Cyclic Subgroups If a subgroup of a group can be generated using the power of an element, the subgroup is called the cyclic subgroup. The term power here means repeatedly applying the group operation to the element:

$$a^n \to a \bullet a \bullet \dots \bullet a$$
 (*n* times)

the set made from this process is referred to as $\langle a \rangle$. Note that the duplicate elements must be discarded to the also that $a^0 = e$.

Example 4.7 Four cyclic subgroups can be made from the group $G = \langle Z_6, + \rangle$. They are $H_1 = \langle \{0\}, + \rangle$ for $H_2 = \langle \{0, 2, 4\}, + \rangle$, $H_3 = \langle \{0, 3\}, + \rangle$, and $H_4 = G$. Note that when the operation is addition, a^n mean multiplying n by a. Note also that in all of these groups, the operation is addition modulo 6. The following now how we find the elements of these cyclic subgroups.

The cyclic subgroup generated from 0 is H_1 , which has only one element, the identity element.

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0^0 \mod 6 = 0 (stop: the process will be repeated)
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The cyclic subgroup generated from 1 is H_4 , which is G itself.

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1<sup>0</sup> mod 6 = 0

1<sup>1</sup> mod 6 = 1

1<sup>2</sup> mod 6 = (1 + 1) mod 6 = 2

1<sup>3</sup> mod 6 = (1 + 1 + 1) mod 6 = 3

1<sup>4</sup> mod 6 = (1 + 1 + 1 + 1) mod 6 = 4

1<sup>5</sup> mod 6 = (1 + 1 + 1 + 1 + 1) mod 6 = 5 (stop: the process will be repeated)
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The cyclic subgroup generated from 2 is H_2 , which has three elements: 0, 2, and 4.

$$2^{0}$$
 mod $6 = 0$
 2^{1} mod $6 = 2$
 2^{2} mod $6 = (2 + 2)$ mod $6 = 4$

(stop: the process will be repeated)

e. The cyclic subgroup generated from 4 is H_2 ; this is not $4^0 \mod 6 = 0$ $4^1 \mod 6 = 4$ $4^2 \mod 6 = (4+4) \mod 6 = 2$ (stop: the following subgroup generated from 5 is H_4 , which is $5^0 \mod 6 = 0$ $5^1 \mod 6 = 5$ $5^2 \mod 6 = 4$ $5^3 \mod 6 = 3$	ne process will be repeated) G itself.
e. The cyclic subgroup generated from 4 is H_2 ; this is not $4^0 \mod 6 = 0$ $4^1 \mod 6 = 4$ $4^2 \mod 6 = (4+4) \mod 6 = 2$ (stop: the following subgroup generated from 5 is H_4 , which is $5^0 \mod 6 = 0$ $5^1 \mod 6 = 5$ $5^2 \mod 6 = 4$ $5^3 \mod 6 = 3$	ne process will be repeated) G itself.
$4^1 \mod 6 = 4$ $4^2 \mod 6 = (4 + 4) \mod 6 = 2$ (stop: the cyclic subgroup generated from 5 is H ₄ , which is $5^0 \mod 6 = 0$ $5^1 \mod 6 = 5$ $5^2 \mod 6 = 4$ $5^3 \mod 6 = 3$	G itself. It is to be repeated) in the process with the repeated) in the repeated of t
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$5^5 \mod 6 = 1$ (stop: t	disable of the state of the sta
Example 4.8 Three cyclic subgroups can be made from elements: 1, 3, 7, and 9. The cyclic subgroups are H ₁ = <{1 lowing show how we find the elements of these subgroups. a. The cyclic subgroup generated from 1 is H ₁ . The selement.	$\{1, \times\}, \times\}, H_2 = \{1, 9\}, \times\}, \text{ and } H_3 = G.$ The first subgroup has only one element, the idea.
	the process will be repeated)
b. The cyclic subgroup generated from 3 is H_3 , which $3^0 \mod 10 = 1$ $3^1 \mod 10 = 3$ $3^2 \mod 10 = 9$	on or purious as refuring to the
$3^3 \mod 10 = 7$ (stop:	rue bioress will be reheared
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7^0 mod $10 = 1$ 7^1 mod $10 = 7$ 7^2 mod $10 = 9$ 7^3 mod $10 = 9$	is Gritself. In tank or is a control of the control
7^0 mod $10 = 1$ 7^1 mod $10 = 7$ 7^2 mod $10 = 9$ 7^3 mod $10 = 9$	the process will be repeated)