Задания для самостоятельного решения

В-1

① $$\lim_{x\to\infty} \frac{3x^{3} - 5x^2 + 2}{2x^{3} + 5x^2 - x} = \left\{\frac{\infty}{\infty}\right\} = \frac{3}{2} = 1{,}5$$

② $$\lim_{x\to\infty} \frac{x^{5} - 2x + 4}{2x^{4} + 3x^2 - x} = \left\{\frac{\infty}{\infty}\right\} = \infty$$

③ $$\lim_{x\to\infty} \frac{2x^4 + \sqrt{3x-5}}{7x^3 - 2x^2 + 1} = \left\{\frac{\infty}{\infty}\right\} = \lim_{x\to\infty} \frac{2x^{4} + 3x^{\frac{1}{2}}}{7x^{3} - 2x^2}$$

$$\frac{-5^{\frac{1}{2}}}{+1} = \infty$$

④ $$\lim_{x\to 2} \frac{x^2 - 5x + 6}{x^2 - 12x + 20} = \left\{\frac{0}{0}\right\} =$$

$$= \left[\begin{array}{l} D = 25 - 24 = 1 \\ x_{1/2} = \frac{5 \pm 1}{2} = 3 \\ \qquad\qquad = 2 \end{array}\right. \Rightarrow (x-3)(x-2)$$

$$D = 144 - 80 = 64$$

$$x_{1/2} = \frac{12 \pm 8}{2} = 10 \qquad \Rightarrow (x-10)(x-2)$$
$$= 2$$

$$= \lim_{x\to 2} \frac{(x-3)\cancel{(x-2)}}{(x-10)\cancel{(x-2)}} = \lim_{x\to 2} \frac{x-3}{x-10} = \frac{2-3}{2-10} =$$

$$= \frac{1}{8}$$

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⑤ $$\lim_{x\to 3} \frac{2x^2+11x+15}{3x^2+5x-12} = \left\{\frac{\infty}{\infty}\right\} =$$

$$= \frac{2\cdot 3^2+11\cdot 3+15}{3\cdot 3^2+5\cdot 3-12} = \frac{66}{30} = \frac{11}{5}$$

⑥ $$\lim_{x\to 3} \frac{x^2+x-12}{\sqrt{x-2}-\sqrt{4-x}} = \left\{\frac{0}{0}\right\} =$$

$$= \lim_{x\to 3} \frac{(x+4)(x-3)(\sqrt{x-2}+\sqrt{4-x})}{x-2-4+x} =$$

$$= \lim_{x\to 3} \frac{(x+4)\cancel{(x-3)}(\sqrt{x-2}+\sqrt{4-x})}{\underbrace{2x-6}_{=2\cancel{(x-3)}}} =$$

$$= \lim_{x\to 3} \frac{(x+4)(\sqrt{x-2}+\sqrt{4-x})}{2} =$$

$$= \frac{(3+4)(\overset{1}{\sqrt{3-2}}+\overset{1}{\sqrt{4-3}})}{2} = 7$$

⑦ $$\lim_{x\to 8} \frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}} = \left\{\frac{0}{0}\right\} =$$

$$= \lim_{x\to -8} \frac{(\sqrt{1-x}-3)(\sqrt{1-x}+3)}{(2+x^{\frac{1}{3}})(\sqrt{1-x}+3)} =$$

$$= \cancel{1-x-9} \quad \cancel{(2-x^{\frac{1}{3}})}$$

$$= \lim_{x\to -8} \frac{(1-x-9)\cdot(4-2\cdot\sqrt[3]{x}+\sqrt[3]{x^2})}{(\sqrt{1-x}+3)(2+\sqrt[3]{x})(4-2\sqrt[3]{x}+\sqrt[3]{x^2})} =$$

$$= \lim_{x\to -8} \frac{(-x-8)(4-2\cdot\sqrt[3]{x}+\sqrt[3]{x^2})}{(\sqrt{1-x}+3)(8+x)} =$$

$$= \lim_{x\to -8} \frac{-\cancel{(x+8)}(4-2\cdot\sqrt[3]{x}+\sqrt[3]{x^2})}{(\sqrt{1-x}+3)\cancel{(8+x)}} =$$

$$= \frac{4-2\cdot\sqrt[3]{-8}+\sqrt[3]{(-8)^2}}{\sqrt{1+8}+3} =$$

$$= \frac{4-2\cdot(-2)+\sqrt[3]{64}}{\sqrt{9}+3} = \frac{4+4+4}{3+3} =$$

$$= \frac{16}{9}$$

8) $$\lim_{x\to -2}\left(\frac{1}{x+2}^{\backslash x^2-4} - \frac{2}{x^2-4}^{\backslash x+2}\right) = \lim_{x\to -2}\left(\frac{x^2-4-2(x+2)}{(x+2)(x^2-4)}\right) =$$

$$= \lim_{x\to -2}\frac{x^2-4-2x-4}{x^3+2x^2-4x-8} = \lim_{x\to 2}\frac{x^2-2x-8}{x^3+2x^2-4x-8} =$$

$$= \left\{\frac{0}{0}\right\} = \lim_{x\to -2}\frac{(x-4)\cancel{(x+2)}}{x^2\cancel{(x+2)}(x+4)(-4)} =$$

$$= \lim_{x\to -2}\frac{x-4}{-4x^2(x+4)} = \lim_{x\to -2}\frac{x^{0}-4}{-4x^{3}-16x^2} = 0$$

9) $\rightarrow$ $\searrow$

$$\lim_{x \to \infty} \left(\sqrt{(x+1)(x+2)} - x\right) =$$

$$= \{\infty - \infty\} = \lim_{x \to \infty} \frac{\left(\sqrt{(x+1)(x+2)} - x\right) \cdot \left(\sqrt{(x+1)(x+2)} + x\right)}{\left(\sqrt{(x+1)(x+2)} + x\right)} = \lim_{x \to \infty} \frac{(x+1)(x+2) - x^2}{\sqrt{(x+1)(x+2)} + x} =$$

$$= \lim_{x \to \infty} \frac{\cancel{x^2} + x + 2x + 3 - \cancel{x^2}}{\sqrt{(x+1)(x+2)} + x} = \lim_{x \to \infty} \frac{3x + 3}{\sqrt{(x+1)(x+2)} + x} =$$

$$\underset{x^2 + 3x + 3}{''}$$

$$= \lim_{x \to \infty} \frac{3x^{①} + 3}{x^{①}\left(\sqrt{1 + \frac{3}{x} + \frac{3}{x^2}} + 1\right)} =$$

$$= \frac{3}{1} = \underline{3}$$