname{ctg}\left(3e^x\right)+c$.

1.33. $\int\frac{3+(\arcsin 3x)^2}{\sqrt{1-9x^2}}dx$. Ответ: $\arcsin 3x+\frac{(\arcsin 3x)^3}{9}+c$.

1.34. $\int\frac{\cos\frac{x}{2}dx}{\sqrt[3]{4+3\sin\frac{x}{2}}}$. Ответ: $\sqrt[3]{\left(4+3\sin\frac{x}{2}\right)^2}+c$.

ПРАКТИЧЕСКОЕ ЗАНЯТИЕ № 2

Тема: Метод интегрирования по частям.

Метод интегрирования по частям описан в § 2.3, формула (3). $\int u dv = uv - \int v du$.

2.1. Найти $\int x \cdot \cos 2x\, dx$.

$$\blacktriangleright \int x \cdot \cos 2x\, dx = \left\{ \begin{array}{l} u = x,\ du = dx \\ dv = \cos 2x\, dx,\ v = \int \cos 2x\, dx = \frac{1}{2}\sin 2x \end{array} \right\} =$$

$$= \frac{1}{2} x \sin 2x - \int \frac{1}{2}\sin 2x\, dx = \frac{1}{2} x \sin 2x + \frac{1}{4}\cos 2x + c.$$

2.2. Найти $\int \left(x^2 + 2x - 1\right) \cdot \sin 3x\, dx$.

$$\blacktriangleright \int \left(x^2 + 2x - 1\right) \cdot \sin 3x\, dx = \left\{ \begin{array}{l} u = x^2 + 2x - 1;\ du = \left(2x + 2\right) dx = 2\left(x + 1\right) dx \\ dv = \sin 3x\, dx;\ v = \int \sin 3x\, dx = -\frac{1}{3}\cos 3x \end{array} \right\} =$$

$$= -\frac{1}{3}\left(x^2 + 2x - 1\right) \cdot \cos 3x + \frac{2}{3}\int \left(x + 1\right) \cdot \cos 3x\, dx =$$

$$= \left\{ \begin{array}{l} u = x + 1,\ du = dx \\ dv = \cos 3x\, dx,\ v = \int \cos 3x\, dx = \frac{1}{3}\sin 3x \end{array} \right\} =$$

$$= -\frac{1}{3}\left(x^2 + 2x - 1\right) \cdot \cos 3x + \frac{2}{3}\left(\frac{1}{3}\left(x + 1\right) \cdot \sin 3x - \frac{1}{3}\int \sin 3x\, dx\right) =$$

$$= -\frac{1}{3}\left(x^2 + 2x - 1\right) \cdot \cos 3x + \frac{2}{9}\left(x + 1\right) \cdot \sin 3x + \frac{2}{27}\cos 3x + c.$$

2.3. Найти $\int \frac{\ln x\, dx}{x^3}$.

$$\blacktriangleright \int \frac{\ln x\, dx}{x^3} = \left\{ \begin{array}{l} u = \ln x,\ du = \frac{dx}{x} \\ dv = \frac{dx}{x^3},\ v = \int \frac{dx}{x^3} = \int x^{-3} dx = \frac{x^{-2}}{-2} = -\frac{1}{2x^2} \end{array} \right\} =$$

$$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\int \frac{dx}{x^3} = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c.$$

2.4. Найти $\int \ln\left(1 + x^2\right) dx$.

$$\blacktriangleright \int \ln\left(1 + x^2\right) dx = \left\{ \begin{array}{l} u = \ln\left(1 + x^2\right),\ du = \frac{2x\, dx}{1 + x^2} \\ dv = dx,\ v = x \end{array} \right\} = x \ln\left(1 + x^2\right) - 2\int \frac{x^2 dx}{x^2 + 1} =$$

$$= x\ln\left(1+x^2\right) - 2\int \frac{\left(x^2+1\right)-1}{\left(x^2+1\right)}dx = x\ln\left(1+x^2\right) - 2\int\left(1-\frac{1}{x^2+1}\right)dx =$$

$$= x\ln\left(1+x^2\right) - 2\left(\int dx - \int \frac{dx}{x^2+1}\right) = x\ln\left(1+x^2\right) - 2x + 2\,\text{arctg}\,x + c\,.$$

2.5. Найти $\int \cos(\ln x)\,dx$.

$$\blacktriangleright \int \cos(\ln x)\,dx = \left\{\begin{matrix} u = \cos(\ln x),\ du = -\sin(\ln x)\cdot\dfrac{dx}{x} \\ dv = dx,\ v = x \end{matrix}\right\} =$$

$$= x\cos(\ln x) + \int \sin(\ln x)\,dx = \left\{\begin{matrix} u = \sin(\ln x),\ du = \cos(\ln x)\dfrac{dx}{x} \\ dv = dx,\ v = x \end{matrix}\right\} =$$

$$= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)dx\,;$$

$$2\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x)$$

$$\int \cos(\ln x)\,dx = \frac{1}{2}x\left(\cos(\ln x) + \sin(\ln x)\right) + c\,.$$

САМОСТОЯТЕЛЬНАЯ РАБОТА

Найти неопределенные интегралы.

2.6. $\int x\,e^{-5x}dx$. Ответ: $-\frac{1}{5}x\,e^{-5x} - \frac{1}{25}e^{-5x} + c$.

2.7. $\int x^2\cdot\cos 3x\,dx$. Ответ: $\frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x + c$.

2.8. $\int x^4\cdot\ln x\,dx$. Ответ: $\frac{x^5}{5}\left(\ln x - \frac{1}{5}\right) + c$.

2.9. $\int \frac{x\cos x\,dx}{\sin^2 x}$. Ответ: $-\frac{x}{\sin x} + \ln\left|\,\text{tg}\,\frac{x}{2}\right| + c$.

2.10. $\int \arcsin x\,dx$. Ответ: $x\arcsin x + \sqrt{1-x^2} + c$.

2.11. $\int \ln(x-3)\,dx$. Ответ: $x\ln(x-3) - x - 3\ln(x-3) + c$.

2.12. $\int \frac{\text{arctg}\,\sqrt{x+1}}{\sqrt{x+1}}dx$. Ответ: $2\sqrt{x+1}\cdot\text{arctg}\,\sqrt{x+1} - \ln|2+x| + c$.

2.13. $\int e^x\cdot\cos x\,dx$. Ответ: $\frac{e^x}{2}(\sin x + \cos x) + c$.

2.14. $\int (3x+1)\cdot\sin 2x\,dx$. Ответ: $-\frac{1}{2}(3x+1)\cdot\cos 2x + \frac{3}{4}\sin 2x + c$.

ПРАКТИЧЕСКОЕ ЗАНЯТИЕ № 3

Тема: Интегрирование рациональных дробей.

3.1. Найти $\int \frac{5x^3+9x^2-22x-8}{x^3-4x}dx$.

Решение. Подынтегральная рациональная дробь неправильная, поэтому выделяем целую часть:

$$\begin{array}{l|l} 5x^3+9x^2-22x-8 & x^3-4x \\ \underline{-\,5x^3-20x} & \hline 5 \\ 9x^2-2x-8 & \end{array}$$

Таким образом, $\int \frac{5x^3+9x^2-22x-8}{x^3-4x}dx = \int\left(5+\frac{9x^2-2x-8}{x^3-4x}\right)dx$.

Полученную правильную дробь методом неопределенных коэффициентов распишем на сумму простейших дробей:

$$\frac{9x^2-2x-8}{x^3-4x} = \frac{9x^2-2x-8}{x(x-2)(x+2)} = \frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+2}.$$

$$9x^2-2x-8 = A(x^2-4)+Bx(x+2)+Cx(x-2).$$

Коэффициенты *A, B, C* найдем методом частных значений.

$$\begin{array}{l|ll} x=0 & -4A=-8, & A=2, \\ x=2 & 8B=24, & B=3, \\ x=-2 & 8C=32, & C=4. \end{array}$$

$$\int \frac{5x^3+9x^2-22x-8}{x^3-4x}dx = \int\left(5+\frac{x}{2}+\frac{3}{x-2}+\frac{4}{x+2}\right)dx =$$

$$= 5x+2\ln|x|+3\ln|x-2|+4\ln|x+2|+c\,.$$

3.2. Найти $\int \frac{3x+2}{x(x+1)^3}dx$.

Решение. Подынтегральная дробь правильная, расписываем ее на сумму простейших дробей:

$$\frac{3x+2}{x(x+1)^3} = \frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}+\frac{D}{(x+1)^3} = \frac{A(x+1)^3+Bx(x+1)^2+Cx(x+1)+Dx}{x(x+1)^3}.$$

$$3x+2 = A(x+1)^3+Bx(x+1)^2+Cx(x+1)+Dx\,.$$

Комбинируя методы частных значений и сравнения коэффициентов, найдем *A, B, C, D*.

$$\begin{array}{l|ll} x=0 & A=2, & \\ x=-1 & -1=-D, & D=1, \\ x^3 & A+B=0, & B=-A, \quad B=-2, \\ x^2 & 3A+2B+C=0, & C=-3A-2B=-2, \quad C=-2; \end{array}$$

$$\int \frac{3x+2}{x(x+1)^3}\,dx = \int\left(\frac{2}{x}-\frac{2}{x+1}-\frac{2}{(x+1)^2}+\frac{1}{(x+1)^3}\right)dx =$$

$$= 2\ln|x| - 2\ln|x+1| + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + c\,.$$

3.3. Найти $\int \frac{2x^2-3x+1}{x^3+1}\,dx$.

Решение. Подынтегральная дробь правильная.

$$\frac{2x^2-3x+1}{x^3+1} = \frac{2x^2-3x+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1}+\frac{Bx+C}{x^2-x+1} = \frac{A(x^2-x+1)+(Bx+C)(x+1)}{(x+1)(x^2-x+1)}.$$

$$2x^2-3x+1 = A(x^2-x+1)+(Bx+C)(x+1)$$

$$\begin{array}{l|ll} x=-1 & 6=3A, & A=2, \\ x^2 & 2=A+B, & B=2-A=0, \quad B=0, \\ x^0 & 1=A+C, & C=1-A, \quad C=-1; \end{array}$$

$$\int \frac{2x^2-3x+1}{x^3+1}\,dx = \int\left(\frac{2}{x+1}-\frac{1}{x^2-x+1}\right)dx = 2\ln|x+1| - \int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} =$$

$$= 2\ln|x+1| - \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{\left(x-\frac{1}{2}\right)\cdot 2}{\sqrt{3}} + c = 2\ln|x+1| - \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}} + c\,.$$

3.4. Найти $\int \frac{x^3-2x+2}{(x-1)^2(x^2+1)}\,dx$.

Решение. Подынтегральная дробь правильная, распишем ее на сумму простейших:

$$\frac{x^3-2x+2}{(x-1)^2(x^2+1)} = \frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+1}.$$

$$x^3-2x+2 = A(x-1)(x^2+1)+B(x^2+1)+(Cx+D)(x-1)^2.$$

$$\begin{array}{l|l} x=1 & 1=2B, \quad B=1/2, \\ x^3 & 1=A+C \\ x^2 & 0=-A+B-2C+D \\ x^0 & 2=-A+B+D \end{array} \Rightarrow \left\{\begin{array}{l} A+C=1 \\ -A+B=2C-D \\ -A+B=2-D \end{array}\right\} \Rightarrow 2C-D=2-D;\ C=1,\ \mathrm{A}=0.$$

$$D=2+A-B=2-\frac{1}{2}=\frac{3}{2},\ D=\frac{3}{2}.$$

$$\int\frac{x^3-2x+2}{(x-1)^2(x^2+1)}dx=\int\left(\frac{1}{2(x-1)^2}+\frac{x+3/2}{x^2+1}\right)dx=-\frac{1}{2(x-1)}+\int\left(\frac{x}{x^2+1}+\frac{3/2}{x^2+1}\right)dx=$$

$$=-\frac{1}{2(x-1)}+\frac{1}{2}\ln|x^2+1|+\frac{3}{2}\operatorname{arctg}\,x+c.$$

САМОСТОЯТЕЛЬНАЯ РАБОТА

Найти неопределенные интегралы.

3.5. $\int\frac{4x-1}{x(x^2+4)}dx.$ Ответ: $-\frac{1}{4}\ln|x|+\frac{1}{8}\ln|x^2+4|+2\operatorname{arctg}\frac{x}{2}+c.$

3.6. $\int\frac{x-4}{x^2-5x+6}dx.$ Ответ: $2\ln|x-2|-\ln|x-3|+c.$

3.7. $\int\frac{x^3+1}{x^3-x^2}dx.$ Ответ: $x+\frac{1}{x}+3\ln|x-1|-\ln|x|+c.$

3.8. $\int\frac{x^2dx}{x^4-1}.$ Ответ: $\frac{1}{2}\operatorname{arctg}\,x+\frac{1}{4}\ln|x-1|-\frac{1}{4}\ln|x+1|+c.$

3.9. $\int\frac{3x+4}{x^2+2x+5}dx.$ Ответ: $\frac{3}{2}\ln|x^2+2x+5|+\frac{1}{2}\operatorname{arctg}\frac{x+1}{2}+c.$

ПРАКТИЧЕСКОЕ ЗАНЯТИЕ № 4

Тема: Интегрирование тригонометрических выражений.

Методы интегрирования тригонометрических выражений описаны в § 4.

4.1. Найти $\int \frac{dx}{8-4\sin x+7\cos x}$.

► Данный интеграл вычисляется с помощью универсальной подстановки $\operatorname{tg}\frac{x}{2}=t$. В этом случае $\cos x=\frac{1-t^2}{1+t^2}$; $\sin x=\frac{2t}{1+t^2}$; $dx=\frac{2dt}{1+t^2}$.

$$\int \frac{dx}{8-4\sin x+7\cos x}=\int \frac{2dt}{\left(1+t^2\right)\left(8-4\cdot\frac{2t}{1+t^2}+7\cdot\frac{1-t^2}{1+t^2}\right)}=\int \frac{2dt}{8+8t^2-8t+7-7t^2}=$$

$$=2\int \frac{dt}{t^2-8t+15}=2\int \frac{dt}{(t-4)^2-1}=\ln\left|\frac{t-4-1}{t-4+1}\right|+c=\{\text{см. ф. 15}\}=\ln\left|\frac{\operatorname{tg}\frac{x}{2}-5}{\operatorname{tg}\frac{x}{2}+3}\right|+c.$$

4.2. Найти $\int \frac{dx}{\sin^2 x-4\sin x\cos x+5\cos^2 x}$.

► Подынтегральная функция четная относительно $\sin x$ и $\cos x$, поэтому применима подстановка $\operatorname{tg} x=t$.

$$\int \frac{dx}{\sin^2 x-4\sin x\cos x+5\cos^2 x}=\left\{\begin{matrix}\operatorname{tg} x=t\\ \frac{dx}{\cos^2 x}=dt\end{matrix}\right\}=\int \frac{dx}{\cos^2 x\left(\operatorname{tg}^2 x-4\operatorname{tg} x+5\right)}=$$

$$=\int \frac{dt}{t^2-4t+5}=\int \frac{dt}{(t-2)^2+1}=\operatorname{arctg}(t-2)+c=\{\text{см. ф. 14}\}=\operatorname{arctg}(\operatorname{tg} x-2)+c.$$

4.3. Найти $\int \frac{\cos^3 x\,dx}{4\sin^2 x-1}$.

► Подынтегральная функция нечетная относительно $\cos x$, поэтому применима подстановка $\sin x=t$.

$$\int \frac{\cos^3 x\,dx}{4\sin^2 x-1}=\left\{\begin{matrix}\sin x=t\\ \cos x\,dx=dt\end{matrix}\right\}=\int \frac{\cos^2 x\cdot\cos x\,dx}{4\sin^2 x-1}=\int \frac{\left(1-\sin^2 x\right)\cdot\cos x\,dx}{4\sin^2 x-1}=$$

$$=\int \frac{\left(1-t^2\right)dt}{4t^2-1}=-\frac{1}{4}\int \frac{4t^2-4}{4t^2-1}dt=-\frac{1}{4}\int \frac{\left(4t^2-1\right)-3}{4t^2-1}dt=-\frac{1}{4}\int\left(1-\frac{3}{4t^2-1}\right)dt=$$

$$= -\frac{1}{4}\left(t - \frac{3}{4}\ln\left|\frac{2t-1}{2t+1}\right|\right) + c = \frac{3}{16}\ln\left|\frac{2\sin x - 1}{2\sin x + 1}\right| - \frac{1}{4}\sin x + c .$$

4.4. Найти $\int \operatorname{ctg}^3 x\, dx$.

► Применяем равенство $1 + \operatorname{ctg}^2 x = \frac{1}{\sin^2 x}$. Отсюда $\operatorname{ctg}^2 x = \frac{1}{\sin^2 x} - 1$.

$$\int \operatorname{ctg}^3 x\, dx = \int \operatorname{ctg}^2 x \cdot \operatorname{ctg} x\, dx = \int \left(\frac{1}{\sin^2 x} - 1 \right) \operatorname{ctg} x\, dx =$$

$$= \int \operatorname{ctg} x \cdot \frac{dx}{\sin^2 x} - \int \operatorname{ctg} x\, dx = -\frac{1}{2}\operatorname{ctg}^2 x - \ln|\sin x| + c .$$

4.5. Найти $\int \cos 3x \cdot \cos^2 4x\, dx$.

► Применим формулу $\cos^2 \frac{\alpha}{2} = \frac{1}{2}(1 + \cos\alpha)$.

$$\int \cos 3x \cdot \cos^2 4x\, dx = \int \cos 3x \cdot \frac{1}{2}(1 + \cos 8x)\, dx = \frac{1}{2}\int (\cos 3x + \cos 3x \cdot \cos 8x)\, dx =$$

$$= \frac{1}{2}\int \cos 3x\, dx + \frac{1}{2}\int \cos 3x \cdot \cos 8x\, dx = \left\{ \cos\alpha \cdot \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)] \right\} =$$

$$= \frac{1}{6}\sin 3x + \frac{1}{2}\int \frac{1}{2}[\cos 5x + \cos 11x]dx = \frac{1}{6}\sin 3x + \frac{1}{4}\left(\frac{1}{5}\sin 5x + \frac{1}{11}\sin 11x \right) + c =$$

$$= \frac{1}{6}\sin 3x + \frac{1}{20}\sin 5x + \frac{1}{44}\sin 11x + c .$$

САМОСТОЯТЕЛЬНАЯ РАБОТА

Найти неопределенные интегралы.

4.6. $\int \frac{dx}{3 + 4\sin x}$. Ответ: $\frac{2}{3}\operatorname{arctg}\frac{5\operatorname{tg}\frac{x}{2} + 4}{3} + c$.

4.7. $\int \frac{dx}{3\cos^2 x + 4\sin^2 x}$. Ответ: $\frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2\operatorname{tg} x}{\sqrt{3}} + c$.

4.8. $\int \sin^2 x \cdot \cos^2 x\, dx$. Ответ: $\frac{1}{8}\left(x - \frac{1}{4}\sin 4x \right) + c$.

4.9. $\int \frac{\sin^3 x\, dx}{\cos x - 3}$. Ответ: $\frac{\cos^2 x}{2} + 3\cos x + 8\ln|\cos x - 3| + c$.

4.10. $\int \cos^3 x \cdot \sin^2 x\, dx$. Ответ: $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c$.

4.11. $\int \operatorname{ctg}^4 x\, dx$. Ответ: $-\frac{1}{3}\operatorname{ctg}^3 x + \operatorname{ctg} x + x + c$.

4.12. $\int \frac{\cos 2x\, dx}{\sqrt{3+4\sin 2x}}$. Ответ: $\frac{1}{4}\sqrt{3+4\sin 2x} + c$.

4.13. $\int \frac{\sin^2 x\, dx}{1+\cos^2 x}$. Ответ: $\sqrt{2}\operatorname{arctg}\left(\frac{\operatorname{tg} x}{\sqrt{2}}\right) - x + c$.

4.14. $\int \frac{\sin x\, dx}{\sin x + 1}$. Ответ: $\frac{2}{1+\operatorname{tg}\frac{x}{2}} + x + c$.

4.15. $\int \frac{\sin 3x\, dx}{\sqrt[3]{(3+2\cos 3x)^2}}$. Ответ: $-\frac{1}{2}\sqrt[3]{3+2\cos 3x} + c$.

4.16. $\int \frac{dx}{\cos^3 x \cdot \sin x}$. Ответ: $\ln\left|\operatorname{tg} x\right| + \frac{1}{2}\operatorname{tg}^2 x + c$.

ПРАКТИЧЕСКОЕ ЗАНЯТИЕ № 5

Тема: Интегрирование иррациональных выражений.

Методы интегрирования иррациональных выражений описаны в § 5.

5.1. Найти $\int \frac{dx}{\sqrt{2x^2-x+3}}$.

$$\blacktriangleright \int \frac{dx}{\sqrt{2x^2-x+3}} = \left\{ \begin{array}{l} 2x^2-x+3 = 2\left(x^2-\frac{x}{2}+\frac{3}{2}\right) = \\ = 2\left[\left(x^2-2\cdot\frac{x}{4}+\frac{1}{16}\right)-\frac{1}{16}+\frac{3}{2}\right] = \\ = 2\left[\left(x-\frac{1}{4}\right)^2+\frac{23}{16}\right] \end{array} \right\} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(x-\frac{1}{4}\right)^2+\frac{23}{16}}} =$$

$$= \frac{1}{\sqrt{2}} \ln\left| x-\frac{1}{4}+\sqrt{x^2-\frac{x}{2}+\frac{3}{2}} \right| + c\,.$$ (См. ф. 18).

5.2. Найти $\int \frac{(x+5)\,dx}{\sqrt{3-6x-x^2}}$. (См. теорию § 5.2)

$$\blacktriangleright \int \frac{(x+5)\,dx}{\sqrt{3-6x-x^2}} = \left\{ \begin{array}{l} \left(3-6x-x^2\right)' = -2x-6 \\ x+5 = -\frac{1}{2}(-2x-6)+2 \end{array} \right\} = \int \frac{-\frac{1}{2}(-2x-6)+2}{\sqrt{3-6x-x^2}}\,dx =$$

$$= -\frac{1}{2}\int \frac{(-2x-6)\,dx}{\sqrt{3-6x-x^2}} \Bigg|_{(\text{см. ф. } 16)} + 2\int \frac{dx}{\sqrt{3-6x-x^2}} = -\frac{1}{2}\cdot 2\sqrt{3-6x-x^2} +$$

$$+ \left\{ \begin{array}{l} 3-6x-x^2 = -\left(x^2+6x-3\right) = \\ = -\left(x^2+6x+9-12\right) = \\ = -\left[(x+3)^2-12\right] = 12-(x+3)^2 \end{array} \right\} + 2\int \frac{dx}{\sqrt{12-(x+3)^2}} \Bigg|_{(\text{см. ф. } 17)} =$$

$$= -\sqrt{3-6x-x^2} + 2\arcsin\frac{x+3}{\sqrt{12}} + c\,.$$

5.3. Найти $\int \frac{dx}{(x+1)\sqrt{x^2-1}}$. (См. теорию § 5.3)

$$\blacktriangleright \int \frac{dx}{(x+1)\sqrt{x^2-1}} = \left\{ \begin{matrix} x+1=\dfrac{1}{t}, & t=\dfrac{1}{x+1} \\ x=\dfrac{1}{t}-1, & dx=-\dfrac{dt}{t^2} \end{matrix} \right\} = \int \frac{-dt}{t^2 \cdot \frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2-1}} =$$

$$= -\int \frac{dt}{t\sqrt{\frac{1}{t^2}-\frac{2}{t}+1-1}} = -\int \frac{dt}{t\sqrt{\frac{1-2t}{t^2}}} = -\int \frac{dt}{\sqrt{1-2t}} = \{\text{см. ф. 16}\} =$$

$$= -\left(-\frac{1}{2}\right)\cdot 2\sqrt{1-2t}+c = \sqrt{1-\frac{2}{x+1}}+c = \sqrt{\frac{x-1}{x+1}}+c.$$

5.4. Найти $\int \dfrac{dx}{x^2\sqrt{9+x^2}}$. (См. теорию § 5.4)

$$\blacktriangleright \int \frac{dx}{x^2\sqrt{9+x^2}} = \left\{ \begin{matrix} x=3\operatorname{tg} t, & \operatorname{tg} t=\dfrac{x}{3} \\ dx=\dfrac{3dt}{\cos^2 t}, & 9+x^2=9+9\operatorname{tg}^2 t=9\left(1+\operatorname{tg}^2 t\right)=\dfrac{9}{\cos^2 t} \end{matrix} \right\} =$$

$$= \int \frac{3dt}{\cos^2 t \cdot 9\operatorname{tg}^2 t \cdot \frac{3}{\cos t}} = \frac{1}{9}\int \frac{dt}{\cos t \cdot \frac{\sin^2 t}{\cos^2 t}} = \frac{1}{9}\int \frac{\cos t\, dt}{\sin^2 t} = \frac{1}{9}\int \sin^{-2} t \cdot \cos t\, dt =$$

$$= \{\text{см. ф. 1}\} = -\frac{1}{9}\sin^{-1} t + c = -\frac{1}{9\sin t}+c = c-\frac{\sqrt{9+x^2}}{9x},$$ т. к.

$$\frac{1}{\sin^2 t} = 1+\operatorname{ctg}^2 t,$$ отсюда $$\sin t = \frac{1}{\sqrt{1+\operatorname{ctg}^2 t}} = \frac{\operatorname{tg} t}{\sqrt{\operatorname{tg}^2 t+1}} = \frac{\frac{x}{3}}{\sqrt{\frac{x^2}{9}+1}} =$$

$$= \frac{\frac{x}{3}}{\frac{\sqrt{x^2+9}}{3}} = \frac{x}{\sqrt{x^2+9}}.$$

5.5. Найти $\int \dfrac{\sqrt{x+2}}{x+3}dx$. (См. теорию § 5.1)

$$\blacktriangleright \int \frac{\sqrt{x+2}}{x+3}dx = \left\{ \begin{matrix} x+2=t^2, & x=t^2-2, \\ dx=2t\,dt, & t=\sqrt{x+2}. \end{matrix} \right\} = \int \frac{t\cdot 2t\,dt}{t^2-2+3} = 2\int \frac{t^2 dt}{t^2+1} =$$

$$= 2\int \frac{\left(t^2+1\right)-1}{t^2+1}dt == 2\int\left(1-\frac{1}{t^2+1}\right)dt = 2\left(t-\operatorname{arctg} t\right)+c = 2\left(\sqrt{x+2}-\operatorname{arctg}\sqrt{x+2}\right)+c.$$

5.6. Найти $\int \frac{\sqrt{x}\,dx}{x-\sqrt[3]{x^2}}$. (См. теорию § 5.1)

$$\blacktriangleright \int \frac{\sqrt{x}\,dx}{x-\sqrt[3]{x^2}} = \left\{\begin{array}{ll} x=t^6, & t=\sqrt[6]{x}, \\ dx=6t^5dt. & \end{array}\right\} = \int \frac{t^3\cdot 6t^5dt}{t^6-t^4} = 6\int \frac{t^8dt}{t^4\left(t^2-1\right)} = 6\int \frac{t^4dt}{t^2-1} =$$

$$= 6\int \frac{\left(t^4-1\right)+1}{t^2-1}dt = 6\int\left(\frac{t^4-1}{t^2-1}+\frac{1}{t^2-1}\right)dt = 6\int\left(t^2+1+\frac{1}{t^2-1}\right)dt =$$

$$= 6\left(\frac{t^3}{3}+t+\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right)+c = 2\sqrt{x}+6\sqrt[6]{x}+3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right|+c.$$

САМОСТОЯТЕЛЬНАЯ РАБОТА

Найти неопределенные интегралы.

5.7. $\int \frac{dx}{\sqrt{x^2+4x+5}}$. Ответ: $\ln\left|x+2+\sqrt{x^2+4x+5}\right|+c$.

5.8. $\int \frac{(3x+4)\,dx}{\sqrt{x^2+6x+13}}$. Ответ: $3\sqrt{x^2+6x+13}-5\ln\left|x+3+\sqrt{x^2+6x+13}\right|+c$.

5.9. $\int \frac{dx}{x\sqrt{x^2+x-1}}$. Ответ: $c-\arcsin\frac{2-x}{x\sqrt{5}}$.

5.10. $\int \frac{\sqrt{2x+1}}{1+\sqrt{2x+1}}dx$. Ответ: $\frac{2x+1}{2}+\sqrt{2x+1}+\ln\left|\sqrt{2x+1}+1\right|+c$.

5.11. $\int \sqrt{4-x^2}\,dx$. Ответ: $2\left(\arcsin\frac{x}{2}+\frac{x}{4}\sqrt{4-x^2}\right)+c$.

5.12. $\int \frac{\sqrt{x}-\sqrt[3]{x^2}}{\sqrt[4]{x}}dx$. Ответ: $\frac{4}{5}\sqrt[4]{x^5}-\frac{12}{17}\sqrt[12]{x^{17}}+c$.

5.13. $\int \frac{dx}{\left(\sqrt{4+x^2}\right)^3}$. Ответ: $\frac{1}{4}\cdot\frac{x}{\sqrt{4+x^2}}+c$.

5.14. $\int \frac{dx}{x^3\sqrt{x^2-1}}$. Ответ: $-\frac{1}{2}\arcsin\frac{1}{x}+\frac{\sqrt{x^2-1}}{2x^2}+c$.

ИНДИВИДУАЛЬНЫЕ ЗАДАНИЯ

Вариант 1.

Проинтегрировать:

1. $\int \frac{5z\,dz}{z^2+4}$,
2. $\int \sqrt{4x-1}\,dx$,
3. $\int \frac{dx}{\sqrt{4-3x^2}}$,
4. $\int \frac{x\,dx}{\sqrt{9+4x^4}}$,
5. $\int \cos t\, e^{\sin t}\,dt$,
6. $\int \frac{\cos(\ln x)}{2x}\,dx$,
7. $\int \frac{dx}{\operatorname{tg} 3x}$,
8. $\int \frac{2x^2-x+1}{x^3}\,dx$,
9. $\int \frac{\sin x\,dx}{\sqrt{1+2\cos x}}$,
10. $\int \frac{e^x dx}{3+4e^x}$,
11. $\int \frac{\sqrt[3]{\operatorname{arctg} x}}{1+x^2}\,dx$,
12. $\int \frac{e^{2x}dx}{\sqrt{9-4e^{4x}}}$,
13. $\int \frac{x\,dx}{\sqrt{49x^2-3}}$,
14. $\int \frac{\sin 3x\,dx}{16-\cos^2 3x}$,
15. $\int \frac{dx}{x(9+\ln^3 x)}$,
16. $\int e^{4y}\cdot\sqrt{e^{4y}-25}\,dy$,
17. $\int \frac{3^{\operatorname{arctg} x}dx}{1+x^2}$,
18. $\int \frac{dx}{\cos^2 x(3\operatorname{tg} x+1)}$,
19. $\int \frac{(x+2)\,dx}{\sqrt{x^2+4x+15}}$,
20. $\int \frac{\operatorname{tg}^3 \frac{x}{4}dx}{\cos^2 \frac{x}{4}}$,
21. $\int \frac{4x+3}{x^2+9}\,dx$,
22. $\int \cos^6 4x\cdot\sin 4x\,dx$,
23. $\int \frac{\cos\frac{x}{2}dx}{4+\sin\frac{x}{2}}$,
24. $\int \frac{x^3dx}{\sqrt{16-x^8}}$,
25. $\int \sin\frac{x}{3}\cdot\sqrt{16+\cos\frac{x}{3}}dx$,
26. $\int \frac{e^{2x}dx}{e^x+3}$,
27. $\int \ln\left(x^2+4\right)dx$,
28. $\int \frac{x\cos x}{\sin^2 x}\,dx$,
29. $\int x\cdot\operatorname{tg}^2 x\,dx$,
30. $\int \frac{\operatorname{arctg}\sqrt{x}}{\sqrt{x}}\,dx$,
31. $\int \frac{\ln\cos 2x}{\sin^2 2x}\,dx$,
32. $\int e^{-2x}\cdot\sin 3x\,dx$,
33. $\int \frac{x^2dx}{x^2+1}$,
34. $\int \frac{x-2}{x^2-4x+7}\,dx$,
35. $\int \frac{x\,dx}{(x-1)(x+1)^2}$,
36. $\int \frac{x^2dx}{x^4-16}$,
37. $\int \frac{2x+1}{x^4-4x^3+4x^2}\,dx$,
38. $\int \frac{3x^5-12x^3-7}{x^2+2x}\,dx$,
39. $\int \frac{4x\,dx}{x^3+8}$,
40. $\int \sin^3 x\cdot\cos^2 x\,dx$,
41. $\int \frac{dx}{\sin^4 x}$,
42. $\int \cos x\cdot\cos 3x\,dx$,
43. $\int \frac{dx}{3+5\cos x}$,
44. $\int \frac{\operatorname{tg}^2 x+2}{\cos 2x}\,dx$,
45. $\int \frac{\cos^3 x+1}{\sin^2 x}\,dx$,
46. $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$,
47. $\int \frac{dx}{x\sqrt{x^2-4}}$,
48. $\int \frac{dx}{\sqrt{x^2+2x+3}}$,
49. $\int \frac{\sqrt{x^2+9}}{x^2}\,dx$,
50. $\int \frac{dx}{(x+1)\sqrt{x^2+2x+2}}$.

Вариант 2.

Проинтегрировать:

1. $\int \frac{4\sqrt{\ln x}\,dx}{x}$,
2. $\int \sin\frac{x}{3}dx$,
3. $\int \frac{(x-3)\,dx}{x^2-6x-10}$,
4. $\int \frac{3\,dx}{\sqrt{1-x^2}\arcsin x}$,
5. $\int 8e^{3x+2}dx$,
6. $\int \frac{x\,dx}{x^4-4}$,
7. $\int \sin^2 4x\cdot\cos 4x\,dx$,
8. $\int \frac{2x-3}{\sqrt{x^2-5}}dx$,
9. $\int \frac{x\,dx}{\sqrt[3]{x^2+2}}$,
10. $\int \frac{\cos x\,dx}{\sin^4 x}$,
11. $\int \frac{dx}{x(5+\ln^2 x)}$,
12. $\int \frac{x^2dx}{\sqrt{4-9x^6}}$,
13. $\int s\cdot\sqrt{3-49s^2}ds$,
14. $\int \frac{\sin 3t\,dt}{\sqrt{16+\cos^2 3t}}$,
15. $\int \frac{dx}{x\sqrt{(4-\ln x)^3}}$,
16. $\int \frac{e^{2x}dx}{\sqrt{25-e^{4x}}}$,
17. $\int e^{\sin^2 x}\cdot\sin 2x\,dx$,
18. $\int \frac{\sin 3x\,dx}{\sqrt[3]{\cos^2 3x}}$,
19. $\int \frac{x+\operatorname{arctg} 2x}{1+4x^2}dx$,
20. $\int \frac{e^{4t}dt}{16+e^{4t}}$,
21. $\int \frac{dx}{\sin 4x}$,
22. $\int \frac{x\,dx}{\cos^2(4-x^2)}$,
23. $\int \frac{\sin\frac{x}{2}dx}{\sqrt[3]{3+2\cos\frac{x}{2}}}$,
24. $\int \frac{dx}{(x+1)\sqrt{\ln^2(x+1)-1}}$,
25. $\int \sin^4\frac{x}{3}\cdot\cos\frac{x}{3}dx$,
26. $\int \frac{dx}{e^{2x}-1}$,
27. $\int (4x-2)\cos 2x\,dx$,
28. $\int \ln(x^2+14)dx$,
29. $\int \frac{x\,dx}{\cos^2 3x}$,
30. $\int \frac{\ln(\cos 2x)\,dx}{\cos^2 2x}$,
31. $\int \sqrt{x}\operatorname{arctg}\sqrt{x}\,dx$,
32. $\int e^{2x}\cdot\cos 4x\,dx$,
33. $\int \frac{x^2dx}{x^2+4}$,
34. $\int \frac{x\,dx}{x^2-7x+13}$,
35. $\int \frac{x\,dx}{x^2-3x+2}$,
36. $\int \frac{x^4dx}{x^4-81}$,
37. $\int \frac{x^6+3x^3-1}{x^2+x}dx$,
38. $\int \frac{x+1}{x^4+4x^2}dx$,
39. $\int \frac{4x\,dx}{x^3-8}$,
40. $\int \frac{dx}{\sin x+8\cos x}$,
41. $\int \cos\frac{x}{2}\cdot\cos\frac{x}{3}dx$,
42. $\int \frac{\sin^3 x\,dx}{\cos^4 x}$,
43. $\int \operatorname{tg}^2 5x\,dx$,
44. $\int \frac{dx}{3\sin^2 x+2\cos^2 x}$,
45. $\int \frac{\cos 2x\,dx}{\sin^4 x}$,
46. $\int \frac{dx}{(x-2)\sqrt{1-x}}$,
47. $\int \frac{dx}{x\sqrt{4-x^2}}$,
48. $\int \frac{dx}{\sqrt{x^2-2x-6}}$,
49. $\int \frac{dx}{\sqrt{(25+x^2)^3}}$,
50. $\int \frac{dx}{x\sqrt{2x^2+2x+1}}$.

Вариант 3.

Проинтегрировать:

1. $\int \frac{2e^t dt}{1-e^t}$,
2. $\int \frac{dx}{\cos^2 5x}$,
3. $\int \frac{5\operatorname{arctg}^3 x dx}{1+x^2}$,
4. $\int \frac{x\,dx}{\sqrt{9-25x^4}}$,
5. $\int 7e^{-x} dx$,
6. $\int \cos^2 3x \cdot \sin 3x\,dx$,
7. $\int \frac{(x-6)\,dx}{\sqrt{x^2-12x+25}}$,
8. $\int \frac{(5x^2-6x+1)dx}{\sqrt{x}}$,
9. $\int \frac{\sqrt{1+\ln x}}{x} dx$,
10. $\int \frac{\sin 4x\,dx}{\cos^5 4x}$,
11. $\int \frac{3\,dx}{\cos^2 x \cdot tgx}$,
12. $\int \frac{e^{\arcsin x} dx}{\sqrt{1-x^2}}$,
13. $\int \frac{e^{4t} dt}{\sqrt{e^{4t}+3}}$,
14. $\int \frac{\sin 2x\,dx}{\sqrt{3\sin^2 x+4}}$,
15. $\int \frac{\sin \frac{x}{2} dx}{\sqrt[4]{3+\cos\frac{x}{2}}}$,
16. $\int \frac{4x+3}{x^2+5} dx$,
17. $\int \frac{s\,ds}{\sqrt{3-49s^2}}$,
18. $\int \frac{\sin 3x\,dx}{\sqrt{16-\cos^2 3x}}$,
19. $\int \frac{e^{4t} dt}{\sqrt{e^{4t}-25}}$,
20. $\int \frac{\ln x\,dx}{x(\ln^2 x-1)}$,
21. $\int \frac{\sqrt{1+\ln(x+1)}}{x+1} dx$,
22. $\int \frac{\sin 4x\,dx}{9+\cos^2 4x}$,
23. $\int \frac{x-\operatorname{arctg}^2 x}{1+x^2} dx$,
24. $\int \frac{e^{-2x} dx}{\left(e^{-2x}+4\right)^6}$,
25. $\int \frac{e^{tg\,2x}}{\cos^2 2x} dx$,
26. $\int \frac{e^{3x} dx}{e^x+2}$,
27. $\int \arccos x\,dx$,
28. $\int (4-3x)e^{-3x} dx$,
29. $\int x \cdot \ln\left(x^2+1\right) dx$,
30. $\int \frac{x \cdot \sin 2x}{\cos^3 2x} dx$,
31. $\int (2x+1)\sin 2x\,dx$,
32. $\int e^{-3x} \cdot \cos 2x\,dx$,
33. $\int \frac{dx}{(x+5)^4}$,
34. $\int \frac{x-5}{x^2-2x+2} dx$,
35. $\int \frac{x+1}{x^2-5x+6} dx$,
36. $\int \frac{dx}{(x+2)^2\left(x^2+1\right)}$,
37. $\int \frac{3x^3+1}{x^2-1} dx$,
38. $\int \frac{dx}{x^4-4x^3+4x^2}$,
39. $\int \frac{x\,dx}{x^3+8}$,
40. $\int \frac{dx}{4\sin x+3\cos x}$,
41. $\int \frac{\cos^2 x\,dx}{\sin^6 x}$,
42. $\int x \cdot \sin^2\left(x^2\right) dx$,
43. $\int \sin 10x \cdot \sin 15x\,dx$,
44. $\int \frac{dx}{1+\operatorname{tg} x}$,
45. $\int \frac{\sin^2 x \cdot \cos^2 x\,dx}{\left(\sin^3 x+\cos^3 x\right)}$,
46. $\int \frac{dx}{\sqrt{2x-1}-\sqrt[4]{2x-1}}$,
47. $\int \frac{dx}{x^2\sqrt{x^2+49}}$,
48. $\int \frac{dx}{\sqrt{4x-3-x^2}}$,
49. $\int \frac{dx}{\left(x^2+2\right)\sqrt{x^2-1}}$,
50. $\int \frac{dx}{x\sqrt{2x^2+x+1}}$.

Вариант 4.

Проинтегрировать:

1. $\int \frac{4x\,dx}{4+x^2}$,
2. $\int \frac{\sqrt{x}-x^3e^x+x^2}{x^3}\,dx$,
3. $\int \frac{x\,dx}{\sqrt{4-x^2}}$,
4. $\int \frac{dx}{x\ln x}$,
5. $\int \frac{x^3dx}{\sqrt{49+9x^4}}$,
6. $\int \frac{dt}{\sin^2\frac{3t}{2}}$,
7. $\int \frac{dz}{\cos^2(1-z)}$,
8. $\int x\sin\left(1-3x^2\right)dx$,
9. $\int \frac{3x+1}{x^2+9}\,dx$,
10. $\int x\sqrt{x^2+4}\,dx$,
11. $\int \sin t\cdot e^{3\cos t}dt$,
12. $\int \frac{3\,dx}{\sqrt{1-x^2}\arcsin^2 x}$,
13. $\int \frac{s\,ds}{49s^2-3}$,
14. $\int \frac{(x-2)\,dx}{\sqrt[3]{x^2-4x+9}}$,
15. $\int \frac{(x-3)dx}{\sqrt{9x^2+5}}$,
16. $\int \operatorname{ctg}\frac{x}{4}dx$,
17. $\int \cos^4 6x\cdot\sin 6x\,dx$,
18. $\int \frac{\sin 3t\,dt}{\sqrt{\cos^2 3t-16}}$,
19. $\int \frac{\sqrt{4+\ln^2 x}}{x}\,dx$,
20. $\int \frac{e^{2x}dx}{e^{4x}+25}$,
21. $\int \frac{\sqrt[3]{\arcsin 2x}\,dx}{\sqrt{1-4x^2}}$,
22. $\int \frac{\cos 4x}{\sqrt[5]{\sin^2 4x}}\,dx$,
23. $\int \frac{e^{4x}dx}{\sqrt{e^{4x}+9}}$,
24. $\int \frac{dx}{x\left(9+\ln^2 x\right)}$,
25. $\int \frac{e^{\operatorname{arctg}2x}dx}{1+9x^2}$,
26. $\int \frac{e^{2x}dx}{e^x-5}$,
27. $\int x\cdot\operatorname{arctg}\,x\,dx$,
28. $\int x^3\cdot e^{x^2}dx$,
29. $\int (3x+4)\,e^{3x}dx$,
30. $\int \frac{\ln\left(x^2-1\right)dx}{x^2}$,
31. $\int (4-16x)\cdot\sin 4x\,dx$,
32. $\int e^{-4x}\cdot\cos 2x\,dx$,
33. $\int \frac{x^2dx}{x^2+5}$,
34. $\int \frac{(5x+3)\,dx}{x^2+10x+29}$,
35. $\int \frac{dx}{x^3-2x^2+x}$,
36. $\int \frac{dx}{x\left(x^2+5\right)}$,
37. $\int \frac{-x^5+9x^3+4}{x^2+3x}dx$,
38. $\int \frac{3x\,dx}{(x-2)\left(x^2+2x+1\right)}$,
39. $\int \frac{2x^2-4x+24}{3x^3-24}\,dx$,
40. $\int \frac{(1-\sin x)\,dx}{\cos x}$,
41. $\int \sin^2 2x\cdot\cos^3 2x\,dx$,
42. $\int \sin 3x\cdot\sin x\,dx$,
43. $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}\,dx$,
44. $\int \operatorname{ctg}^3 x\,dx$,
45. $\int \frac{\sin^3 4x\,dx}{\cos^5 4x}$,
46. $\int \frac{dx}{\sqrt{x+3}+\sqrt{(x+3)^3}}$,
47. $\int \frac{dx}{x^2\sqrt{x^2-9}}$,
48. $\int \frac{dx}{\sqrt{8+6x-9x^2}}$,
49. $\int \frac{dx}{x\sqrt{10x^2-6x+1}}$,
50. $\int \frac{dx}{\sqrt{\left(25-x^2\right)^3}}$.

Вариант 5.

Проинтегрировать:

1. $\int \frac{x\,dx}{\sqrt{4+x^2}}$,
2. $\int \frac{x\,dx}{x^2+16}$,
3. $\int 3t\cdot e^{t^2}\,dt$,
4. $\int \frac{dx}{\sqrt{1-4x^2}}$,
5. $\int \frac{5\,dx}{\left(1+x^2\right)\operatorname{arctg}^3 x}$,
6. $\int \operatorname{ctg}\frac{x}{2}\,dx$,
7. $\int x\cdot\sin\left(x^2+1\right)dx$,
8. $\int \sqrt[3]{(8-3x)^6}\,dx$,
9. $\int \frac{(2-x)^2\,dx}{\sqrt{x}}$,
10. $\int \sin^3 5x\cdot\cos 5x\,dx$,
11. $\int \frac{x^2dx}{x^6+9}$,
12. $\int \frac{\left(2x-\sqrt{\arcsin\ x}\right)dx}{\sqrt{1-x^2}}$,
13. $\int \frac{(x+4)\,dx}{x^2+8x-10}$,
14. $\int \frac{(x-8)\,dx}{\sqrt{16x^2-9}}$,
15. $\int \frac{ds}{\sqrt{49s^2-9}}$,
16. $\int \sin 6x\sqrt{16-\cos^2 3x}dx$,
17. $\int \frac{e^{4t}dt}{\sqrt{25-e^{4t}}}$,
18. $\int \frac{dx}{x\left(\ln^2 x-1\right)}$,
19. $\int \frac{\sqrt[3]{16+\ln x}\,dx}{x}$,
20. $\int \frac{e^{6x}dx}{\left(e^{6x}+9\right)^5}$,
21. $\int \frac{dx}{\cos^2 3x\left(\operatorname{tg} 3x+4\right)}$,
22. $\int \frac{dx}{(x-5)\ln^2(x-5)}$,
23. $\int \frac{e^{3x}dx}{\sqrt{4-e^{6x}}}$,
24. $\int \frac{\sin 2x\,dx}{9+\cos^2 2x}$,
25. $\int \frac{\sqrt[5]{\operatorname{arctg}\ 2x}\,dx}{1+4x^2}$,
26. $\int \frac{dx}{e^{2x}+1}$,
27. $\int x^3\ln x\,dx$,
28. $\int (1-6x)\,e^{2x}dx$,
29. $\int \frac{arctg\sqrt{x}}{x\sqrt{x}}\,dx$,
30. $\int \frac{x\,dx}{\cos^2 4x}$,
31. $\int \frac{x\arcsin\ x\,dx}{\sqrt{1-x^2}}$,
32. $\int e^{2x}\cdot\sin 4x\,dx$,
33. $\int \frac{dx}{(1-x)^5}$,
34. $\int \frac{dx}{x^2+2x+5}$,
35. $\int \frac{dx}{x(x-2)^2}$,
36. $\int \frac{x^2dx}{1-x^4}$,
37. $\int \frac{2x^3+5}{x^2-x-2}\,dx$,
38. $\int \frac{x^4+x+4}{x^4+4x^2}\,dx$,
39. $\int \frac{x\,dx}{x^3+27}$,
40. $\int \frac{dx}{\cos x+2\sin\ x+3}$,
41. $\int \sin 3x\cdot\cos 5x\,dx$,
42. $\int \frac{\sin^2 2x\,dx}{\cos^4 2x\,dx}$,
43. $\int \frac{(11-3\operatorname{tg} x)dx}{\operatorname{tg}\ x+3}$,
44. $\int \sin^5 4x\,dx$,
45. $\int \frac{dx}{2-5\sin^2 x}$,
46. $\int \frac{dx}{1+\sqrt{4+x}}$,
47. $\int \frac{dx}{\sqrt{x^2+4x-3}}$,
48. $\int \frac{dx}{x\sqrt{x^2+x-1}}$,
49. $\int \frac{dx}{\left(2-x^2\right)\sqrt{2-x^2}}$,
50. $\int \frac{dx}{\sqrt{\left(36+x^2\right)^3}}$.

Вариант 6.

Проинтегрировать:

1. $\int \frac{3x\,dx}{x^2-6}$,
2. $\int \sin\frac{x}{2} e^{\cos\frac{x}{2}} dx$,
3. $\int \sqrt[3]{6x+5}\,dx$,
4. $\int \frac{\sin(\ln x)\,dx}{6x}$,
5. $\int \frac{5\varphi\,d\varphi}{\sqrt{16+9\varphi^2}}$,
6. $\int \frac{x\,dx}{\sqrt{25-9x^4}}$,
7. $\int \operatorname{ctg} 6x\,dx$,
8. $\int \frac{x^3+2x^2-3}{x^4}dx$,
9. $\int \frac{\cos 6x\,dx}{\sqrt[3]{4+2\sin 6x}}$,
10. $\int \frac{e^{6t}dt}{4-e^{6t}}$,
11. $\int \frac{x-\operatorname{arctg}^3 3x}{1+9x^2}dx$,
12. $\int \frac{5x+6}{x^2-16}dx$,
13. $\int \frac{e^{\frac{x}{2}}dx}{\sqrt{4-e^x}}$,
14. $\int \frac{tdt}{\sqrt{49t^2+3}}$,
15. $\int \frac{\sin 6x\,dx}{16+\cos^2 6x}$,
16. $\int \frac{dx}{x\sqrt{6-\ln x}}$,
17. $\int \frac{e^{4x}dx}{\sqrt{25-e^{8x}}}$,
18. $\int \frac{4^{\operatorname{arctg} 2x}}{1+4x^2}dx$,
19. $\int e^{6x}\sqrt{25+e^{6x}}\,dx$,
20. $\int \frac{(3x-5)\,dx}{\sqrt{3x^2-10x+4}}$,
21. $\int \cos^6\frac{x}{2}\cdot\sin\frac{x}{2}dx$,
22. $\int \frac{dx}{\cos^2 4x(5\operatorname{tg} 4x+3)}$,
23. $\int \frac{\sin 3x\,dx}{(9+4\cos 3x)^3}$,
24. $\int \frac{x^3dx}{\sqrt{x^3+16}}$,
25. $\int \frac{\sqrt{16-\operatorname{tg} 6x}}{\cos^2 6x}dx$,
26. $\int \sqrt{1-e^{2x}}dx$,
27. $\int (5x-2)e^{3x}dx$,
28. $\int \operatorname{arctg}\sqrt{4x-1}\,dx$,
29. $\int \ln(4x^2+1)\,dx$,
30. $\int \frac{x\cos 2x\,dx}{\sin^3 2x}$,
31. $\int x\cdot\arcsin\frac{1}{x}dx$,
32. $\int e^{-4x}\cdot\cos 3x\,dx$,
33. $\int \frac{x^2dx}{x^2+6}$,
34. $\int \frac{(x+2)\,dx}{x^2+2x+2}$,
35. $\int \frac{dx}{81-x^4}$,
36. $\int \frac{x^3+1}{x^2-x}dx$,
37. $\int \frac{x^3dx}{x^3-8}$,
38. $\int \frac{8+4x-2x^3}{(x^2+4x+4)(x-2)}dx$,
39. $\int \frac{x^3+x+2}{(x^2-4)(x^2-2x)}dx$,
40. $\int \frac{\cos x\,dx}{1+3\cos x}$,
41. $\int \frac{\cos^3 x\,dx}{\sin^4 x}$,
42. $\int \cos 2x\cdot\cos 3x\,dx$,
43. $\int \sin^4 x\,dx$,
44. $\int \frac{1+\operatorname{tg} x}{\sin 2x}dx$,
45. $\int \operatorname{tg}^5 2x\,dx$,
46. $\int \frac{\sqrt{x}\,dx}{x-\sqrt[3]{x^2}}$,
47. $\int \frac{dx}{x^4\sqrt{x^2-1}}$,
48. $\int \frac{dx}{\sqrt{x^2+8x-3}}$,
49. $\int \frac{x^2dx}{\sqrt{16-x^2}}$,
50. $\int \frac{x^3dx}{\sqrt{x^2+4}}$.

Вариант 7.

Проинтегрировать:

1. $\int \cos\frac{x}{7}\,dx$,
2. $\int \frac{\sqrt[3]{\ln(x+1)}}{x+1}\,dx$,
3. $\int \frac{(3x+2)\,dx}{3x^2+4x-10}$,
4. $\int x\,e^{7x^2}\,dx$,
5. $\int \frac{dx}{\sqrt{1-x^2}\,\arcsin^5 x}$,
6. $\int \frac{x\,dx}{x^4+16}$,
7. $\int \frac{(3x-4)\,dx}{\sqrt{5-x^2}}$,
8. $\int \sin^4 7x\cdot\cos 7x\,dx$,
9. $\int \frac{x\,dx}{\sqrt[4]{\left(4-x^2\right)^3}}$,
10. $\int \frac{\cos\frac{x}{2}}{\sin^7\frac{x}{2}}\,dx$,
11. $\int \frac{dx}{x\left(9+\ln^2 x\right)}$,
12. $\int \frac{x^2dx}{\sqrt{9-16x^6}}$,
13. $\int \frac{\sin 3x\,dx}{\sqrt{16+\cos 3x}}$,
14. $\int s\sqrt[4]{9s^2-49}\,ds$,
15. $\int \frac{dt}{t\sqrt{16-\ln^2 t}}$,
16. $\int \frac{e^{2x}dx}{e^{4x}+25}$,
17. $\int 7^{\sin^2 2x}\cdot\sin 4x\,dx$,
18. $\int \frac{\cos 5x\,dx}{\sqrt[3]{\sin^2 5x}}$,
19. $\int \frac{1-\operatorname{arctg}^3 3x}{1+9x^2}\,dx$,
20. $\int \frac{e^{\frac{x}{7}}dx}{\sqrt{9+e^{\frac{x}{7}}}}$,
21. $\int \frac{dx}{\sin 6x}$,
22. $\int \frac{x^2dx}{\cos^2\left(4+x^3\right)}$,
23. $\int \frac{\cos\frac{x}{4}}{\sqrt{16+\sin^2\frac{x}{4}}}\,dx$,
24. $\int \frac{dx}{(x+7)\sqrt{4-\ln^2(x+7)}}$,
25. $\int \frac{\operatorname{tg}^4\frac{x}{5}}{\cos^2\frac{x}{5}}\,dx$,
26. $\int \frac{dx}{\sqrt{1-e^{4x}}}$,
27. $\int \frac{\ln x\,dx}{x^3}$,
28. $\int \left(1-4x^2\right)\sin 4x\,dx$,
29. $\int \operatorname{arctg}\sqrt{6x-1}\,dx$,
30. $\int \cos(\ln x)\,dx$,
31. $\int \frac{x\,dx}{\sin^2 6x}$,
32. $\int e^{3x}\sin 4x\,dx$,
33. $\int \frac{dx}{(3x-1)^5}$,
34. $\int \frac{3x-1}{x^2-4x+8}\,dx$,
35. $\int \frac{x-1}{x^2-4}\,dx$,
36. $\int \frac{x^5-x^3+1}{x^2-x}\,dx$,
37. $\int \frac{dx}{x\left(x^2+1\right)(x+1)}$,
38. $\int \frac{x^3+2x^2+10x}{(x+1)^2\left(x^2-x+1\right)}\,dx$,
39. $\int \frac{3x\,dx}{(x-2)\left(x^2+2x+1\right)}$,
40. $\int \frac{1-\sin x}{1+\sin x}\,dx$,
41. $\int \sin\frac{x}{3}\cdot\sin\frac{2x}{3}\,dx$,
42. $\int \frac{\cos^2 x\,dx}{\sin^6 x}$,
43. $\int \sin^5 x\,dx$,
44. $\int \frac{dx}{3+\cos^2 x}$,
45. $\int \frac{\cos x+\sin x}{\sin 2x}\,dx$,
46. $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}$,
47. $\int \frac{dx}{x^4\sqrt{9+x^2}}$,
48. $\int \frac{x\,dx}{\sqrt{x^2-2x+5}}$,
49. $\int \sqrt{4-x^2}\,dx$,
50. $\int \frac{dx}{x\sqrt{9x^2+6x+8}}$.

Вариант 8.
Проинтегрировать:

1. $\int \frac{e^{8t}dt}{1-e^{8t}}$,
2. $\int \frac{dx}{\sin^2 \frac{x}{6}}$,
3. $\int \frac{7\operatorname{arctg}^4 x\, dx}{1+x^2}$,
4. $\int \frac{x\, dx}{\sqrt{25+9x^4}}$,
5. $\int x\, e^{-\frac{x^2}{7}} dx$,
6. $\int \frac{x^2-4x+3}{\sqrt{x}} dx$,
7. $\int \frac{(2x-3)\, dx}{\sqrt{2x^2-6x+8}}$,
8. $\int \cos^3 8x \cdot \sin 8x\, dx$,
9. $\int \frac{\sqrt[3]{8+\ln x}\, dx}{x}$,
10. $\int \frac{\cos\frac{x}{8} dx}{\sin^5 \frac{x}{8}}$,
11. $\int \frac{dx}{\cos^2 4x \cdot \operatorname{tg} 4x}$,
12. $\int \frac{e^{\arcsin 4x}}{\sqrt{1-16x^2}} dx$,
13. $\int \frac{e^{7x}dx}{\sqrt{e^{7x}+3}}$,
14. $\int \frac{\sin 4x\, dx}{\sqrt{3\cos^2 2x+4}}$,
15. $\int \frac{\cos 5x\, dx}{\sqrt[3]{3+\sin 5x}}$,
16. $\int \frac{(6x+5)dx}{x^2+8}$,
17. $\int \frac{tdt}{\sqrt{25-16t^2}}$,
18. $\int \frac{\cos 8x\, dx}{\sqrt{25-\sin^2 8x}}$,
19. $\int \frac{e^{5x}dx}{\sqrt{e^{5x}-36}}$,
20. $\int \frac{\ln x\, dx}{x\left(\ln^2 x-9\right)}$,
21. $\int \frac{\sqrt{1+\ln(x+8)}}{x+8} dx$,
22. $\int \frac{\cos 8x\, dx}{81+\sin^2 8x}$,
23. $\int \frac{3x-\operatorname{arctg}^4 2x}{1+4x^2} dx$,
24. $\int \frac{e^{-\frac{x}{2}}dx}{\left(e^{-\frac{x}{2}}+9\right)^5}$,
25. $\int \frac{2^{\operatorname{tg} x}dx}{\cos^2 x}$,
26. $\int \sqrt{e^{4x}-1}\, dx$,
27. $\int \ln\left(x^2-9\right)dx$,
28. $\int \frac{x\cos 4x\, dx}{\sin^3 4x}$,
29. $\int e^{-3x}(2-9x)\, dx$,
30. $\int \operatorname{arctg}\sqrt{2x-1}\, dx$,
31. $\int (5x+6)\cos 2x\, dx$,
32. $\int e^{-x}\cdot \cos 5x\, dx$,
33. $\int \frac{x^2dx}{x^2+9}$,
34. $\int \frac{(x+2)\, dx}{x^2+2x+5}$,
35. $\int \frac{x\, dx}{(x-2)(x-1)^2}$,
36. $\int \frac{x^2dx}{x^4-81}$,
37. $\int \frac{3x\, dx}{x^3+x^2-4x-4}$,
38. $\int \frac{x^3dx}{\left(x^2-4\right)(x+2)}$,
39. $\int \frac{3x\, dx}{x^3+1}$,
40. $\int \sin 2x \cdot \sin 5x\, dx$,
41. $\int \frac{dx}{1+\sin x+\cos x}$,
42. $\int \frac{dx}{\cos x \cdot \sin^3 x}$,
43. $\int \frac{dx}{\sin^2 x+\sin x\cos x-2\cos^2 x}$,
44. $\int \sin^2 3x \cdot \cos^2 3x\, dx$,
45. $\int \frac{\sin 2x\, dx}{\cos^3 x}$,
46. $\int \frac{\sqrt[3]{x^2}-\sqrt[4]{x}}{\sqrt{x}} dx$,
47. $\int \frac{dx}{x\sqrt{x^2-9}}$,
48. $\int \frac{dx}{\sqrt{x^2+2x+5}}$,
49. $\int \frac{\sqrt{4+x^2}}{x^6} dx$,
50. $\int \frac{dx}{x\sqrt{10+6x-x^2}}$.

Вариант 9.

Проинтегрировать:

1. $\int \frac{x\,dx}{x^2+81}$,
2. $\int \frac{dx}{\sqrt{25-81x^2}}$,
3. $\int \frac{\sqrt{x}-x^4e^{-x}+3^3}{x^4}dx$
4. $\int \frac{dx}{(x+9)\ln(x+9)}$,
5. $\int \frac{dx}{\sqrt{16+25x^2}}$,
6. $\int \frac{dx}{\sin 7x}$,
7. $\int \frac{dx}{\cos^2\left(\frac{\pi}{3}-x\right)}$,
8. $\int x^2\cdot\sin\left(1+5x^3\right)dx$,
9. $\int \frac{5x-3}{\sqrt{x^2+49}}dx$,
10. $\int x^3\sqrt{10+x^4}dx$,
11. $\int \cos 2t\cdot e^{\sin 2t}dt$,
12. $\int \frac{v\,dv}{81v^2-4}$,
13. $\int \frac{4dx}{\sqrt{1-x^2}\,\arcsin^4 x}$,
14. $\int \frac{(4x+3)dx}{\sqrt{4x^2+6x+9}}$,
15. $\int \frac{(x+7)dx}{49x^2+81}$,
16. $\int \operatorname{ctg}\frac{x}{7}dx$,
17. $\int \cos^6 9x\cdot\sin 9x\,dx$,
18. $\int \frac{\cos\frac{x}{3}dx}{\sqrt{\sin^2\frac{x}{3}+49}}$,
19. $\int \frac{\sqrt{9-\ln^2 x}\cdot\ln x}{x}dx$,
20. $\int \frac{e^{-2x}dx}{e^{-4x}+81}$,
21. $\int \frac{x-\arcsin^3 5x}{\sqrt{1-25x^2}}dx$,
22. $\int \frac{\sin 6x\,dx}{\sqrt[3]{\cos^2 6x}}$,
23. $\int \frac{e^{9x}dx}{\sqrt{e^{9x}+25}}$,
24. $\int \frac{dx}{x(81+\ln^2 x)}$?
25. $\int \frac{e^{\operatorname{arctg}\,7x}dx}{1+49x^2}$,
26. $\int \frac{dx}{\sqrt{e^{2x}-4}}$,
27. $\int (x+2)\cos 2x\,dx$,
28. $\frac{\ln(\cos 3x)\,dx}{\sin^2 3x}$,
29. $\int \sqrt{x}\cdot\ln^2 x\,dx$,
30. $\int \operatorname{arctg}\sqrt{3x-1}\,dx$,
31. $\int \frac{x\,dx}{\sin^2 8x}$,
32. $\int e^{5x}\cos 2x\,dx$,
33. $\int \frac{dx}{(9x-1)^6}$,
34. $\int \frac{(2x-3)\,dx}{x^2+2x-7}$,
35. $\int \frac{x\,dx}{x^2+3x+2}$,
36. $\int \frac{x^4dx}{x^4-81}$,
37. $\int \frac{x^3dx}{(x^2-4)(x+1)}$,
38. $\int \frac{3x\,dx}{(x-2)(x^2+2x+1)}$,
39. $\int \frac{(x^2+x-2)dx}{(x-2)(x^2-2x+4)}$,
40. $\int \frac{dx}{5+\sin x}$,
41. $\int \sin 2x\cdot\cos 5x\,dx$,
42. $\int \frac{\sin^4 x\,dx}{\cos^2 x}$,
43. $\int \cos^3 x\cdot\sin^4 x\,dx$,
44. $\int \frac{dx}{2\sin^2 x-3\cos^2 x}$,
45. $\int \frac{dx}{\sin^4 x\cdot\cos^4 x}$,
46. $\int \frac{dx}{x\sqrt{2x-9}}$,
47. $\int \frac{dx}{x\sqrt{16-x^2}}$,
48. $\int \frac{dx}{(x^2+9)\sqrt{x^2+9}}$,
49. $\int \frac{dx}{\sqrt{x^2+6x+10}}$,
50. $\int \frac{dx}{x\sqrt{x^2+2x-5}}$.

Вариант 10.

Проинтегрировать:

1. $\int \frac{x\,dx}{\sqrt{25+x^2}}$,
2. $\int \frac{x\,dx}{x^2+100}$,
3. $\int \frac{dx}{\sqrt{25-81x^2}}$,
4. $\int 4t^2 \cdot e^{t^3}\,dt$,
5. $\int \frac{2dx}{\left(1+9x^2\right)\operatorname{arctg}^3 3x}$,
6. $\int x^2 \cdot \cos\left(x^3+\frac{\pi}{6}\right)dx$,
7. $\int \sqrt[6]{(4-10x)^6\,dx}$,
8. $\int \operatorname{ctg}\frac{x}{10}\,dx$,
9. $\int \frac{(3+x)^2\,dx}{\sqrt[3]{x}}$,
10. $\int \frac{x^2 dx}{x^6+9}$,
11. $\int \sin^6\frac{5x}{2}\cdot\cos\frac{5x}{2}\,dx$,
12. $\int \frac{3x-\arccos^3 4x}{\sqrt{1-16x^2}}\,dx$,
13. $\int \frac{(x+5)\,dx}{x^2+10x-16}$,
14. $\int \frac{(4x-3)\,dx}{\sqrt{4x^2+25}}$,
15. $\int \frac{ds}{81s^2+16}$,
16. $\int \sin 8x\sqrt{81-\sin^2 4x}\,dx$,
17. $\int \frac{e^{\frac{x}{10}}dx}{\sqrt{16-e^{\frac{x}{10}}}}$,
18. $\int \frac{dx}{x\left(\ln^2 x-100\right)}$,
19. $\int \frac{\sqrt[4]{81-\ln x}\,dx}{x}$,
20. $\int \frac{e^{10x}dx}{\left(e^{10x}+13\right)^7}$,
21. $\int \frac{dx}{(x+10)\ln^2(x+10)}$,
22. $\int \frac{dx}{\cos^2 6x(\operatorname{tg} 6x+10)}$,
23. $\int \frac{e^{5x}dx}{\sqrt{81-e^{10x}}}$,
24. $\int \frac{\cos 7x\,dx}{16+\sin^2 7x}$,
25. $\int \frac{\sqrt[6]{\operatorname{arctg}\,3x}\,dx}{1+9x^2}$,
26. $\int \frac{e^{4x}dx}{e^{2x}+1}$,
27. $\int (2-4x)\sin 2x\,dx$,
28. $\int \operatorname{arctg}\sqrt{5x-1}\,dx$,
29. $\int e^{-2x}(4x-3)\,dx$,
30. $\int \frac{x\,dx}{\cos^2 8x}$,
31. $\int \frac{\ln\cos 6x\,dx}{\cos^2 6x}$,
32. $\int e^{10x}\cdot\cos 2x\,dx$,
33. $\int \frac{x^2 dx}{x^2+8}$,
34. $\int \frac{(4x-1)\,dx}{4x^2-4x+5}$,
35. $\int \frac{3x\,dx}{(x-2)(x+2)^2}$,
36. $\int \frac{dx}{(x+3)^2\left(x^2+1\right)}$,
37. $\int \frac{x^3+4}{x^2-4}\,dx$,
38. $\int \frac{dx}{x^4+2x^3+x^2}$,
39. $\int \frac{(8+4x)\,dx}{(x-2)\left(x^2+2x-4\right)}$,
40. $\int \frac{dx}{2\sin x+3\cos x-5}$,
41. $\int \sin^4\frac{x}{2}\cdot\cos^5\frac{x}{2}\,dx$,
42. $\int \cos x\cdot\cos 3x\,dx$,
43. $\int \frac{\sin^2 2x}{\cos^6 2x}\,dx$,
44. $\int \frac{dx}{(2\sin x+3\cos x)^2}$,
45. $\int \operatorname{tg}^4\frac{x}{2}\,dx$,
46. $\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}\,dx$,
47. $\int \frac{dx}{x\sqrt{6x+8-x^2}}$,
48. $\int \frac{x^3 dx}{\sqrt{\left(9+x^2\right)^3}}$,
49. $\int x^3\sqrt{\left(25-x^2\right)^3}\,dx$,
50. $\int \frac{x\,dx}{\sqrt{x^2-2x-5}}$.

Вариант 11.

Проинтегрировать:

1. $\int \frac{5x^2 dx}{x^3+6}$,
2. $\int \cos 10x \cdot e^{\sin 10x} dx$,
3. $\int \sqrt[4]{7x+10}\, dx$,
4. $\int \frac{\cos(\ln x) dx}{x}$,
5. $\int \frac{x\, dx}{\sqrt{81+16x^4}}$,
6. $\int \frac{t\, dt}{\sqrt{100-81t^4}}$,
7. $\int \operatorname{tg} 11x\, dx$,
8. $\int \frac{x^3-x^2+4}{\sqrt{x}} dx$,
9. $\int \frac{\sin 10x\, dx}{\sqrt[3]{9+3\cos 10x}}$,
10. $\int \frac{e^x dx}{47+e^x}$,
11. $\int \frac{x+\operatorname{arctg}^4 5x\, dx}{1+25x^2}$,
12. $\int \frac{4t-3}{t^2-25} dt$,
13. $\int \frac{e^{-5x} dx}{\sqrt{16-e^{-10x}}}$,
14. $\int \frac{v\, dv}{121v^2+3}$,
15. $\int \frac{\sin 10x\, dx}{1+\sin^2 5x}$,
16. $\int \frac{dx}{x\sqrt{10+\ln x}}$,
17. $\int \frac{e^{7y} dy}{\sqrt{49-e^{14y}}}$,
18. $\int \frac{5^{\operatorname{arctg}\, 3x}}{1+9x^2} dx$,
19. $\int e^{11x} \sqrt{5-e^{11x}}\, dx$,
20. $\int \frac{(2x-3) dx}{\sqrt{2x^2-6x+10}}$,
21. $\int \sin^8 7x \cdot \cos 7x\, dx$,
22. $\int \frac{\cos 9x\, dx}{(10-\sin 9x)^4}$,
23. $\int \frac{dx}{\cos^2 11x(3\operatorname{tg} 11x+4)}$,
24. $\int \frac{x^3 dx}{\sqrt{9x^3+100}}$,
25. $\int \frac{\sqrt{3-\operatorname{tg}\frac{x}{2}}}{\cos^2 \frac{x}{2}} dx$,
26. $\int \frac{e^{4x} dx}{\sqrt{e^{2x}-1}}$,
27. $\int (3x-2)\cos 5x\, dx$,
28. $\int \ln(x^2+9) dx$,
29. $\int (x^2+4) e^{-x} dx$,
30. $\int \operatorname{arctg} \sqrt{16x-1}\, dx$,
31. $\int \frac{x \sin 6x\, dx}{\cos^3 6x}$,
32. $\int e^{2x} \cdot \sin 10x\, dx$,
33. $\int \frac{dx}{(1-10x)^7}$,
34. $\int \frac{(5x+1) dx}{x^2-4x+1}$,
35. $\int \frac{x^3+2}{x^4+3x^2} dx$,
36. $\int \frac{x^4+2x+2}{x^4-1} dx$,
37. $\int \frac{dx}{(x+3)^2(x^2+4)}$,
38. $\int \frac{(2x-1) dx}{x^4+4x^3+4x^2}$,
39. $\int \frac{4x\, dx}{(x-1)(x^2+4x+1)}$,
40. $\int \frac{dx}{2\sin x+3\cos x+4}$,
41. $\int \sin x \cdot \sin 3x\, dx$,
42. $\int \frac{\sin^5 3x\, dx}{\cos^2 3x}$,
43. $\int 2\cos^4 2x\, dx$,
44. $\int \frac{dx}{5\sin^2 x-4\cos^2 x}$,
45. $\int \frac{\sin 2x\, dx}{7+3\cos^2 2x}$,
46. $\int \frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}} dx$,
47. $\int \frac{dx}{x^2\sqrt{25-x^2}}$,
48. $\int \frac{3x\, dx}{\sqrt{x^2+4x-10}}$,
49. $\int \frac{(x+1) dx}{\sqrt{(9+x^2)^3}}$,
50. $\int \frac{dx}{x\sqrt{x^2-2x+7}}$.

Вариант 12.

Проинтегрировать:

1. $\int \sin \frac{x}{12}\,dx,$
2. $\int \frac{\sqrt[4]{\ln(x+3)}\,dx}{x+3},$
3. $\int \frac{(5x+4)\,dx}{5x^2+8x+4},$
4. $\int x^2 \cdot e^{10x^3}\,dx,$
5. $\int \frac{dx}{\sqrt{1-4x^2}\,\arcsin^2 2x},$
6. $\int \frac{x\,dx}{x^4+144},$
7. $\int \frac{(6x-5)\,dx}{\sqrt{16+x^2}},$
8. $\int \sin^7 10x \cdot \cos 10x\,dx,$
9. $\int \frac{x\,dx}{\sqrt[5]{(16+x^2)^3}},$
10. $\int \frac{\sin 12x\,dx}{\cos^8 12x},$
11. $\int \frac{dx}{x(121+\ln^2 x)},$
12. $\int \frac{x^2 dx}{\sqrt{64-36x^6}},$
13. $\int \frac{\cos 15x\,dx}{\sqrt{1+\sin 15x}},$
14. $\int s\sqrt[3]{81s^2-41}\,ds,$
15. $\int \frac{dt}{t\sqrt{36-\ln^2 t}},$
16. $\int \frac{e^{-10x}dx}{e^{-20x}+16},$
17. $\int 8^{\sin^2 3x} \cdot \sin 6x\,dx,$
18. $\int \frac{\sin \frac{x}{10}\,dx}{\sqrt[4]{\cos^3 \frac{x}{10}}},$
19. $\int \frac{4+\operatorname{arctg}^5 4x}{1+16x^2}\,dx,$
20. $\int \frac{e^{11x}dx}{\sqrt{5+e^{11x}}},$
21. $\int \frac{dx}{\cos 4x},$
22. $\int \frac{x^4 dx}{\cos^2(4+x^5)},$
23. $\int \frac{\sin 8x\,dx}{\sqrt{49+\cos^2 8x}},$
24. $\int \frac{\operatorname{ctg}^2 x}{\sin^2 x}\,dx,$
25. $\int \frac{dx}{(x+9)\sqrt{4-\ln^2(x+9)}},$
26. $\int \frac{dx}{e^{2x}-36},$
27. $\int \ln(x^2+16)\,dx,$
28. $\int x \cdot \sin^2 x\,dx,$
29. $\int \operatorname{arctg}\sqrt{7x-1}\,dx,$
30. $\int \frac{\ln(\cos 8x)}{\sin^2 8x}\,dx,$
31. $\int (5x^2+2)e^{3x}dx,$
32. $\int e^{-4x} \cdot \cos 5x\,dx,$
33. $\int \frac{x^2 dx}{x^2+10},$
34. $\int \frac{3x+4}{x^2+6x+13}\,dx,$
35. $\int \frac{dx}{x^2(x+3)^2},$
36. $\int \frac{(8x+8)\,dx}{x^4-16},$
37. $\int \frac{x^4 dx}{x^4+9x^2},$
38. $\int \frac{(x^2+2)\,dx}{(x^2-9)(x^2-2x)},$
39. $\int \frac{dx}{x^3-8},$
40. $\int \frac{dx}{5-4\cos x+3\sin x},$
41. $\int \sin x \cdot \cos 5x\,dx,$
42. $\int \operatorname{ctg}^4 x\,dx,$
43. $\int \frac{\sin^5 2x\,dx}{\cos^2 2x},$
44. $\int (\sin^4 x \cdot \cos^2 x + \cos^3 x)dx,$
45. $\int \frac{dx}{(3\sin x - 2\cos x)^2},$
46. $\int \frac{\sqrt{x+5}\,dx}{1+\sqrt[3]{x+5}},$
47. $\int \frac{dx}{x\sqrt{x^2-16}},$
48. $\int \frac{x\,dx}{\sqrt{x^2-4x-12}},$
49. $\int \frac{\sqrt{x^2+81}\,dx}{x^2},$
50. $\int \frac{dx}{(81-x^2)^{\frac{3}{2}}}.$

Вариант 13.

Проинтегрировать:

1. $\int \frac{7x\,dx}{x^2+8}$,
2. $\int \cos\left(\frac{t}{2}\right) e^{\sin\frac{t}{2}} dt$,
3. $\int \sqrt{14x-1}\,dx$,
4. $\int \frac{\cos(\ln x)}{5x} dx$,
5. $\int \frac{d\varphi}{\sqrt{14-3\varphi^2}}$,
6. $\int \frac{x\,dx}{\sqrt{9+8x^4}}$,
7. $\int \frac{dx}{tg11x}$,
8. $\int \frac{2x^3-4x^4+1}{x^4} dx$,
9. $\int \frac{\sin x\,dx}{\sqrt{11+2\cos x}}$,
10. $\int \frac{e^x dx}{13+4e^x}$,
11. $\int \frac{\sqrt[5]{\operatorname{arctg}\,2x}}{1+4x^2} dx$,
12. $\int \frac{e^{2x} dx}{\sqrt{19+4e^{4x}}}$,
13. $\int \frac{x\,dx}{\sqrt{49x^2-13}}$,
14. $\int \frac{\sin 3x\,dx}{25-\cos^2 3x}$,
15. $\int \frac{dx}{x\left(10+\ln^2 x\right)}$,
16. $\int e^{4x}\cdot\sqrt{e^{4x}+25}\,dx$,
17. $\int \frac{6^{\operatorname{arctg} y}}{1+y^2} dy$,
18. $\int \frac{dx}{\cos^2 x\left(13\,tgx+4\right)}$,
19. $\int \frac{5\operatorname{tg}^5 \frac{x}{4}}{\cos^2 \frac{x}{4}} dx$,
20. $\int \frac{(x-2)\,dx}{\sqrt{x^2-4x+15}}$,
21. $\int \cos^3 4x\cdot\sin 4x\,dx$,
22. $\int \frac{(4x+3)\,dx}{x^2+4}$,
23. $\int \frac{\cos\frac{x}{2} dx}{3-\sin\frac{x}{2}}$,
24. $\int \frac{x^3 dx}{\sqrt{16-9x^8}}$,
25. $\int \sin\frac{x}{3}\cdot\sqrt{\left(16-\cos\frac{x}{3}\right)^3} dx$,
26. $\int \frac{e^{2x} dx}{3e^{2x}+4}$,
27. $\int \ln\left(x^2+3\right) dx$,
28. $\int \frac{x\cdot\cos 5x\,dx}{\sin^2 5x}$,
29. $\int x\cdot\operatorname{tg}^2 3x\,dx$,
30. $\int \frac{\operatorname{arctg}\sqrt{x+1}}{\sqrt{x+1}} dx$,
31. $\int \frac{\ln\cos 12x}{\sin^2 12x} dx$,
32. $\int e^{-6x}\cdot\sin 3x\,dx$,
33. $\int \frac{x^2 dx}{x^2+10}$,
34. $\int \frac{(x+4)\,dx}{x^2-4x-7}$,
35. $\int \frac{3x\,dx}{(x-1)^2(x+1)}$,
36. $\int \frac{x^2 dx}{x^4-81}$,
37. $\int \frac{(x+1)\,dx}{x^4+4x^3+4x^2}$,
38. $\int \frac{\left(x^5+2x^3+7\right)dx}{x^3+2x}$,
39. $\int \frac{3x\,dx}{x^3+8}$,
40. $\int \sin^3\frac{x}{2}\cdot\cos^2\frac{x}{2} dx$,
41. $\int \frac{dx}{\sin^4 2x}$,
42. $\int \cos x\cdot\cos 5x\,dx$,
43. $\int \frac{dx}{3+7\cos x}$,
44. $\int \frac{\operatorname{tg}^2 x-2}{\cos 2x} dx$,
45. $\int \frac{\cos^3 x-4}{\sin^2 x} dx$,
46. $\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$,
47. $\int \frac{dx}{x\cdot\sqrt{x^2-64}}$,
48. $\int \frac{dx}{\sqrt{x^2-2x-3}}$,
49. $\int \frac{\sqrt{x^2+16}}{x^2} dx$,
50. $\int \frac{dx}{(x-1)\sqrt{x^2-2x+2}}$.

Вариант 14.

Проинтегрировать:

1. $\int \left(x^2+3\right)^4 x\,dx$,
2. $\int \frac{x^3 dx}{\sqrt[3]{x^4+1}}$,
3. $\int \frac{\cos t\,dt}{\sqrt{2\sin t+3}}$,
4. $\int \frac{3-\sqrt{5+x^2}}{5+x^2}dx$,
5. $\int \sin\left(2x-1\right)dx$,
6. $\int \cos\frac{x}{3}dx$,
7. $\int e^{x^3+1}\cdot x^2\,dx$,
8. $\int \frac{\left(3t^2-1\right)dt}{\sqrt{1-t+t^3}}$,
9. $\int \frac{e^x dx}{\sqrt{e^{2x}+5}}$,
10. $\int \frac{dx}{3x^2-7}$,
11. $\int \frac{s^2 ds}{\sqrt{2-s^6}}$,
12. $\int 2^{\operatorname{tg} 2x}\cdot\frac{dx}{\cos^2 2x}$,
13. $\int \frac{dx}{2x-1}$,
14. $\int \frac{e^{2t}dt}{3e^{2t}+4}$,
15. $\int \frac{dx}{\sqrt{x}\cos^2\sqrt{x}}$,
16. $\int \frac{3x\,dx}{2+x^2}$,
17. $\int \frac{\sin z\,dz}{3+\cos^2 z}$,
18. $\int \frac{dx}{\sin^2 4x}$,
19. $\int \frac{dz}{\sqrt{1-5z^2}}$,
20. $\int \frac{x\,dx}{\sqrt{x^4-3}}$,
21. $\int \operatorname{tg} 6x\,dx$,
22. $\int \frac{\ln^3 2x\,dx}{x}$,
23. $\int \frac{\left(\sin x-\cos x\right)dx}{\left(\cos x+\sin x\right)^4}$,
24. $\int \frac{dx}{x\left(\ln^2 x-4\right)}$,
25. $\int \left(\operatorname{tg} x+\operatorname{ctg} x\right)^2 dx$,
26. $\int \frac{dx}{1-e^x}$,
27. $\int e^{2x}\left(3+9x\right)dx$,
28. $\int \left(1+5x^2\right)\cos 2x\,dx$,
29. $\int e^{-x}\sin x\,dx$,
30. $\int \operatorname{arctg}\sqrt{12x-1}\,dx$,
31. $\int \ln^2 x\,dx$,
32. $\int \frac{x\sin x}{\cos^3 x}dx$,
33. $\int \frac{dx}{\left(x-1\right)^2}$,
34. $\int \frac{\left(x+1\right)dx}{x^2+4x+5}$,
35. $\int \frac{3x^2 dx}{x^2+1}$,
36. $\int \frac{3x^3+x^2+5x+1}{x^3+x}dx$,
37. $\int \frac{\left(11x+16\right)dx}{\left(x-1\right)\left(x+2\right)^2}$,
38. $\int \frac{3x\,dx}{x^3+2x^2+2x+1}$,
39. $\int \frac{\left(2x-1\right)dx}{x^4+4x^3+4x^2}$,
40. $\int \sin 9x\cdot\sin x\,dx$,
41. $\int \frac{\sin^5 x\,dx}{\cos^2 x}$,
42. $\int \frac{dx}{\cos x+2\sin x+3}$,
43. $\int \frac{\operatorname{tg} x\,dx}{\sin^2 x-5\cos^2 x+4}$,
44. $\int \frac{dx}{\cos^4 x}$,
45. $\int \frac{\cos^3 x\,dx}{\sin x-3}$,
46. $\int \frac{\left(1-3x\right)dx}{\sqrt{4x^2+4x+17}}$,
47. $\int \frac{dx}{\sqrt[3]{2x-1}+\sqrt{2x-1}}$,
48. $\int \frac{\left(1+\sqrt[3]{x^2}\right)^{-1}}{\sqrt[3]{x^2}}dx$,
49. $\int \frac{\sqrt{x^2-4}}{x^4}dx$,
50. $\int \frac{dx}{\left(x+1\right)^2\sqrt{2x^2+2x+1}}$.

Вариант 15.

Проинтегрировать:

1. 1. $\int \frac{x^2 dx}{\sqrt{x^3+1}}$,
2. $\int t^3\sqrt{2t^4-7}\,dt$,
3. $\int 7e^{x^2+2x+3}(x+1)\,dx$,
4. $\int 2^x \cdot 3^x dx$,
5. $\int \sin^7 2x \cdot \cos 2x\,dx$,
6. $\int (\sin 2x + \cos 2x)^2 dx$,
7. $\int \cos\frac{1}{x}\cdot\frac{dx}{x^2}$,
8. $\int \frac{e^{2x}dx}{5-e^{2x}}$,
9. $\int \operatorname{ctg} 2x\,dx$,
10. $\int \frac{dx}{\sin^2(9x+1)}$,
11. $\int \frac{dx}{7-9x}$,
12. $\int \frac{dx}{\sqrt{4+3x^2}}$,
13. $\int \frac{x\,dx}{\sqrt{5x^2-3}}$,
14. $\int \frac{dx}{\cos^2 7x}$,
15. $\int \frac{dx}{x^2-3}$,
16. $\int \frac{dx}{\sqrt{9-25x^2}}$,
17. $\int \frac{dx}{(x-1)\sqrt{4-\ln^2(x-1)}}$,
18. $\int \frac{e^x dx}{2+e^{2x}}$,
19. $\int \frac{\sin x \cdot \cos x\,dx}{\sqrt{\cos^2 x - \sin^2 x}}$,
20. $\int \frac{(x^2+1)dx}{(x^3+3x+1)^5}$,
21. $\int \frac{\sin 2x\,dx}{9-\cos^2 2x}$,
22. $\int \frac{xdx}{3x^2+2}$,
23. $\int \frac{\sqrt{3-x^2+2\sqrt{3+x^2}}}{\sqrt{9-x^4}}dx$,
24. $\int \frac{dx}{5x^2+6}$,
25. $\int \frac{3dx}{\sqrt{1-4x^2}\arcsin 2x}$,
26. $\int \frac{dz}{1+e^z}$,
27. $\int \frac{xdx}{\cos^2 x}$,
28. $\int e^{3x}(5x-2)\,dx$,
29. $\int (x^2+3x)\cos x\,dx$,
30. $\int e^{-x}\cos 2x\,dx$,
31. $\int \frac{x\arcsin 2x}{\sqrt{1-4x^2}}dx$,
32. $\int \ln\left(x+\sqrt{4+x^2}\right)dx$,
33. $\int \frac{dx}{(x+1)^3}$,
34. $\int \frac{(3x-1)dx}{x^2-4x+5}$,
35. $\int \frac{x^2 dx}{x^2-3}$,
36. $\int \frac{5x^4+9x^2-22x-8}{x^3-4x}dx$,
37. $\int \frac{3x^3+x+46}{(x-1)^2(x^2+9)}dx$,
38. $\int \frac{4x\,dx}{x^3-5x-12}$,
39. $\int \frac{dx}{x^2(x+1)^2}$,
40. $\int \sin 12x \cdot \sin 13x\,dx$,
41. $\int \frac{\cos^3 x\,dx}{\sqrt{\sin x}}$,
42. $\int \frac{dx}{8-4\sin x+7\cos x}$,
43. $\int \frac{(6+tgx)dx}{9\sin^2 x+4\cos^2 x}$,
44. $\int \operatorname{tg}^3 x\,dx$,
45. $\int \frac{\sin^2 x\,dx}{\cos^6 x}$,
46. $\int \frac{(1-x)dx}{\sqrt{x^2-x+1}}$,
47. $\int \frac{dx}{\sqrt{1-3x}-\sqrt[4]{1-3x}}$,
48. $\int \frac{dx}{x^4\sqrt{1+x^4}}$,
49. $\int \frac{dx}{x\sqrt{3x^2-2x+1}}$,
50. $\int \frac{dx}{\sqrt{(1+x^2)^3}}$.

Вариант 16.

Проинтегрировать:

1. $\int \left(2x^3+1\right)^4 x^2 dx$,
2. $\int \frac{\sqrt[4]{\operatorname{arctg}\ x}\, dx}{1+x^2}$,
3. $\int \frac{dx}{\sin^2 6x}$,
4. $\int \sin\left(3x-2\right) \mathrm{dx}$,
5. $\int \left(e^{-2x}+e^{5x}\right) dx$,
6. $\int \frac{2\cdot 3^x dx}{1+3^x}$,
7. $\int \frac{\left(4x+1\right) dx}{\sqrt{2+x+2x^2}}$,
8. $\int \frac{dx}{\cos^2\left(4x+3\right)}$,
9. $\int \frac{dx}{\sqrt{1-x^2}\ \arcsin\ x}$,
10. $\int \frac{dz}{2-5z^2}$,
11. $\int \cos\sqrt{x}\cdot\frac{dx}{\sqrt{x}}$,
12. $\int \frac{ds}{\sqrt{9s^2-4}}$,
13. $\int \frac{\sin\ 3x\, dx}{\cos^3 3x}$,
14. $\int 2^{\ln x}\frac{dx}{x}$,
15. $\int \frac{dt}{1-3t^2}$,
16. $\int \frac{3x-4}{x^2+6} dx$,
17. $\int \frac{\cos 2x\, dx}{16+\sin^2 2x}$,
18. $\int \frac{e^x dx}{\sqrt{25-e^{2x}}}$,
19. $\int \frac{3x\, dx}{x^4-3}$,
20. $\int \frac{2dt}{\left(t+2\right)\sqrt{3+\ln^2\left(t+2\right)}}$,
21. $\int \frac{\operatorname{arctg}\ 2x}{1+4x^2} dx$,
22. $\int \frac{dx}{\sin^2 3x\sqrt{ctg^2 3x-2}}$,
23. $\int \frac{\left(2\cos x+3\sin\ x\right) dx}{\left(2\sin\ x-3\cos x\right)^3}$,
24. $\int \frac{\sqrt{4+x^2}-3\sqrt{4-x^2}}{\sqrt{16-x^4}}$,
25. $\int \frac{3x^2}{x^3+1}\cdot\ln\left(x^3+1\right) dx$,
26. $\int \frac{e^{2x}dx}{e^x+3}$,
27. $\int \left(3-4x\right)e^{5x} dx$,
28. $\int x^2\cdot\ln\left(1+x\right) dx$,
29. $\int \left(x^2+1\right)\cos x\, dx$,
30. $\int e^{-2x}\cos x\, dx$,
31. $\int \operatorname{arctg}\sqrt{x-1}\, dx$,
32. $\int \frac{\arcsin\ \sqrt{x}}{\sqrt{1-x}} dx$,
33. $\int \frac{dx}{\left(x-2\right)^3}$,
34. $\int \frac{\left(3x+1\right) dx}{x^2+x+2}$,
35. $\int \frac{\left(x^2-1\right) dx}{x^2+1}$,
36. $\int \frac{x^4-3x^2-3x+2}{x^3-x^2-2x} dx$,
37. $\int \frac{\left(x^3-6\right) dx}{x^4+6x^2+8}$,
38. $\int \frac{\left(2x+1\right) dx}{\left(x-2\right)^2\left(x^2+4\right)}$,
39. $\int \frac{x^2 dx}{\left(x+2\right)^2\left(x+4\right)^2}$,
40. $\int \sin\ 3x\cdot\cos 5x\, dx$,
41. $\int \frac{dx}{2\sin\ x+3\cos x-5}$,
42. $\int \frac{\left(4+tgx\right) dx}{2\sin^2 x+18\cos^2 x}$,
43. $\int \frac{dx}{\sqrt[3]{\sin^4 x\cos x}}$,
44. $\int \frac{\cos^5 x\, dx}{\sin^2 x}$,
45. $\int \frac{\sin^3 x\, dx}{2\cos x-1}$,
46. $\int \frac{\left(2x-5\right) dx}{\sqrt{x^2+x+1}}$,
47. $\int \frac{2-\sqrt{x-1}}{\sqrt{x-1}+x-1} dx$,
48. $\int \frac{\sqrt[3]{1-\sqrt[3]{x}}}{\sqrt[3]{x^2}} dx$,
49. $\int x^2\sqrt{1-x^2}\ dx$,
50. $\int \frac{dx}{x^2\sqrt{x^2+5}}$.

Вариант 17.

Проинтегрировать:

1. $\int \sqrt{2x-1}\,dx$,
2. $\int \cos^4 3x \cdot \sin 3x\,dx$,
3. $\int \frac{x\,dx}{\cos^2\left(x^2+1\right)}$,
4. $\int \frac{dx}{\sqrt{1-9x^2}\arccos 3x}$,
5. $\int \cos\left(2e^x\right)e^x dx$,
6. $\int \left(3^{2x}+2^{3x}\right)dx$,
7. $\int \frac{dt}{t\sin^2(\ln t)}$,
8. $\int \sin\sqrt{2x-1}\cdot\frac{dx}{\sqrt{2x-1}}$,
9. $\int \frac{2\ln x+5}{x\sqrt{\ln x}}dx$,
10. $\int \frac{dt}{2-5t}$,
11. $\int \sin t \cdot e^{2\cos t}dt$,
12. $\int \frac{dt}{7+3t^2}$,
13. $\int \frac{\sin 2x\,dx}{\sqrt{9-\cos^4 x}}$,
14. $\int \frac{dt}{x\sqrt{\ln^2 x-16}}$,
15. $\int \frac{(x+1)\,dx}{\sqrt{3x^2+5}}$,
16. $\int \operatorname{ctg} 10x\,dx$,
17. $\int \frac{dx}{3x^2-4}$,
18. $\int (1+2\sin x)^2 dx$,
19. $\int \frac{x\,dx}{2x^2+3}$,
20. $\int \frac{dt}{\left(\operatorname{tg}^2 t+9\right)\cos^2 t}$,
21. $\int \frac{x\,dx}{\sqrt{9-x^4}}$,
22. $\int \frac{s^3 ds}{\sqrt{41+8s^4}}$,
23. $\int \frac{e^{2x}dx}{5-e^{4x}}$,
24. $\int \frac{x^3 dx}{\sqrt{x^8+3}}$,
25. $\int \frac{dx}{\operatorname{ctg} x\ln^2\cos x}$,
26. $\int \frac{\left(1-\sqrt{x}\right)dx}{\sqrt{x}(1+x)}$,
27. $\int (2x-1)e^{-2x}dx$,
28. $\int \left(4x^2-2\right)\cos 2x\,dx$,
29. $\int \arcsin 2x\,dx$,
30. $\int x^2 \ln x\,dx$,
31. $\int \frac{\sqrt{1+x^2}}{x^2}dx$,
32. $\int e^{2x}\cos x\,dx$,
33. $\int \frac{dx}{(x+2)^2}$,
34. $\int \frac{(4x+3)\,dx}{x^2+10x+29}$,
35. $\int \frac{(1+x)^2}{x^2+1}dx$,
36. $\int \frac{x^4+12}{x^3-5x^2+6x}dx$,
37. $\int \frac{4x^2+3x+2}{(x+1)^2\left(x^2+1\right)}dx$,
38. $\int \frac{x^3+x+2}{(x+2)x^3}dx$,
39. $\int \frac{dx}{1+x^3}$,
40. $\int \cos(2x+1)\cdot\cos(2x-1)dx$,
41. $\int \sin^3 x\cdot\cos^{7/2} x\,dx$,
42. $\int \frac{dx}{2+3\sin x}$,
43. $\int \frac{6\sin^2 x\,dx}{3\cos 2x-4}$,
44. $\int \operatorname{tg}^4 x\,dx$,
45. $\int \frac{\cos^3 4x\,dx}{\sin^5 4x}$,
46. $\int \frac{x\,dx}{\sqrt{3-2x-x^2}}$,
47. $\int \frac{dx}{\sqrt{1+3x}-\sqrt[4]{1+3x}}$,
48. $\int \frac{\sqrt[3]{1-\sqrt[4]{x}}\,dx}{\sqrt{x}}$,
49. $\int \frac{\sqrt{x^2+4}}{x^3}dx$,
50. $\int \frac{dx}{x\sqrt{7x^2+6x+1}}$.

Вариант 18.

Проинтегрировать:

1. $\int \frac{(x+2)\,dx}{\sqrt{x^2+2}}$,
2. $\int \frac{2^x dx}{3+2^x}$,
3. $\int e^{t^3+3t+1}\left(t^2+1\right)dt$,
4. $\int \frac{dx}{\sqrt{3-25x^2}}$,
5. $\int \frac{3\arcsin^2 x\,dx}{\sqrt{1-x^2}}$,
6. $\int \operatorname{ctg}\frac{x}{3}dx$,
7. $\int \sin(\ln x)\frac{dx}{x}$,
8. $\int \frac{x^5 dx}{\sqrt{3x^6+2}}$,
9. $\int \frac{(2+3x)^2 dx}{\sqrt{x}}$,
10. $\int \frac{dx}{x\sin^2(\ln x)}$,
11. $\int \frac{dx}{3+4x^2}$,
12. $\int \frac{x dx}{3-4x^4}$,
13. $\int \frac{(x+1)\,dx}{x^2+2x+10}$,
14. $\int \frac{x+3}{\sqrt{4x^2-5}}dx$,
15. $\int \frac{\sin x\cdot\cos x\,dx}{\sqrt{3+\sin^4 x}}$,
16. $\int (1-2\cos x)^2 dx$,
17. $\int \frac{\sin 2t\,dt}{\sqrt{25-\cos^2 2t}}$,
18. $\int 3^{\operatorname{ctg} x}\frac{dx}{\sin^2 x}$,
19. $\int \frac{dz}{(\operatorname{ctg}^2 z-4)\sin^2 z}$,
20. $\int \frac{dx}{\cos^2 x(\operatorname{tg} x+4)}$,
21. $\int \frac{dx}{\cos^2 7x}$,
22. $\int \frac{x^3 dx}{\sqrt{9-x^8}}$,
23. $\int \frac{x+\cos x}{x^2+2\sin x}dx$,
24. $\int \frac{e^{5t}dt}{25+e^{10t}}$,
25. $\int \frac{e^x\sqrt{\operatorname{arctg}\, e^x}}{1+e^{2x}}dx$,
26. $\int \frac{dx}{e^{2x}-1}$,
27. $(1+6x)e^{3x}dx$,
28. $\left(3x^2+5\right)\cos 3x\,dx$,
29. $\int \operatorname{arctg}\, 2x\,dx$,
30. $\int (x+1)\ln^2(x+1)\,dx$,
31. $\int \cos\ln(x+1)\,dx$,
32. $\int \frac{\sqrt{1-x^2}}{x^2}dx$,
33. $\int \frac{dx}{(x-3)^3}$,
34. $\int \frac{x\,dx}{2x^2+6x+25}$,
35. $\int \frac{x^3+1}{x^2-x}dx$,
36. $\int \frac{2x^4-5x^2-8x+8}{x^3-4x}dx$,
37. $\int \frac{(5x-7)\,dx}{x^2\left(x^2-4x+5\right)}$,
38. $\int \frac{2x^3+x+1}{(x+1)x^3}dx$,
39. $\int \frac{x+2}{x^3+27}dx$,
40. $\int \cos 2x\cdot\cos 3x\,dx$,
41. $\int \cos^3 x\cdot\sin^{-5} x\,dx$,
42. $\int \frac{dx}{\sin x+2\cos x-1}$,
43. $\int \frac{12+tgx}{3\sin^2 x+12\cos^2 x}dx$,
44. $\int \frac{dx}{\cos x\cdot\sin^3 x}$,
45. $\int \frac{2\sin^3 x\,dx}{3\cos x-1}$,
46. $\int \frac{(3-4x)\,dx}{\sqrt{x^2-4x+13}}$,
47. $\int \frac{1+\sqrt[3]{x}}{2-\sqrt[3]{x}}dx$,
48. $\int \sqrt[3]{x}\left(2+\sqrt[3]{x^2}\right)^{1/4}dx$,
49. $\int \frac{\sqrt{x^2-9}}{x^3}dx$,
50. $\int \frac{dx}{x\sqrt{x^2+3x-1}}$.

Вариант 19.

Проинтегрировать:

1. $\int \frac{dx}{4-3x}$,
2. $\int e^{-x^3+1}x^2dx$,
3. $\int \sqrt[3]{x^3-7}\,x^2dx$,
4. $\int \sin\sqrt[3]{x}\,\frac{dx}{\sqrt[3]{x^2}}$,
5. $\int \frac{\ln x-3}{x\sqrt{\ln x}}dx$,
6. $\int \frac{dx}{\sqrt{9-2x^2}}$,
7. $\int tg2x\,dx$,
8. $\int \frac{3+(\operatorname{arctg}\,x)^3}{1+x^2}dx$,
9. $\int \frac{\cos x\,dx}{\sin^2 x}$,
10. $\int \frac{x^2dx}{4-x^6}$,
11. $\int \left(2^{4x}+e^{-2x}\right)dx$,
12. $\int \frac{3x+4}{4x^2-3}dx$,
13. $\int \frac{\sin xdx}{\sqrt{\cos^2 x-3}}dx$,
14. $\int \frac{e^xdx}{e^x+5}$,
15. $\int \frac{x\,dx}{\sin^2\left(x^2+3\right)}$,
16. $\int \frac{\sin 2t\,dt}{\sqrt{2+\cos^2 t}}$,
17. $\int \frac{e^{3t}dt}{\sqrt{4-e^{3t}}}$,
18. $\int (1+\sin 2x)^2dx$,
19. $\int \frac{dz}{7+5z^2}$,
20. $\int \frac{dx}{x^2\cos^2\left(\frac{1}{x}\right)}$,
21. $\int \frac{ds}{\sqrt{4s^2+8}}$,
22. $\int \cos\left(y^2+1\right)y\,dy$,
23. $\int \frac{\sin 3x\,dx}{(7+2\cos 3x)^2}$,
24. $\int \frac{x\,dx}{3+4x^4}$,
25. $\int \frac{dx}{\operatorname{tg}\,x\ln^2\sin x}$,
26. $\int \frac{dx}{e^x+3}$,
27. $\int \left(x^2+1\right)e^xdx$,
28. $\int x\cos 3x\,dx$,
29. $\int \ln\left(x^2+1\right)dx$,
30. $\int \frac{x\,dx}{\sin^2 x}$,
31. $\int \operatorname{arctg}\sqrt{8x-1}\,dx$,
32. $\int e^{-x}\sin 2x\,dx$,
33. $\int \frac{dx}{(x-4)^4}$,
34. $\int \frac{xdx}{2x^2+2x+3}$,
35. $\int \frac{3x^3+1}{x^2-1}dx$,
36. $\int \frac{3x^4+3x^3-5x^2+2}{x^3+x^2-2x}dx$,
37. $\int \frac{x+4}{x^4+4x^2}dx$,
38. $\int \frac{(8+4x)\,dx}{\left(x^2-4x+4\right)(x+2)}$,
39. $\int \frac{\left(x^2+3\right)dx}{x^3+x^2+2x-4}$,
40. $\int \cos 2x\cdot\cos 5x\,dx$,
41. $\int \sin^2 x\cdot\cos^4 x\,dx$,
42. $\int \frac{\cos x\,dx}{2+\cos x}$,
43. $\int \frac{5\operatorname{tg}x+2}{2\sin 2x+5}dx$,
44. $\int \frac{\cos^3 xdx}{\sqrt[3]{\sin^4 x}}$,
45. $\int \operatorname{ctg}^4 3x\,dx$,
46. $\int \frac{(2-3x)\,dx}{\sqrt{3-2x-x^2}}$,
47. $\int \frac{2+\sqrt{x}}{1-\sqrt{x}}dx$,
48. $\int \frac{dx}{x\left(1+\sqrt[3]{x}\right)^2}$,
49. $\int x^3\sqrt{4-x^2}\,dx$,
50. $\int \sqrt{1+e^{2x}}\,dx$.

Вариант 20.

Проинтегрировать:

1. $\int \cos\left(4-x^2\right) x\,dx$,
2. $\int \left(2x^3+10\right)^5 x^2 dx$,
3. $\int \frac{dx}{10x-1}$,
4. $\int t\,e^{-t^2}\,dt$,
5. $\int \sqrt[3]{\sin^2 2x}\cos 2x\,dx$,
6. $\int \frac{x\,dx}{x^4+1}$,
7. $\int \frac{2x-1}{\sqrt{25-4x^2}}dx$,
8. $\int \frac{\sin x}{\cos^3 x}dx$,
9. $\int \frac{x^5}{\sqrt{3-x^6}}dx$,
10. $\int 2^{\sqrt{2x-1}}\frac{dx}{\sqrt{2x-1}}$,
11. $\int \frac{e^{2x}dx}{5+e^{4x}}$,
12. $\int \frac{dx}{x\sqrt{49-\ln^2 x}}$,
13. $\int \frac{dx}{\cos^2 6x}$,
14. $\int \frac{2dt}{3+4t^2}$,
15. $\int \operatorname{tg} 8x\,dx$,
16. $\int \sin\left(2x-3\right)dx$,
17. $\int \frac{x\,dx}{2x^2+1}$,
18. $\int \frac{e^x dx}{\sqrt{e^{2x}-3}}$,
19. $\int \frac{\cos 2x\,dx}{\sqrt{3+\sin^2 2x}}$,
20. $\int \frac{5x^3 dx}{x^8-9}$,
21. $\int \frac{dx}{\sqrt{3x^2-7}}$,
22. $\int \frac{2-\cos^2 3x}{\sin^2 3x}dx$,
23. $\int \frac{3x+5}{\sqrt{x^2+5}}dx$,
24. $\int \frac{dx}{36-5x^2}$,
25. $\int \left(2\operatorname{tg} x+\operatorname{ctg} x\right)^2 dx$,
26. $\int \frac{x^3+x}{x^4+1}dx$,
27. $\int x^2 e^{-x}dx$,
28. $\int \left(2x+1\right)\cos\frac{x}{2}\,dx$,
29. $\int \operatorname{arctg}\sqrt{9x-1}\,dx$,
30. $\int e^x \sin 2x\,dx$,
31. $\int x\operatorname{ctg}^2 x\,dx$,
32. $\int \frac{\ln\left(\ln x\right)dx}{x}$,
33. $\int \frac{dx}{\left(x+3\right)^4}$,
34. $\int \frac{\left(3x+2\right)dx}{x^2-4x+9}$,
35. $\int \frac{2x^3+5}{x^2-x-2}dx$,
36. $\int \frac{x^4 dx}{x^4+5x^2+4}$,
37. $\int \frac{x^2-2x+1}{\left(x+1\right)^3\left(x-4\right)}dx$,
38. $\int \frac{x^3-3x^2-12}{x^3-9x^2+26x-24}dx$,
39. $\int \frac{2x+1}{x^4-x^3+x^2}dx$,
40. $\int \sin 2x\cdot\sin 5x\,dx$,
41. $\int \frac{\sin^3 x\,dx}{\sqrt[4]{\cos^7 x}}$,
42. $\int \frac{dx}{1+3\sin x-2\cos x}$,
43. $\int \frac{\operatorname{tg}^2 x\,dx}{3\sin^2 x+4\cos^2 x-7}$,
44. $\int \sin^4 x\,dx$,
45. $\int \frac{\cos 2x-\sin 2x}{\sin 4x}dx$,
46. $\int \frac{\left(x+1\right)dx}{\sqrt{5-4x-x^2}}$,
47. $\int \frac{dx}{\left(\sqrt[3]{x^2}+\sqrt[3]{x}\right)^2}$,
48. $\int x^5\left(1+x^2\right)^{2/3}dx$,
49. $\int \frac{\sqrt{x^2-25}}{x^2}dx$,
50. $\int \frac{dx}{x\sqrt{4x^2+x+5}}$.

Вариант 21.

Проинтегрировать:

1. $\int \frac{dx}{1-4x}$,
2. $\int \frac{dx}{\cos^2(2x+1)}$,
3. $\int \frac{dx}{(\arccos x)\sqrt[5]{1-x^2}}$,
4. $\int \frac{dx}{\sin^2 x \cdot \cos^2 x}$,
5. $\int e^{\operatorname{arctg} 3x} \cdot \frac{dx}{1+9x^2}$,
6. $\int \frac{x^2 + \sqrt[3]{x} - 2\ln x}{x} dx$,
7. $\int \frac{(x+3)dx}{\sqrt{x^2+6x-8}}$,
8. $\int (2x+1)^{20} dx$,
9. $\int \frac{dx}{4+3x^2}$,
10. $\int \cos(\pi x+1) dx$,
11. $\int \frac{dx}{\cos^2 2x\sqrt{1+\operatorname{tg} 2x}}$,
12. $\int \left(e^{-5x} + 2^{5x}\right) dx$,
13. $\int \frac{2x + (\operatorname{arctg}\ 2x)^3}{1+4x^2} dx$,
14. $\int \frac{dx}{\sin^2 4x\sqrt{2+\operatorname{ctg}^2 4x}}$,
15. $\int \frac{(2x-5)dx}{10x^2-7}$,
16. $\int \frac{x^2 dx}{\sqrt{4-x^6}}$,
17. $\int \frac{e^{\frac{t}{2}} dt}{\sqrt{e^t - 1}}$,
18. $\int \operatorname{ctg} 5x\, dx$,
19. $\int \sin\left(\frac{1}{x}\right)\frac{dx}{x^2}$,
20. $\int \frac{dx}{\sqrt{x}(4+x)}$,
21. $\int \frac{\ln x\, dx}{x(\ln^2 x + 7)}$,
22. $\int \frac{\sin x \cdot \cos x\, dx}{16 + \sin^4 x}$,
23. $\int \frac{dx}{x\sqrt{11 - \ln^2 x}}$,
24. $\int \frac{\sin 4x\, dx}{4 - \cos^2 2x}$,
25. $\int \frac{dx}{\sqrt{1+x} - \sqrt{x}}$,
26. $\int \frac{2^{\sin t} \cos t\, dt}{9 - 4^{\sin t}}$,
27. $\int (4-3x)e^{-3x} dx$,
28. $\int (4x^2 - 1)\sin 4x\, dx$,
29. $\int \arccos 2x\, dx$,
30. $\int \frac{\ln x}{x^5} dx$,
31. $\int \sin(\ln x) dx$,
32. $\int \frac{x \arcsin x\, dx}{\sqrt{1+x^2}}$,
33. $\int \frac{7dx}{(x-5)^2}$,
34. $\int \frac{(3x+2)dx}{x^2+2x+10}$,
35. $\int \frac{(2x^3+1)dx}{x^2+4}$,
36. $\int \frac{x^5 - x^4 6x^3 + 13x + 6}{x^3 - x^2 - 6x} dx$,
37. $\int \frac{x^3 + 2x^2 + 10x}{(x+1)^2(x^2 - x + 1)} dx$,
38. $\int \frac{(x+2)dx}{(x+1)^3 x}$,
39. $\int \frac{x^3 + 3x + 1}{(x^2+1)(x^2+3)} dx$,
40. $\int \frac{\cos 2x\, dx}{\sin^2 x}$,
41. $\int \cos 2x \cdot \sin 5x\, dx$,
42. $\int \frac{(1+\sin x)dx}{1 + \cos x + \sin x}$,
43. $\int \sin^4 x \cdot \cos^4 x\, dx$,
44. $\int \frac{6\sin^2 x\, dx}{4 + \cos 2x}$,
45. $\int \frac{dx}{\sqrt{\sin x \cdot \cos^3 x}}$,
46. $\int \frac{x\, dx}{\sqrt{12 + 6x - x^2}}$,
47. $\int \frac{dx}{\sqrt[3]{x+2} + \sqrt[3]{(x+2)^2}}$,
48. $\int x^3\sqrt{1+x^2}\, dx$,
49. $\int \frac{1}{(1-x)^2}\sqrt{\frac{1-x}{1+x}} dx$,
50. $\int \frac{dx}{x\sqrt{7x^2 + 2x + 5}}$.

Вариант 22.

Проинтегрировать:

1. $\int \frac{dx}{2+3x}$,
2. $\int \frac{dt}{\sqrt{9-4t^2}}$,
3. $\int \frac{\sqrt{x-1}+\ln(x-1)}{x-1}dx$,
4. $\int \frac{t\,dt}{\sqrt{25+16t^4}}$,
5. $\int \frac{dx}{(1+25x^2)\operatorname{arctg}\,5x}$,
6. $\int \frac{dx}{\sin^2(3x-1)}$,
7. $\int e^x \cos(3e^x+1)dx$,
8. $\int 2^{\cos^2 x}\cdot \sin 2x\,dx$,
9. $\int \frac{3x-5}{2x^2+9}dx$,
10. $\int (3x-1)^{10}dx$,
11. $\int e^{\arcsin 2x}\cdot \frac{dx}{\sqrt{1-4x^2}}$,
12. $\int \sin(2x-1)dx$,
13. $\int \frac{dx}{\operatorname{tg}\,4x}$,
14. $\int \frac{x^5dx}{\sqrt{7-2x^6}}$,
15. $\int \frac{\cos 2x\,dx}{(2\sin 2x+3)^3}$,
16. $\int \frac{dx}{3-4x^2}$,
17. $\int \frac{x\,dx}{7+9x^2}$,
18. $\int \operatorname{tg}^2 2x\,dx$,
19. $\int x^3\cdot\sqrt[4]{9+x^4}\,dx$,
20. $\int \frac{e^t dt}{3+4e^{2t}}$,
21. $\int \frac{dx}{x\sqrt{1-\ln^2 x}}$,
22. $\int \frac{\sin 3t\,dt}{\sqrt[5]{\cos 3t}}$,
23. $\int \frac{e^{2x}dx}{9-e^{4x}}$,
24. $\int \frac{dt}{\cos^2 t\sqrt{2+5\operatorname{tg}^2 t}}$,
25. $\int \operatorname{tg}\,x\cdot\ln(\cos x)dx$,
26. $\int \frac{x\,dx}{x^4+2x^2+5}$,
27. $\int (x-1)\cos\frac{x}{3}dx$,
28. $\int (x^2+3)\cdot e^{2x}dx$,
29. $\int \sqrt{x}\cdot\ln x\,dx$,
30. $\int (2+x)\operatorname{tg}^2 x\,dx$,
31. $\int \operatorname{arctg}\,\sqrt{12x-1}\,dx$,
32. $\int e^{-2x}\cdot\sin x\,dx$,
33. $\int \frac{dx}{(x+4)^6}$,
34. $\int \frac{(x+2)dx}{3x^2-6x+16}$,
35. $\int \frac{(4x^3-1)dx}{x^2-4}$,
36. $\int \frac{3x^3+2x^2+1}{x^3-x^2-4x+4}dx$,
37. $\int \frac{dx}{(x+1)^2(x^2+1)}$,
38. $\int \frac{x^3+6x^2+18x-4}{(x^2-4)(x^2+4x+4)}dx$,
39. $\int \frac{(x+4)dx}{(x^2+2)(x^2+x+2)}$,
40. $\int \cos x\cdot\cos 6x\,dx$,
41. $\int \frac{dx}{\sin^4 x}$,
42. $\int \frac{dx}{4+\sin x+\cos x}$,
43. $\int \frac{36\,dx}{(6-\operatorname{tg}\,x)\sin 2x}$,
44. $\int \cos^5 x\cdot\sqrt{\sin x}\,dx$,
45. $\int \frac{\sin x\,dx}{(1-\cos x)^2}$,
46. $\int \frac{(3x-2)dx}{\sqrt{9x^2+6x+2}}$,
47. $\int \frac{dx}{x\sqrt{x^2+5x+1}}$,
48. $\int \frac{\sqrt[3]{2x-3}-1}{\sqrt[3]{2x-3}+2}dx$,
49. $\int \frac{dx}{x^4\sqrt{1+x^2}}$,
50. $\int \frac{\sqrt{1+\sqrt[3]{x}}\,dx}{\sqrt{x^3}}$.

Вариант 23.

Проинтегрировать:

1. $\int \left(5x^2+1\right)^7 x\,dx$,
2. $\int \frac{dx}{x\sqrt{\ln x}}$,
3. $\int \frac{dx}{\sqrt{2-25x^2}}$,
4. $\int e^{\operatorname{ctg} 2x} \frac{dx}{\sin^2 2x}$,
5. $\int \sqrt[4]{1+3\cos 2x}\cdot \sin 2x\,dx$,
6. $\int e^{2x}\cdot \sin\left(3e^{2x}-1\right)dx$,
7. $\int \frac{dx}{7+8x}$,
8. $\int \operatorname{ctg} 4x\,dx$,
9. $\int \frac{dx}{\cos^2 5x}$,
10. $\int 4^{\sqrt{x}} \frac{dx}{\sqrt{x}}$,
11. $\int \frac{dx}{5-9x^2}$,
12. $\int \frac{x+(\arccos 3x)^3}{\sqrt{1-9x^2}}dx$,
13. $\int \frac{dx}{3x^2+16}$,
14. $\int \frac{dt}{\sqrt{1-4t^2}\ \arcsin\ 2t}$,
15. $\int \frac{e^{-x}dx}{\sqrt{4-e^{-2x}}}$,
16. $\int \frac{\sin 3x\,dx}{25+\cos^2 3x}$,
17. $\int \frac{ds}{\sqrt{81s^2+4}}$,
18. $\int \frac{dt}{t\sqrt{\ln^2 t-1}}$,
19. $\int \sin^5 \frac{3x}{2}\cdot \cos\frac{3x}{2}dx$,
20. $\int \cos^2 3x\,dx$,
21. $\int \frac{dx}{\sqrt{x}\cdot(x+4)}$,
22. $\int \frac{e^t dt}{e^{2t}-4}$,
23. $\int \frac{(x+2)\,dx}{\sqrt{x^2+4x-3}}$,
24. $\int \frac{x^4 dx}{\sqrt{2+x^{10}}}$,
25. $\int \frac{(1-\cos x)\,dx}{(x-\sin\ x)^2}$,
26. $\int \frac{(1+\ln x)\,dx}{3+x\ln x}$,
27. $\int \frac{1}{2}x\cdot \sin\ 2x\,dx$,
28. $\int e^{-x}\left(2x^2-1\right)dx$,
29. $\int e^{3x}\cos x\,dx$,
30. $\int \operatorname{arctg}\ \sqrt{10x-1}\,dx$,
31. $\int \sin\ x\cdot \ln(\cos x)\,dx$,
32. $\int \frac{\arcsin\ x}{\sqrt{1+x}}dx$,
33. $\int \frac{dx}{(x+5)^3}$,
34. $\int \frac{(5x+3)\,dx}{2x^2+10x+29}$,
35. $\int \frac{2x^3+1}{x^2+x}dx$,
36. $\int \frac{2x^4-x^2+1}{x^3-x}dx$,
37. $\int \frac{x\,dx}{(x-1)^2\left(x^2+2x+2\right)}$,
38. $\int \frac{2x^3+2x+1}{\left(x^2-x+1\right)\left(x^2+1\right)}dx$,
39. $\int \frac{7x\,dx}{(x+2)(x-1)^3}$,
40. $\int \sin\frac{x}{4}\cdot\cos\frac{3x}{4}dx$,
41. $\int \frac{dx}{3+5\cos x}$,
42. $\int \frac{12dx}{(6+5\ \operatorname{tg}\ x)\cdot\sin\ 2x}$,
43. $\int \frac{dx}{\cos^4 x}$,
44. $\int \cos^3\frac{x}{2}\cdot\sqrt[4]{\sin\frac{x}{2}}\,dx$,
45. $\int \frac{\cos x\,dx}{(1-\cos x)^2}$,
46. $\int \frac{(5x-1)\,dx}{\sqrt{4x^2+4x+2}}$,
47. $\int \frac{dx}{\sqrt[4]{2+x}+\sqrt{2+x}}$,
48. $\int \sqrt[3]{x}\cdot\sqrt[7]{1+\sqrt[3]{x^4}}\,dx$,
49. $\int \frac{dx}{\sqrt{\left(x^2+2x+5\right)^3}}$,
50. $\int \frac{e^x\sqrt{e^x-1}}{e^x+3}dx$.

Вариант 24.

Проинтегрировать:

1. $\int \sqrt[3]{2\sin 2x+3}\cos 2x\,dx$,
2. $\int e^{\arccos 2x}\cdot\frac{dx}{\sqrt{1-4x^2}}$,
3. $\int\frac{dx}{2x+3}$,
4. $\int\sin(5x-1)\,dx$,
5. $\int\frac{x+3}{\sqrt{x^2+3}}dx$,
6. $\int\frac{dx}{\sqrt{1-9x^2}}$,
7. $\int\operatorname{ctg}10x\,dx$,
8. $\int\left(2^x+3^x\right)^2dx$,
9. $\int\frac{(2\ln x+3)^4}{x}dx$,
10. $\int\frac{e^{5t}dt}{4+e^{5t}}$,
11. $\int\frac{3x-4}{x^2-16}dx$,
12. $\int\frac{dx}{\cos^2 x\sqrt{4-25\operatorname{tg}^2 x}}$,
13. $\int\frac{dt}{3-4t^2}$,
14. $\int\frac{3\,ds}{4s^2+25}$,
15. $\int\frac{dx}{\sqrt{1-x^2}\arccos x}$,
16. $\int\frac{\left(x^2-1\right)dx}{\sqrt{x^3-3x+1}}$,
17. $\int\frac{dx}{\sin^2 10x}$,
18. $\int\frac{2^x dx}{9+4^x}$,
19. $\int\frac{e^t dt}{\sqrt{e^t+3}}$,
20. $\int\frac{e^x dx}{\sqrt{9-e^{2x}}}$,
21. $\int\sin^7 4x\cdot\cos 4x\,dx$,
22. $\int\sin^2 3x\,dx$,
23. $\int\frac{\operatorname{tg}(x+1)\,dx}{\cos^2(x+1)}$,
24. $\int\frac{x^2dx}{x^6-4}$,
25. $\int\frac{x^4dx}{\sqrt{4x^{10}-3}}$,
26. $\int\frac{\left(2+3x^2\right)dx}{x^2\left(1+x^2\right)}$,
27. $\int(3x+1)\cdot e^{4x}dx$,
28. $\int\left(1-2x^2\right)\cdot\sin 2x\,dx$,
29. $\int\operatorname{arctg}\sqrt{11x-1}\,dx$,
30. $\int\sqrt[3]{x}\cdot\ln x\,dx$,
31. $\int\frac{x\sin x}{\cos^2 x}dx$,
32. $\int e^{-3x}\cdot\cos x\,dx$,
33. $\int\frac{dx}{(x+6)^4}$,
34. $\int\frac{(x+1)\,dx}{5x^2+2x+1}$,
35. $\int\frac{3x^3-2}{x^3-4x}dx$,
36. $\int\frac{2x^4-2x^3+7x^2+5}{x^3-x^2-x-2}dx$,
37. $\int\frac{x^2-2}{x^3(x+2)^2}dx$,
38. $\int\frac{x\,dx}{(x-1)^{10}}$,
39. $\int\frac{3x^3+4x^2+6x}{\left(x^2+2\right)\left(x^2+2x+2\right)}dx$,
40. $\int\cos\frac{x}{4}\cdot\sin\frac{3x}{8}dx$,
41. $\int\sin^5 x\,dx$,
42. $\int\frac{dx}{\sin x+\cos x}$,
43. $\int\frac{\operatorname{tg}^2 x\,dx}{4+3\cos 2x}$,
44. $\int\sin^2 2x\cdot\cos^4 2x\,dx$,
45. $\int\frac{1-\cos x}{1+\cos x}dx$,
46. $\int\frac{(x+1)\,dx}{\sqrt{x^2-2x+10}}$,
47. $\int\frac{3+\sqrt[4]{x}}{1+\sqrt[3]{x}}dx$,
48. $\int\frac{dx}{x^3\sqrt[5]{1+x^{-1}}}$,
49. $\int\frac{dx}{x^2\sqrt{1+x^2}}$,
50. $\int\frac{e^{6x}dx}{\sqrt{e^{3x}+1}}$.

Вариант 25.

Проинтегрировать:

1. $\int \sin\frac{t}{3}\,dt$,
2. $\int \frac{\sqrt[3]{\ln(x+2)}}{x+2}\,dx$,
3. $\int \frac{5t\,dt}{5t^2+8}$,
4. $\int x^3\cdot e^{-5x^4}\,dx$,
5. $\int \frac{dx}{\sqrt{1-9x^2}\,(\arcsin 3x)^6}$,
6. $\int \frac{t\,dt}{t^4+121}$,
7. $\int \frac{8x-3}{\sqrt{4+x^2}}\,dx$,
8. $\int \sin^5 9t\cdot\cos 9t\,dt$,
9. $\int \cos^2 3t\,dt$,
10. $\int \frac{dx}{2x^2-9}$,
11. $\int \frac{e^x dx}{\cos^2(3e^x)}$,
12. $\int \frac{ds}{\sqrt{3-49s^2}}$,
13. $\int \frac{dx}{2x+3}$,
14. $\int \frac{x^3 dx}{\sin^2(3+x^4)}$,
15. $\int \frac{(1+x-x^2)\,dx}{\sqrt{(1-x^2)^3}}$,
16. $\int \frac{2x^5-3x^2}{1+3x^3-x^6}\,dx$,
17. $\int 5^{\operatorname{ctg}3x}\frac{dx}{\sin^2 3x}$,
18. $\int \frac{\cos 4t\,dt}{\sqrt{\sin^2 4t-7}}$,
19. $\int \frac{2dx}{x(3+\ln^2 x)}$,
20. $\int \frac{dx}{e^x(3+e^{-x})}$,
21. $\int \operatorname{tg}\frac{x}{6}\,dx$,
22. $\int \frac{3x+(\arccos 3x)^2}{\sqrt{1-9x^2}}\,dx$,
23. $\int \frac{\cos 2x-\sin 2x}{(\sin 2x+\cos 2x)^3}\,dx$,
24. $\int \frac{dx}{(x+9)(4-\ln^2(x+9))}$,
25. $\int \frac{e^{-t}dt}{\sqrt{4-e^{-2t}}}$,
26. $\int \frac{x\cos x+\sin x}{(x\sin x)^2}\,dx$,
27. $\int (1-3x)\cdot e^{-4x}dx$,
28. $\int (x^2-3x)\cdot\sin x\,dx$,
29. $\int e^{4x}\cdot\cos 2x\,dx$,
30. $\int \operatorname{arctg}\sqrt{13x-1}\,dx$,
31. $\int (x^2-2x+3)\ln x\,dx$,
32. $\int \frac{x\cos x}{\sin^3 x}\,dx$,
33. $\int \frac{dx}{(x+2)^{10}}$,
34. $\int \frac{(2x+3)\,dx}{2x^2+2x+5}$,
35. $\int \frac{3x^3+2x^2+1}{x^3-x^2-4x+4}\,dx$,
36. $\int \frac{x^3+3}{(x+1)(x^2+1)}\,dx$,
37. $\int \frac{x^2+1}{(x-1)(x+1)^2}\,dx$,
38. $\int \frac{x\,dx}{(x+1)^7}$,
39. $\int \frac{(x^2-x)\,dx}{(x^2+5)(x^2+2x+4)}$,
40. $\int \sin 3x\cdot\sin^2 x\,dx$,
41. $\int \operatorname{tg}^5 3x\,dx$,
42. $\int \frac{\cos x\,dx}{1+\cos x}$,
43. $\int \frac{8\operatorname{tg} x\,dx}{3\cos^2 x+8\sin^2 x-7}$,
44. $\int \frac{dx}{\sqrt[4]{\sin^3 x\cdot\cos^5 x}}$,
45. $\int \frac{\sin^3 x\,dx}{1+\cos^2 x}$,
46. $\int \frac{(5x-7)\,dx}{\sqrt{x^2+3x+8}}$,
47. $\int \frac{dx}{\left(\sqrt[3]{(x-1)^2}+\sqrt[3]{x-1}\right)^2}$,
48. $\int \frac{\sqrt[3]{1+\sqrt{x}}}{\sqrt[3]{x^5}}\,dx$,
49. $\int \frac{\sqrt{x^2+1}}{x^2}\,dx$,
50. $\int \frac{dx}{x\sqrt{x^2-16x}}$.