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# ASSIGNMENT 1

## I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING

1.) $\mathcal{L}\{3-e^{-3t}+5\sin 2t\} = F(s)$

$\mathcal{L}\{3\} \Rightarrow 3\mathcal{L}\{1\} = 3\left(\frac{1}{s}\right) = \frac{3}{s}$

$\mathcal{L}\{e^{-3t}\} = \frac{1}{s+3}$ ; $a=3$

$\mathcal{L}\{5\sin 2t\} \Rightarrow 5\mathcal{L}\{\sin 2t\} = 5\left(\frac{2}{s^2+2^2}\right) = \frac{10}{s^2+4}$ ; $\omega=2$

$\boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$

2.) $\mathcal{L}\{3+12t+42t^3-3e^{2t}\} = F(s)$

$\mathcal{L}\{3\} \Rightarrow 3\mathcal{L}\{1\} = 3\left(\frac{1}{s}\right) = \frac{3}{s}$

$\mathcal{L}\{12t\} \Rightarrow 12\mathcal{L}\{t\} = 12\left(\frac{1}{s^2}\right) = \frac{12}{s^2}$

$\mathcal{L}\{42t^3\} \Rightarrow 42\mathcal{L}\{t^3\}$ ; $n=3$

$\hookrightarrow 42\left(\frac{3!}{s^{3+1}}\right) \Rightarrow 42\left(\frac{6}{s^4}\right) = \frac{252}{s^4}$

$\mathcal{L}\{3e^{2t}\} \Rightarrow 3\mathcal{L}\{e^{2t}\} = 3\left(\frac{1}{s-2}\right) = \frac{3}{s-2}$ ; $a=2$

$\boxed{F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$

3.) $\mathcal{L}\{(t+1)(t+2)\} = F(s)$

$\mathcal{L}\{t^2+3t+2\} = F(s)$

$\mathcal{L}\{t^2\}$ ; $n=2 \Rightarrow \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$\mathcal{L}\{3t\} \Rightarrow 3\mathcal{L}\{t\} = 3\left(\frac{1}{s^2}\right) = \frac{3}{s^2}$

$\mathcal{L}\{2\} \Rightarrow 2\mathcal{L}\{1\} = 2\left(\frac{1}{s}\right) = \frac{2}{s}$

$\boxed{F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}}$

## II. SOLVE FOR THE INVERSE LAPLACE TRANSFER OF THE FOLLOWING:

1.) $\mathcal{L}^{-1}\left\{\frac{8-3s+s^2}{s^3}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3} - \frac{3}{s^2} + \frac{1}{s}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3}\right\} \Rightarrow 4\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} = 4t^2 u(t)$

$\mathcal{L}^{-1}\left\{\frac{3}{s^2}\right\} \Rightarrow 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = 3t\, u(t)$

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$ $\boxed{\therefore f(t) = (4t^2 - 3t + 1)\, u(t)}$

2.) $\mathcal{L}^{-1}\left\{\frac{5}{s-2} - \frac{4s}{s^2+9}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{5}{s-2}\right\} \Rightarrow 5\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$

$\hookrightarrow 5e^{2t}u(t)$

$\mathcal{L}^{-1}\left\{\frac{4s}{s^2+9}\right\} \Rightarrow 4\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}$

$\hookrightarrow 4\cos 3t \; u(t)$

$\boxed{f(t) = (5e^{2t} - 4\cos 3t)u(t)}$

3.) $\mathcal{L}^{-1}\left\{\frac{7}{s^2+6}\right\} = f(t)$

$7\mathcal{L}^{-1}\left\{\frac{1}{s^2+6}\right\} = \frac{7}{\sqrt{6}}\mathcal{L}^{-1}\left\{\frac{\sqrt{6}}{s^2+6}\right\}$

$\hookrightarrow \left[\frac{7}{\sqrt{6}}(\sin\sqrt{6}t)u(t)\right] \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$\boxed{f(t) = \frac{7\sqrt{6}}{6} \sin\sqrt{6}t\, u(t)}$

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ASSIGNMENT 2

1.) $F(s) = \dfrac{1}{s(s^2+2s+2)}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$1 = A(s^2+2s+2) + s(Bs+C)$$

if $s=0$;

$$1 = A(2)+0$$

$$\frac{1}{2} = A$$

SUBSTITUTING VALUE OF A

$$\left[1 = \frac{1}{2}(s^2+2s+2) + Bs^2 + Cs\right]2$$

$$2 = s^2 + 2s + 2 + 2Bs^2 + 2Cs$$

$$2 = s^2(2B+1) + s(2C+2) + 2$$

$$B = -\frac{1}{2} \quad ; \quad C = -1$$

$$\mathcal{L}^{-1}\left\{\frac{\frac{1}{2}}{s} - \frac{(\frac{1}{2}s+1)}{s^2+2s+2}\right.$$

① $\mathcal{L}^{-1}\left\{\frac{\frac{1}{2}}{s}\right\} = \frac{1}{2}$

② $\mathcal{L}^{-1}\left\{\frac{(\frac{1}{2}s+1)}{s^2+2s+2}\right\} \Rightarrow \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+2s+2}\right\}$

$$= \frac{(s+1)+1}{(s^2+2s+2)+1}$$

$$= \frac{(s+1)+1}{(s+1)^2+1} \quad ; a=-1, \ \omega = \sqrt{1} = 1$$

USING FORMULA

$$\mathcal{L}^{-1}\left\{\frac{(s+a)+\omega}{(s+a)^2+\omega^2}\right\} = e^{-at}\left[\cos\omega t + \sin\omega t\right]u(t)$$

$$\frac{1}{2}e^{-t}(\cos t + \sin t)\,u(t)$$

$$f(t) = \frac{1}{2} - \frac{1}{2}e^{-t}(\cos t + \sin t)$$

$$\boxed{f(t) = \frac{1}{2}\left[1 - e^{-t}(\cos t + \sin t)\right]}$$

2.) $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)}$

$$\mathcal{L}^{-}\left\{\frac{5(s+2)}{s^2(s+1)(s+3)}\right\} = \frac{A}{s^2} + \frac{B}{s+1} + \frac{C}{s+3} + \frac{D}{s}$$

$$5(s+2) = A(s+1)(s+3) + B(s^2)(s+3) + C(s^2)(s+1) + D(s)(s+1)(s+3)$$

if $s = 0$;

$10 = 3A + 0 + 0 + 0$

$A = \frac{10}{3}$

if $s = -1$

$5 = 0 + 2B + 0 + 0$

$B = \frac{5}{2}$

if $s = -3$

$-5 = 0 + 0 - 18C + 0$

$C = \frac{5}{18}$

if $s = -2$, SUBSTITUTE A, B, C

$$5((-2)+2) = \frac{10}{3}((-2)+1)((-2)+3) + \frac{5}{2}(-2)^2((-2)+3) + \frac{5}{18}(-2)^2((-2)+1) + D(-2)((-2)+1)((-2)+3)$$

$$0 = -\frac{10}{3} + 10 - \frac{10}{9} + 2D$$

$$0 = \frac{50}{9} + 2D$$

$$\left[2D = \frac{50}{9}\right]\frac{1}{2}$$

$$D = -\frac{25}{9}$$

$$\mathcal{L}^{-}\left\{\frac{\frac{10}{3}}{s^2} + \frac{\frac{5}{2}}{s+1} + \frac{\frac{5}{18}}{s+3} - \frac{25/9}{s}\right\}$$

$$\boxed{f(t) = \frac{10t}{3} + \frac{5e^{-t}}{2} + \frac{5e^{-3t}}{18} - \frac{25}{9}}$$

3.) $F(s) = \dfrac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

$$\begin{array}{r|l} & s^2 + s + 2 \\ \hline s^2 + s & s^4 + 2s^3 + 3s^2 + 4s + 5 \\ & -\; s^4 + s^3 \\ \hline & s^3 + 3s^2 + 4s + 5 \\ & -\; s^3 + s^2 \\ \hline & 2s^2 + 4s + 5 \\ & -\; 2s^2 + 2s \\ \hline & 2s + 5 \end{array}$$

$$\mathcal{L}^{-}\left\{s^2 + s + 2 + \frac{2s+5}{s^2+s}\right\}$$

① $y''$

② $y'$

③ $2\delta(t)$

④ $\mathcal{L}^{-}\left\{\frac{2s+5}{s^2+s}\right\} = \frac{A}{s} + \frac{B}{s+1}$

$$2s + 5 = A(s+1) + Bs$$

if $s = -1$;

$3 = 0 - B$

$B = -3$

if $s = 0$;

$5 = A + 0$

$A = 5$

$$\mathcal{L}^{-}\left\{\frac{5}{s} - \frac{3}{s+1}\right\} = 5(1) + \left[-3e^{-t}\right]$$

$$= 5 - 3e^{-t}$$

$$\boxed{f(t) = y'' + y' + 2\delta(t) + 5 - 3e^{-t}}$$