

Introduction to Probability  
Tutorial 9

1. A player pays \$1 to play a game where three fair dice are rolled. If three 6s are obtained, the player wins \$500. Otherwise the player wins nothing. What are the expected net winnings of this game? Would you want to play this game? Does your answer depend upon how many times you can play the game?
2. Suppose that the random variable  $X$  takes the values -2, 1, 4 and 6 with probability values  $1/3$ ,  $1/6$ ,  $1/3$  and  $1/6$  respectively.
  - (a) Find the expectation of  $X$ .
  - (b) Find the variance of  $X$  using the formula
$$\text{Var}(X) = E((X - E(X))^2).$$
  - (c) Find the variance of  $X$  using the formula
$$\text{Var}(X) = E(X^2) - (E(X))^2.$$
3. Suppose that you are organizing the game described in slide 7 of Lecture 9, where you charge players \$2 to roll two dice, and then you pay them the difference in the score.
  - (a) What is the variance in your profit from each game? If you are playing a game in which you have positive expected winnings, would you prefer a small or a large variance in the winnings?
  - (b) If you fix the dice so that each die has a probability of 0.2 of scoring a 3 and equal probability of 0.16 of scoring the other five numbers, do your expected winnings increase beyond 6 cents per game? Is it a surprise?
4. A random variable  $X$  has a probability density function  $f(x) = A/\sqrt{x}$  for  $3 \leq x \leq 4$ .
  - (a) What is the value of  $A$ ?
  - (b) What is the cumulative distribution function of  $X$ ?
  - (c) What is the expected value of  $X$ ?
  - (d) What is the standard deviation of  $X$ ?
  - (e) What is the median of  $X$ ?