Assignment3

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10/18/2022

1) Formulating Transportation using R

```
library(lpSolve)
```

```
## Warning: package 'lpSolve' was built under R version 4.1.3
```

```
library(lpSolveAPI)
```

```
## Warning: package 'lpSolveAPI' was built under R version 4.1.3
```

```
library(tinytex)
```

Converting the data into a table format:

```
#setting up cost matrix
cost_1 <- matrix(c(22,14,30,600,100,
                  16,20,24,625,120,
                  80,60,70,"-","-","210/220"),ncol = 5,nrow = 3,byrow = TRUE)
```

```
## Warning in matrix(c(22, 14, 30, 600, 100, 16, 20, 24, 625, 120, 80, 60, : data
## length [16] is not a sub-multiple or multiple of the number of rows [3]
```

```
colnames(cost_1) <- c("Warehouse_1", "Warehouse_2","Warehouse_3","ProductionCost","Production Capacity")

rownames(cost_1) <- c("Plant_A","Plant_B","Demand")

cost_1
```

```
##         Warehouse_1 Warehouse_2 Warehouse_3 ProductionCost Production Capacity
## Plant_A "22"        "14"        "30"        "600"          "100"
## Plant_B "16"        "20"        "24"        "625"          "120"
## Demand  "80"        "60"        "70"        "-"            "-"
```

The Objective function is to Minimize the TC

$$\text{Min } TC = 622x_{11} + 614x_{12} + 630x_{13} + 641x_{21} + 645x_{22} + 649x_{23}$$

Subject to the following constraints : Supply

$$X_{11} + X_{12} + X_{13} >= 100$$

$$X_{21} + X_{22} + X_{23} >= 120$$

Subject to the following constraints : Demand

$$X_{11} + X_{21} >= 80$$

$$X_{12} + X_{22} >= 60$$

$$X_{13} + X_{23} >= 70$$

Non-Negativity Constraints

$$X_{ij} >= 0$$

Where i = 1,2 and j= 1,2,3

```
#The capacity = 220 and Demand = 210. We will add a "Dummy" row for Warehouse_4.
trans.cost_1 <- matrix(c(622,614,630,0,100,
                         641,645,649,0,120,
                         80,60,70,10,220), ncol = 5, nrow = 3, byrow = TRUE)
trans.cost_1
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  622  614  630    0  100
## [2,]  641  645  649    0  120
## [3,]   80   60   70   10  220
```

```
#Defining names for the rows and columns
colnames(trans.cost_1) <- c("Warehouse_1","Warehouse_2","Warehouse_3","Dummy","Production Capacity")

rownames(trans.cost_1) <- c("Plant_1", "Plant_2","Monthly Demand")
trans.cost_1
```

```
##                Warehouse_1 Warehouse_2 Warehouse_3 Dummy Production Capacity
## Plant_1                622         614         630     0                 100
## Plant_2                641         645         649     0                 120
## Monthly Demand          80          60          70    10                 220
```

```
#costs matrix
costs <- matrix(c(622,614,630,0,
                  641,645,649,0), nrow = 2, byrow = TRUE)
costs
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  622  614  630    0
## [2,]  641  645  649    0
```

```
#setting up constraint signs and right-hand sides(supply side)
row.signs <- rep("<=",2)
row.rhs <- c(100,120)
#Supply function cannot be greater than the specified units

#Demand side constraints#
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)
#Demand function can be greater than the specified units
```

```r
#solve the model
lptrans <- lp.transport(costs, "min", row.signs,row.rhs,col.signs,col.rhs)
```

```r
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

80 AEDs in Plant 2 - Warehouse_1

60 AEDs in Plant 1 - Warehouse_2

40 AEDs in Plant 1 - Warehouse_3

30 AEDs in Plant 2 - Warehouse_3

The above mentioned should be the production in each plant and distribution to the three wholesaler warehouses to minimize the overall cost of production as well as shipping

```r
lptrans$objval
```

```
## [1] 132790
```

The combined cost of production and shipping for the defibrilators is $132,790

```r
lptrans$duals
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

2)Formulate the dual of the transportation problem

Since the primal was to minimize the transportation cost the dual of it would be to maximize the value added(VA). u and v will be the variables for the dual.

```r
cost_2 <- matrix(c(622,614,630,100,"u1",
                   641,645,649,120,"u2",
                   80,60,70,220,"-",
                   "v1","v2","v3","-","-"),ncol = 5,nrow = 4,byrow = TRUE)
colnames(cost_2) <- c("Warehouse_1", "Warehouse_2","Warehouse_3","Production Capacity","Supply(Dual)")

rownames(cost_2) <- c("Plant_A","Plant_B","Demand","Demand(Dual)")
```

$$\text{Max } VA = 100P_1 + 120P_2 + 80W_1 + 60W_2 + 70W_3$$

Subject to the following constraints Total Profit Constraints

$$u_1 + v_1 <= 622$$

$$u_1 + v_2 <= 614$$

$$u_1 + v_3 <= 630$$

$$u_2 + v_1 <= 641$$

$$u_2 + v_2 <= 645$$

$$u_2 + v_3 <= 649$$

These are taken from the transposed matrixof the Primal of the LP. These are unrestricted where

$$u_k, v_l$$

where u=1,2 and v=1,2,3

```
#Objective function

f.obj <- c(100,120,80,60,70)

#transposed from the constraints matrix in the primal
f.con <- matrix(c(1,0,1,0,0,
                  1,0,0,1,0,
                  1,0,0,0,1,
                  0,1,1,0,0,
                  0,1,0,1,0,
                  0,1,0,0,1), nrow = 6, byrow = TRUE)

f.dir <- c("<=",
           "<=",
           "<=",
           "<=",
           "<=",
           "<=")

f.rhs <- c(622,614,630,641,645,649)
lp("max",f.obj,f.con,f.dir,f.rhs)
```

```
## Success: the objective function is 139120
```

```
lp("max",f.obj,f.con,f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

Z=139,120 and variables are:

$$u_1 = 614$$

$$u_2 = 633$$

$$v_1 = 8$$

$$v_3 = 16$$

So Z = $139,120 and variables are

$$u_1 = 614$$

which represents Plant A

$$u_2 = 633$$

which represents Plant B

$$v_1 = 8$$

which represents Warehouse_1

$$v_2 = 16$$

which represents Warehouse_3

3) Economic Interpretation of the dual

From the above, the minimal Z(Primal) = 132790 and the maximum Z(Dual) = 139120. We understood that we should not be shipping from Plant(A/B) to all the three Warehouses. We should be shipping from:

$$60X_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40X_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80X_{13}$$

which is 60 Units from Plant B to Warehouse 1.

$$30X_{13}$$

which is 60 Units from Plant B to Warehouse 3.

We will Max the profit from each distribution to the respective capacity.

We have the following:

$$u_1^0 - v_1^0 \leq 622$$

then we subtract

$$v_1^0$$

to the other side to get

$$u_1^0 \leq 622 - v_1^0$$

To compute it would be $614 <= (-8+622) which is correct. we would continue to evaluate these equations:

$$u_1 \leq 622 - v_1 => 614 \leq 622 - 8 = 614 => correct$$

$$u_1 \leq 614 - v_2 => 614 \leq 614 - 0 = 614 => correct$$

$$u_1 \leq 630 - v_3 => 614 \leq 630 - 16 = 614 => correct$$

$$u_2 \leq 641 - v_1 => 633 \leq 614 - 8 = 633 => correct$$

$$u_2 \leq 645 - v_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

$$u_2 \leq 649 - v_3 => 633 \leq 649 - 16 = 633 => correct$$

Now from the Duality and Sensitivity we can test the shadow price.

Change 100 to 101 and 120 to 121 in our LP Transport.

```r
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=",2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=",4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=",2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=",4)

lp.transport(costs,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

```r
lp.transport(costs,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)
```

```
## Success: the objective function is 132771
```

```r
lp.transport(costs,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)
```

```
## Success: the objective function is 132790
```

Here we are taking the min of the specific function and observing the number going down by 19 nd this is an indication that the shadow price is 19, was found from the primal and adding 1 to each of the Plants. Plant B does not have a shadow price. From the dual variable

$$v_1$$

where Marginal Revenue <= Marginal Cost. The equation was

$$u_2 \leq 645 - v_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

and this was found by using

$$u_1^0 - v_1^0 \leq 622$$

then we subtract

$$v_1^0$$

to the other side to get

$$u_1^0 \leq 622 - v_1^0$$

The economic interpretation of the dual follows the universal rule of profit maximization i.e. MR >= MC where "MR" is the Marginal Revenue and "MC" is the Marginal Cost.

```r
lp("max", f.obj,f.con, f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

Warehouse1 >= Plant1 + 621 i.e. MR1 >= MC1

Marginal Revenue i.e. The revenue generated for each additional unit sold relative to Marginal Cost (MC) i.e. The change in cost at Plant 1 by inducing an increase in the supply function should be greater than or equal to the revenue generated for each additional unit distributed to Warehouse 1.

$$60X_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40X_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80X_{13}$$

which is 60 Units from Plant B to Warehouse 1.

$$30X_{13}$$

which is 60 Units from Plant B to Warehouse 3. from the dual

We want MR=MC. Out of six five of them had MR <= MC. Plant B to Warehouse_2 does not satisfy the requirement. Hence, there will not be any AED device shipment.