Introduction to Probability

Tutorial 12

1. Suppose that $X \sim B(10, 0.12)$. Calculate

   (a) $P(X = 3)$ (b) $P(X = 6)$ (c) $P(X \leq 2)$

   (d) $P(X \geq 7)$ (e) $E(X)$ (f) $\text{Var}(X)$

2. Draw line graphs of the probability mass functions of a $B(6, 0.5)$ distribution and a $B(6, 0.7)$ distribution. Mark the expected values of the distributions on the line graphs and calculate the standard deviations of the two distributions.

3. A fair die is rolled 8 times. Calculate the probability that there are:

   (a) Exactly 5 even numbers

   (b) Exactly one 6

   (c) No 4s

4. Consider 2 independent binomial random variables $X_1 \sim B(n_1, p)$ and $X_2 \sim B(n_2, p)$. If $Y = X_1 + X_2$, explain why $Y \sim B(n_1 + n_2, p)$.

5. If X has a geometric distribution with parameter $p = 0.7$, calculate

   (a) $P(X = 4)$ (b) $P(X = 1)$ (c) $P(X \leq 5)$ (d) $P(X \geq 8)$

6. Suppose that $X_1, \ldots, X_r$ are independent random variables, each with a geometric distribution with parameter p. Explain why

   $$Y = X_1 + \ldots + X_r$$

   has a negative binomial distribution with parameters p and r. Use this relationship to establish the mean and variance of a negative binomial distribution.

7. If X has a negative binomial distribution with parameters $p = 0.6$ and $r = 3$, calculate:

   (a) $P(X = 5)$

   (b) $P(X = 8)$

   (c) $P(X \leq 7)$

   (d) $P(X \geq 7)$

8. An archer hits a bull's-eye with a probability of 0.09 and the results of different attempts can be taken to be independent of each other.

   (a) If the archer shoots a series of arrows, what is the probability that the first bull's-eye is scored with the fourth arrow?

   (b) What is the probability that the third bull's-eye is scored with the tenth arrow?

   (c) What is the expected number of arrows shot before the first bull's-eye is scored?

   (d) What is the expected number of arrows shot before the third bull's-eye is scored?