PANGANIBAN, JOHN KENNETH A.
ME - 4203 20-07429

ASSIGNMENT 1

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING

1.) $\mathcal{L}\{3-e^{-3t}+5\sin 2t\} = F(s)$

$\mathcal{L}\{3\} \Rightarrow 3\mathcal{L}\{1\} = 3\left(\frac{1}{s}\right) = \frac{3}{s}$

$\mathcal{L}\{e^{-3t}\} = \frac{1}{s+3}$; $a=3$

$\mathcal{L}\{5\sin 2t\} \Rightarrow 5\mathcal{L}\{\sin 2t\} = 5\left(\frac{2}{s^2+2^2}\right) = \frac{10}{s^2+4}$; $\omega=2$

$\boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$

2.) $\mathcal{L}\{3+12t+42t^3-3e^{2t}\} = F(s)$

$\mathcal{L}\{3\} \Rightarrow 3\mathcal{L}\{1\} = 3\left(\frac{1}{s}\right) = \frac{3}{s}$

$\mathcal{L}\{12t\} \Rightarrow 12\mathcal{L}\{t\} = 12\left(\frac{1}{s^2}\right) = \frac{12}{s^2}$

$\mathcal{L}\{42t^3\} \Rightarrow 42\mathcal{L}\{t^3\}$; $n=3$

$\hookrightarrow 42\left(\frac{3!}{s^{3+1}}\right) \Rightarrow 42\left(\frac{6}{s^4}\right) = \frac{252}{s^4}$

$\mathcal{L}\{3e^{2t}\} \Rightarrow 3\mathcal{L}\{e^{2t}\} = 3\left(\frac{1}{s-2}\right) = \frac{3}{s-2}$; $a=2$

$\boxed{F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$

3.) $\mathcal{L}\{(t+1)(t+2)\} = F(s)$

$\mathcal{L}\{t^2+3t+2\} = F(s)$

$\mathcal{L}\{t^2\}$; $n=2 \Rightarrow \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$\mathcal{L}\{3t\} \Rightarrow 3\mathcal{L}\{t\} = 3\left(\frac{1}{s^2}\right) = \frac{3}{s^2}$

$\mathcal{L}\{2\} \Rightarrow 2\mathcal{L}\{1\} = 2\left(\frac{1}{s}\right) = \frac{2}{s}$

$\boxed{F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}}$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFER OF THE FOLLOWING:

1.) $\mathcal{L}^{-1}\left\{\frac{8-3s+s^2}{s^3}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3} - \frac{3}{s^2} + \frac{1}{s}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3}\right\} \Rightarrow 4\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} = 4t^2 u(t)$

$\mathcal{L}^{-1}\left\{\frac{3}{s^2}\right\} \Rightarrow 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = 3t\, u(t)$

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$ $\boxed{\therefore f(t) = (4t^2 - 3t + 1)\, u(t)}$

2.) $\mathcal{L}^{-1}\left\{\frac{5}{s-2} - \frac{4s}{s^2+9}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{5}{s-2}\right\} \Rightarrow 5\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$

$\rightarrow 5e^{2t}u(t)$

$\mathcal{L}^{-1}\left\{\frac{4s}{s^2+9}\right\} \Rightarrow 4\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}$

$\rightarrow 4\cos 3t \; u(t)$

$\boxed{f(t) = (5e^{2t} - 4\cos 3t)\,u(t)}$

3.) $\mathcal{L}^{-1}\left\{\frac{7}{s^2+6}\right\} = f(t)$

$7\mathcal{L}^{-1}\left\{\frac{1}{s^2+6}\right\} = \frac{7}{\sqrt{6}}\mathcal{L}^{-1}\left\{\frac{\sqrt{6}}{s^2+6}\right\}$

$\rightarrow \left[\frac{7}{\sqrt{6}}(\sin\sqrt{6})\,u(t)\right]\cdot\frac{\sqrt{6}}{\sqrt{6}}$

$\boxed{f(t) = \frac{7\sqrt{6}}{6}\sin\sqrt{6}\;u(t)}$