Huynh Le An Khanh
Vietnamese-German University
Binh Duong, Vietnam

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Tutorial 8 - Mathematics for CS

1. Consider a random variable measuring the following quantities. In each case, state with reasons whether you think it is more appropriate to define the random variable as discrete or as continuous.

   (a) A person's height: Discrete. It is possible to measuring a person's height continuously, however, a person's height continuous changes, and hence measurements with units smaller than year quickly becomes meaningless.

   (b) A student's course grade: Discrete. It is impossible grade a student's ability with precision.

   (c) The thickness of a metal plate: Continuous. Measuring with continuity gives a better picture for how thickness changes overtime.

   (d) The purity of a chemical solution: Continuous. Measuring with continuity gives a better picture for how the purity changes overtime.

   (e) The type of personal computer a person owns: Discrete. There is a finite numbers of computer types.

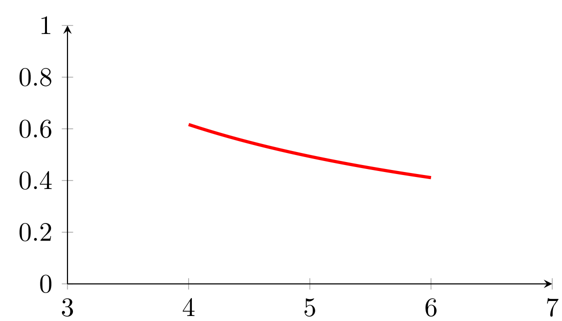
   (f) A person's age: Discrete. It is possible to measuring a person's age continuously, however, one's age continuously changes, and hence measurements with units smaller than year quickly becomes meaningless.

2. A random variable X takes values between 4 and 6 with a probability density function

$$f(x) = \frac{1}{x \ln(1.50)}$$

   for $4 \leq x \leq 6$ and $f(x) = 0$ elsewhere.

   (a) Make a sketch of the probability density function.



   (b) Check that the total area under the probability density function is equal to 1.

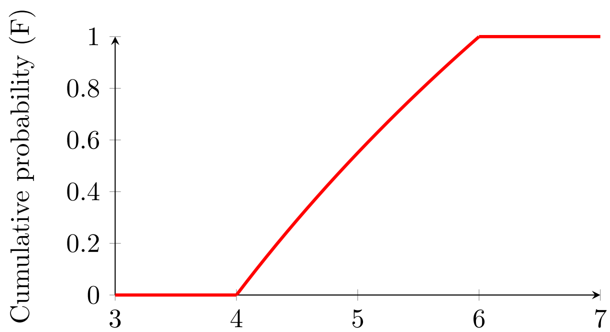
$$\int_{-\infty}^{\infty} f(x)\, dx = \int_{4}^{6} \frac{1}{x \ln(1.5)}\, dx = \left. \frac{\ln(x)}{\ln(1.5)} \right|_{x=4}^{x=6} = 1$$

(c) What is $P(4.5 \le X \le 5.5)$?

$$\int_{4.5}^{5.5} \frac{1}{x\ln(1.5)}\,dx = \left.\frac{\ln(x)}{\ln(1.5)}\right|_{x=4.5}^{x=5.5} = \ln(5.5/4.5)/\ln(1.5) \approx 0.4949148310$$

(d) Construct and sketch the cumulative distribution function.
For $x < 4, F(x) = 0$. For $x > 6, F(x) = 1$.
For $4 \le x \le 6$,

$$F(x) = \int_4^x \frac{1}{t\ln(1.5)}\,dt = \left.\frac{\ln(t)}{\ln(1.5)}\right|_{t=4}^{t=x} = \frac{\ln(x)-\ln(4)}{\ln(1.5)}$$



(e) What is the expected value of this random variable?

$$E(X) = \int_4^6 \frac{x}{x\ln(1.5)}\,dx = \left.\frac{x}{\ln(1.5)}\right|_{x=4}^{x=6} = \frac{2}{\ln(1.5)} \approx 4.93260692475$$

(f) What is the median of this random variable?

$$F(x) = 0.5 \Leftrightarrow \frac{\ln(x)-\ln(4)}{\ln(1.5)} = 0.5 \Leftrightarrow \ln(x) = 0.5\ln(1.5)+\ln(4) \Leftrightarrow x = 4\sqrt{1.5} = 2\sqrt{6}$$

(g) What is the variance and the standard deviation of this random variable?

$$\begin{aligned}\mathrm{Var}(X) &= \int_4^6 \frac{x^2}{x\ln(1.5)}\,dx - E(X)^2 = \int_4^6 \frac{x}{\ln(1.5)}\,dx - E(X)^2 \\ &= \left.\frac{x^2}{2\ln(1.5)}\right|_{x=4}^{x=6} - E(X)^2 = \frac{10}{\ln(1.5)} - E(X)^2 \approx 0.332423549644\end{aligned}$$

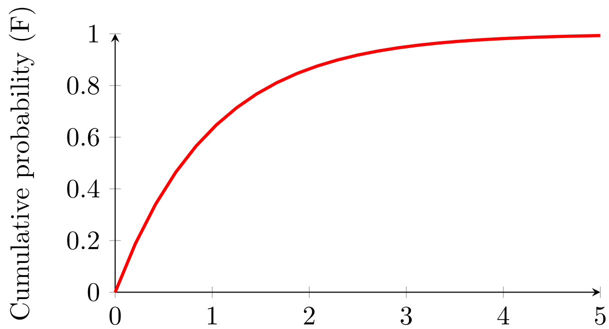
$$\sigma = \sqrt{\mathrm{Var}(X)} \approx 0.576561835057$$

3. A random variable X takes values between 0 and ∞ with a cumulative distribution function

$$F(x) = A + Be^{-x}$$

for $0 \le x < \infty$.

(a) Find the values of A and B and sketch the cumulative distribution function.
Since $1 = \lim_{x\to\infty} F(x)$,$1 = \lim_{x\to\infty} A + Be^{-x} = A$.
Hence $A = 1$.
Since X takes values between 0 and ∞, $F(0) = 0$ (X does not takes 0).
Hence $F(0) = A + Be^{-x} = A + B = 0 \Rightarrow B = -1$.
Therefore, $F(x) = 1 - e^{-x}$

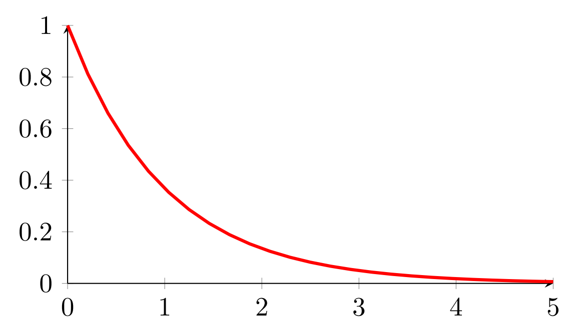


(b) What is $P(2 \le X \le 3)$?

$$P(2 \le X \le 3) = F(3) - F(2) = e^{-2} - e^{-3} \approx 0.0855482149$$

(c) Construct and sketch the probability density function.
For $0 < x < \infty$,

$$f(x) = \frac{d}{dx}\left(1 - e^{-x}\right) = e^{-x}$$



Done by Huynh Le An Khanh on November 6, 2025.