2001 Thorup, Zwick - Compact routing schemes

Weighted undirected graphs  
  
Improving routing table size (inside vertice information) for various stretch and small headers  
  
Tree Covers approach is well-known before this paper  
  
Section 2 - Routing in trees via heavy-light decomposition

Balls and clusters first used in L. Cowen. “Compact routing with minimum stretch.” ? (named here bunches and clusters)

Algorithm to construct landmarks W: E[|W|] <= 2s log n and |B\_W(v)| < 4n/s

Possible to derandomize it? (2005 Approximate Distance Oracles) - reference to future paper :)

Focus on routing schemes, expanding work:

L. Cowen. Compact routing with minimum stretch. Journal of Algorithms, pages 170–183, 2001. Special issue for SODA’99.

2005 Thorup, Zwick - Approximate Distance Oracles  
  
Weighted undirected graphs with non-negative edge weight

No (2,1) stretch oracle here

For each k there is algorithm preprocessing graph in expected time O(km(n^(1/k)) producing data structure of size O(n^(1 + 1/k)), query time O(k) answering approximate distance between two nodes, if real distance is d then returned in between d and d\*(2k - 1), paths no longer then estimate can be retrieved in constant time per edge.

Space requirements optimal.

This produces steach 3, space O(n^(3/2)) Oracle.

D. Dor, S. Halperin, and U. Zwick. All pairs almost shortest paths.

SIAM J. Computing, 29:1740–1759, 2000.

Earlier paper about oracles

Introducing Tree Covers to Distance Oracles?

Approximate Distance Oracles for Metric Spaces

Preprocessing:

A\_0 = V. A\_k = {}

for i <- 1 to k - 1

let A\_i contain each element of A\_(i-1) with probability n^(-1/k)

p\_i(v): p\_i(v) in A\_i, dist(v, p\_i(v) = dist(v, A\_i))

B(v) <- sum i=0 ->k-1 {w in A\_i - A\_(i+1) | dist(w, v) < dist(A\_(i+1), v)}

Oracle\_dist(u,v):

w <- u, i <- 0

while w not in B(v)

i <- i+1

(u,v) <- (v,u)

w <- p\_i(u)

return dist(w, u) + dist(w, v)

Derandomization (Works only for metric spaces):  
LEMMA 3.6. Let N\_1,..., N\_n subsets of U be a collection of sets with

|U| = u and |N\_i| >= s, for 1 <= i <= n. Then, a set A of size at most u/s

s (ln (ns/u) + 1) <= u/s (ln n + 1) such that elements in both N\_i an A =/= 0 for 1 <= i <= n,

can be found, deterministically, in O(u + sum i= ->n (|N\_i|)) time.

The set A, whose existence is claimed in Lemma 3.6, is obtained

by repeatedly adding to A elements of U that hit as many unhit sets

as possible, until only u/s

sets are unhit. The construction of A is

then completed by adding an element from each one of the unhit

sets.

More info at

N. Alon and J. Spencer. The probabilistic method. Wiley, 1992.  
  
2006 Approximate Distance Oracles for Unweighted Graphs in Expected O(n^2) time

Here oracles are (3, 0) distance oracles

Claims that Thorup and Zwick first came up with Approximate Distance Oracle idea

Here is also presented first linear time algorithm for computing an optimal size (2, 1) -spanner of an unweighted graph.

2008 paper in known here

Description of work done in other papers, improvements to Thorup and Zwick oracle - preprocessing time

Very nicely written

2008 Baswana, Goyal, Sen - All-Pairs Nearly 2-Approximate Shortest Paths in O(n^2 polylog n) Time∗

(2,1) distance oracle on undirected unweighted graph

Inexplicitly first (2, 1) distance oracle of subquadratic space?