

# Binary Trees

+ (BST)

~Arbind

# Trees

## Binary Tree

```
struct Node {
    int data;
    Node* left;
    Node* right;
    Node(int val) {
        data = val;
        left = right = NULL;
    }
}
```

```
main() {
    struct Node* root =
        new Node(1);
    root->left = new Node(2);
    root->right = new Node(3);
}
```

## Traversal Techniques (BFS/DFS)

### DFS

#### Inorder Traversal (Left root right)

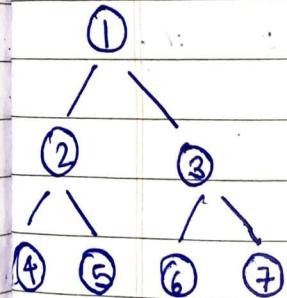
4 2 5 1 6 3 7

#### Preorder Traversal (Root Left right)

1 2 4 5 3 6 7

#### Postorder Traversal (Left right Root)

4 5 2 6 7 3 1



#### BFS (Level Order)

1 2 3 4 5 6 7

## Pre-order Traversal

TC  $\rightarrow$  O(N)  
SC  $\rightarrow$  O(N)  
↳ auxiliary space

```
void preorder(node)
```

```
{ if (node == NULL)
```

```
    return;
```

```
(1) print node
```

```
cout << node -> data << "left";
```

```
preorder (node -> left);
```

```
preorder (node -> right);
```

```
}
```

## In-order Traversal

```
void inorder (node)
```

```
{
```

```
if (node == NULL)
```

```
return;
```

```
(1) print node
```

```
inorder (node -> left);
```

```
cout << node -> data << " ";
```

```
inorder (node -> right);
```

```
}
```

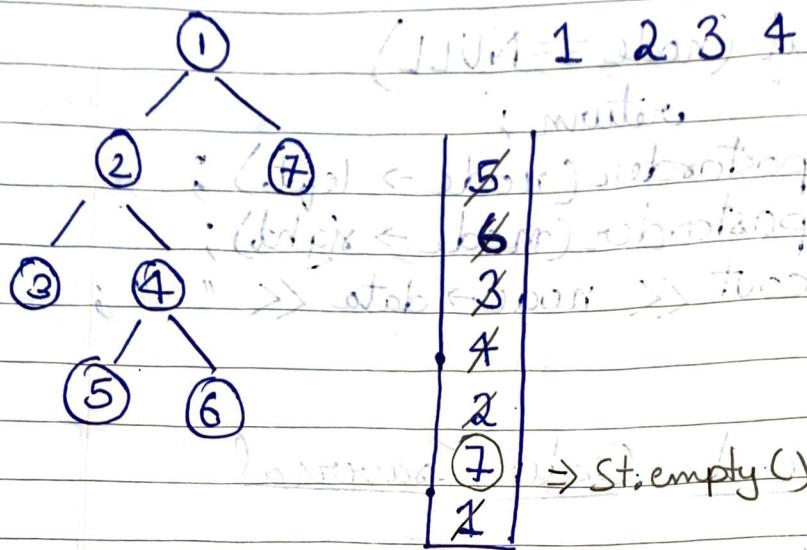
## Post-order Traversal

```
void postorder(node)
{
    if (node == NULL)
        return;
    postorder(node->left);
    postorder(node->right);
    cout << node->data << " ";
}
```

## Level Order Traversal

```
vector<vector<int>> levelOrder(TreeNode *root) {
    vector<vector<int>> ans;
    if (root == NULL)
        return ans;
    queue<TreeNode*> q;
    q.push(root);
    while (!q.empty()) {
        int size = q.size();
        vector<int> level;
        for (int i = 0; i < size; i++) {
            TreeNode *node = q.front();
            q.pop();
            level.push_back(node->val);
            if (node->left) q.push(node->left);
            if (node->right) q.push(node->right);
        }
        ans.push_back(level);
    }
    return ans;
}
```

## Iterative Preorder Traversal (Root Left Right)



public:

```
vector<int> preorder1(TreeNode* root) {
    vector<int> preorder;
    if (root == NULL) return {};
    stack<TreeNode*> st;
    st.push(root);
    while (!st.empty()) {
        TreeNode* curr = st.top();
        st.pop();
        cout << curr->val;
        if (curr->right) st.push(curr->right);
        if (curr->left) st.push(curr->left);
        preorder.push_back(curr->val);
    }
    return preorder;
}
```

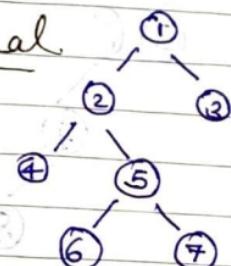
```

if (root->right != NULL)
    st.push(root->right);
if (root->left != NULL)
    st.push(root->left);
}
return preorder;
}

```

### Iterative Inorder Traversal (Left Root Right)

4 2 6 5 7 1 3



### Code

```

3
7
6
5
4
2
1
stack<TreeNode*> st;
TreeNode* node = root;
vector<int> inorder;
while (true) {
    if (node != NULL) {
        st.push(node);
        node = node->left;
    }
    else {
        if (st.empty()) break;
        node = st.top();
        st.pop();
        inorder.push_back(node->val);
        node = node->right;
    }
}
return inorder;
}

```

4 2 6 5 7 1 3

```

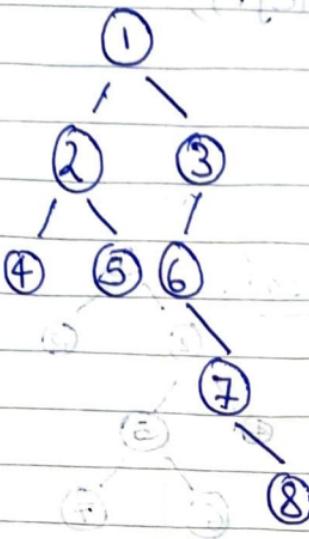
4 2 6 5 7 1 3
if (st.empty())
    return inorder;
    node = node->right;
}
return inorder;
}

```

## Iterative Postorder Traversal

2 Stack

4 5 2 8 7 6 3 1



vector<int> = postorder();
if (root == NULL)

return postorder;

stack<TreeNode\*> st1, st2;

st1.push(root);

while(!st1.empty()) {

root = st1.top();

st2.push(root);

st1.pop();

if (root->left != NULL)

st1.push(root->left);

if (root->right != NULL)

st1.push(root->right);

while (!st.empty())

postorder.push\_back (st2.top() →  
val);

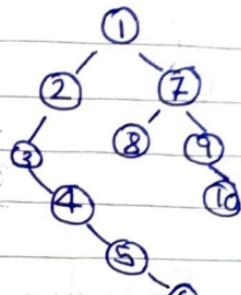
st2.pop();

return postorder;

time space

1 stack (dn) O(N)

65 4 3 2 8 10 971



while (cur != null || !st.empty())

{

if (cur != null)

st.push (cur)

cur = cur → left;

else

temp = st.top () → right;

if (temp == Null)

temp = st.top ()

st.pop()

ans.push\_back (temp);

while (!st.empty ()) { if (temp == st.top () → right)

temp = st.top (), st.pop ()

ans.push\_back (temp - val);

else

cur = temp;

Skew Tree  
all node  
at max have only 1 child

Max Dept in Binary Tree  $TC \rightarrow O(N)$   $SC \rightarrow O(1)$

(1) (2) (3) (4) (5) (6)

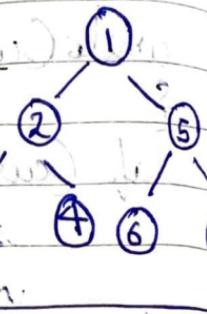
Code:

```
int maxDepth (TreeNode* root){  
    if (root == NULL)  
        return 0;  
    int lh = maxDepth (root -> left);  
    int rh = maxDepth (root -> right);  
    return 1 + max (lh, rh);  
}
```

Preorder (Inorder)  $\downarrow$  Postorder

$TC \rightarrow O(3N)$   $SC \rightarrow O(4N)$

|        |   |
|--------|---|
| (7, 1) | Preorder $\rightarrow 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7$  |
| (6, 1) | Inorder $\rightarrow 3 \ 2 \ 4 \ 1 \ 6 \ 5 \ 7$   |
| (5, 1) | Postorder $\rightarrow 3 \ 4 \ 2 \ 6 \ 7 \ 5 \ 1$ |
| (4, 1) |   |
| (3, 1) |   |
| (2, 1) |   |
| (1, 1) |   |



if  $num = 1$  then  $i \leftarrow 1$   $q[1] \leftarrow \text{graph}$   
 $preOrder$   $\leftarrow \{\text{graph}\}$   $q[1] = \{\text{graph}\}$   $i = 1$   
 $++i$   $q[2] \leftarrow \text{graph}$   $i = 2$   
 $left$   $q[3] \leftarrow \text{graph}$   $i = 3$

$(node, num)$   $\leftarrow$  if  $num \neq 2, 3$  then  
 $(\text{graph}, 1)$

$inOrder$   $\leftarrow \{\text{graph}\}$   $get\_t \leftarrow \text{graph}$   
 $++i$   $q[4] \leftarrow \text{graph}$   $i = 4$   
 $right$   $q[5] \leftarrow \text{graph}$   $i = 5$

if  $num == 3$   
 $postOrder$

$q[6] \leftarrow \text{graph}$   $i = 6$

## Code:

```
stack<pair<TreeNode*, int>> st;
st.push({root, 1});
vector<int> pre, in, post
```

```
if (root == NULL)
```

```
    return;
```

```
while (!st.empty()) {
```

```
    auto it = st.top();
    st.pop();
```

```
    if (it.second == 1) {
```

```
        pre.push_back(it.first->val);
```

```
        it.second++;
```

```
        st.push(it);
```

```
    else if (it.first->left == NULL) {
```

```
        st.push({it.first->right, 1});
```

```
}
```

```
(left <-> right) st.push(it);
```

```
else if (it.second == 2) {
```

```
    in.push_back(it.first->val);
```

```
    it.second++;
```

```
    st.push(it);
```

```
    if (it.first->right != NULL) {
```

```
        st.push({it.first->right, 1});
```

else {

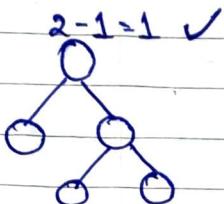
post.push\_back(it->val);

}

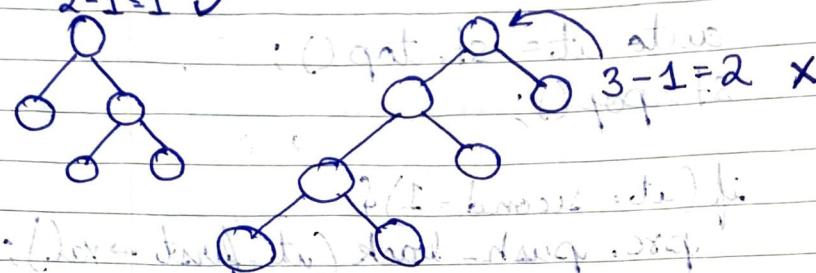
Time O(N)  
Space O(N)

### Check for Balanced Binary Tree

Balanced BT  $\rightarrow$  for every node, height(left) - height(right)  $\leq 1$



$$3-1=2 \times$$



```
int check(node){  
    if (node == NULL) return 0;  
    if (lh == -1) return -1;  
    if (rh == -1) return -1;
```

lh = check(node->left)

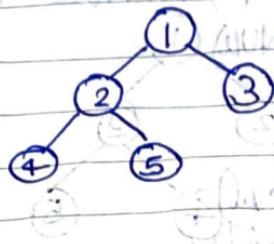
rh = check(node->right)

```
if (lh == -1 || rh == -1) return -1;  
if (abs(lh-rh) > 1) return -1;
```

```
if (lh != -1 & rh != -1) return max(lh, rh)+1;
```

```
else return max(lh, rh)+1;
```

## Diameter of Binary Tree

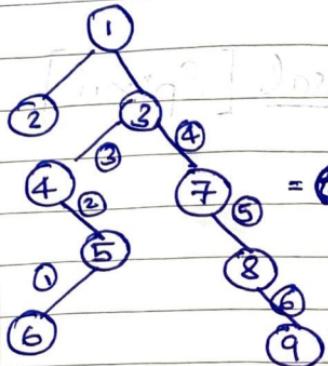


$\rightarrow$  longest path between 2 nodes  
 $\rightarrow$  path does not need to pass via root.

Code:

TreeNode\*

```
int ( . node, int dia)
if (!node) {
    return 0;
}
```

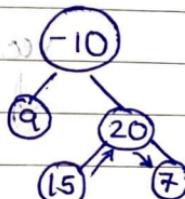


$= \text{dia}$   
 $\text{int lh} = \text{ht}(\text{node} \rightarrow \text{left}, \text{dia})$   
 $\text{int rh} = \text{ht}(\text{node} \rightarrow \text{right}, \text{dia})$   
 $\text{dia} = \max(\text{dia}, \text{lh} + \text{rh})$   
 $\text{return } 1 + \max(\text{lh}, \text{rh});$

## Max Path Sum

any given node =  $\text{val} + (\max L + \max R)$

Code : int manPath ( . node, int maxi )  
 if (node == NULL) return 0;



$\text{int left} = \max(0, \text{manPath}(\text{node} \rightarrow \text{left}))$   
 $\text{int right} = \max(0, \text{manPath}(\text{node} \rightarrow \text{right}), \text{maxi})$

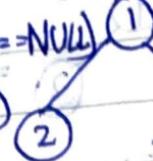
$\text{maxi} = (\text{maxi}, \text{left} + \text{right} + \text{node} \rightarrow \text{val})$   
 $\text{return } \max(\text{left}, \text{right}) + \text{node} \rightarrow \text{val};$

Check if two trees are Identical

bool isSame(TreeNode\* p, TreeNode\* q)

if ( $p == \text{NULL} \text{ || } q == \text{NULL}$ ) return ( $p == q$ )

else if ( $p \rightarrow \text{val} == q \rightarrow \text{val}$ )



return isSame( $p \rightarrow \text{left}, q \rightarrow \text{left}$ ) & isSame( $p \rightarrow \text{right}, q \rightarrow \text{right}$ );

if ( $p \rightarrow \text{val} == q \rightarrow \text{val}$ )

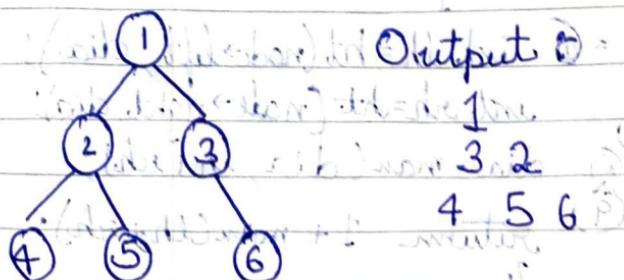
$p \rightarrow \text{right}, q \rightarrow \text{right}$ ;

else if ( $p \rightarrow \text{left}, q \rightarrow \text{left}$ )

else if ( $p \rightarrow \text{right}, q \rightarrow \text{right}$ )

return isSame( $p \rightarrow \text{left}, q \rightarrow \text{left}$ ) & isSame( $p \rightarrow \text{right}, q \rightarrow \text{right}$ );

## Zig-Zag Traversal [Spiral]



vector<int> ans;  
if ( $\text{Node} == \text{NULL}$ ) return ans;  
return ans;

queue<TreeNode\*> q;  
q.push(root);

bool flag = true;

while (!q.empty()) {

(current\_level\_size < flag) ? current\_level\_size++ : current\_level\_size--;

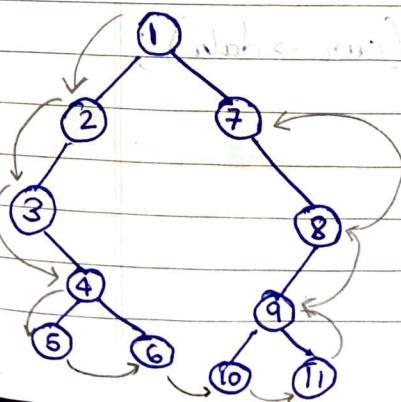
(current\_level\_size < flag) ? current\_level\_size++ : current\_level\_size--;

```

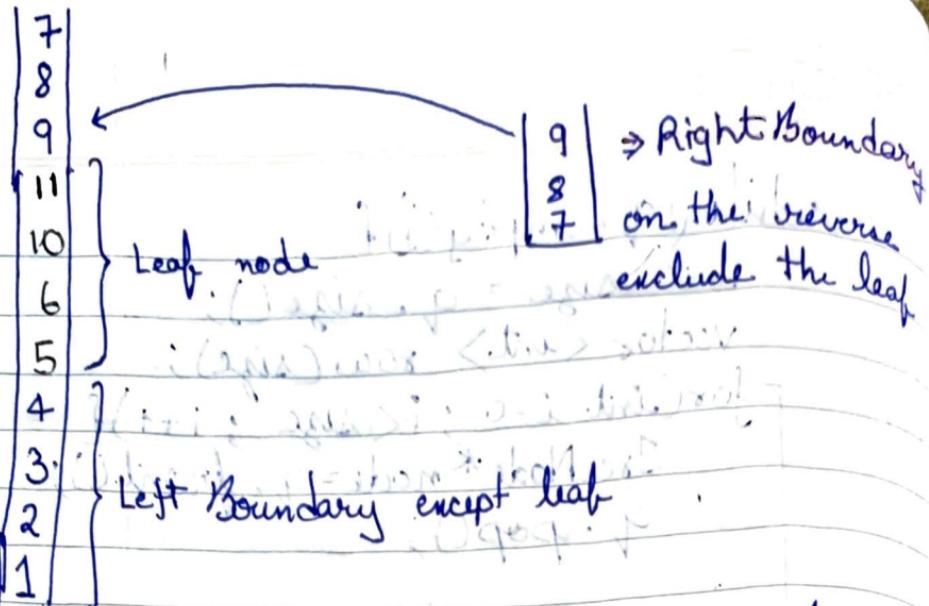
while (!q.empty()) {
    int size = q.size();
    vector<int> row(size);
    for (int i = 0; i < size; i++) {
        TreeNode* node = q.front();
        q.pop();
        int index = (flag) ? i : (size - 1 - i);
        row[index] = node->val;
        if (node->left)
            q.push(node->left);
        if (node->right)
            q.push(node->right);
    }
    flag = !flag;
    ans.push_back(row);
}
return ans;

```

## Boundary Traversal



- Leaf Boundary excluding leaf
- Leafnode (left + right)
- Right Boundary in the reverse exclude the leaf



ds-1-21) Leaf node  $\Rightarrow$  in-order traversal  
 $\therefore$  low  $\leftarrow$  high : [cyclic] preorder

Code :  $(\text{if}(\text{curr} \neq \text{NULL})) \{\;$

```
bool isleaf (Node* root){  

    if(!root  $\rightarrow$  left & !root  $\rightarrow$  right)  

        return true;  

    return false; ; null != root  

}
```

void addleftBoundary (Node\* root, vector<int>& res)  
{

```
Node* curr = root  $\rightarrow$  left;  

while(curr){  

    if(!isleaf (curr))  

        res.push_back (curr->data);  

    if (curr  $\rightarrow$  left)  

        curr = curr  $\rightarrow$  left;  

    else  

        curr = curr  $\rightarrow$  right;  

}
```

```
void addRightBoundary(Node* root, vector<int>& res)
```

```
{  
    Node* curr = root->right;  
    vector<int> temp;  
    while (curr) {  
        if (!isLeaf(curr))  
            temp.push_back(curr->data);  
        if (curr->right) right  
            curr = curr->  
        else left  
            curr = curr->left;  
    }  
}
```

```
for (int i = temp.size() - 1; i >= 0; i--) {  
    res.push_back(temp[i])
```

```
void addLeaves(Node* root, vector<int>& res)
```

```
{  
    if (isLeaf(root)) {  
        res.push_back(root->data);  
        return;  
    }  
    if (root->left)  
        addLeaves(root->left, res);  
    if (root->right)  
        addLeaves(root->right, res);  
}
```

root to vector<int> printBoundary

(by this)

(Node\* root)

{

vector<int> res;

if (!root) { res <= this; return; }

return res;

if (!isLeaf(root))

(left <-> res).push\_back(root->data);

addLeftBoundary (root, res);

addLeaves (root, res);

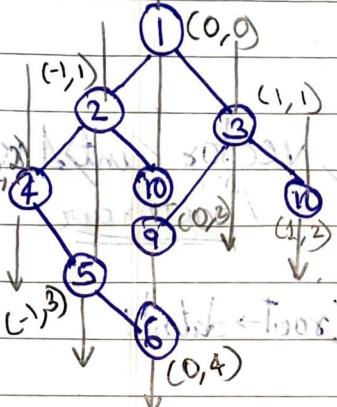
addRightBoundary (root, res);

return res;

}

if (-1 < i < 1 & (i != 0 || (i == 0 & !isLeaf(root)))) {

Vertical Order Traversal Left (-1, +1)  
right (+1, +1)



4

2 5

1

2 3

3

10

11

right (+1, +1)

smaller value 1st

node = 1 true (0,0)

node = 2 false (-1,1)

(1,0,0) (2,-1,1) (3,1,1) (4,-2,2) (10,0,2) (9,0,2) (10,2,1)

Q (node, v, level)

(data <- toos) true | data

: (data <- toos) true | data

vertical  
↓  
 $\text{map} < \text{int}, \text{map} < \text{int}, \text{multiset} < \text{int} \rangle$

level  
↓

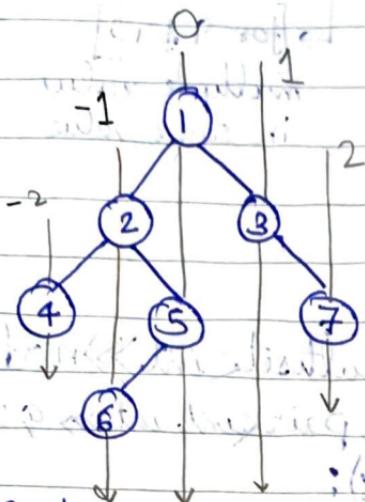
↳ [for 9 & 10]

multiple value  
in same value

### Code :

```
map<int, map<int, multiset<int>> nodes;
queue<pair<TreeNode*, pair<int, int>> q;
q.push({root, {0, 0}});
while(!q.empty()) {
    auto p = q.front();
    q.pop();
    TreeNode* node = p.first;
    int x = p.second.first, y = p.second.second;
    nodes[x][y].insert(node->val);
    if(node->left)
        q.push({node->left, {x-1, y+1}});
    if(node->right)
        q.push({node->right, {x+1, y+1}});
}
vector<vector<int>> ans;
for(auto &p: nodes) {
    vector<int> col;
    for(auto &q: p.second) {
        col.insert(col.end(), q.second.begin(),
                   q.second.end());
    }
    ans.push_back(col);
}
```

## Top view of a Binary Tree



4 2 1 3 7  
 $(5, 0) \Rightarrow$  will not be considered as we have 1

|         |
|---------|
| (4, -2) |
| (3, 1)  |
| (2, 1)  |
| (1, 0)  |

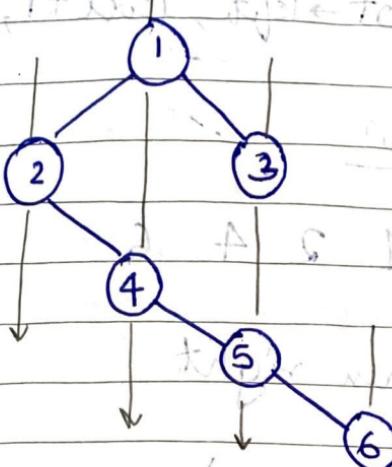
Code :

```
vector<int> ans;
if (root == NULL)
    return ans;
map<int, int> mp;
queue<pair<Node*, int>> q;
q.push({root, 0});
while (!q.empty()) {
    auto it = q.front(); q.pop();
    Node* node = it.first;
    int line = it.second;
    if (mp.find(line) == mp.end())
        mp[line] = node->data;
    if (node->left)
        q.push({node->left, line - 1});
    if (node->right)
        q.push({node->right, line + 1});
```

```
for (auto it : mp) ans.push_back(it.second);
```

```
return ans;
```

### Bottom View



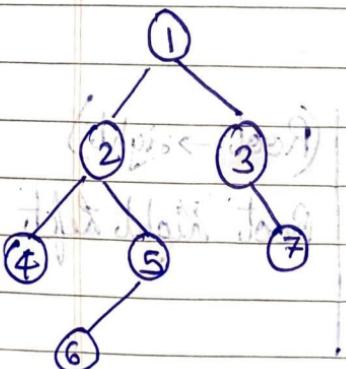
ans[]: 2 4 5 6

### Code diff

```
if(mp.find(line) == mp.end()) { }  
mp[line] = node->data; } } } } }
```

Right Side O(N) Space O(H)  $\rightarrow$  height

ans[]: 1 3 7 6



```
if (1 node) return  
if (level == ds.size()) ds.push_back(node);  
if (node->right, level+1);  
if (node->left, level+1);
```

Code

```

void helper(TreeNode* root, int level, vector<int>& res) {
    if (root == NULL)
        return;
    if (res.size() == level)
        res.push_back(root->val);
    helper(root->right, level + 1, res);
    helper(root->left, level + 1, res);
}

```

### Left view

ans[]: 1 2 4 6

⇒ Left then right

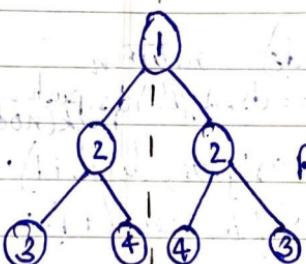
Code:

```

if (root == NULL)
    return;
if (ans.size() == level)
    ans.push_back(root->val);
helper(root->left, level + 1);
helper(root->right, level + 1);

```

### Symmetrical Binary Tree



Mirror

(Root → left) | (Root → right)

Root Left Right

Root Right Left

```

class Solution {
public:
    bool isSymmetric(TreeNode* root) {
        if (root == NULL) return true;
        helper(root, root);
    }
};

bool helper(TreeNode* left, TreeNode* right) {
    if (left == NULL && right == NULL) return true;
    if (left == NULL || right == NULL) return false;
    if (left->val != right->val) return false;
    helper(left->left, right->right);
    helper(left->right, right->left);
}

```

```

{
    if (left == NULL && right == NULL) return true;
    if (left == NULL || right == NULL) return false;
    if (left->val != right->val) return false;
    helper(left->left, right->right);
    helper(left->right, right->left);
}

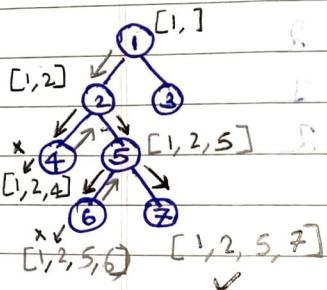
```

```

return helper(left->left, right->right) &&
       helper(left->right, right->left);
}

```

Root to Node Path



```

A = (vector<int>) arr;
B = (D)
if (A == NULL) C = (0, 0); return arr;
getPaths(A, arr, B);
return arr;

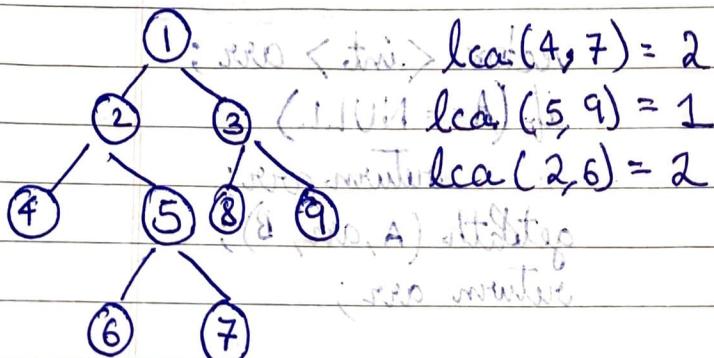
```

```

bool getPath(TreeNode* root, vector<int> &arr, int n) {
    if (!root) return false;
    arr.push_back(root->val);
    if (root->val == n) return true;
    if (getPath(root->left, arr, n) || getPath(root->right, arr, n))
        return true;
    arr.pop_back();
    return false;
}

```

## Lowest Common Ancestor



TreeNode\* lca(TreeNode\* root, TreeNode\* p, TreeNode\* q)

if (root == NULL || root == p || root == q)  
return root;

TreeNode\* left = lca(root -> left, p, q);  
TreeNode\* right = lca(root -> right, p, q);

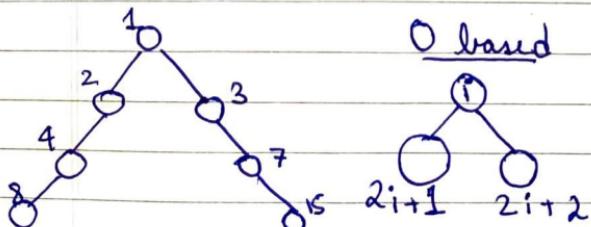
if (!left)  
return right;

else if (!right)  
return left;

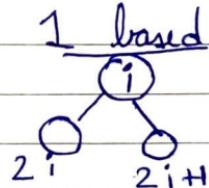
else  
return root;

## Max Width of Binary Tree

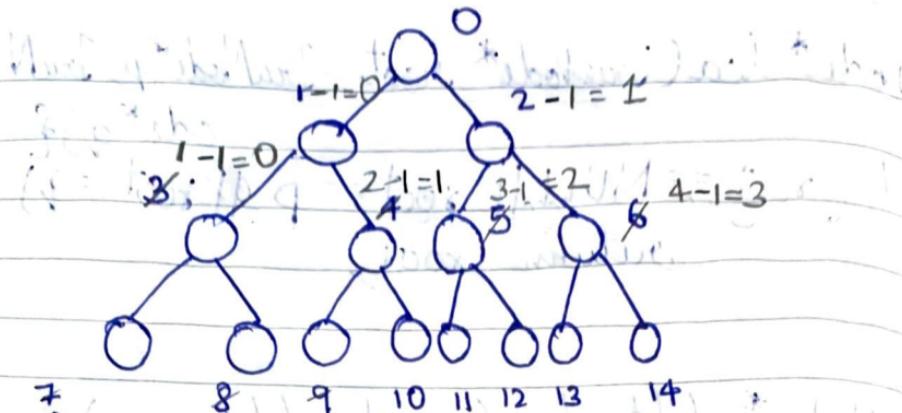
width = no. of nodes in a level between  
any 2 nodes.



0 based



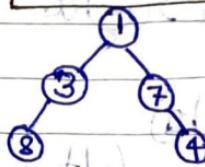
1 based



$$i(i - \min)$$

$$2i+1 \quad 2i+2$$

$$(last - first + 1)$$



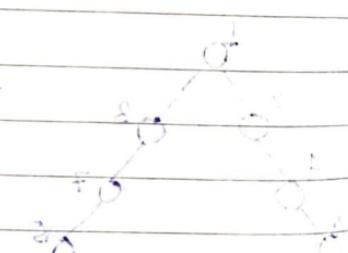
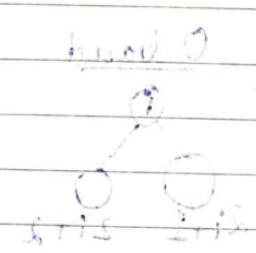
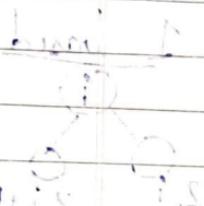
- $(4, 4)$
- $(8, 1)$
- $(7, 2)$
- $(3, 1)$
- $(1, 0)$

$$2 * (2 - 1) + 1 + 1 = 2 * 1 + 2 = 4$$

$$2 * (1 - 1) + 1$$

$$= 0 + 1 = 1$$

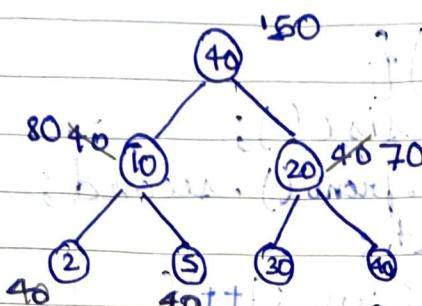
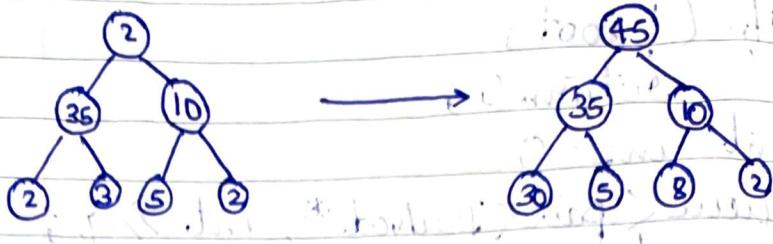
$$4 - 1 + 1 = 4$$



Code:

```
if (!root)
    return 0;
int ans = 0;
queue<pair<TreeNode*, int>>q;
q.push({root, 0});
while (!q.empty()){
    int size = q.size();
    int mn = q.front().second;
    int first, last;
    for (int i = 0; i < size; i++) {
        int curid = q.front().second - mn;
        TreeNode* node = q.front().first;
        q.pop();
        if (i == 0)
            first = curid;
        if (i == n - 1)
            second = curid;
        if (node->left)
            q.push({node->left, curid * 2 + 1});
        if (node->right)
            q.push({node->right, curid * 2 + 2});
    }
    ans = max(ans, last - first + 1);
}
return ans;
```

## Children Sum Property O(n)



Code :

```

if (!root) return;
int child = 0;
if (root->left)
    child += root->left->data;
if (root->right)
    child += root->right->data;
    
```

if (child  $\geq$  root  $\rightarrow$  data)

(uproot child,  $\rightarrow$  root  $\rightarrow$  data = child),

else

if (root  $\rightarrow$  left)

root  $\rightarrow$  left  $\rightarrow$  data = root  $\rightarrow$  data;

else if (root  $\rightarrow$  right)

root  $\rightarrow$  right  $\rightarrow$  data = root  $\rightarrow$  data;

change tree (root  $\rightarrow$  left)

change tree (root  $\rightarrow$  right)

int tot = 0;

if (root  $\rightarrow$  left)

tot += root  $\rightarrow$  left  $\rightarrow$  data;

if (root  $\rightarrow$  right)

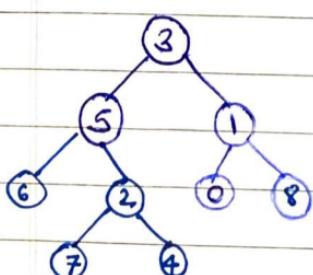
tot += root  $\rightarrow$  right  $\rightarrow$  data;

if (root  $\rightarrow$  left || root  $\rightarrow$  right)

root  $\rightarrow$  data = tot;

Nodes at a distance K

K=2, target = 5



```

void markparents(TreeNode* root, unordered_map<TreeNode*, TreeNode*> &parents)
{
    queue<TreeNode*> q; // stack
    q.push(root);
    while (!q.empty()) {
        TreeNode* current = q.front();
        q.pop();
        if (current->left) {
            parent[current->left] = current;
            q.push(current->left);
        }
        if (current->right) {
            parent[current->right] = current;
            q.push(current->right);
        }
    }
}

```

```

public <int> distanceK(TreeNode* root, TreeNode* target, int k) {
    unordered_map<TreeNode*, TreeNode*> parent;
    markparents(root, parent);
}

```

```

unordered_map<TreeNode*, bool> visited;
queue<TreeNode*> q;
q.push(target);
visited[target] = true;

```

```
while (!q.empty()) {
```

```
    int size = q.size();  
    if (curr_level++ == k):
```

```
        break;
```

```
    for (int i = 0; i < size; i++) {
```

```
        TreeNode* current = q.front();  
        q.pop();
```

```
        if (current->left && !visited[current->left])
```

```
{
```

```
            q.push(current->left);
```

```
            visited[current->left] = true;
```

```
}
```

```
        if (current->right && !visited[current->right])
```

```
{
```

```
            q.push(current->right);
```

```
            visited[current->right] = true;
```

```
}
```

```
        if (parent[current] && !visited[parent[current]])
```

```
{
```

```
            q.push(parent[current]);
```

```
            visited[parent[current]] = true;
```

```
}
```

```
        cout <<
```

```
        endl;
```

```

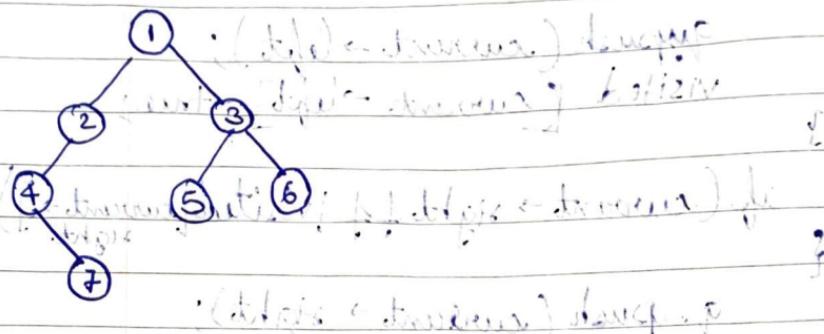
vector<int> ans;
while (!queue.empty()) {
    TreeNode* current = queue.front();
    queue.pop();
    ans.push_back(current->val);
}

```

3

return ans;

## Minimum Time taken to Burn the BT



(1, 2, 3) burn 1st  
4, 5, 6 burn 2nd

(7) burns 3rd

same as before

to find TreeNode  $\Rightarrow$  in parent

while ( $!q$ .empty())

BinaryTreeNode\* node = q.front();

if [ $node \rightarrow data == target$ ] {

res = node;

To find  
TreeNode

```
int findMaxDistance (map<TreeNode*, TreeNode*> &parents, BinaryTreeNode * target)
```

```
{
```

```
queue < TreeNode*> q;
```

```
q.push(target);
```

```
map <TreeNode*, int> vis;
```

```
vis[target] = 1;
```

```
int ans = 0;
```

```
[ while (!q.empty()) {
```

```
int n = q.size();
```

```
int fl = 0;
```

```
[ for (int i = 0; i < n; i++) {
```

```
TreeNode * temp = q.top();
```

```
q.pop();
```

```
if (node->left && !vis[node->left]) {
```

```
fl = 1;
```

```
vis[node->left] = 1;
```

```
q.push(node->left);
```

```
}
```

```
if (node->right && !vis[node->right]) {
```

```
fl = 1;
```

```
vis[node->right] = 1;
```

```
q.push(node->right);
```

```
}
```

```
parent
```

```
if (parent[node] && !vis[parent[node]]) {
```

```
fl = 1;
```

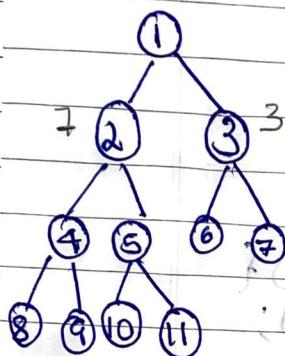
```
vis[parent[node]] = 1;
```

```
q.push(parent[node]); }
```

BottomUpBFS(fl) : given complete binary tree  
 returns ans[bottom up] present in tree.

3 returns ans[bottom up] present in tree.

## Total Nodes in a Complete BT



$$\{ \text{if } 7 + 3 = 10 \neq 11 \} \text{ if } \\ = 11$$

(6)  $\Rightarrow$   $p = \infty$  true  
 $\therefore 0 \cdot 11$  true

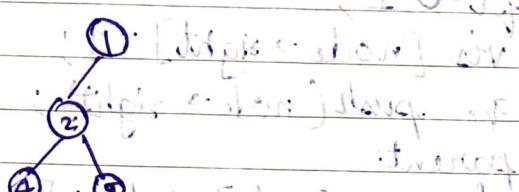
$$\{ \text{if } i < l_h = 30 - 1 \text{ then } 2^{l_h^3 - 1} \leq 7 \}$$

(7)  $\Rightarrow p = 3^2 = 9$  shall work

$$\{ \{ \text{if } \} \leftarrow \text{short} \} \text{ if } l_h = 2 \text{ then } 2^{l_h^2 - 1} \leq 31 \\ r_h = 2^l_h - 1 \text{ then } \{ \text{if } \} \leftarrow \text{short} \} \text{ fi } \\ \{ \text{if } \} \leftarrow \text{short} \} \text{ fi }$$

if ( $l_h = r_h$ )  $\leftarrow$  [if  $\{ \} \leftarrow \text{short} \}$  in  
 $n^{2^l_h - 1} \leftarrow \text{short} \} \text{ then } p$

else  $\{ \{ \text{if } \} \leftarrow \text{root} \text{ then } \text{root} \rightarrow \text{left} \text{ then } \text{root} \rightarrow \text{right} \}$



$\{ l = \{ \{ \text{if } \} \leftarrow \text{short} \} \text{ if } \}$

$\{ \} \leftarrow \{ \{ \text{if } \} \leftarrow \text{short} \} \text{ then } p$

code in  $\text{CountN}(\text{TreeNode}^* \text{root})$  {  
if ( $! \text{root}$ )  
    return 0;

int lh = find height left (root);  
int rh = find height right (root);

if ( $lh == rh$ )  
    return  $(1 \ll lh) - 1$

$\rightarrow 2^{lh}$

return  $1 + \text{countN}(\text{root} \rightarrow \text{left}) +$   
 $\text{countN}(\text{root} \rightarrow \text{right});$

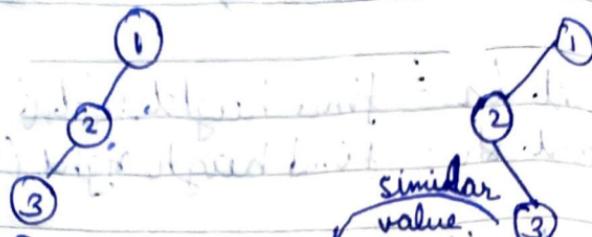
}

int find height left ( $\text{TreeNode}^* \text{root}$ ) {  
int hg = 0;  
while ( $\text{root} \neq \text{NULL}$ ) {  
    hg++;  
     $\text{root} = \text{root} \rightarrow \text{left};$   
}  
return hg;

int find height right ( $\text{TreeNode}^* \text{root}$ ) {  
int hg = 0;  
while ( $\text{root} \neq \text{NULL}$ ) {  
    hg++;  
     $\text{root} = \text{root} \rightarrow \text{right};$   
}  
return hg;

↳ If two different trees have same Pre-Order

Q) Can you construct Binary Tree?  
Ans)



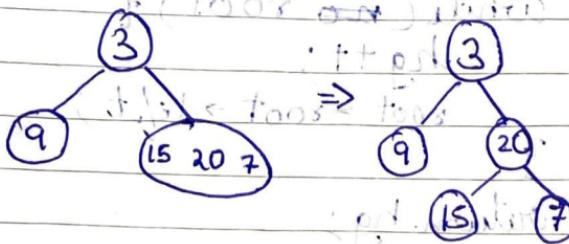
Pre-Order  $\rightarrow 123$

Post-Order  $\rightarrow 321$

So unique can't be constructed using  
post & pre (Inorder is missing)

Inorder = [9 3 15 20 7]

Pre-order = [3 9 15 20 7]



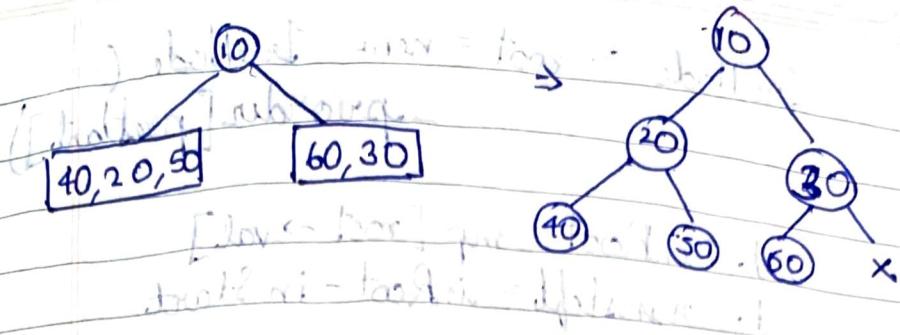
↳ Construct Binary Tree from Preorder & Inorder

inorder  $\rightarrow [40 20 50 10 60 30]$

preorder  $\rightarrow [10 20 40 50 30 60]$

Left is  $\leftarrow$  Root = Root

Post order



Code

```
TreeNode* buildTree(vector<int> &preorder,
                     vector<int> &inorder)
```

```
{ // Create a map (inorder, index) with index = index - tree
    map<int, int> mp;
    for (int i=0; i<inorder.size(); i++) {
        mp[inorder[i]] = i; // to store in-order
    }
```

```
TreeNode* root = buildTree(preorder, 0,
                           preorder.size() - 1, inorder, 0, inorder.size() - 1,
                           mp);
```

```
return root;
```

```
TreeNode* buildTree(vector<int> &preorder, int
                     prestart, int preend, vector<int> &inorder, int
                     instart, int inend, map<int, int> mp){
```

```
if (prestart > preend || instart > inend)
    return NULL;
```

```
TreeNode * root = new TreeNode (  
    preorder[prestart])
```

```
int inRoot = mp[root → val]  
int numLeft = inRoot - inStart
```

```
root → left = buildTree (preorder, prestart + 1,  
    prestart + numLeft, inorder, inStart,  
    inRoot - 1, mp);  
root → right = buildTree (preorder, prestart + 1 +  
    numLeft, inorder, inRoot + 1, inEnd, mp);  
return root;
```

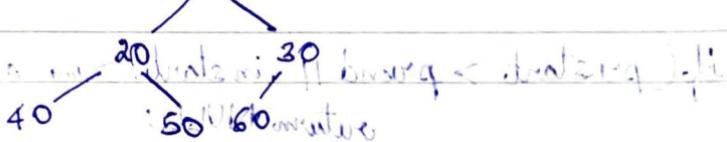
Construct Binary Tree from Inorder and Postorder

Inorder Traversal

Inorder: [40 20 50 10 60 30]

Postorder: [40 50 20 60 30 10]

40, 20, 50, 10, 60, 30



Code:

```
{ if (inorder.size() != postorder.size())
    return NULL;
```

```
map<int, int> mp;
```

```
for (int i = 0; i < inorder.size(); i++)
    mp[inorder[i]] = i
```

```
return buildTreePostIn (inorder, 0,
    inorder.size() - 1, postorder, 0, postorder.size() - 1,
    mp);
```

```
TreeNode* buildTreePostIn (vector<int>& inorder,
    int start, int end, vector<int>& postorder,
    int poststart, int postend, map<int, int>
    &mp) {
    if (poststart > postend || start > end)
        return NULL;
```

```
TreeNode* root = new TreeNode (
    postorder[postend]);
```

```
int inRoot = mp[postorder[postend]];
int numLeft = inRoot - start;
```

- $\text{root} \rightarrow \text{left} = \text{buildTreePostIn}(\text{inorder}, \text{instart}, \text{inRoot}-1, \text{postorder}, \text{poststart} + t, \text{poststart} + \text{numsLeft}-1, \text{mp})$
- $\text{root} \rightarrow \text{right} = \text{buildTreePostIn}(\text{inorder}, \text{inRoot}+1, \text{inEnd}, \text{postorder}, \text{postorder} + \text{numsLeft}, \text{postend}-1, \text{mp})$

return root;

}

return root; // BST will be created.

Serialize + DeSerialize.

root

Serialize

De Serialize

root

1

2

13

4

5

1, 2, 3, #, #, 4, 5,

, #, #, #, #

Code:

```
string serialize(TreeNode *root) {
    if (!root)
        return "";
    string s = "#,"; // null
    queue<TreeNode*> q;
    q.push(root);
    while (!q.empty()) {
        TreeNode *curr = q.front();
        q.pop();
        if (curr)
            s.append(to_string(curr->val) + ",");
        else
            s.append("#,");
        if (curr)
            q.push(curr->left);
        q.push(curr->right);
    }
}
```

if ( $s_1 < s_2$ ) with  $s_1$   
if ( $s_1 == s_2$ ) if  
if ( $s_1 > s_2$ ) with  $s_2$

```
TreeNode * deserialize (string data) {
    if (data.size() == 0)
        return NULL;
    stringstream s(data);
    string str;
    getline(s, str, ',');
    TreeNode *root = new TreeNode(stoi(str));
    queue<TreeNode*> q;
    q.push(root);
    while (!q.empty()) {
        TreeNode *node = q.front();
        q.pop();
        getline(s, str, ',');
        if (str == "#") {
            node->left = NULL;
        } else {
            TreeNode *leftNode = new TreeNode(stoi(str));
            node->left = leftNode;
            q.push(leftNode);
        }
        getline(s, str, ',');
        if (str == "#") {
            node->right = NULL;
        }
    }
}
```

```
getline(s, str, ',');
if (str == "#") {
    node->right = NULL;
```

```

    close {
        with string help as -
        TreeNode * rightn = new TreeNode
        if (not same (stbi(str))) {
            node->right = rightn;
            q.push (rightn);
        }
    }

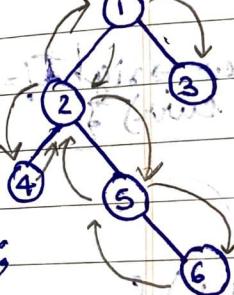
```

3. ~~3. (3 \* subTree).Leaves <= 1000 > solve  
return root; // leaves <= 1000  
too <= 1000 & shall not~~

### Morris Traversal

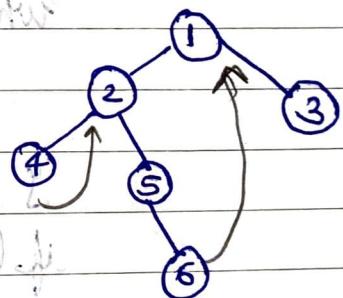
3. (1) Inorder  $O(N)$  Time  $O(1)$  Space

$\Rightarrow$  Threaded Binary Tree



case (i)

if (drivs == left == null)  
print()



Case (ii)

$\rightarrow$  Before going to left, the rightmost will be connected to cur from left subtree

if (drivs == right == null)

→ if thread exists then remove  
that thread  
curr to right

Code:

```
vector <int> getInorder(TreeNode* root){  
    vector <int> inorder  
    TreeNode * curr = root;
```

```
while (curr != NULL) {  
    if (curr->left == NULL) {  
        inorder.push_back(curr->val);  
        curr = curr->right;  
    }  
    else {
```

```
        TreeNode * prev = curr->left;  
        while (prev->right == prev) {  
            prev = prev->right;
```

```
            if (prev->right == NULL) {  
                prev->right = curr;  
                curr = curr->left;  
            }  
        }  
    }
```

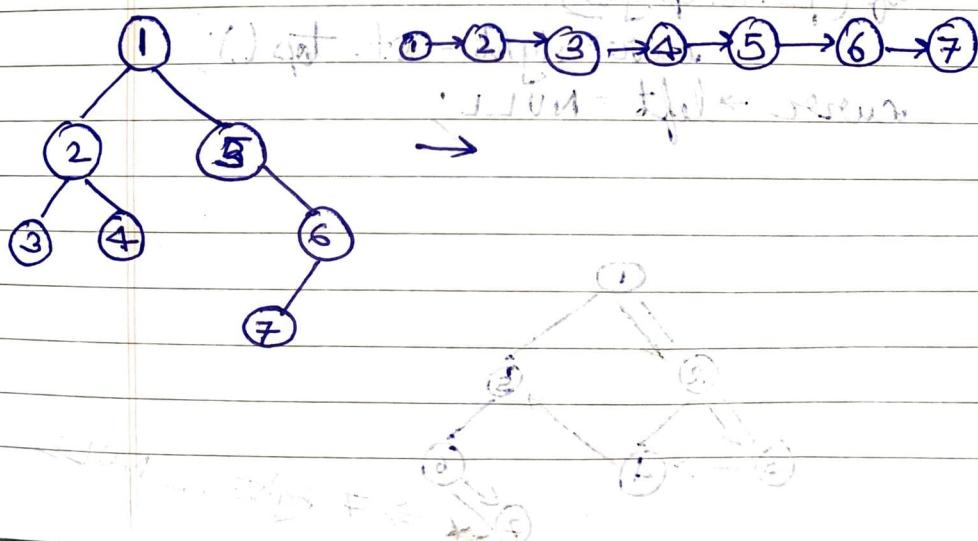
prv->right = NULL;

inorder.push\_back(curr->val);

curr = curr->right;

}  
 }  
 return inorder;  
 }  
for preorder  
 if (prev → right == NULL)  
 {  
 prev → right = curr;  
 preorder.push\_back(curr.val);  
 curr = curr → next; // ST  
 }  
 else {  
 prev → right = NULL;  
 cur = curr → right; // fi
 }

Flatten a Binary Tree



(i)  $TC \rightarrow O(N)$   $SC \rightarrow O(N)$

prev = null  
flatten(node){  
if (node == NULL)  
return;

flatten(node->right);  
flatten(node->left);  
node->right = prev;  
(node->left == NULL);  
prev = node;

(ii)  $TC \rightarrow O(N)$   $SC \rightarrow O(N)$

st.push(root)

while(!st.empty){

curr = st.top(); st.pop();

if (curr->right)

st.push(curr->right)

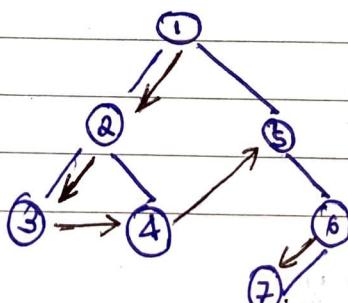
if (curr->left)

st.push(curr->left);

if (!st.empty())

curr->right = st.top();

curr->left = NULL;



TC  $\rightarrow O(N)$

SC  $\rightarrow O(1)$

(11) curr = root;

while (curr){

if (curr  $\rightarrow$  left){

prev = curr  $\rightarrow$  left

while (prev  $\rightarrow$  right){

prev = prev  $\rightarrow$  right;

prev  $\rightarrow$  right = curr  $\rightarrow$  right;

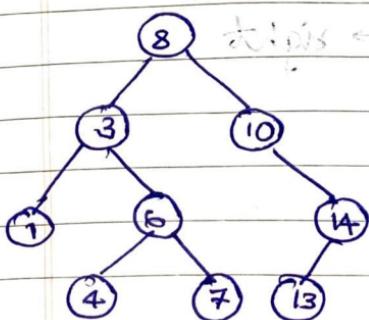
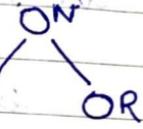
curr = curr  $\rightarrow$  left;

}

curr = curr  $\rightarrow$  right;

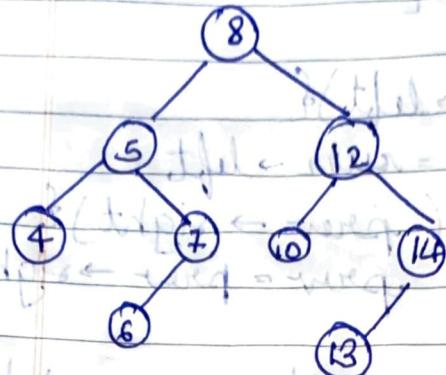
}

## Binary Search Tree



To store duplicate values  
node we can use  
(node, freq)

## Search in a BST



Code:

```
while (root != NULL) {
```

```
    if (root->val == val)
```

```
        return root;
```

```
    else if (root->val > val)
```

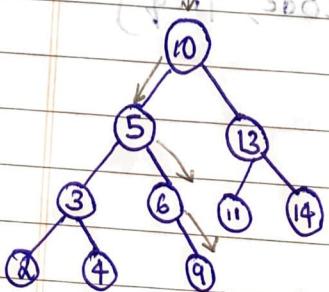
```
        root = root->left
```

```
    else
```

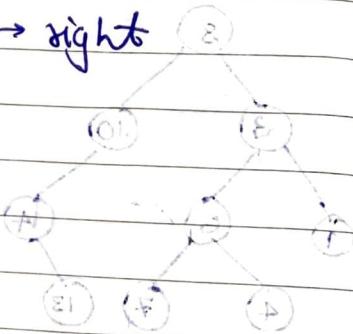
```
        root = root->right
```

return NULL;

Ceil in a BST



Key = 8



Code:

int ceil = -1;

while (root != null) {

if ( $\text{root} \rightarrow \text{data} == \text{key}$ )

ceil = root  $\rightarrow$  data; return key;

else if (key > root  $\rightarrow$  data)

root = root  $\rightarrow$  right;

else if (key < root  $\rightarrow$  data)

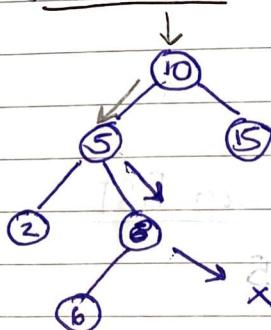
ceil = root  $\rightarrow$  data;

root = root  $\rightarrow$  left;

return ceil;

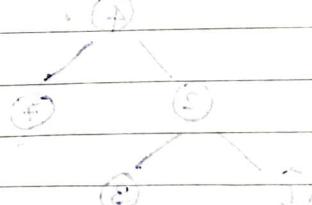
}

Floor in BST



key = 9

ans = 8

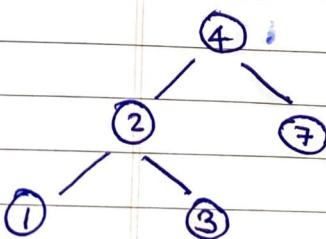


## Code

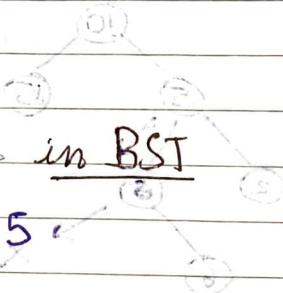
```
int floor=-1; dis sta  
while (floor) {  
    if (root->val == key) {  
        floor = root->val  
        return floor  
    } else if (key > root->val) {  
        floor = root->val;  
        root = root->right;  
    } else {  
        floor = root->left;  
        root = root->left;  
    }  
}
```

return floor;

Insert a given Node in BST



node = 5



Code:

if (node == NULL)

return new TreeNode(val);

TreeNode\* curr = root;

while (true) {

if (curr->val <= val) {

if (curr->right)

curr = curr->right;

else

(tobis->tobis) if (curr->right = new TreeNode(val));  
break;

else { (tobis->tobis) if

if (curr->left) {

curr = curr->left;

(tobis->tobis) else (tobis->tobis) if (curr->left = new TreeNode(val));

break;

} (tobis->tobis) if (curr->left = new TreeNode(val));

(tobis->tobis) if (curr->left = new TreeNode(val));

: return tooroot, int;

{ (tobis->tobis) if (curr->left = new TreeNode(val));

: t1->tobis, int;

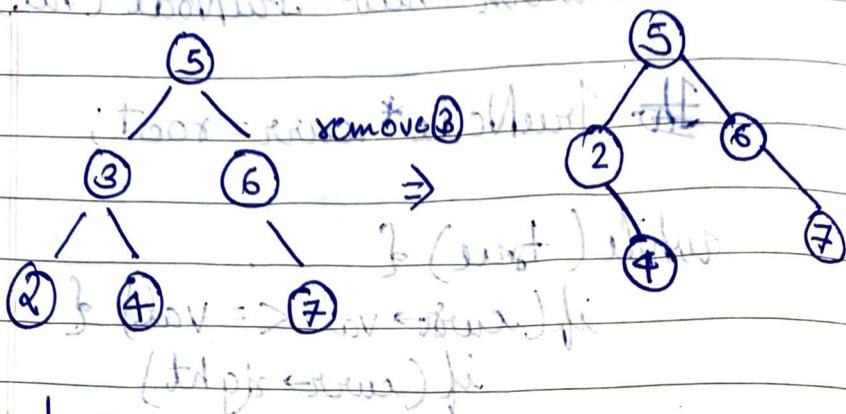
: t1->tobis = blist \* shant;

: t1->tobis->tobis = t1->tobis \* shant;

: t1->tobis = t1->tobis - t1->tobis;

: t1->tobis = t1->tobis - t1->tobis;

## Delete a Node in Binary Tree



Codes:

TreeNode\* findLastRight(TreeNode\* root)

```
if (root -> right)
    return root;
```

```
return findLastRight(root -> right);
```

```
TreeNode* helper(TreeNode* root) {
    if (!root -> left)
        return root -> right;
    else if (!root -> right)
        return root -> left;
```

```
TreeNode* child = root -> right;
TreeNode* lastRight = findLastRight(root -> lastRight -> right = child);
return root -> left;
```

function to delete node

```
TreeNode* deleteNode(TreeNode* root, int key)
```

{

```
if (!root)
```

```
return NULL;
```

```
if (root->val == key)
```

```
return helper(root);
```

```
TreeNode* temp = root;
```

```
while (root) {
```

```
if (root->val > key) {
```

```
if (root->left && root->left->val == key)
```

```
root->left = helper(root->left);
```

else break;

root = root->left;

```
else {
```

```
if (root->right && root->right->val == key)
```

```
root->right = helper(root->right);
```

```
else break; or = root
```

```
else {
```

```
root = root->right;
```

(6) val.2 = tons

(6) val.2

(N = tons).fi.

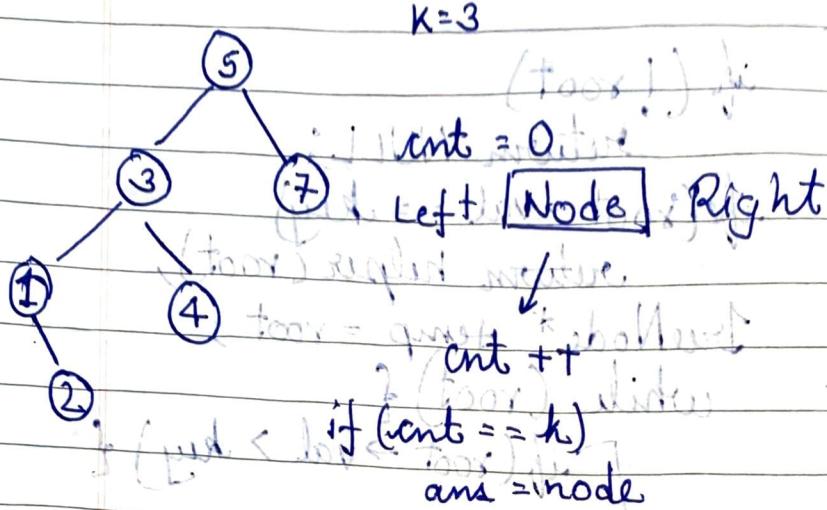
iterations written

```
return temp;
```

}

⇒ Inorder of a BST is always in a sorted order

toos \* find  $k^{\text{th}}$  smallest element in BST  
using in-order traversal



Code

stack <TreeNode\*> s;

int cnt = 0;

while (true) {

if (root) {

s.push(root);

root = root->left;

}

else {

if (s.empty()) break;

root = s.top();

s.pop();

cnt++

if (cnt == k)

return root->val;

$\text{vroot} = \text{vroot} \rightarrow \text{right};$

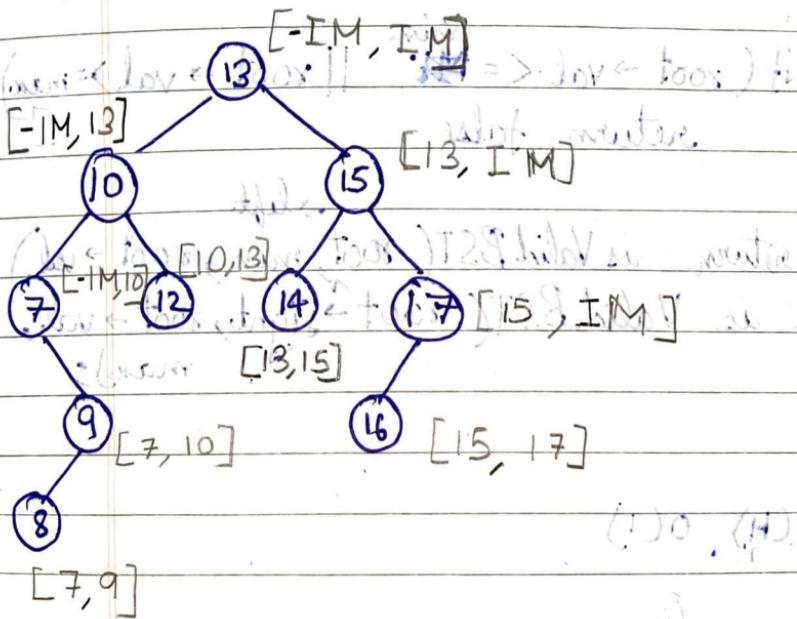
}  
return -1;

(,308 short note) T28. bisection random  
largest (,308 short note) T29. bisection written

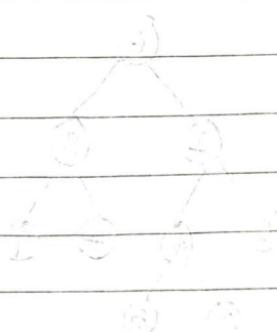
$k^{\text{th}}$  largest =  $(N - k^{\text{th}})$  smallest  
then

for the 1st traversal account the no. of nodes = N

Check if a tree is a BST



Set (2, 0)



Code: ~~size true - false~~

public:

boolean is Valid BST (TreeNode\* root) {  
 return is Valid BST (root, ~~long~~ MIN, ~~long~~ MAX);

if (ans == N - 1) return 1;

boolean is Valid BST (TreeNode\* root, ~~long~~ min, ~~long~~ max) {

if (!root)

return true;

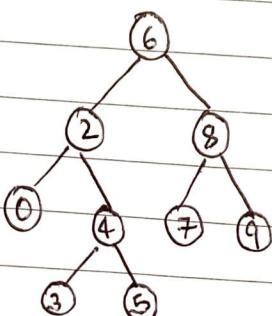
if (root->val <= ~~min~~) || (root->val >= ~~max~~)

return false

return is Valid BST (root, min, ~~root->val~~)

if is Valid BST (~~root->right~~, ~~root->val~~, max);

LCA O(H) O(1)



(0, 5)  $\Rightarrow$  2

TreeNode lCA(TreeNode\* root, TreeNode\* p, TreeNode\* q) {

if (root == null)

return NULL;

int curr = root.val;

if (curr < p.val && curr < q.val)

return lCA(root->right, p, q);

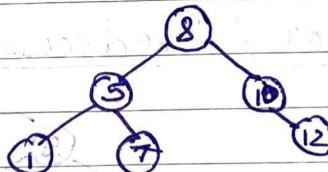
if (curr > p.val && curr > q.val)

return lCA(root->left, p, q);

return root;

Construct BST from preorder traversal

①  $\Rightarrow$  preorder  $\rightarrow [8, 5, 1, 7, 10, 12]$



$\Leftarrow$  Insert one by one  $TC \Rightarrow O(N \times N)$   
 $SC \Rightarrow O(1)$

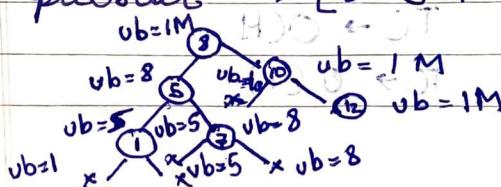
②  $\Rightarrow$  Sorting  $= [1, 5, 7, 8, 10, 12]$   
 & gives inorder

$TC \Rightarrow O(n \log n) + O(n)$   
 $SC \Rightarrow O(N)$

③

preorder

$\rightarrow [8, 5, 1, 7, 10, 12]$



In worst case we have to visit node 3 times  
 $TC \Rightarrow O(3N) \approx O(1)$   
 $SC \Rightarrow O(1)$

Method: Code: shall we do it straight

```
TreeNode* bstFromPreorder(vector<int>& A)
{
    int i=0; (base case)
    return Build(A,i, INT_MAX);
}

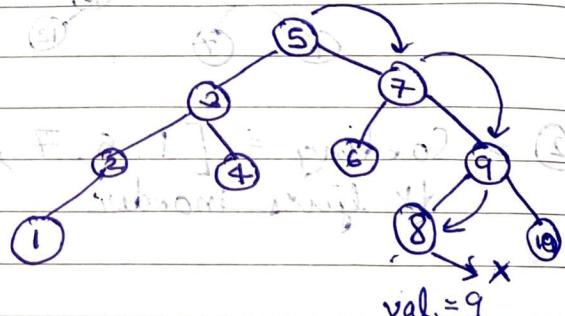
TreeNode* build(vector<int>& A, int l, int r)
{
    if(i==A.size() || A[i] > max)
        return NULL;
    TreeNode* root = new TreeNode(A[i++]);
    root->left = build(A, i, root->val);
    root->right = build(A, i, max);
    return root;
}
```

Inorder Successor / Predecessor in BST

first val > 8

TC  $\rightarrow$  O(N)

SC  $\rightarrow$  O(1)



TC  $\rightarrow$  O(CH)

TC  $\rightarrow$  O(N)

Code :

```

TreeNode* inOrderS(TreeNode* root, TreeNode* p) {
    TreeNode* successor = NULL;
    while (root) {
        if (p->val >= root->val)
            root = root->right;
        else {
            successor = root;
            root = root->left;
        }
    }
    return successor;
}

```

Predecessor :-

```

if (p->val <= root->val) {
    root = root->left;
}
else {
    predecessor = root;
    root = root->right;
}

```

BSTIterator :-

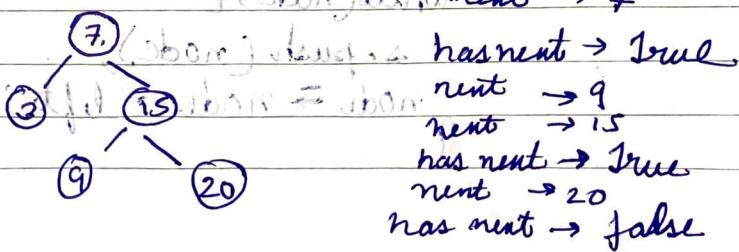
```

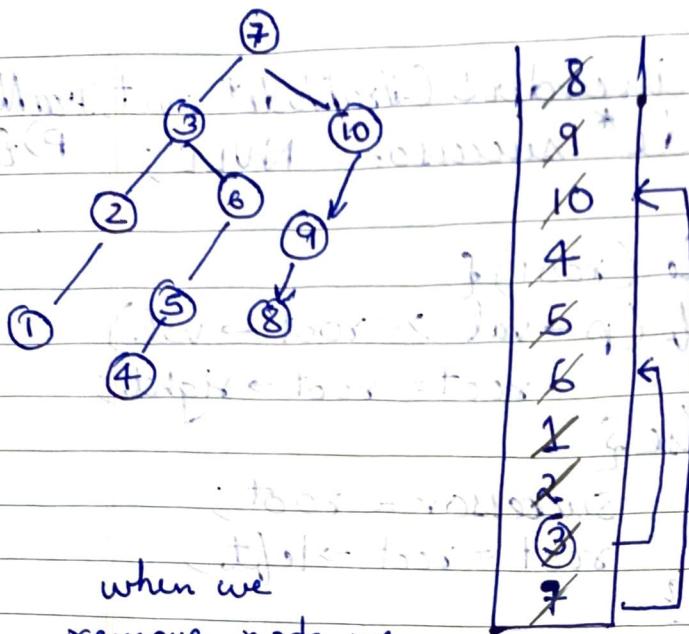
class BSTIterator {
public:
    int next() {
        node = node->right;
        return node->val;
    }

    bool hasNext() {
        return node != NULL;
    }

private:
    node = NULL;
}

```





when we  
remove node we  
go to its right

1 2 3 4 5 6 7 8 9 10

$SC \rightarrow O(H)$

$TC \rightarrow O(N/N) \rightarrow O(1)$

$N$  push in stack in  $N$  calls

Code:

stack <TreeNode\*> s;

void push\_all(TreeNode\* node) {

while(node) {

s.push(node);

node = node->left;

} } }

what's main code

public:

BSTIterator (TreeNode\* root) {

    push\_all(root);

int next() {

    TreeNode\* temp = s.top();

    s.pop();

    push\_all(root->right);

    return temp->val;

}

bool hasNext() {

    return !s.empty();

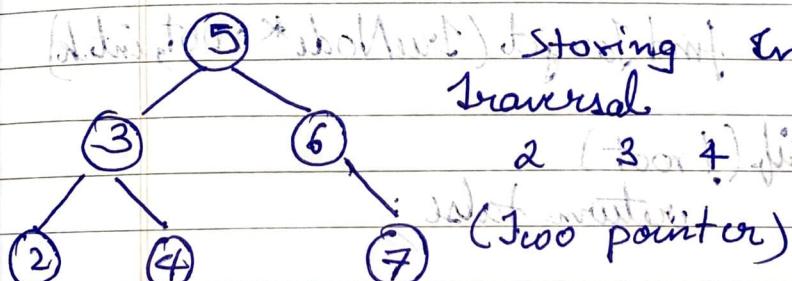
}

Two Sum IV

K = 9

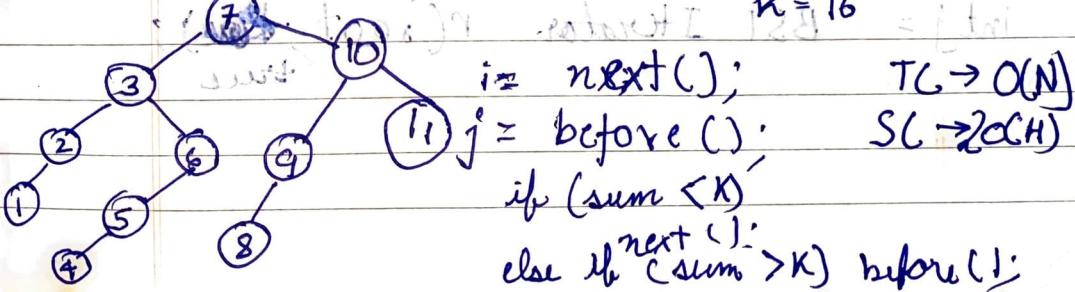
Inorder  
↓

TC → O(N)  
SC → O(N) + O(N)



2 (3 + 4) < 5 6 7

(Two pointer)



i = next();

j = before();

if (sum < K)

else if (sum > K)

TC → O(N)

SC → 2O(H)

Code:

```
int next() {  
    if (!reverse) return T28  
    if ((tmpNode == root) || !tmpNode) return  
    if (!reverse)  
        pushAll(tmpNode->right);  
    else  
        pushAll(tmpNode->left);  
    if (tmpNode->right) return  
    if (tmpNode->left) return  
    pushAll(*node);
```

```
if (reverse) node = node->right;  
else node = node->left;
```

```
absent  
(ii) C++ ST  
(main) + 32 P = X  
out  
return  
bool findTarget(TreeNode* root, int k)
```

```
if (!root) return false;  
BST Iterator l(root, false);  
BST Iterator r(root, true);
```

```
int i = l.next();  
int j = r.next();
```

```
if (i < j) {
```

while ( $i < j$ ) {

if ( $i + j \geq k$ ) {

return true;

else if ( $i + j < k$ )

$i = l.\text{next}();$

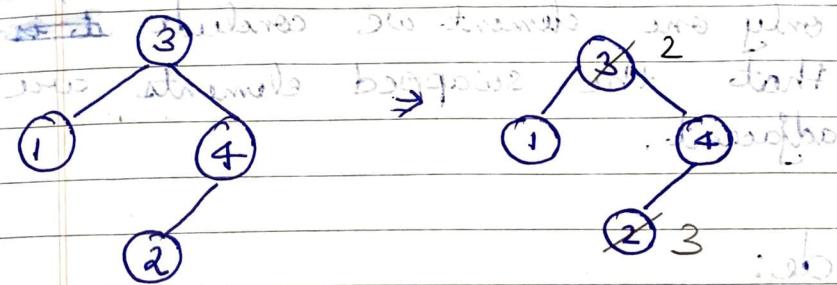
else

$j = r.\text{next}();$

return false;

Recover BST

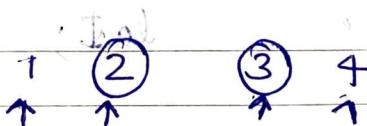
Two nodes swapped



But force  $SC = O(N)$  work out

→ Inorder Traversal

→ sort them

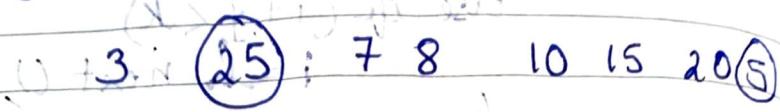


TC  $\rightarrow O(N)$

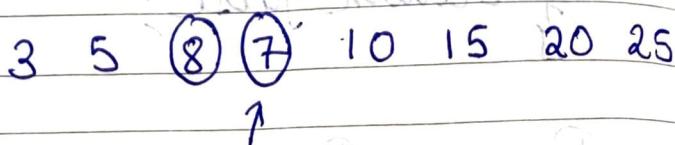
SC  $\rightarrow O(1)$

Swap can have 2 cases

1. Swapped nodes are not adjacent.



2. They are adjacent.



begin with slow and fast pointers. If slow == fast then we find only one element. So when we find only one element we conclude that the swapped elements are adjacent.

Code:

```
Tree Node *first;
```

```
" " previous; last = -
```

```
" " middle; last = -
```

```
" " last;
```

```
void inorder (TreeNode* root) {
```

```
    if (!root)  
        return;
```

```
    inorder (root->left);
```

```
    if (prev && (root->val < prev->val))
```

```
        if (!first) {  
            first = prev;  
            middle = root;
```

```
}
```

```
else
```

```
    last = root;
```

```
}
```

```
    prev = root;
```

```
    inorder (root->right);
```

```
}
```

```
public:
```

```
void recoverTree (TreeNode* root) {
```

```
    first = middle = last = NULL;
```

```
    prev = new TreeNode (INT_MIN);
```

```
    inorder (root);
```

```
    if (first && last)
```

```
        swap
```

```
    else if (first && middle)
```

```
        swap (first->val, middle->val);
```

```
}
```

```
}
```

largest BST is BT

[3, 8, Min]

[2, 8] 10

5

15

[1, Min, Max]

1

8

7

[1, 7, 7]

[1, 1]

[1, 8, 8]

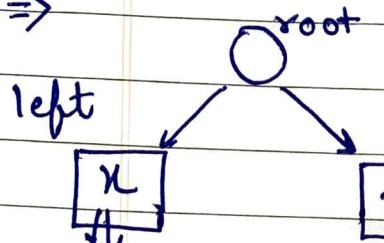
Brute force ((young)) to every node  
+ check if its valid BST

$$TC \rightarrow O(N) \times O(N) \approx O(N^2)$$

validate a BST

↳ going to every node

⇒



largest < root

smallest > root

largest < root & smallest

$$\boxed{\text{size} = 1 + x + y}$$

postorder  
left

right

root, save root & go up

$$TC \rightarrow O(N)$$

$$SC \rightarrow O(1)$$

Code :

```
class NodeValue {
```

public:

```
    int maxNode, minNode, maxSize;
```

```
    NodeValue(int minNode, int maxNode,  
              int maxSize) {
```

```
        this->maxNode = maxNode;
```

```
        this->minNode = minNode;
```

```
        this->maxSize = maxSize;
```

```
}
```

```
}
```

```
NodeValue LBST (TreeNode* root) {
```

```
if (!root)
```

```
    return NodeValue (INT_MAX, INT_MIN  
                      0);
```

```
auto left = LBST (root -> left);
```

```
auto right = LBST (root -> right);
```

```
if (left. maxNode < root -> val &&  
    root -> val < right. minNode) {
```

in case  
of 1 root

```
    return NodeValue (min (root -> val, left.  
                          minNode),  
                     max (root -> val, right. maxNode),
```

```
                     left. maxSize + right. maxSize + 1);
```

```
return (INT_MIN, INT_MAX, max (left. maxSize,  
                               right. maxSize));
```

```
}
```

```
int largestBSTSubtree(TreeNode* root){  
    return largestBSTSubtree(root)·maxsize;
```

3

: size,

size more than one, shall we, shall we, tree.

shall we write shall we, shall we, shall we, shall we,