Introduction to Probability

Tutorial 13

1. Suppose that $X \sim U(-3, 8)$. Find:

   (a) $E(X)$

   (b) The standard deviation of X

   (c) $P(0 \leq X \leq 4)$

2. A new battery supposedly with a charge of 1.5 volts actually has a voltage with a uniform distribution between 1.43 and 1.6 volts.

   (a) What is the expectation of the voltage?

   (b) What is the standard deviation of the voltage?

   (c) What is the cumulative distribution function of the voltage?

   (d) What is the probability that a battery has a voltage less than 1.48 volts?

   (e) If a box contains 50 batteries, what are the expectation and variance of the number of batteries in the box with a voltage less than 1.5 volts?

3. Use integration by parts to show that if X has an exponential distribution with parameter λ, then

   (a) $E(X) = \frac{1}{\lambda}$ (b) $E(X^2) = \frac{2}{\lambda^2}$

4. Suppose that you are waiting for a friend to call you and that the time you wait in minutes has an exponential distribution with parameter $\lambda = 0.1$.

   (a) What is the expectation of your waiting time?

   (b) What is the probability that you will wait longer than 10 minutes?

   (c) What is the probability that you will wait less than 5 minutes?

   (d) Suppose that after 5 minutes, you are still waiting for the call. What is the distribution of you *additional* waiting time?
   In this case, what is the probability that your total waiting time is longer than 15 minutes?

   (e) Suppose now that the time you wait in minutes for the call has $U(0, 20)$ distribution. What is the expectation of your waiting time? If after 5 minutes you are still waiting for the call, what is the distribution of your *additional* waiting time?