Assignment4

Arbuda Sivani Majeti

10/30/2022

knitr::opts_chunk$set(message = FALSE)knitr :: opts_chunkset(warning = FALSE)

```
library(Benchmarking)
```

```
## Warning: package 'Benchmarking' was built under R version 4.1.3

## Loading required package: lpSolveAPI

## Warning: package 'lpSolveAPI' was built under R version 4.1.3

## Loading required package: ucminf

## Warning: package 'ucminf' was built under R version 4.1.3

## Loading required package: quadprog

##
## Loading Benchmarking version 0.30h, (Revision 244, 2022/05/05 16:31:31) ...

## Build 2022/05/05 16:31:40
```

```
library(lpSolveAPI)
library(ucminf)
library(quadprog)
```

#Formulate and perform DEA analysis under all DEA assumptions FDH, CRS,VRS,IRS,DRS and FRH. Determine Peers and Lambdas under each of the above assumption

```
#Defining Matrix
Health_Care<- matrix(c("F1","F2","F3","F4","F5","F6",
  150,400,320,520,350,320,
  0.2,0.7,1.2,2.0,1.2,0.7,
  14000,14000,42000,28000,19000,14000,
  3500,21000,10500,42000,25000,15000),ncol = 5,byrow = F)

colnames(Health_Care) <- c("DMU","Staff Hours Per day","Supplies per day","Reimbursed Patient Days","Pr

table.Health_Care <- as.table(Health_Care)
table.Health_Care
```

```
##   DMU Staff Hours Per day Supplies per day Reimbursed Patient Days
## A F1  150                 0.2              14000
## B F2  400                 0.7              14000
## C F3  320                 1.2              42000
## D F4  520                 2                28000
## E F5  350                 1.2              19000
## F F6  320                 0.7              14000
##   Privately Paid Patient Days
## A 3500
## B 21000
## C 10500
## D 42000
## E 25000
## F 15000
```

```
X<- matrix(c(150,400,320,520,350,320,
             0.2,0.7,1.2,2.0,1.2,0.7),ncol = 2)

Y<- matrix(c(14000,14000,42000,28000,19000,14000,
             3500,21000,10500,42000,25000,15000), ncol = 2)

colnames(X) <- c("Staff Hours Per day", "Supplies Per day")

colnames(Y) <- c("Reimbursed Patient Days", "Privatley Paid Patient Days")

X
```

```
##      Staff Hours Per day Supplies Per day
## [1,]                 150              0.2
## [2,]                 400              0.7
## [3,]                 320              1.2
## [4,]                 520              2.0
## [5,]                 350              1.2
## [6,]                 320              0.7
```

```
Y
```

```
##      Reimbursed Patient Days Privatley Paid Patient Days
## [1,]                   14000                        3500
## [2,]                   14000                       21000
## [3,]                   42000                       10500
## [4,]                   28000                       42000
## [5,]                   19000                       25000
## [6,]                   14000                       15000
```

```
#1)CRS Assumption
CRS <- dea(X,Y,RTS = "crs")
CRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

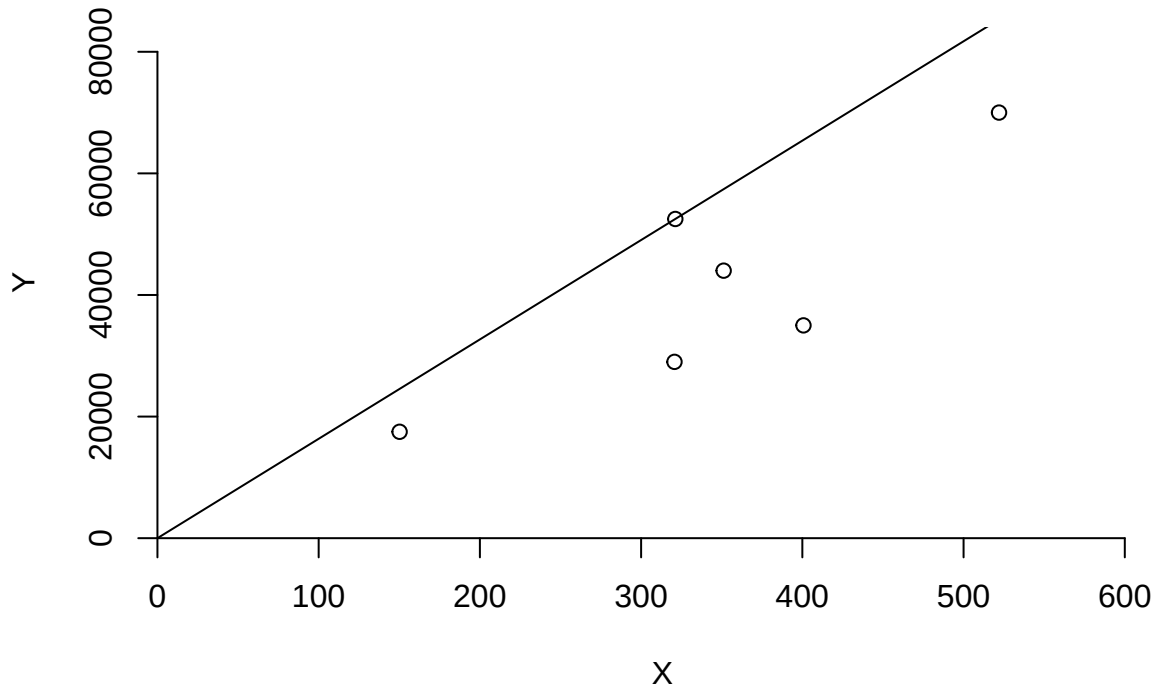
```
peers(CRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```
lambda(CRS)
```

```
##             L1         L2 L3        L4
## [1,] 1.0000000 0.00000000  0 0.0000000
## [2,] 0.0000000 1.00000000  0 0.0000000
## [3,] 0.0000000 0.00000000  1 0.0000000
## [4,] 0.0000000 0.00000000  0 1.0000000
## [5,] 0.2000000 0.08048142  0 0.5383307
## [6,] 0.3428571 0.39499264  0 0.1310751
```

```
dea.plot.frontier(X,Y,RTS = "crs")
```



```r
summary(CRS, digits = 4)
```

```
## Summary of efficiencies
## CRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 4 out of 6
## Mean efficiency: 0.974
## ---
##   Eff range      #  %
##   0.8<= E <0.9   1 17
##   0.9<= E <1     1 17
##         E ==1    4 67
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8675  0.9831  1.0000  0.9742  1.0000  1.0000
```

```r
#b)DRS Assumptions

DRS <- dea(X,Y,RTS = "DRS")
DRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

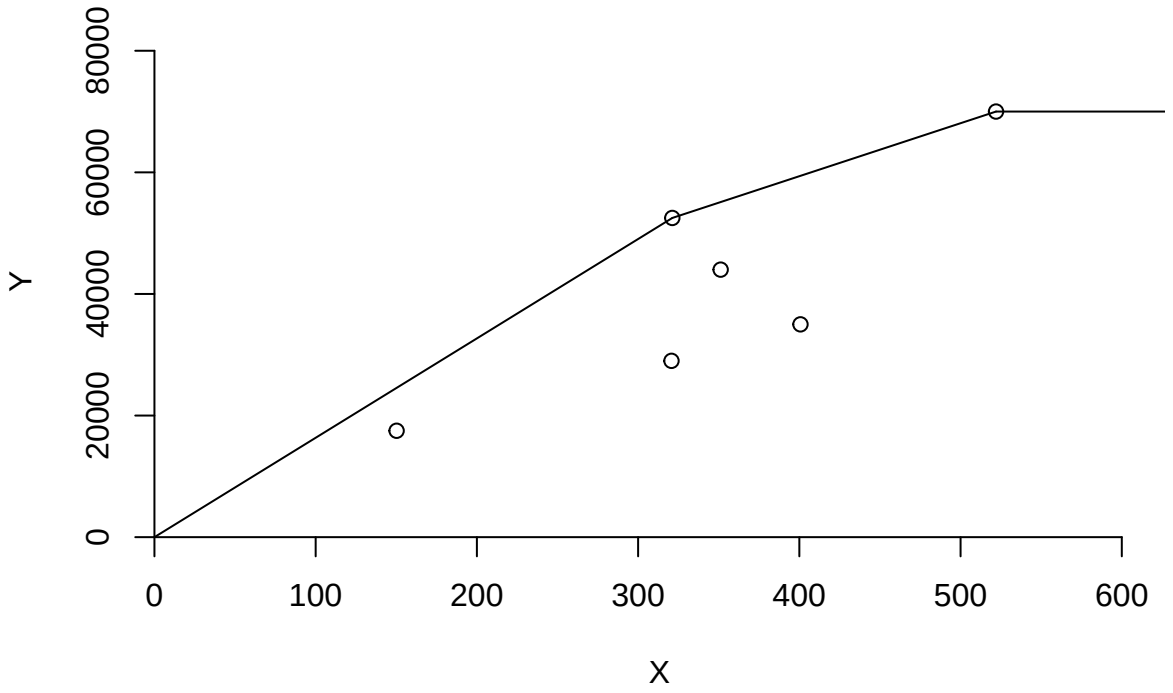
```r
peers(DRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```r
lambda(DRS)
```

```
##             L1         L2 L3        L4
## [1,] 1.0000000 0.00000000  0 0.0000000
## [2,] 0.0000000 1.00000000  0 0.0000000
## [3,] 0.0000000 0.00000000  1 0.0000000
## [4,] 0.0000000 0.00000000  0 1.0000000
## [5,] 0.2000000 0.08048142  0 0.5383307
## [6,] 0.3428571 0.39499264  0 0.1310751
```

```r
dea.plot.frontier(X,Y,RTS = "DRS")
```




```
summary(DRS,digits = 4)
```

```
## Summary of efficiencies
## DRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 4 out of 6
## Mean efficiency: 0.974
## ---
##   Eff range      #  %
##   0.8<= E <0.9   1 17
##   0.9<= E <1     1 17
##         E ==1    4 67
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8675  0.9831  1.0000  0.9742  1.0000  1.0000
```

```
#c)IRS Assumptions

IRS <- dea(X,Y,RTS = "irs")
IRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

```
peers(IRS)
```

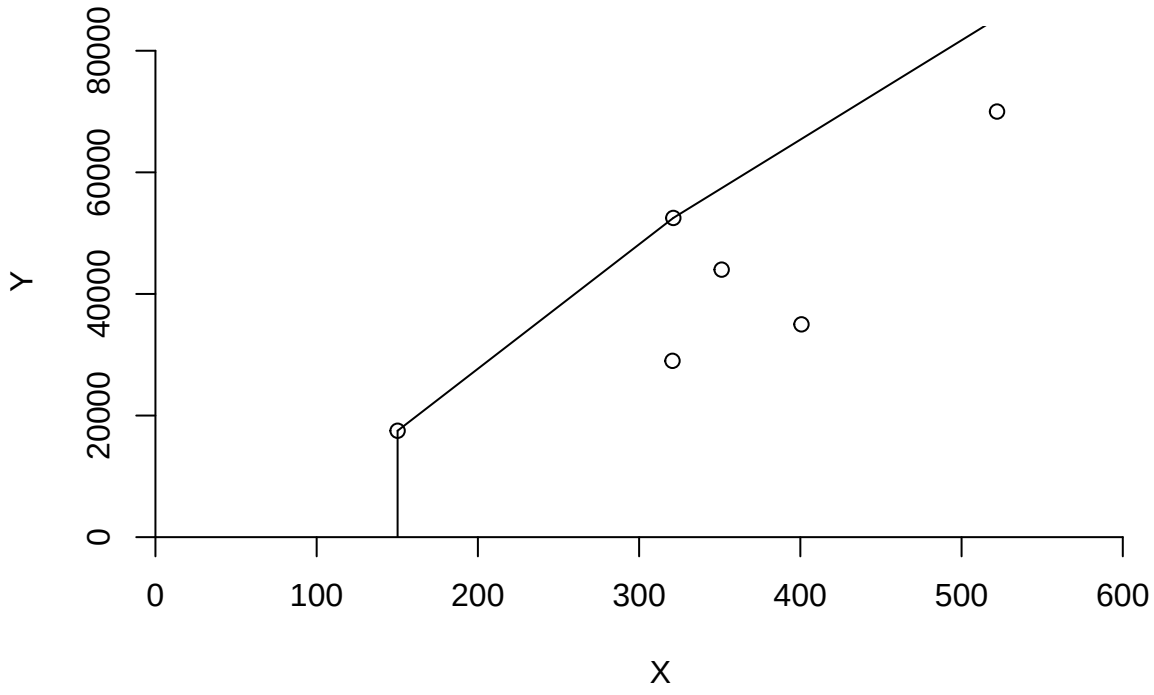
```
##      peer1 peer2 peer3
```

```
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5
```

```
lambda(IRS)
```

```
##             L1        L2 L3 L4        L5
## [1,] 1.0000000 0.0000000  0  0 0.0000000
## [2,] 0.0000000 1.0000000  0  0 0.0000000
## [3,] 0.0000000 0.0000000  1  0 0.0000000
## [4,] 0.0000000 0.0000000  0  1 0.0000000
## [5,] 0.0000000 0.0000000  0  0 1.0000000
## [6,] 0.4014399 0.3422606  0  0 0.2562995
```

```
dea.plot.frontier(X,Y,RTS = "irs")
```




```
summary(IRS,digits = 4)
```

```
## Summary of efficiencies
## IRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 5 out of 6
```

```
## Mean efficiency: 0.983
## ---
##   Eff range      #  %
##   0.8<= E <0.9   1 17
##   0.9<= E <1     0  0
##         E ==1    5 83
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8963  1.0000  1.0000  0.9827  1.0000  1.0000
```

```
#d)VRS Assumptions

VRS <- dea(X,Y,RTS = "VRS")
VRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
```

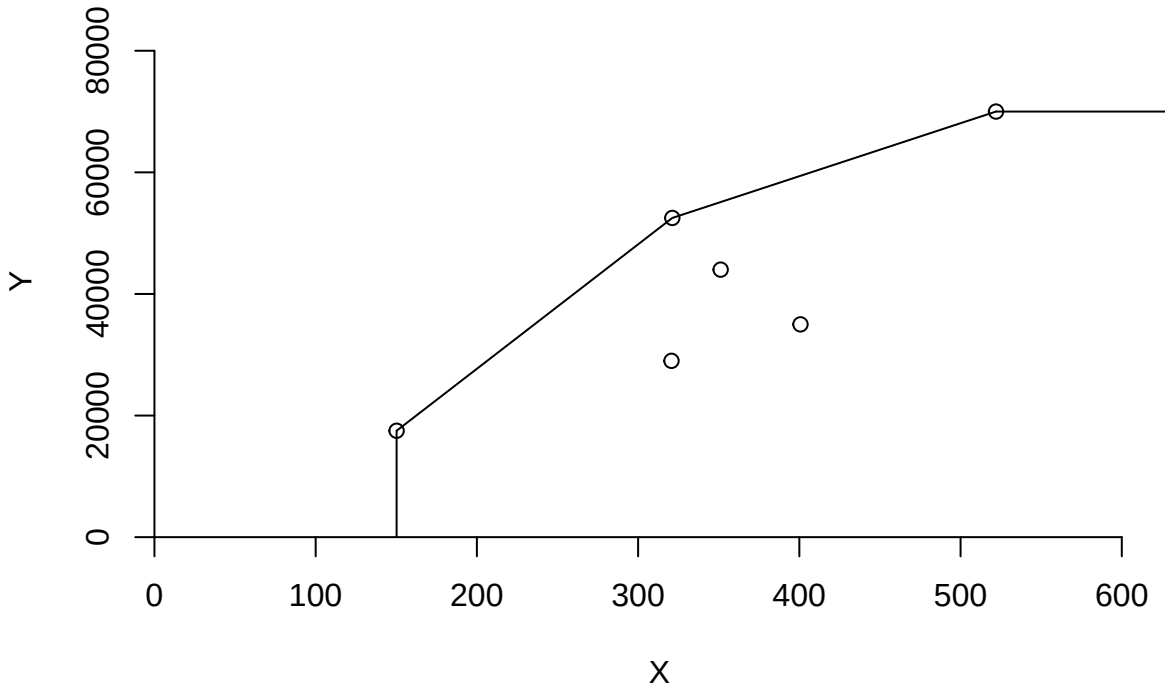
```
peers(VRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5
```

```
lambda(VRS)
```

```
##             L1        L2 L3 L4        L5
## [1,] 1.0000000 0.0000000  0  0 0.0000000
## [2,] 0.0000000 1.0000000  0  0 0.0000000
## [3,] 0.0000000 0.0000000  1  0 0.0000000
## [4,] 0.0000000 0.0000000  0  1 0.0000000
## [5,] 0.0000000 0.0000000  0  0 1.0000000
## [6,] 0.4014399 0.3422606  0  0 0.2562995
```

```
dea.plot.frontier(X,Y,RTS = "VRS")
```




```
summary(VRS, digits = 4)
```

```
## Summary of efficiencies
## VRS technology and input orientated efficiency
## Number of firms with efficiency==1 are 5 out of 6
## Mean efficiency: 0.983
## ---
##   Eff range      #  %
##   0.8<= E <0.9   1 17
##   0.9<= E <1     0  0
##         E ==1    5 83
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.8963  1.0000  1.0000  0.9827  1.0000  1.0000
```

```
#f) FDH Assumptions
FDH <- dea(X,Y,RTS = "FDH")
FDH
```

```
## [1] 1 1 1 1 1 1
```

```
peers(FDH)
```

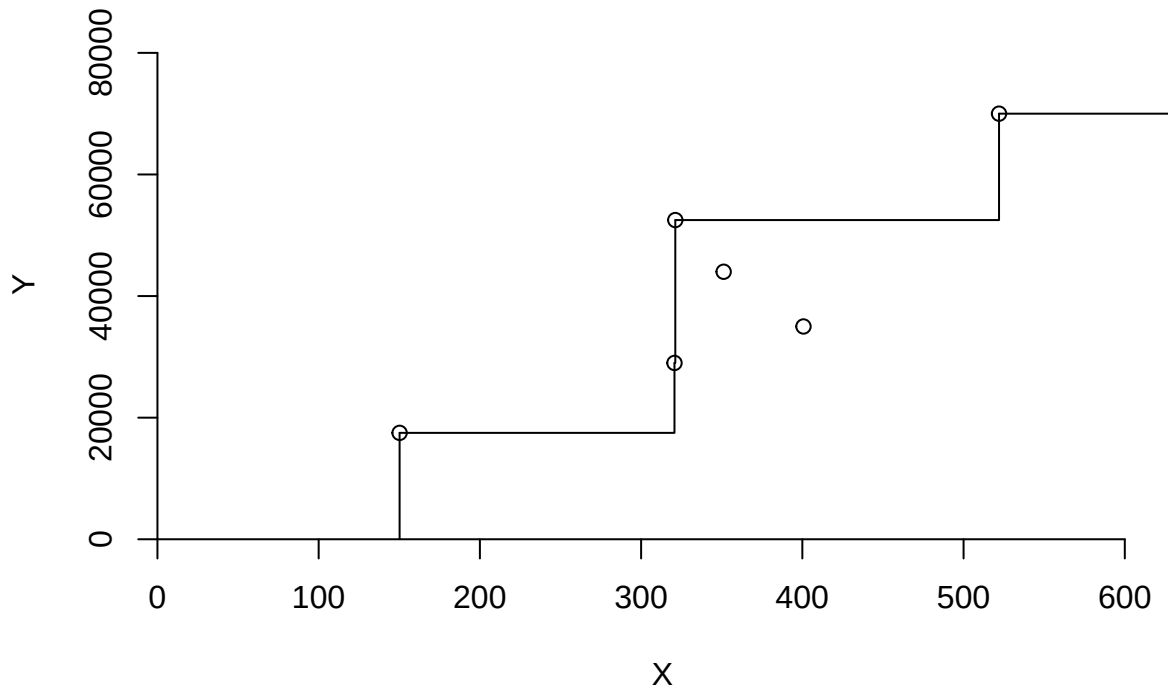
```
##      peer1
## [1,]     1
```

```
## [2,]     2
## [3,]     3
## [4,]     4
## [5,]     5
## [6,]     6
```

```
lambda(FDH)
```

```
##      L1 L2 L3 L4 L5 L6
## [1,]  1  0  0  0  0  0
## [2,]  0  1  0  0  0  0
## [3,]  0  0  1  0  0  0
## [4,]  0  0  0  1  0  0
## [5,]  0  0  0  0  1  0
## [6,]  0  0  0  0  0  1
```

```
dea.plot.frontier(X,Y,RTS = "FDH")
```




```
summary(FDH, digits = 4)
```

```
## Summary of efficiencies
## FDH technology and input orientated efficiency
## Number of firms with efficiency==1 are 6 out of 6
## Mean efficiency: 1
```

```
## ---
##   Eff range      #   %
##         E ==1    6 100
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       1       1       1       1       1       1
```

```
#e)FRH Assumptions

FRH <- dea(X,Y,RTS = "add") #specifying RTS = "add"
FRH
```

```
## [1] 1 1 1 1 1 1
```

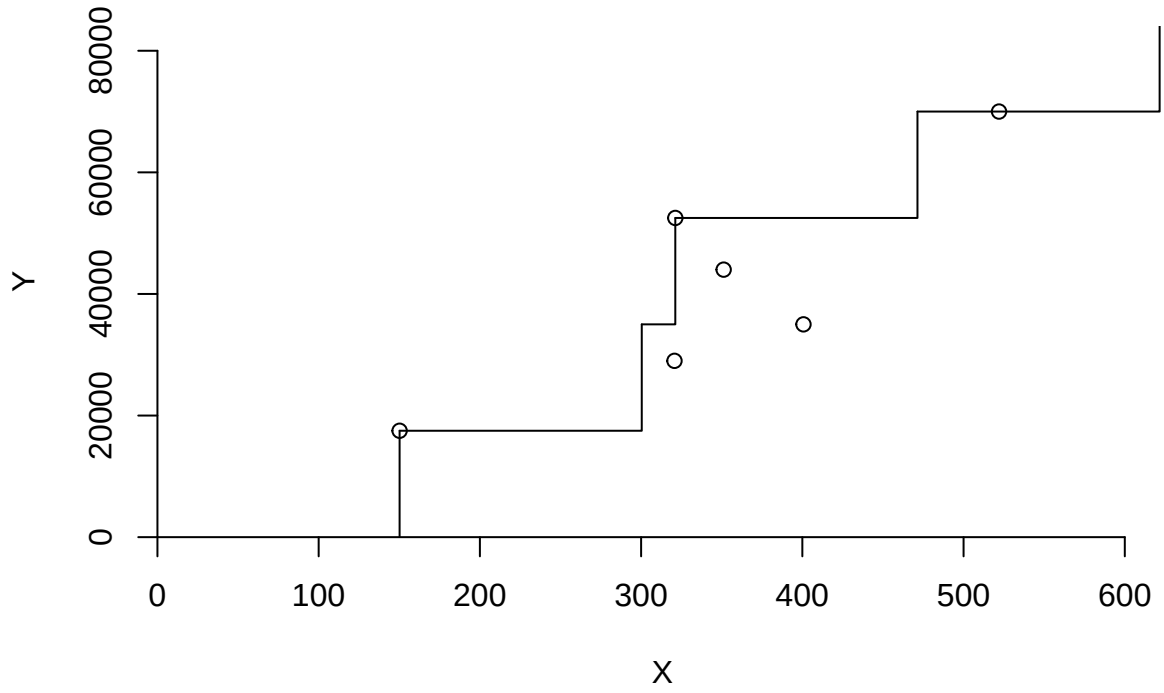
```
peers(FRH)
```

```
##      peer1
## [1,]     1
## [2,]     2
## [3,]     3
## [4,]     4
## [5,]     5
## [6,]     6
```

```
lambda(FRH)
```

```
##      L1 L2 L3 L4 L5 L6
## [1,]  1  0  0  0  0  0
## [2,]  0  1  0  0  0  0
## [3,]  0  0  1  0  0  0
## [4,]  0  0  0  1  0  0
## [5,]  0  0  0  0  1  0
## [6,]  0  0  0  0  0  1
```

```
dea.plot.frontier(X,Y,RTS = "add")
```




```
summary(FRH, digits = 4)
```

```
## Summary of efficiencies
## ADD technology and input orientated efficiency
## Number of firms with efficiency==1 are 6 out of 6
## Mean efficiency: 1
## ---
##   Eff range      #   %
##         E ==1    6 100
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       1       1       1       1       1       1
```

#Summarize the results in a tabular format

```
#Inefficient DMU's
Health_Care.inefficient <- matrix(c("CRS","DRS","IRS","VRS","FDH","FRH",
2,2,1,1,0,0,
"Facility 5&6", "Facility 5&6", "Facility 6", "Facility 6", "-", "-",
"97.75% & 86.7%", "97.75% & 86.7%", "89.63%","89.63%","-","-",
"Facility 1,2 & 4", "Facility 1,2 & 4","Facility 1,2 & 5", "Facility 1,2 & 5","-","-",
"0.2,0.08,0.54 and 0.34,0.4,0.13","0.2,0.08,0.54 and 0.34,0.4,0.13", "0.4,0.34 and 0.26", "0.4,0.34 and

colnames(Health_Care.inefficient) <- c("RTS", "Count_Inefficient", "Name","Inefficiency %", "Peers", "La

as.table(Health_Care.inefficient)
```

```
##   RTS Count_Inefficient Name         Inefficiency % Peers
## A CRS 2                 Facility 5&6 97.75% & 86.7% Facility 1,2 & 4
## B DRS 2                 Facility 5&6 97.75% & 86.7% Facility 1,2 & 4
## C IRS 1                 Facility 6   89.63%         Facility 1,2 & 5
## D VRS 1                 Facility 6   89.63%         Facility 1,2 & 5
## E FDH 0                 -            -              -
## F FRH 0                 -            -              -
##   Lambda
## A 0.2,0.08,0.54 and 0.34,0.4,0.13
## B 0.2,0.08,0.54 and 0.34,0.4,0.13
## C 0.4,0.34 and 0.26
## D 0.4,0.34 and 0.26
## E -
## F -
```

```
#Efficient DMUs
Health_Care.efficient <- matrix(c("CRS", "DRS", "IRS", "VRS", "FDH", "FRH",
"Facility 1, 2, 3 & 4", "Facility 1, 2, 3, & 4", "Facility 1, 2, 3, 4 & 5", "Facility 1, 2, 3, 4 & 5",

colnames(Health_Care.efficient) <- c("RTS", "Efficient DMUs")

as.table(Health_Care.efficient)
```

```
##   RTS Efficient DMUs
## A CRS Facility 1, 2, 3 & 4
## B DRS Facility 1, 2, 3, & 4
## C IRS Facility 1, 2, 3, 4 & 5
## D VRS Facility 1, 2, 3, 4 & 5
## E FDH All DMUs
## F FRH All DMUs
```

#Compare and contrast the above results

1. CRS - Constant Return to Scale:

From the results obtained above it is clear that DMUs 1, 2 , 3 and 4 are efficient and DMU 5 is 97.75% and DMU 6 is 86.7% respectively efficient. The peer units for DMU 4 are 1,2 and 4 with weights of 0.2, 0.08 and 0.54 and for DMU 6 peer units are 1, 2 and 4 with weights of 0.34, 0.4 and 0.13 respectively. CRS helps us to find out if any DMUs can be scaled up or down. DMUs 1, 2,3 and 4 can be scaled up

2. DRS - Decreasing Return to Scale:

From the results obtained above it is clear that DMUs 1, 2 , 3 and 4 are efficient and DMU 5 is 97.75% and DMU 6 is 86.7% respectively efficient. The peer units for DMU 4 are 1,2 and 4 with weights of 0.2, 0.08 and 0.54 and for DMU 6 peer units are 1, 2 and 4 with weights of 0.34, 0.4 and 0.13 respectively. CRS helps us to find out if any DMUs can be scaled up or down. DMUs 5 and 6 can be scaled down.

3. IRS - Increasing Return to Scale:

From the results obtained above it is clear that DMUs 1, 2 , 3, 4 and 5 are efficient and DMU 6 is only 89.63%. The peer units for DMU 6 are 1,2 and 5 with weights of 0.4, 0.34 and 0.26 respectively. IRS helps in finding out if the firm can increase the scale of operations by checking the efficiency scores.

4. VRS- Variable Returns to Scale:

From the results obtained above it is clear that DMUs 1, 2 , 3, 4 and 5 are efficient and DMU 6 is only 89.63%. The peer units for DMU 6 are 1,2 and 5 with weights of 0.4, 0.34 and 0.26 respectively. VRS helps in understanding the scale of operations with any variations towards inputs and outputs by increasing , decreasing or both.

5. FDH - Free Disposability Hull:

From the above results it is clear that all the DMUs are efficient. This method allows the scale to capture all the levels of efficiency.

6. FRH - Free Replicability Hull:

From the above results it is clear that all the DMUs are efficient. This method allows the scale to capture all the levels of efficiency which is free from replication and disposal.

Conclusion:

a) Inefficient peers will have peers and lambda weights whereas, efficient peers will not have peers and lambda weights.
b) DEA is a benchmarking tool that evaluates a population of DMUs in their performance in converting input to outputs.
c) DEA allows the firm in deciding the best DMU and also in analyzing which DMU can be maximized which will lead to increase, decrease or any variations.
d) A firm can also decide on which RTS to employ based on their requirements.