Solving problems by searching

Steps in Problem Solving

- Goal Formulation: based on current situation and the agent's performance measure is the first step in Problem Solving
- Problem Formulation-Deciding what actions and states to consider given a goal.
- Search for different possible actions and choose the best.
- A search algorithm takes problem as input and returns a solution in the form of an action sequence.

Steps in Problem Solving

- The action recommended by the solution can be carried out, known as Execution.

Problem-solving agents

```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then do
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH( problem)
    action ← FIRST(seq)
    seq ← REST(seq)
    return action
```

Example: Romania

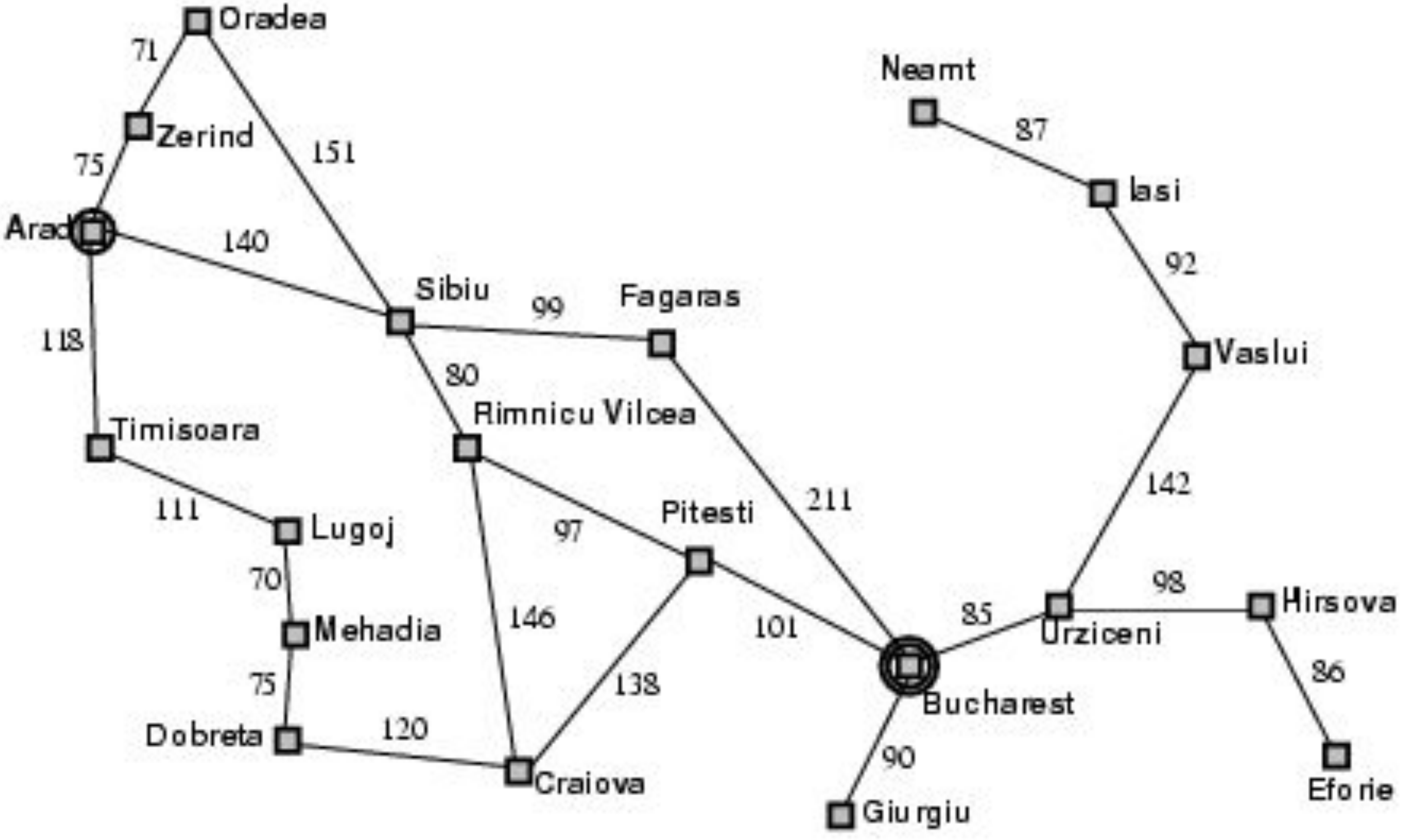
- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest

- Formulate goal:
  - be in Bucharest

- Formulate problem:
  - states: various cities
  - actions: drive between cities

- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania

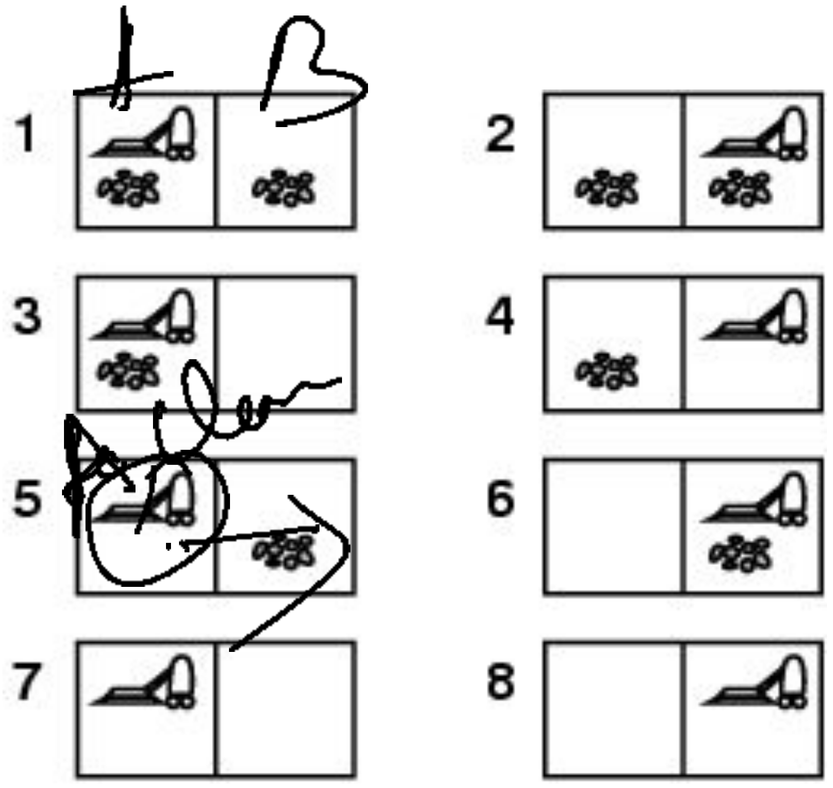


Problem types

- Deterministic, fully observable □ single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable □ sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable □ contingency problem
  - percepts provide new information about current state
  - often interleave} search, execution
- Unknown state space □ exploration problem

Example: vacuum world

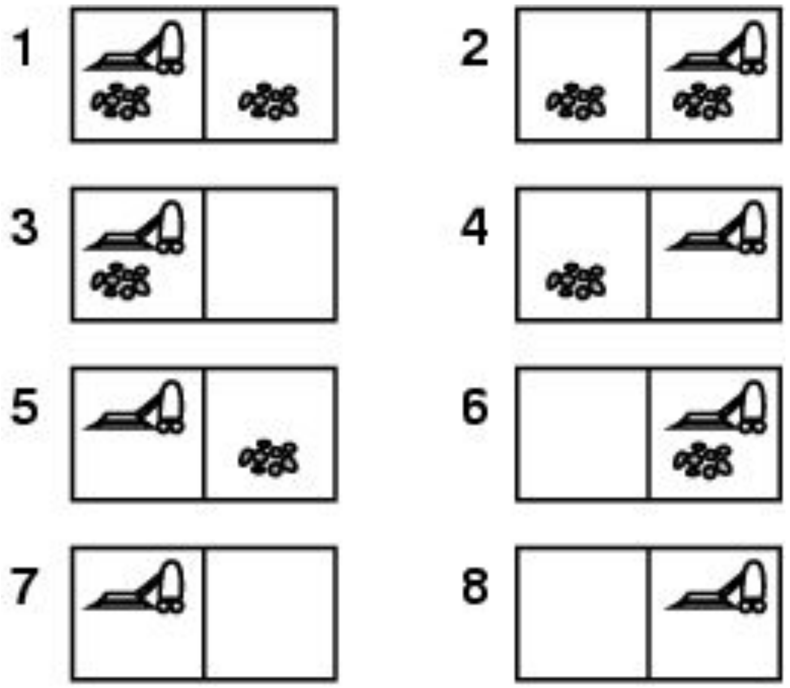
- Single-state, start in #5.
  Solution?



Example: vacuum world

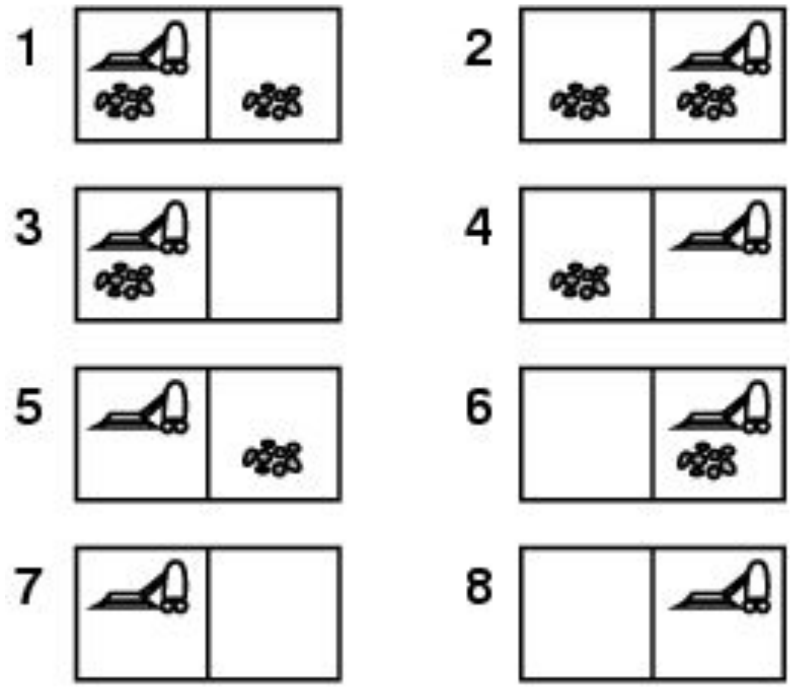
- Single-state, start in #5.
  Solution? *[Right, Suck]*

- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g.,
  Right goes to {*2,4,6,8*}
  Solution?



Example: vacuum world

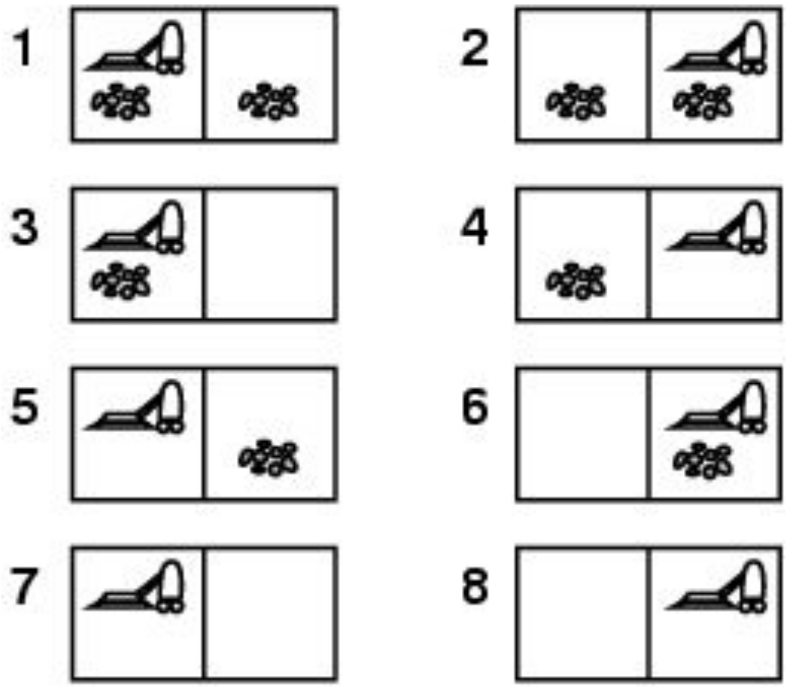
- Sensorless, start in *{1,2,3,4,5,6,7,8}* e.g., *Right* goes to *{2,4,6,8}* Solution? *[Right,Suck,Left,Suck]*



- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean]*, i.e., start in #5 or #7 Solution?

Example: vacuum world

- Sensorless, start in *{1,2,3,4,5,6,7,8}* e.g., *Right* goes to *{2,4,6,8}* Solution? *[Right,Suck,Left,Suck]*



- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean]*, i.e., start in #5 or #7
    Solution? *[Right, **if** dirt **then** Suck]*

Assumptions for the Environment

- Observable : Knows the current state .
- Discrete: Only finitely many actions
- Known : Result of action is known.
- Deterministic : Each action has only one action.

Single-state problem formulation

A problem is defined by four items:

1. initial state e.g., "at Arad"
2. actions or successor function *S(x)* = set of action–state pairs
   - e.g., *S(Arad) = {<Arad □ Zerind, Zerind>, … }*
3. goal test, can be
   - explicit, e.g., *x* = "at Bucharest"
   - implicit, e.g., *Checkmate(x)*
4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - *c(x,a,y)* is the step cost, assumed to be ≥ 0

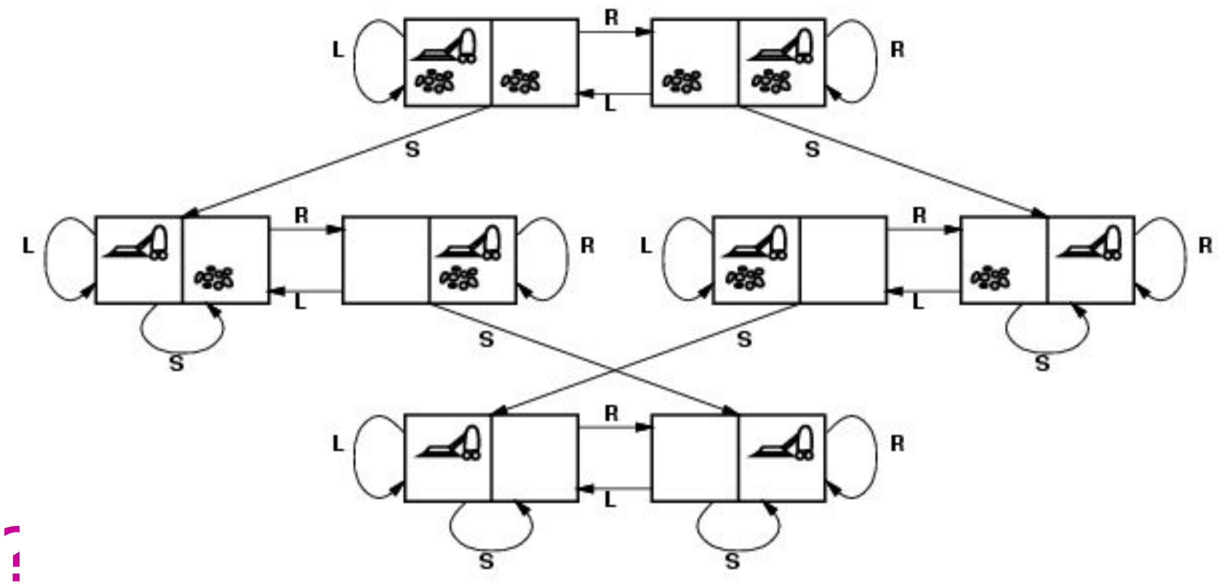
- A solution is a sequence of actions leading from the initial state to a goal state

Selecting a state space

- Real world is absurdly complex
  □ state space must be abstracted for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  - e.g., "Arad □ Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad‟ must get to some real state "in Zerind"
- (Abstract) solution =
  - set of real paths that are solutions in the real world

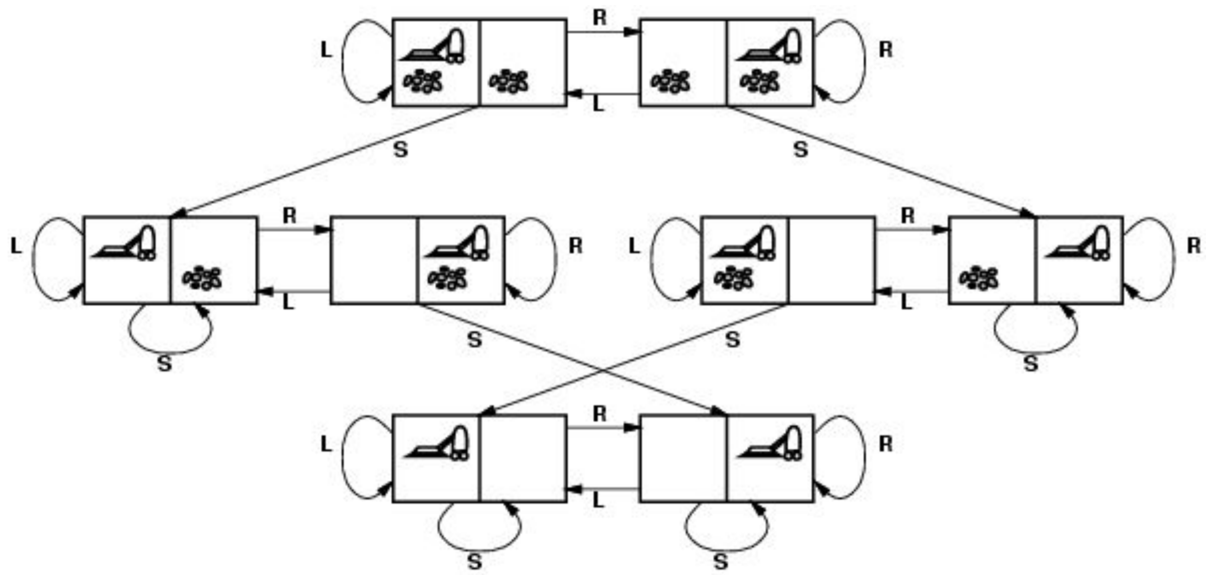
- Each abstract action should be "easier" than the original

Vacuum world state space graph



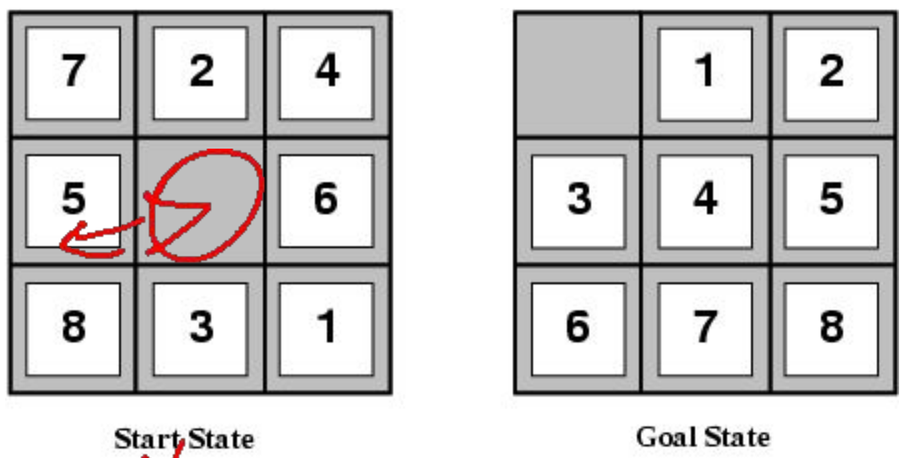
- states?
- actions?
- goal test?
- path cost?

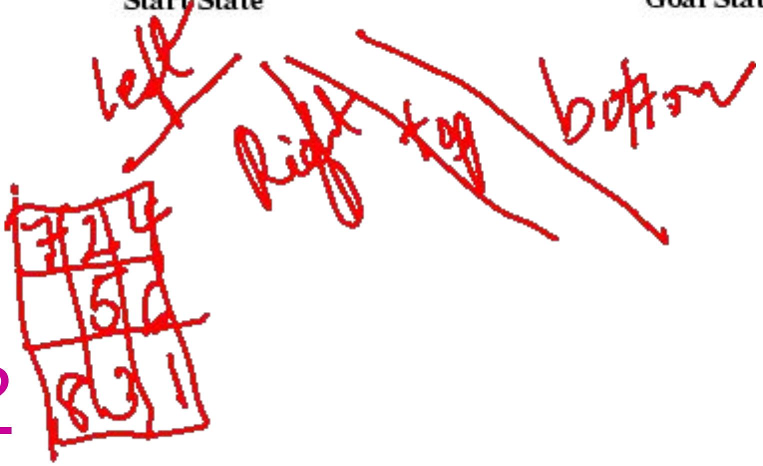
Vacuum world state space graph



- states? integer dirt and robot location
- actions? *Left*, *Right*, *Suck*
- goal test? no dirt at all locations
- path cost? 1 per action

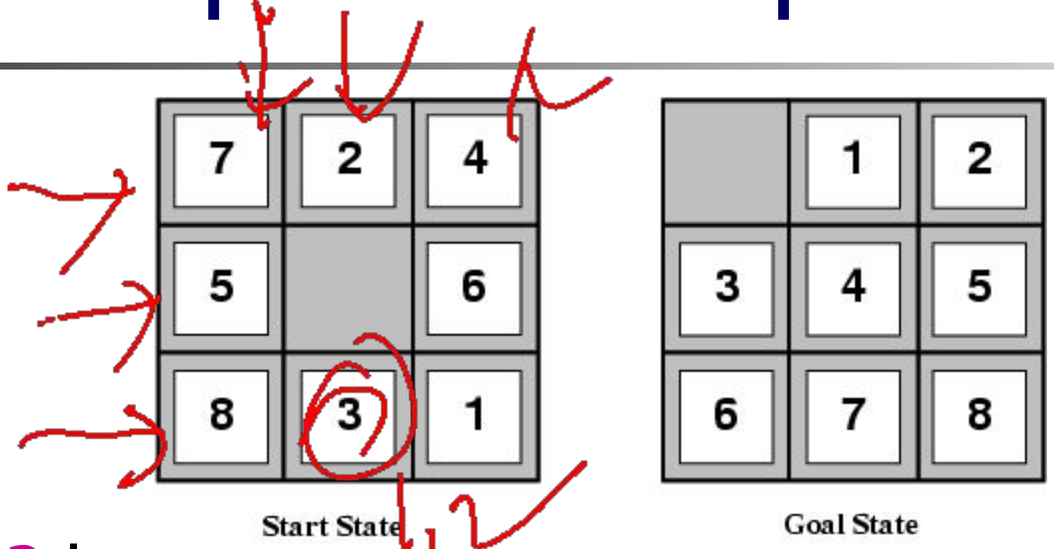
Example: The 8-puzzle





- states?
- actions?
- goal test?
- path cost?

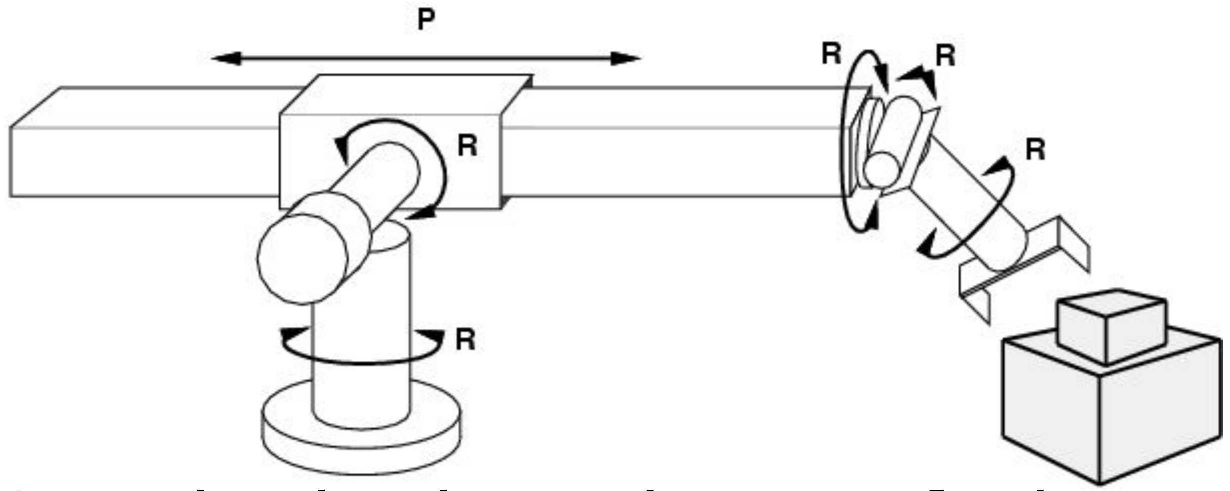
Example: The 8-puzzle



- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move

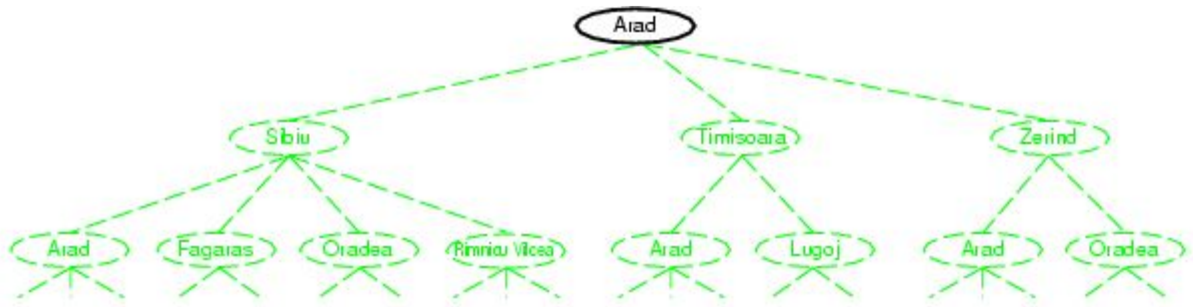
[Note: optimal solution of *n*-Puzzle family is NP-hard]

Example: robotic assembly

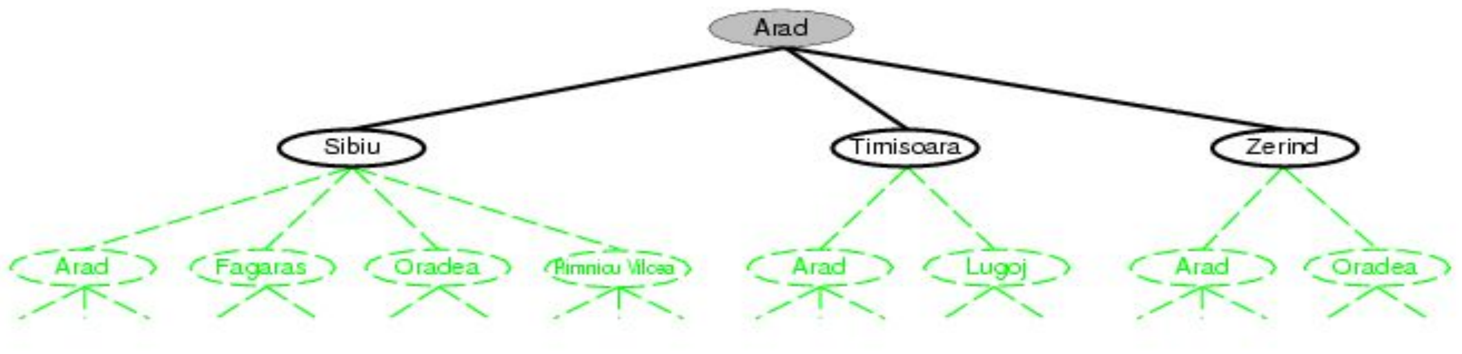


- states?: real-valued coordinates of robot joint angles parts of the object to be assembled
- actions?: continuous motions of robot joints
- goal test?: complete assembly
- path cost?: time to execute

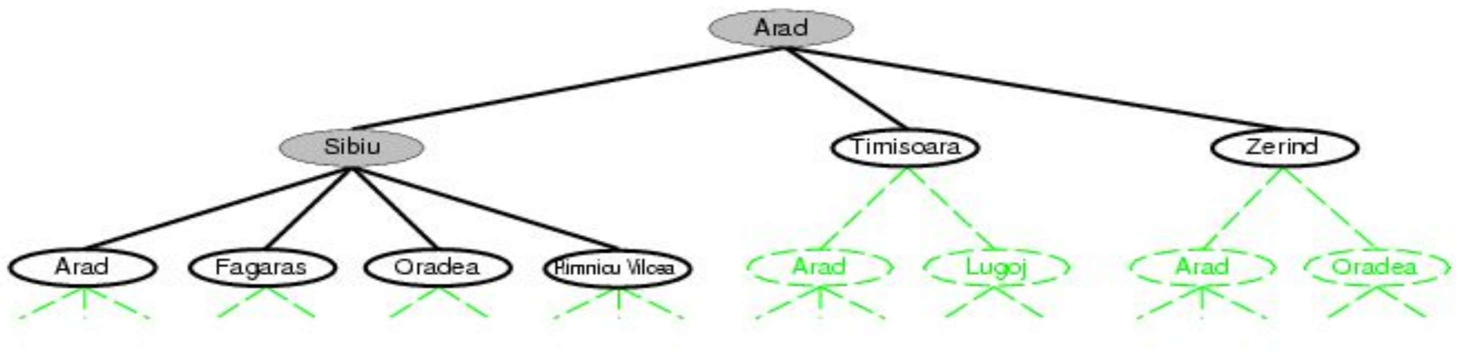
Tree search example



Tree search example



Tree search example



- function TREE-SEARCH(problem) returns a solution, or failure
- initialize the frontier using the initial state of problem
- loop do
- if the frontier is empty then return failure
- choose a leaf node and remove it from the frontier
- if the node contains a goal state then return the corresponding solution
- expand the chosen node, adding the resulting nodes to the frontier

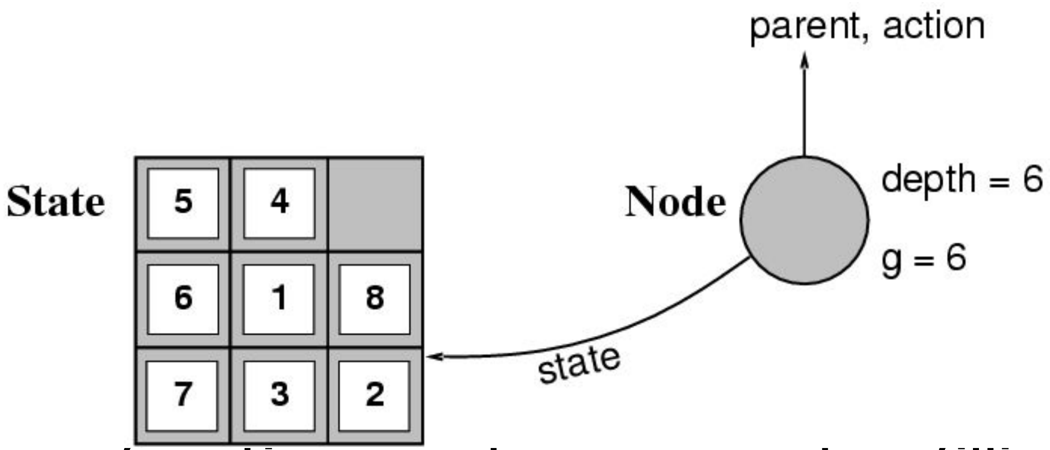
- * frontier –The set of all leaf nodes available for expansion at any given point

- function GRAPH-SEARCH(problem) returns a solution, or failure
- initialize the frontier using the initial state of problem
- initialize the explored set to be empty
- loop do

- if the frontier is empty then return failure
- choose a leaf node and remove it from the frontier
- if the node contains a goal state then return the corresponding solution
- add the node to the explored set
- expand the chosen node, adding the resulting nodes to the frontier
- only if not in the frontier or explored set

Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



- The `Expand` function creates new nodes, filling in the various fields and using the `SuccessorFn` of the problem to create the corresponding states.

- function CHILD-NODE(problem, parent, action) returns a node
- return a node with
- STATE = problem.RESULT(parent.STATE, action),
- PARENT = parent, ACTION = action,
- PATH-COST = parent.PATH-COST + problem.STEP-COST(parent.STATE, action)

Search strategies

- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?

- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree// no.of children at each node
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

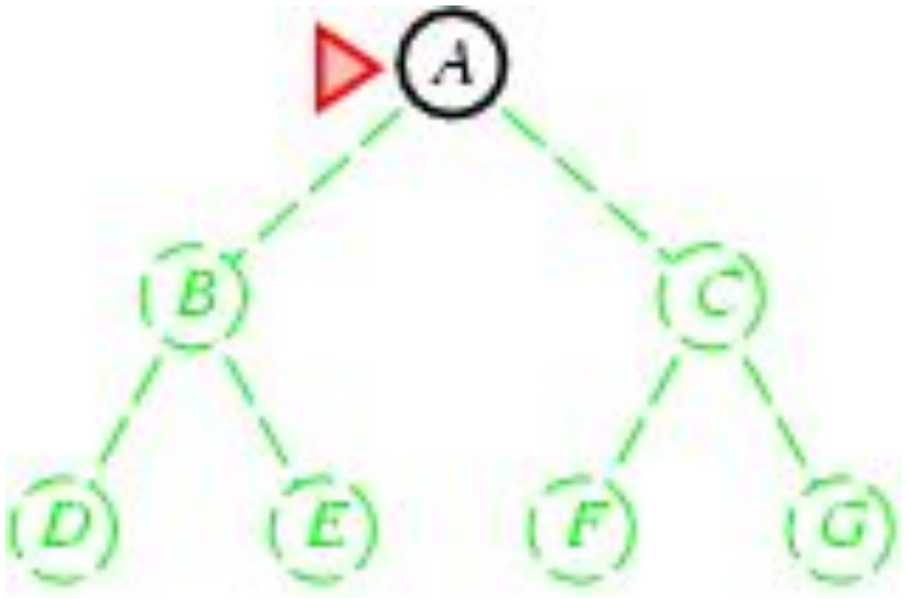
Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Breadth-first search

- Expand shallowest unexpanded node

- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



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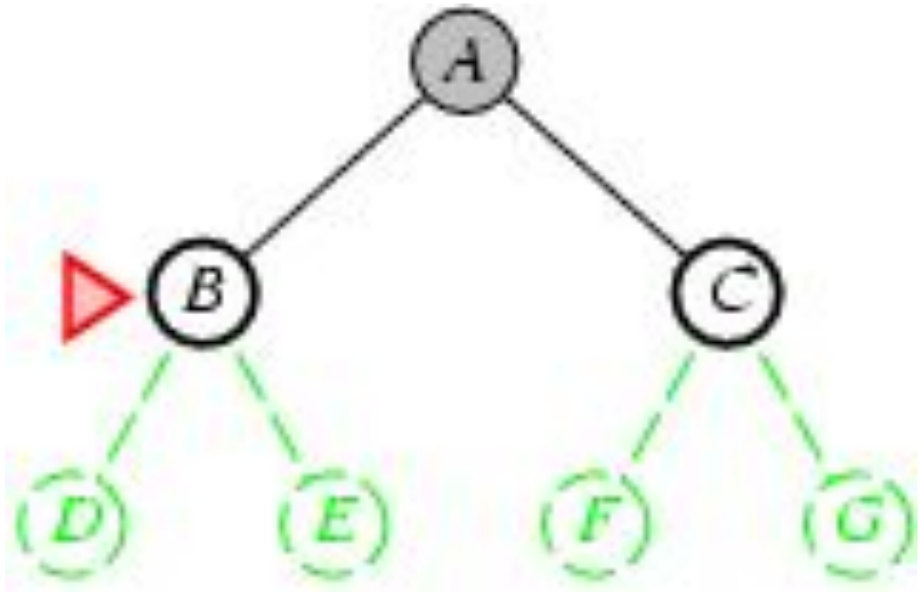

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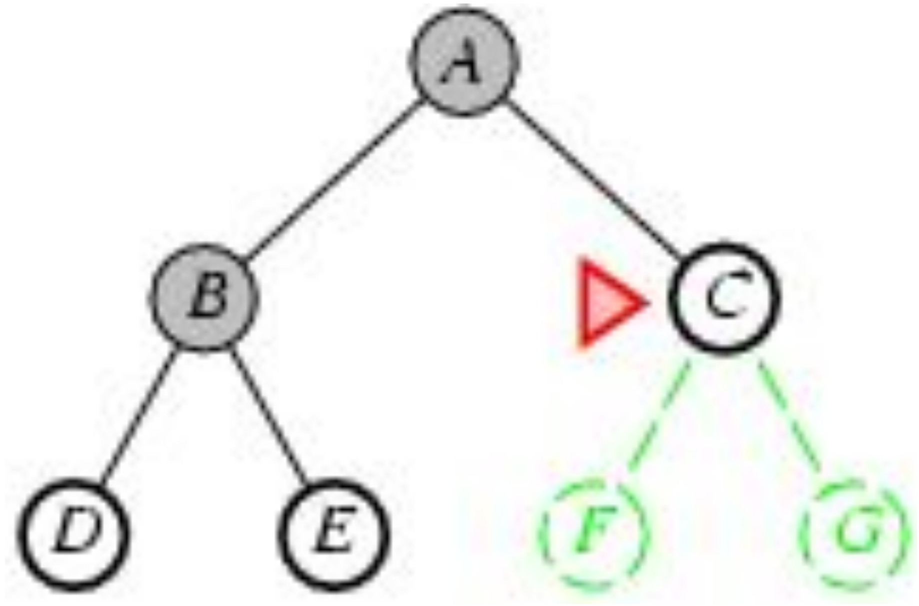

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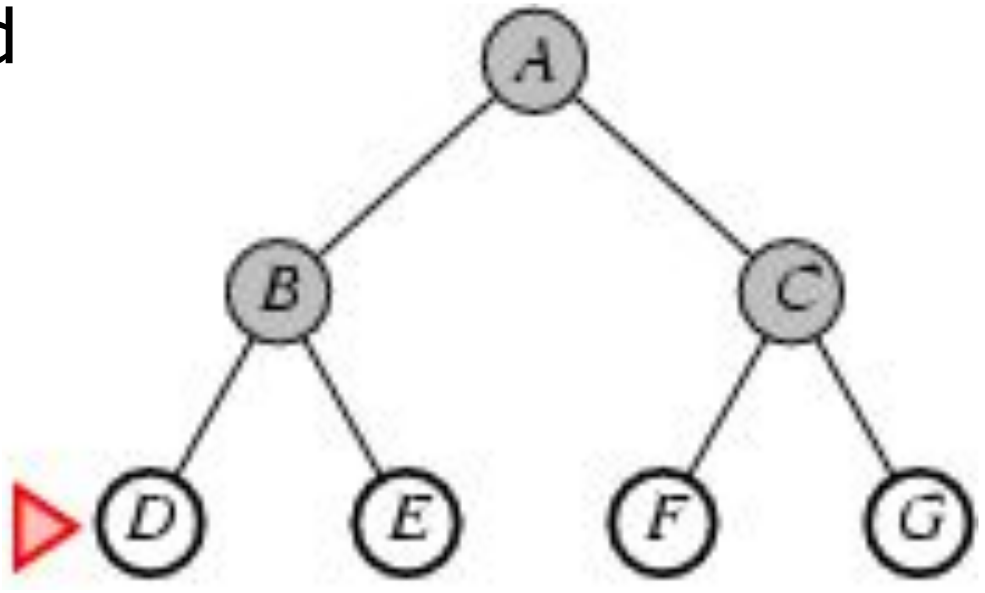


function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure

- node ←a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
- if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
- frontier ← a FIFO queue with node as the only element
- explored ← an empty set
- loop do if EMPTY?(frontier) then return failure
- node ← POP(frontier) /* chooses the shallowest node in frontier */

- add node.STATE to explored
- for each action in problem.ACTIONS(node.STATE) do
- child ← CHILD-NODE(problem, node, action)
- if child.STATE is not in explored or frontier then
- if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
- frontier ← INSERT(child,frontier)

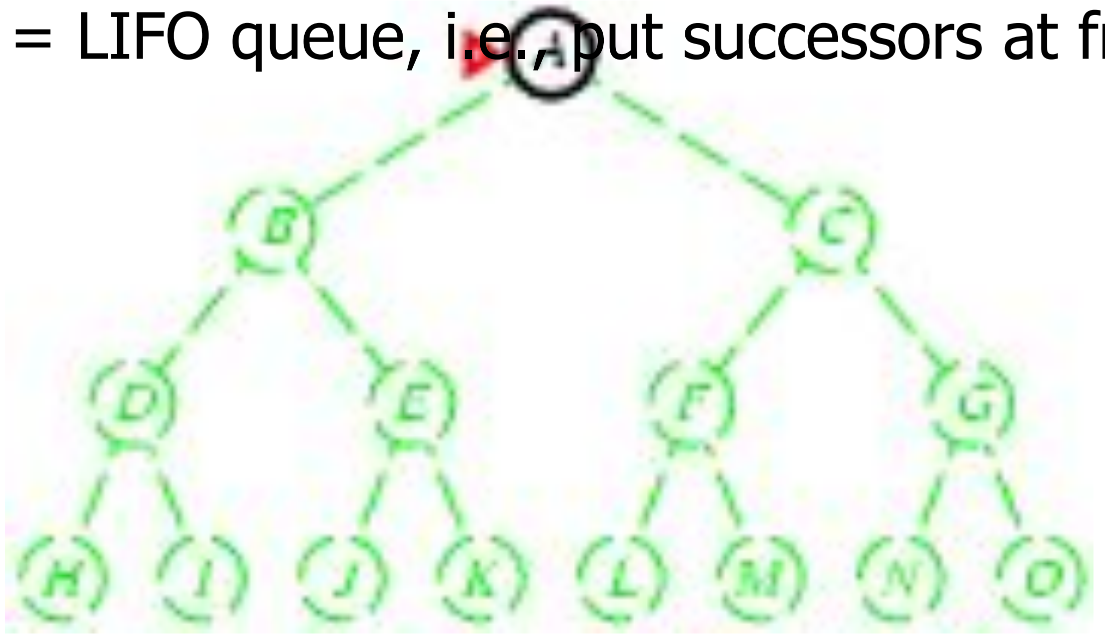
Properties of breadth-first search

- Complete? Yes (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots+b^d + b(b^d-1) = O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)
- Space is the bigger problem (more than time)

Depth-first search

- Expand deepest unexpanded node

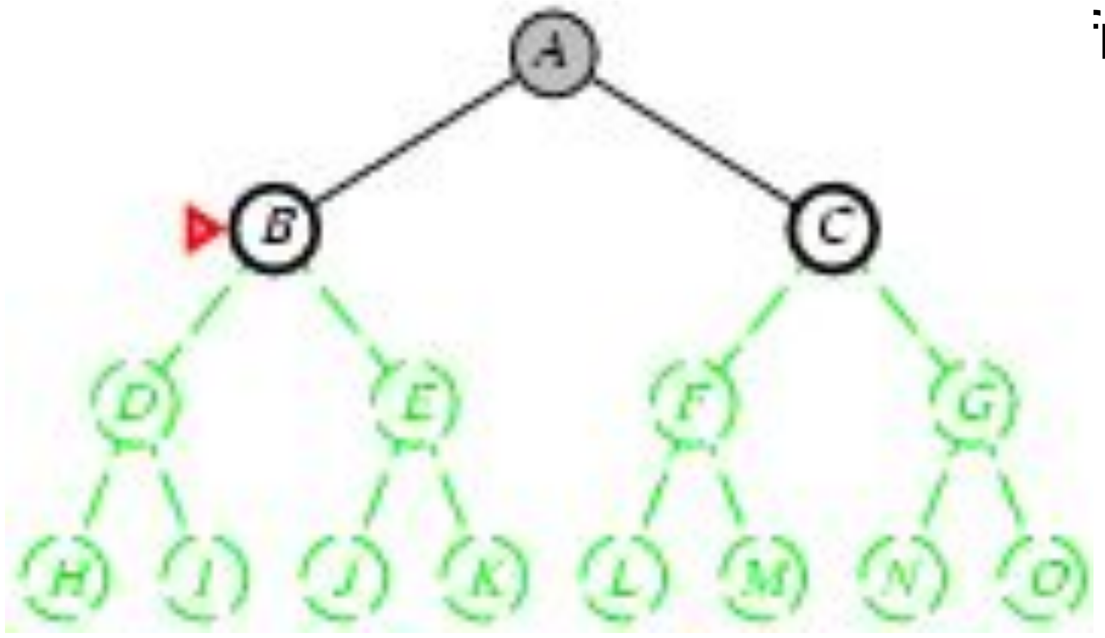
- Implementation:
    - *fringe* = LIFO queue, i.e., put successors at front



Depth-first search

- Expand deepest unexpanded node

- Implementation:
  - *fringe* ront



Depth-first search

- Expand deepest unexpanded node

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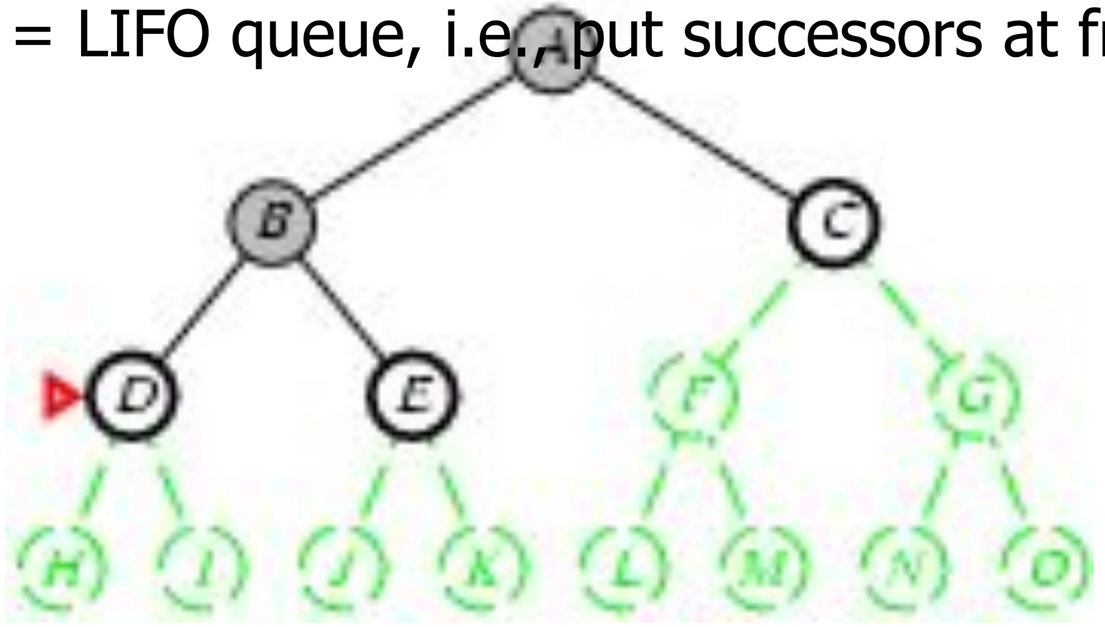

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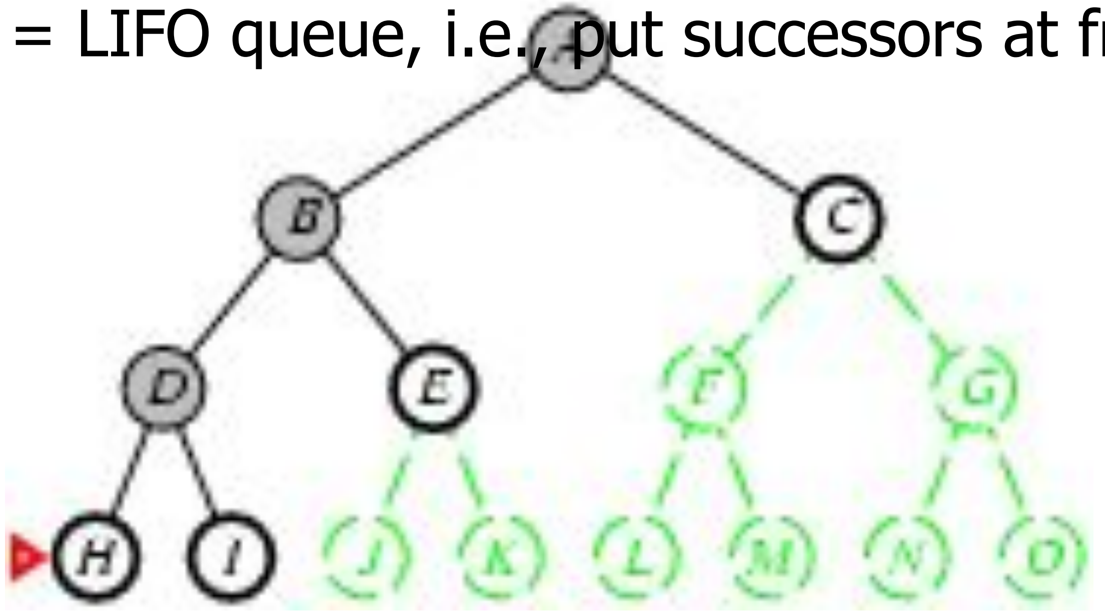

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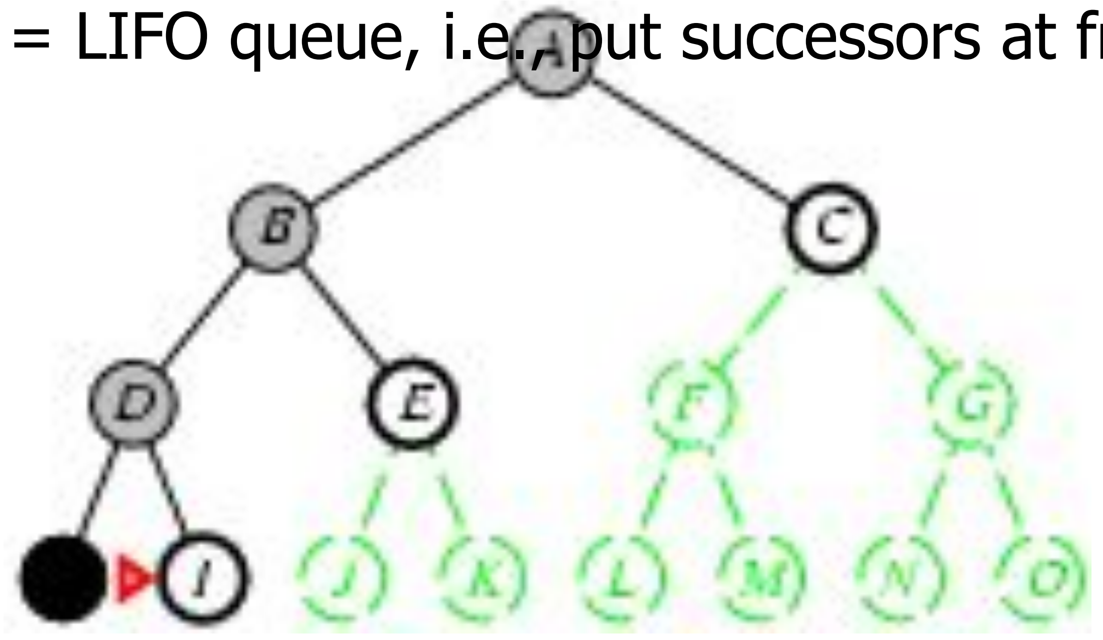

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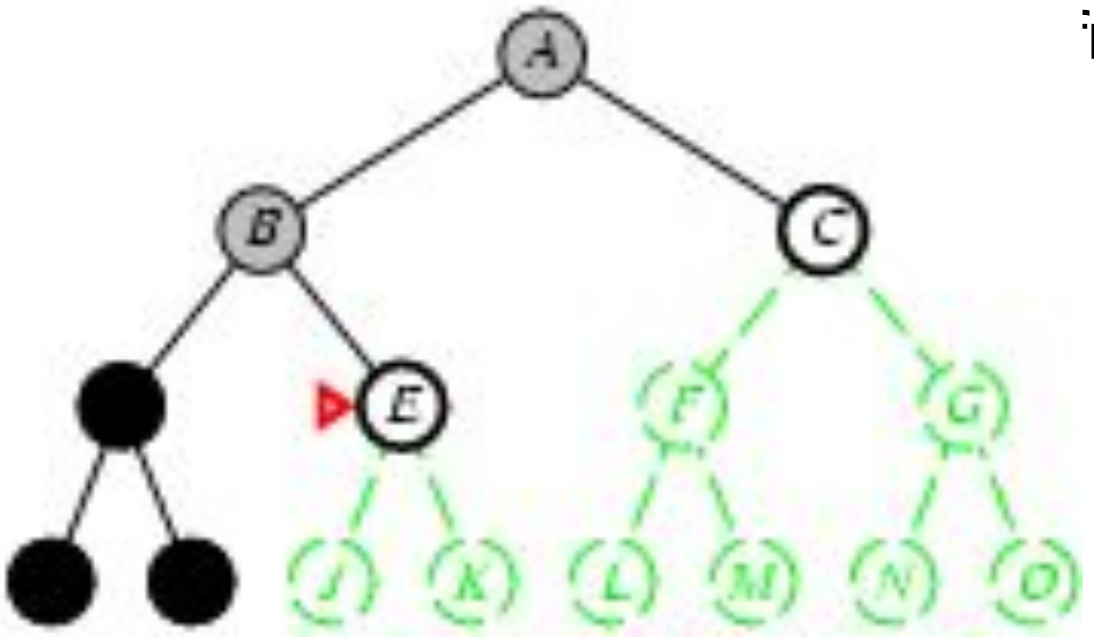
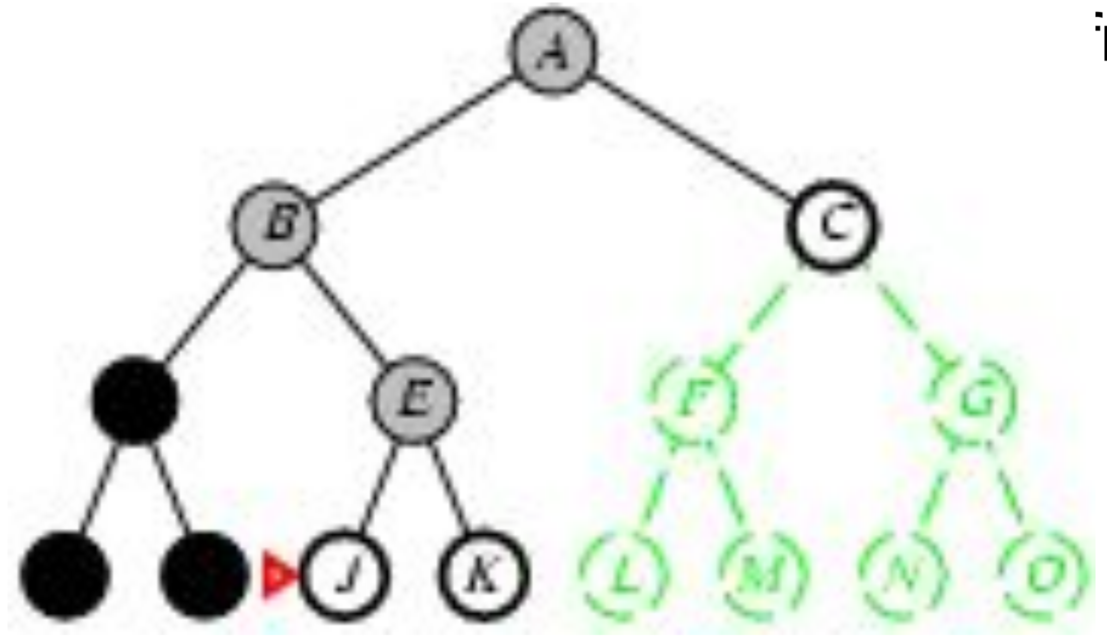


Depth-first search

- Expand deepest unexpanded node

- Implementation:
  - *fringe*



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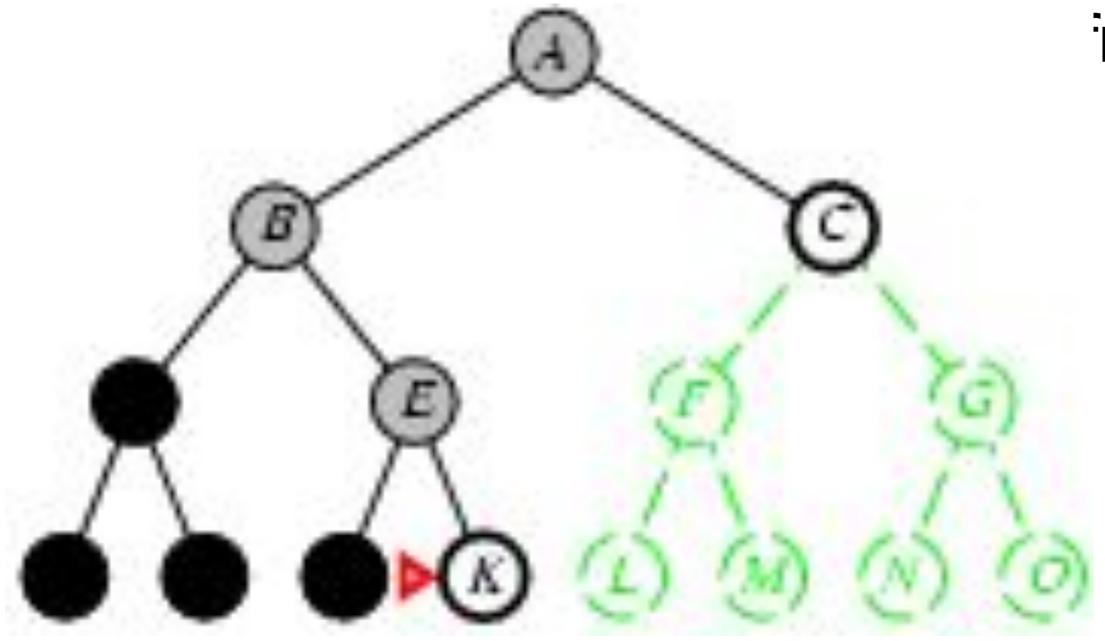
Depth-first search

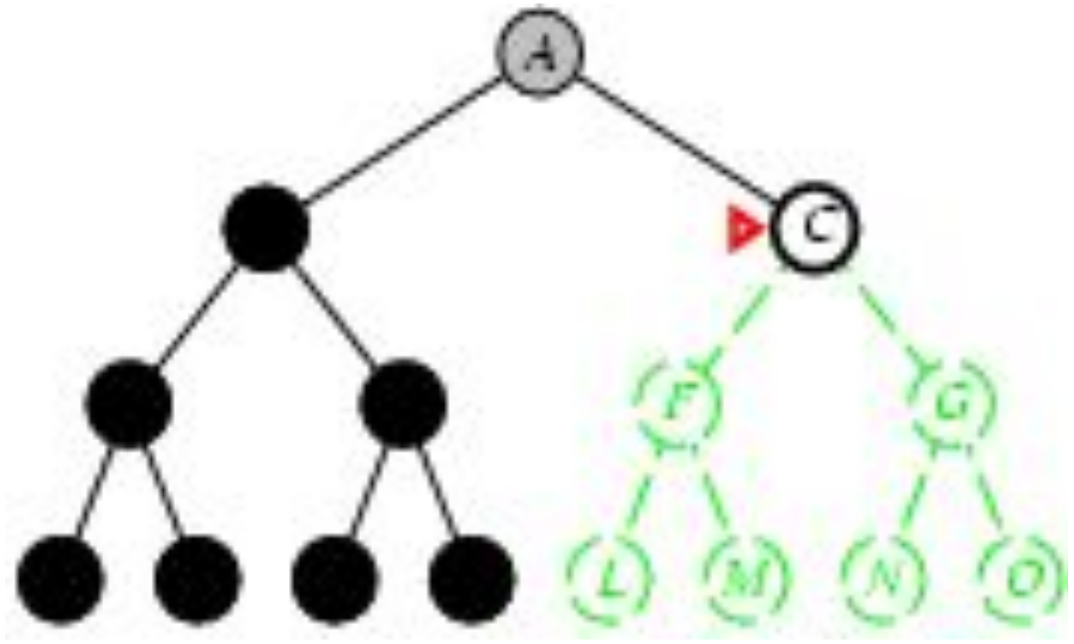
- Expand deepest unexpanded node
- Implementation:
  - *fringe* ront



Depth-first search

- Expand deepest unexpanded node

- Implementation:
  - *fringe* front



Depth-first search

- Expand deepest unexpanded node

- Implementation:
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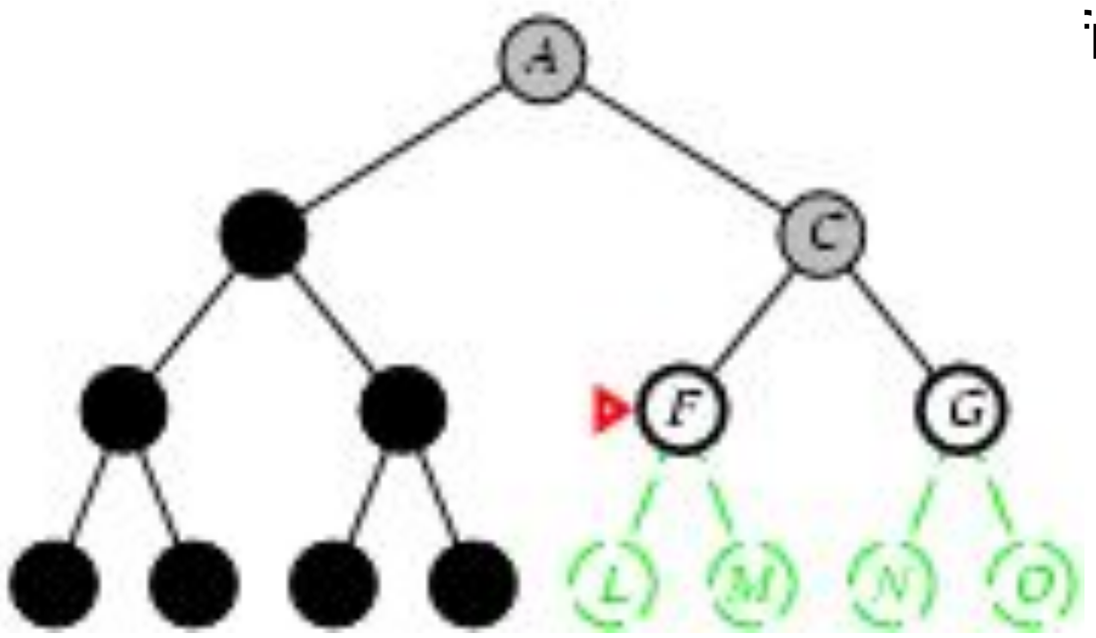

Depth-first search

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- Implementation:
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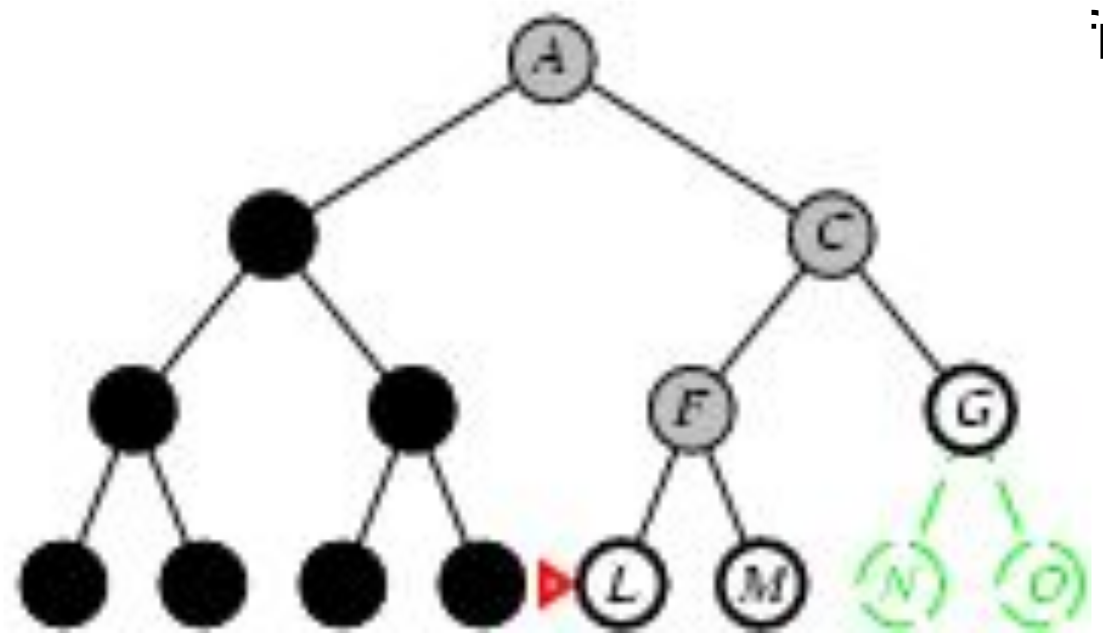

Depth-first search

- Expand deepest unexpanded node

- Implementation:
  - *fringe*

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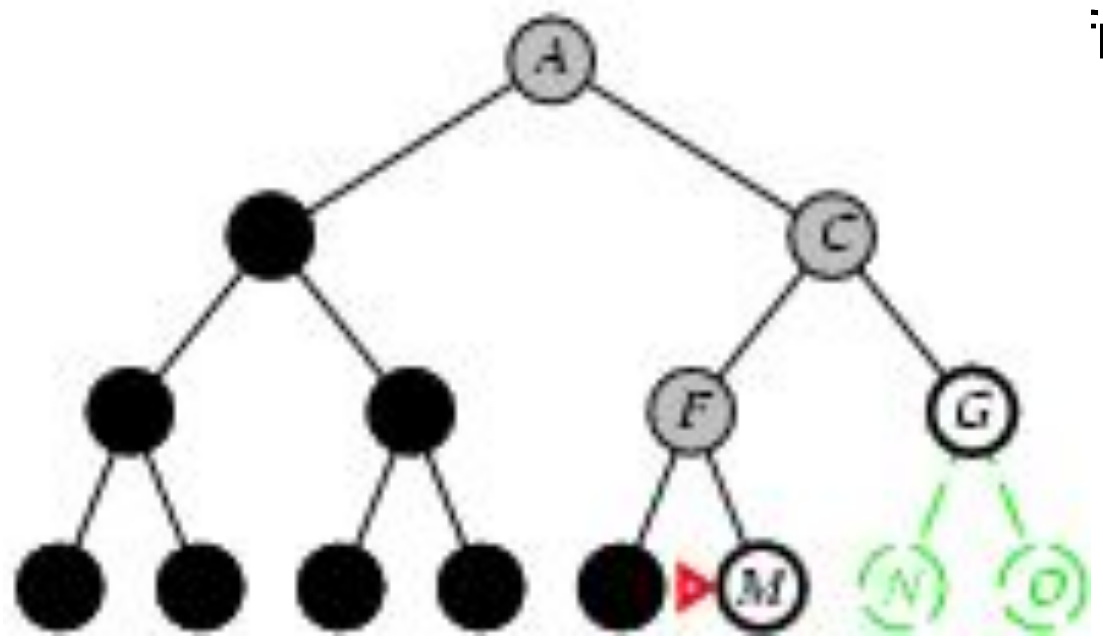


Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path

      ☐ complete in finite spaces

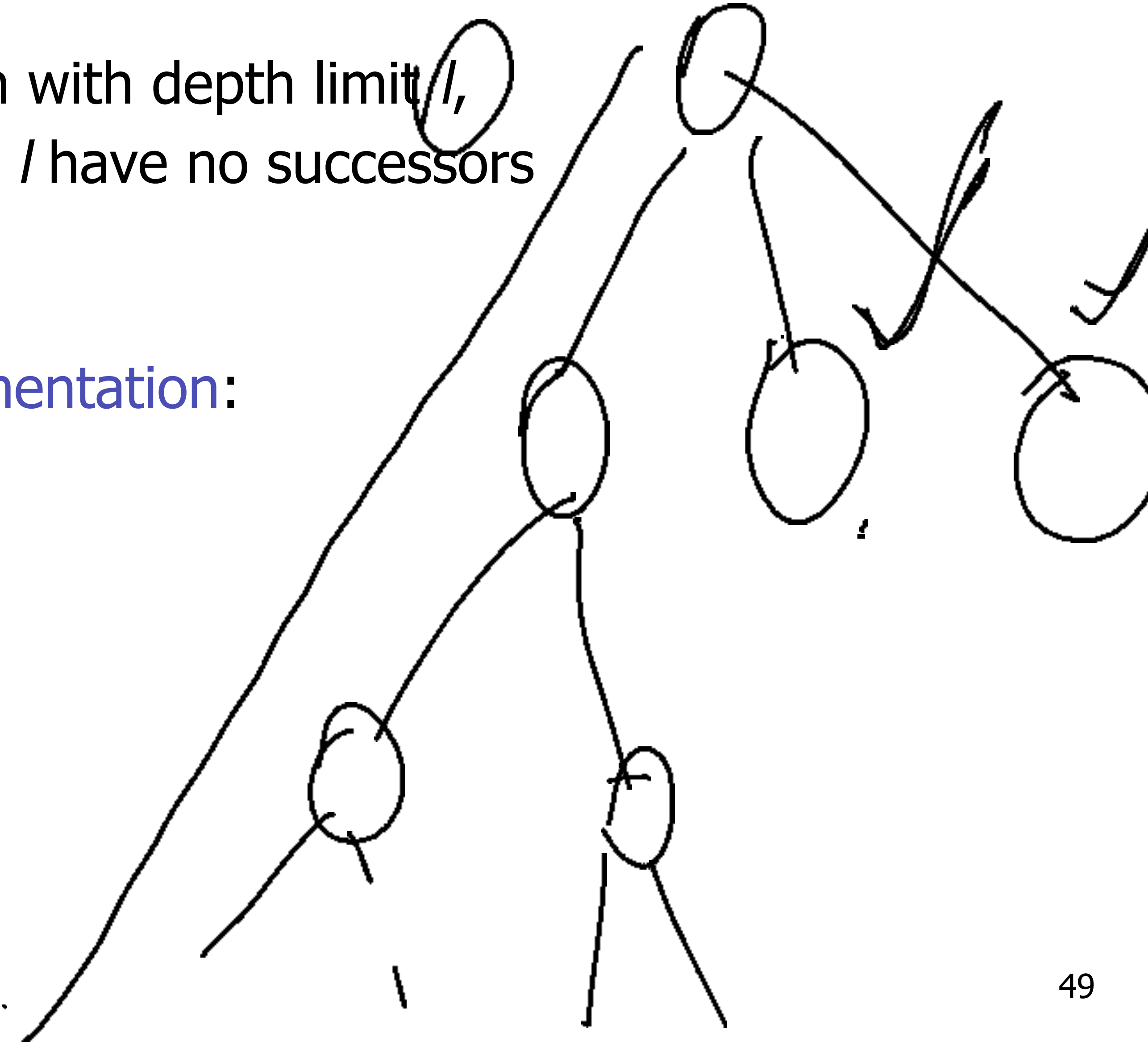
- Time? $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first

- Space? $O(bm)$, i.e., linear space!

Depth-limited search

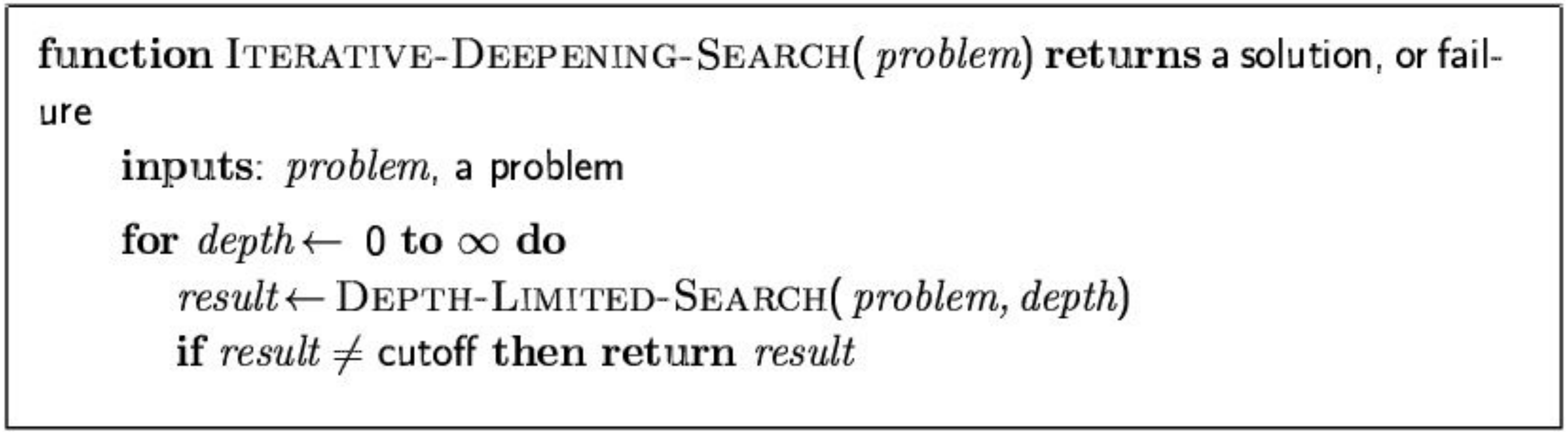
= depth-first search with depth limit *l*,
i.e., nodes at depth *l* have no successors

- Recursive implementation:



```
function DEPTH-LIMITED-SEARCH(problem,limit) returns a solution, or
 failure/cutoff
  return RECURSIVE-DLS(MAKE-NODE(problem.INITIAL-STATE),problem,limit
function RECURSIVE-DLS(node,problem,limit) returns a solution, or
 failure/cutoff
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  else if limit = 0 then return cutoff
  else
    cutoff occurred? ← false
    for each action in problem.ACTIONS(node.STATE) do
      child ← CHILD-NODE(problem,node,action)
      result ← RECURSIVE-DLS(child,problem,limit − 1)
      if result = cutoff then
        cutoffoccurred? ← true
      else if result ≠failure then return result
    if cutoff occurred? then return cutoff else return failure
```

Iterative deepening search

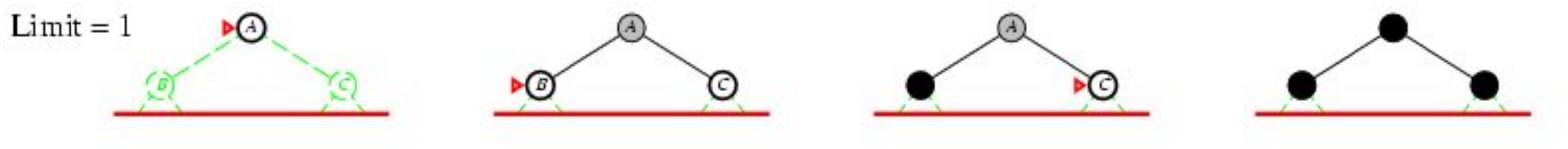


```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH(problem, depth)
        if result ≠ cutoff then return result
```

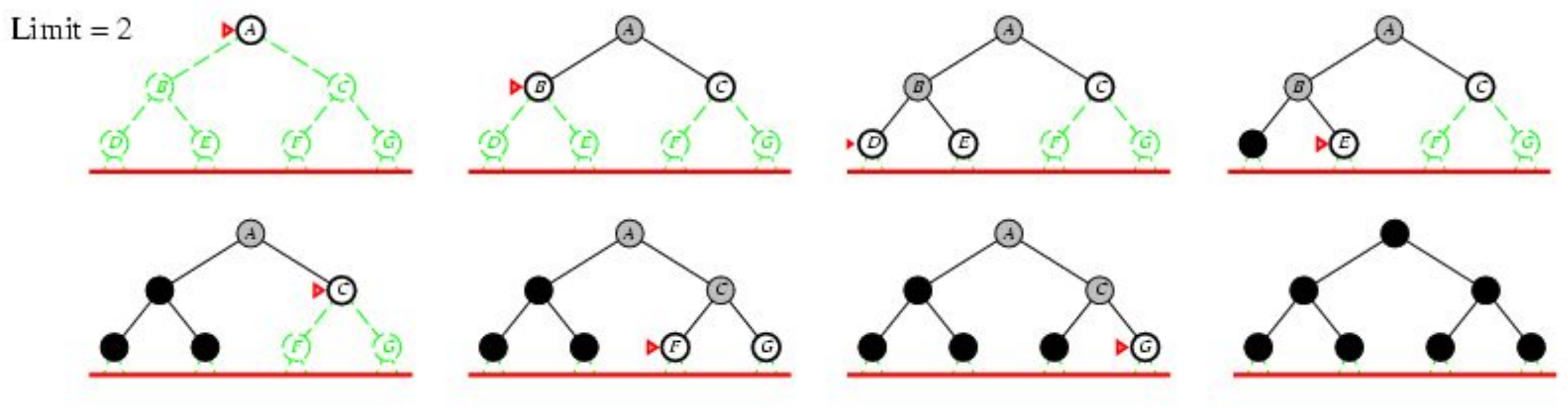
Iterative deepening search l =0

Limit = 0

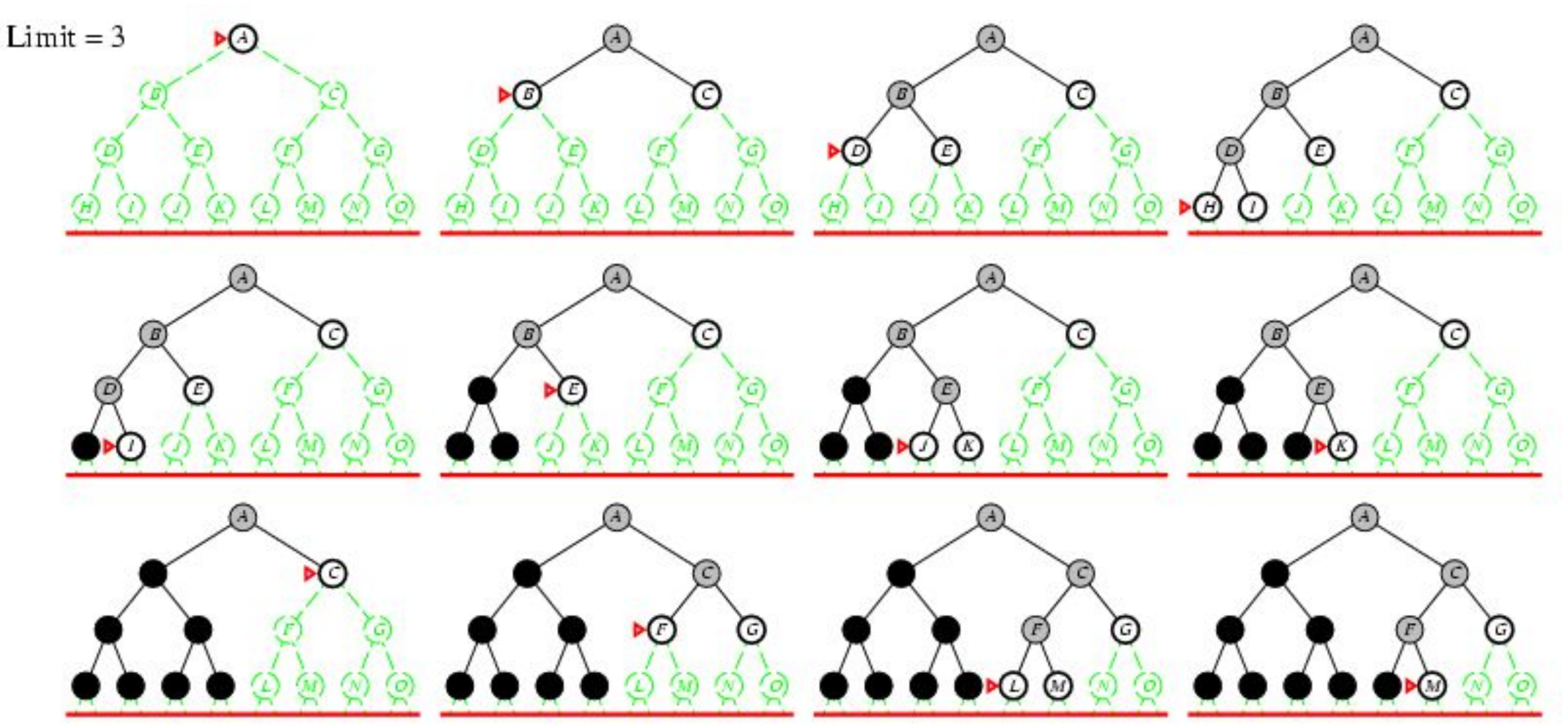
Iterative deepening search *l* =1



Iterative deepening search *l* =2



Iterative deepening search *l* =3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\ b\text{^}^1 + (d-1)b\text{^}^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

//last term is bottom level

- For *b = 10, d = 5*,
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$

Properties of iterative deepening search

- Complete? Yes
- Time? $(d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1