Assignment 2: Module 2 – The LP Model

Problem: 1

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

a. Clearly define the decision variables

b. What is the objective function?

c. What are the constraints?

d. Write down the full mathematical formulation for this LP problem.

Solution:

a) Decision Variables:
   The following are the decision variables:
   - Collegiate as 'X'
   - Mini as 'Y'

b) Objective Function:
   - It was given that each Collegiate generates $32profit and Mini generates $24. The profit function is:
     - Z = $(32X + 24Y)
   - The other vital information is each Collegiate requires 3 square feet and each Mini requires 2 square feet and total supply of Nylon sheet required is 5000 square feet. The total material required would be:
     - 3X + 24Y <= 5000 square feet

c) Constraints:
   - The sales forecast indicate that at most 1000 Collegiates and 1200 Mini can be sold.
     - X <= 1000

- $Y <= 1200$

- The total labor time required from each Collegiate is 45 minutes each Mini is 40 minutes:
    - $(45X + 40Y)$minutes

  They have 35 laborers that each provides 40 hours per week:

    - $35 * 40*60 = 84,000$minutes

  Total labor required : $45X+ 40Y <= 84,000$

d) Mathematical formulation:

$$\text{Maximize } Z = 32X + 24Y$$
$$3X + 2Y <= 5000$$
$$X <= 1000 \ \& \ Y <= 1200$$
$$45X + 40Y <= 84,000$$
$$X,Y >= 0$$

Problem:2

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced

by each of the plants to maximize profit.

a. Define the decision variables

b. Formulate a linear programming model for this problem.

Solution:

a) Decision Variables:
Here we have three plants in three different sizes. Hence, we will have 9 decision variables as follows:
Plant – 1:
Large units produced per day : P1L
Medium units produced per day: P1M
Small units produced per day: P1S

Plant – 2:
Large units produced per day : P2L
Medium units produced per day: P2M
Small units produced per day: P2S

Plant – 3:
Large units produced per day : P3L
Medium units produced per day: P3M
Small units produced per day: P3S

b) Formulating a Linear Programming Model:
- The product can be made in three sizes – large, medium and small and yield a net profit of $420,$360,$300
  - Total Net profit is denoted by Z:
  - Maximize Z = 420(P1L+P2L+P3L) + 360(P1M+P2M+P3M) + 300(P1S+P2S+P3S)
- All the three plants have the excess capacity to produce 750,900 and 450 units per day
  - P1L +P1M + P1S <= 750
  - P2L + P2M + P2S <= 900
  - P3L + P3M + P3S <= 450
- All the three plants have 13000, 12,000 and 5,000 square feet and each unit produces 20,15 and 12 square feet respectively.
  - 20(P1L) + 15(P1M) + 12(P1S) <= 13,000
  - 20(P2L) + 15(P2M) + 12(P2S) <= 12,000
  - 20(P3L) + 15(P3M) + 12(P3S) <= 5,000
- Sales forecast indicate that 900, 1,200 and 750 units would be sold per day by all the three plants
  - P1L + P1M + P1S <= 900
  - P2L + P2M + P2S <= 1,200
  - P3L + P3M + P3S <= 750

- Lay-Off’s
    - 1/750(P1L + P1M +P1S) – 1/900(P2L+P2M+P2S) = 0
    - 1/750(P1L + P1M +P1S) – 1/450(P3L+P3M+P3S) = 0

  Hence, P1L,P1M,P1S,P2L,P2M,P2S,P3L,P3M,P3S >= 0