

Adhinar B. Logarithm CPE 3B

 $E$  = modulus of Elasticity

 $I$  = Moment of Inertia

 $h$  = effective length

 $L$  = column length

$$P_{cr} = \frac{\pi^2 EI}{(hL)^2}$$

$$f(P) = P - P_{cr} \cos\left(\frac{P}{P_{cr}}\right) = 0$$

Given:

$$E = 210 \times 10^9 \text{ Pa}$$

$$I = 8.1 \times 10^{-6} \text{ m}^4$$

$$L = 3 \text{ m}$$

$$h = 1.0$$

$$P_{cr} = \frac{\pi^2 (210 \times 10^9) (8.1 \times 10^{-6})}{(1.3 \times 3.0)^2}$$

$$= 9.8696 \times \frac{(210 \times 10^9) (8.1 \times 10^{-6})}{9} = 1.87 \times 10^6 \text{ N}$$

$$f(P) = P - P_{cr} \cos\left(\frac{P}{P_{cr}}\right) \rightarrow P = P_{cr} \cos\left(\frac{P}{P_{cr}}\right)$$

$$\frac{d}{dP} [P] = 1$$

$$\frac{d}{dP} \left[ -P_{cr} \cos\left(\frac{P}{P_{cr}}\right) \right] = -P_{cr} \left[ -\sin\left(\frac{P}{P_{cr}}\right) \right] \times \frac{1}{P_{cr}} = \sin\left(\frac{P}{P_{cr}}\right) \therefore f'(P) = 1 + \sin\left(\frac{P}{P_{cr}}\right)$$

~~Newton Raphson Method~~

$$P_{new} = P_{old} - \frac{f(P_{old})}{f'(P_{old})}$$

$$\text{initial guess} \rightarrow P_0 = 0.5 P_{cr} = 0.5 \times (1.87 \times 10^6)$$

$$P_0 = 9.35 \times 10^5$$

Iteration 1:

$$f(P_0) = (9.35 \times 10^5) - (1.87 \times 10^6) \cos\left(\frac{9.35 \times 10^5}{1.87 \times 10^6}\right) = -705,000$$

$$f'(P_0) = 1 + \sin\left(\frac{9.35 \times 10^5}{1.87 \times 10^6}\right) = 1.4794$$

$$P_1 = P_0 - \frac{f(P_0)}{f'(P_0)} = 935000 - \frac{-705000}{1.4794} \approx \underline{\underline{1.412 \times 10^6}}$$

Iteration 2:

$$f(P_1) = (1.412 \times 10^6) - (1.87 \times 10^6) \cos\left(\frac{1.412 \times 10^6}{1.87 \times 10^6}\right) = 52,000$$

$$f'(P_1) = 1 + \sin\left(\frac{1.412 \times 10^6}{1.87 \times 10^6}\right) = 1.684$$

$$P_2 = P_1 - \frac{f(P_1)}{f'(P_1)} = 1.412 \times 10^6 - \frac{52000}{1.684} \approx \underline{\underline{1.381 \times 10^6}}$$

Iteration 3:

$$f(P_2) = (1.381 \times 10^6) - (1.87 \times 10^6) \cos\left(\frac{1.381 \times 10^6}{1.87 \times 10^6}\right) = -3000$$

$$f'(P_2) = 1 + \sin\left(\frac{1.381 \times 10^6}{1.87 \times 10^6}\right) = 1.673$$

$$P_3 = P_2 - \frac{f(P_2)}{f'(P_2)} = 1.381 \times 10^6 - \frac{-3000}{1.673} \approx \underline{\underline{1.383 \times 10^6}}$$

Iteration 4:

$$f(P_3) = (1.383 \times 10^6) - (1.87 \times 10^6) \cos\left(\frac{1.383 \times 10^6}{1.87 \times 10^6}\right) = -10$$

$$f'(P_3) = 1 + \sin\left(\frac{1.383 \times 10^6}{1.87 \times 10^6}\right) = 1.673$$

$$P_4 = P_3 - \frac{f(P_3)}{f'(P_3)} = 1.383 \times 10^6 - \frac{-10}{1.673} \approx \underline{\underline{1.383 \times 10^6}}$$

$\therefore$  The critical buckling load considering nonlinear effects  
is  $1.383 \times 10^6 \text{ N}$  or  $1.38 \text{ MN}$ .