В-1.

1) $A_1\ (4;6;3)$

$A_2\ (-5;2;6)$

$A_3\ (4;-4;-3)$

$A_4\ (1;2;4)$

а) модуль в-ра $4\overrightarrow{A_3A_2} - \overrightarrow{A_1A_3} \Rightarrow |4\overrightarrow{A_3A_2} - \overrightarrow{A_1A_3}|$

$\overrightarrow{A_3A_2}\ (-9;6;9)$

$4\overrightarrow{A_3A_2}\ (-36;24;36)$

$\overrightarrow{A_1A_3}\ (0;-10;-6)$

$$4\overrightarrow{A_3A_2} - \overrightarrow{A_1A_3} = (-36;34;42)$$

$$|4\overrightarrow{A_3A_2} - \overrightarrow{A_1A_3}| = \sqrt{(-36)^2+(34)^2+(42)^2} =$$

$$= \sqrt{1296+1156+1764} = \sqrt{4216} = 2\sqrt{1054}$$

б) $\text{пр.}_{\overrightarrow{A_4A_2}}(\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4}) = |\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4}| \cdot \cos(\overrightarrow{A_4A_2} \wedge (\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4}))$

$$\cos(\overrightarrow{A_4A_2} \wedge (\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4})) = \frac{\overrightarrow{A_4A_2} \cdot (\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4})}{|\overrightarrow{A_4A_2}| \cdot |\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4}|}$$

$\overrightarrow{A_1A_4}\ (-3;-4;1)$

$\overrightarrow{A_3A_4}\ (-3;6;7) \quad 2\overrightarrow{A_3A_4}\ (-6;12;14)$

$$\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4} = (3;-16;-13)$$

$$|\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4}| = \sqrt{(3)^2+(-16)^2+(-13)^2} =$$

$$= \sqrt{9+256+169} = \sqrt{434}$$

$\overrightarrow{A_4A_2}\ (-6;0;2) \qquad |\overrightarrow{A_4A_2}| = \sqrt{(-6)^2+2^2} = \sqrt{36+4} = \sqrt{40}$

$$\overrightarrow{A_4A_2} \cdot (\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4}) = (-6)\cdot 3 + 0\cdot(-16) + 2\cdot(-13) =$$

$$= -18 - 26 = -44$$

$$|\overrightarrow{A_4A_2}| \cdot |\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4}| = \sqrt{40} \cdot \sqrt{434} = \sqrt{17360} =$$

$$= 2\sqrt{4340} = 4\sqrt{1085}$$

$$\cos(\overrightarrow{A_4A_2} \wedge (\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4})) = \frac{-44}{4\sqrt{1085}} = -\frac{11}{\sqrt{1085}}$$

$$\text{пр.}_{\overrightarrow{A_4A_2}}(\overrightarrow{A_1A_4} - 2\overrightarrow{A_3A_4}) = \sqrt{434} \cdot \left(-\frac{11}{\sqrt{1085}}\right) = -\frac{11\sqrt{434}}{\sqrt{1085}}$$

$$= -\frac{11\sqrt{2}}{\sqrt{5}} = -\frac{11\sqrt{10}}{5}$$

в) угол м/у. в-и $\overrightarrow{A_3A_2}$ и $\overrightarrow{A_3A_4} \Rightarrow \overrightarrow{A_3A_2} \wedge \overrightarrow{A_3A_4}$

$$\cos(\overrightarrow{A_3A_2} \wedge \overrightarrow{A_3A_4}) = \frac{\overrightarrow{A_3A_2} \cdot \overrightarrow{A_3A_4}}{|\overrightarrow{A_3A_2}| \cdot |\overrightarrow{A_3A_4}|}$$

$\overrightarrow{A_3A_2}\ (-9;6,9)$

$\overrightarrow{A_3A_4}\ (-3;6;7)$ $\quad \overrightarrow{A_3A_2}\cdot\overrightarrow{A_3A_4} = (-9)\cdot(-3)+6\cdot6+9\cdot7 =$

$= 27+36+63 = 126$

$|\overrightarrow{A_3A_2}| = \sqrt{(-9)^2+(6)^2+(9)^2} = \sqrt{81+36+81} = \sqrt{198} = 3\sqrt{22}$

$|\overrightarrow{A_3A_4}| = \sqrt{(-3)^2+(6)^2+(7)^2} = \sqrt{9+36+49} = \sqrt{94}$

$|\overrightarrow{A_3A_2}|\cdot|\overrightarrow{A_3A_4}| = 3\sqrt{22}\cdot\sqrt{94} = 3\sqrt{2068} = 6\sqrt{517}$

$\cos(\overrightarrow{A_3A_2}\wedge\overrightarrow{A_3A_4}) = \frac{\overset{21}{\cancel{126}}}{\cancel{6}\sqrt{517}} = \frac{21}{\sqrt{517}}$

$\overrightarrow{A_3A_2}\wedge\overrightarrow{A_3A_4} = \arccos\left(\frac{21\sqrt{517}}{517}\right)$

2) $S_{\triangle A_2A_3A_4}$

$\overrightarrow{A_2A_3} = (9;-6;-9)$ $\quad S = \frac{1}{2}\,|\underbrace{\overrightarrow{A_2A_3}\times\overrightarrow{A_2A_4}}_{=\vec{c}}|$

$\overrightarrow{A_2A_4} = (6;0;-2)$

$$\overrightarrow{A_2A_3}\times\overrightarrow{A_2A_4} = \begin{vmatrix} i & j & k \\ 9 & -6 & -9 \\ 6 & 0 & -2 \end{vmatrix} = i\cdot(-6)\cdot(-2) + 9\cdot0\cdot k + 6\cdot j\cdot(-9) - k\cdot(-6)\cdot6 - i\cdot0\cdot(-9) - j\cdot9\cdot(-2) =$$

$$\begin{matrix} i & j & k \\ 9 & -6 & -9 \end{matrix}$$

$= 12i - 54j + 36k + 18j =$

$= 12i - 36j + 36k$

$=> (12;-36;36)$

$|\vec{c}| = \sqrt{12^2+(-36)^2+36} = \sqrt{144+1296+1296} =$

$= \sqrt{2736} = 12\sqrt{19}$

$S = \frac{1}{2}\cdot12\sqrt{19} = 6\sqrt{19}$

3) объём пирамиды $\quad V_{пирам.\,A_1A_2A_3A_4} = \frac{1}{6}\,|\overrightarrow{A_1A_2}\cdot(\overrightarrow{A_1A_3}\times\overrightarrow{A_1A_4})|$

$\overrightarrow{A_1A_2}\ (-9;-4;3)$

$\overrightarrow{A_1A_3}\ (0;-10;-6)$ $\quad \overrightarrow{A_1A_2}\cdot(\overrightarrow{A_1A_3}\times\overrightarrow{A_1A_4}) = \begin{vmatrix} -9 & -4 & 3 \\ 0 & -10 & -6 \\ -3 & -4 & \boxed{1} \end{vmatrix} = \begin{vmatrix} 0 & 8 & 3 \\ -18 & -34 & -6 \\ 0 & 0 & 1 \end{vmatrix} =$

$\overrightarrow{A_1A_4}\ (-3;-4;1)$

$$\begin{matrix} 3 & 4 \end{matrix} \qquad \begin{matrix} 0 & 8 & 3 \\ -18 & -34 & -6 \end{matrix}$$

$= 12\cdot8 = 144$

$V_{\text{пирам } A_1A_2A_3A_4} = \frac{1}{6} \cdot 144 = 24.$

Ответ:
а) $2\sqrt{1054}$
б) $-\frac{11\sqrt{10}}{5}$
в) $\arccos \frac{21\sqrt{577}}{577}$
г) $6\sqrt{19}$
д) 24

② На оси ординат найти точку M, равноуд. от точек $A(1;-4;7)$ и $B(5;6;5)$

$M \in O_y \quad (0;y;0) \quad AM = MB$

$AM = \sqrt{1^2 + (-4-y)^2 + 7^2} = \sqrt{1 + (16 + y^2 + 8y) + 49} =$
$= \sqrt{y^2 + 8y + 66}$

$BM = \sqrt{5^2 + (6-y)^2 + 5^2} = \sqrt{25 + (36 + y^2 - 12y) + 25} =$
$= \sqrt{y^2 - 12y + 86}$

$\sqrt{y^2 + 8y + 66} = \sqrt{y^2 - 12y + 86}$

$\cancel{y^2} + 8y + 66 = \cancel{y^2} - 12y + 86$

$20y = 20$

$y = 1$

$M(0;1;0)$

Ответ: $M(0;1;0)$

③ Найти работу силы $\vec{F}(2;2;5)$, затраченную на перем-е тела по прямой, у точки $A(1;2;3)$ в точку $B(5;6;-5)$

$\vec{AB}(4;4;-8)$

$\vec{A} = \vec{F} + \vec{AB} = (6;6;-3)$

$|\vec{A}| = \sqrt{6^2 + 6^2 + (-3)^2} = \sqrt{36+36+9} = \sqrt{81} = 9.$

Ответ: 9.