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Tutorial 5 - Mathematics for CS

1. An item of clothing is picked at random from one of two drawers in a dark room. The first drawer (D_1) contains 6 socks and 6 ties, and the other (D_2) contains 2 socks and 4 handkerchiefs. What is the probability that the item comes from the first drawer if it turns out to be a sock? Assuming the drawers are equally likely to be picked.

$$P(\text{sock}|D_1) = \frac{6}{6+6} = \frac{1}{2} \qquad P(\text{sock}|D_2) = \frac{2}{2+4} = \frac{1}{3}$$

$$P(\text{sock}) = P(\text{sock}|D_1) \times P(D_1) + P(\text{sock}|D_2) \times P(D_2) = 1/2 \times 1/2 + 1/2 \times 1/3 = \frac{5}{12}$$

Hence,

$$P(D_1|\text{sock}) = \frac{P(\text{sock} \cap D_1)}{P(\text{sock})} = \frac{1/2 \times 1/2}{5/12} = \frac{3}{5}$$

2. Suppose it is known that 1% of the population suffers from a particular disease (D). A blood test has a 97% chance of identifying the disease for diseased individuals, but also has a 6% chance of falsely indicating that a healthy person has the disease.
To summary:

$$P(D) = 0.01 \qquad P(+|D) = 0.97 \qquad P(+|D') = 0.06$$

(a) What is the probability that a person will have a positive blood test?

$$P(+) = P(+|D)P(D) + P(+|D')P(D') = 0.01 \times 0.97 + 0.99 \times 0.06 = 0.0691$$

(b) If your blood test is positive, what is the chance that you have the disease?

$$P(D|+) = \frac{P(+|D)P(D)}{P(+)} = \frac{0.01 \times 0.97}{0.0691} = \frac{97}{691} \approx 0.1404$$

(c) If your blood test is negative, what is the chance that you do not have the disease?

$$P(-|D') = 1 - 0.06 = 0.94 \qquad P(-) = 0.01 \times 0.03 + 0.99 \times 0.94 = 0.9309$$

Hence

$$P(D'|-) = \frac{0.99 \times 0.94}{0.9309} = \frac{0.9306}{0.9309} = \frac{9306}{9309} \approx 0.99968$$

3. The weather on a particular day is classified as either cold, warm or hot. There is a probability of 0.15 that it is cold and a probability of 0.25 that it is warm. In addition, on each day it may either rain or not rain. On cold days there is a probability of 0.3 that it will rain, on warm days there is a probability of 0.4 that it will rain, and on hot days there is a probability of 0.5 that it will rain. If it is not raining on a particular day, what is the probability that it is cold?

$$P(\text{no rain}) = 0.15 \times (1-0.3) + 0.25 \times (1-0.4) + 0.60 \times (1-0.5)$$
$$= 0.15 \times 0.7 + 0.25 \times 0.6 + 0.60 \times 0.5 = 0.105 + 0.15 + 0.30 = 0.555$$

Hence,

$$P(\text{cold}|\text{no rain}) = \frac{P(\text{no rain}|\text{cold})P(\text{cold})}{P(\text{no rain})} = \frac{0.7 \times 0.15}{0.555} = \frac{0.105}{0.555} = \frac{7}{37} \approx 0.18919.$$

4. You have two coins, a fair one with probability of heads $\frac{1}{2}$ and an unfair one with probability of heads $\frac{1}{3}$, but otherwise identical. A coin is selected at random and tossed, falling heads up. How likely is it that it is the fair one?

$$P(\text{fair}|\text{head}) = \frac{P(\text{head}|\text{fair})P(\text{fair})}{P(\text{head}|\text{fair})P(\text{fair}) + P(\text{head}|\text{unfair})P(\text{unfair})} = \frac{1/2 \times 1/2}{1/2 \times 1/2 + 1/3 \times 1/2} = \frac{3}{5}$$

5. Are the following statements true or false?

   (a) If a fair coin is tossed three times, the probability of obtaining two heads and one tail is the same as the probability of obtaining one head and two tails.
   True. Each outcome has probability 1/8. Number of outcome with two heads $= \binom{3}{2} = 3$, for one head $= \binom{3}{1} = 3$. Thus both probabilities are 3/8.

   (b) If a card is drawn at random from a deck of cards, the probability that it is a heart increases if it is conditioned on the knowledge that it is an ace.
   False $P(\text{heart}) = 1/4$. $P(\text{heart}|\text{ace}) = \frac{P(\text{heart}\cap\text{ace})}{P(\text{ace})} = \frac{1/52}{4/52} = \frac{1}{4}$.

   (c) If two events are independent, then the probability that they both occur can be calculated by multiplying their individual probabilities.
   True. This is the definition of independence: $P(A \cap B) = P(A)P(B)$.

   (d) It is always true that $P(A|B) + P(A'|B) = 1$.
   True. $P(A|B) + P(A'|B) = P((A \cup A')|B) = P(\Omega|B) = 1$.

   (e) It is always true that $P(A|B) + P(A|B') = 1$.
   False. If A never occurs and $B \neq \emptyset$ and $B \neq \Omega$ then $P(A|B) + P(A|B') = 0$.

   (f) It is always true that $P(A|B) \leq P(A)$.
   False. If B is A and $0 < P(A) < 1$ then $P(A|B) = P(A|A) = 1 > P(A)$

Done by Huynh Le An Khanh on November 6, 2025.