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Tutorial 14 - Mathematics for CS

1. Suppose $Z \sim N(0,1)$. Find

   (a) $P(Z \leq 1.34) = \Phi(1.34) \approx 0.909877327536$

   (b) $P(Z \geq -0.22) = 1 - \Phi(-0.22) \approx 0.587064422648$

   (c) $P(-2.19 \leq Z \leq 0.43) = \Phi(0.43) - \Phi(-2.19) \approx 0.652140060994$

   (d) $P(0.09 \leq Z \leq 1.76) = \Phi(1.76) - \Phi(0.09) \approx 0.424939704127$

   (e) $P(|Z| \leq 0.38) = \Phi(0.38) - \Phi(-0.38) \approx 0.296054584848$

   (f) $P(|Z| \geq 0.91) = \Phi(-0.91) + 1 - \Phi(0.91) \approx 0.362822509784$

   (g) The value of x for which $P(Z \leq x) = 0.55 \Rightarrow x \approx 0.125661346855$

   (h) The value of x for which $P(Z \geq x) = 0.72 \Rightarrow x = -0.582841507271$

   (i) The value of x for which $P(|Z| \leq x) = 0.31$ Since $P(|X| \leq x) > 0 \Rightarrow x \geq 0$

   $$\Rightarrow 0.31 = P(|X| \leq x) = \Phi(x) - \Phi(-x) = 1 - 2\Phi(-x) \Rightarrow \Phi(-x) = 0.345 \Rightarrow x = 0.398855$$

   (j) The value of x for which $P(|Z| \geq x) = 0.42$ Since $P(|X| \geq x) < 1 \Rightarrow x \geq 0$

   $$\Rightarrow 0.21 = P(|Z| \geq x) = \Phi(-x) + 1 - \Phi(x) = 2\Phi(-x) \Rightarrow \Phi(-x) = 0.21 \Rightarrow x = 0.8064212$$

2. Suppose $X \sim N(10,2)$. Find

   (a) $P(X \leq 10.34) = \Phi(\frac{10.34-10}{2}) \approx 0.567494931675$

   (b) $P(X \geq 11.98) = 1 - \Phi(\frac{11.98-10}{2}) \approx 0.161087059511$

   (c) $P(7.67 \leq X \leq 9.90) = \Phi(\frac{9.90-10}{2}) - \Phi(\frac{7.67-10}{2}) \approx 0.358051697742$

   (d) $P(10.88 \leq X \leq 13.22) = \Phi(\frac{13.22-10}{2}) - \Phi(\frac{10.88-10}{2}) \approx 0.276269625512$

   (e) $P(|X-10| \leq 3) = P(7 \leq X \leq 13) = \Phi(\frac{13-10}{2}) - \Phi(\frac{7-10}{2}) \approx 0.866385597462$

   (f) $P(|X-10| \geq 8) = \Phi(\frac{18-10}{2}) - \Phi(\frac{2-10}{2}) \approx 0.999936657516$

   (g) The value of x for which $P(X \leq x) = 0.81$

   $$\Rightarrow x = 10 + 2\Phi^{-1}(0.81) \approx 11.7557925901$$

   (h) The value of x for which $P(X \geq x) = 0.04$

   $$\Rightarrow x = 10 + 2\Phi^{-1}(1 - 0.04) \approx 13.5013721425$$

   (i) The value of x for which $P(|X-10| \geq x) = 0.63$ Since $P(|X-10| \geq x) < 1 \Rightarrow x \geq 0$

   $$P(|X-10| \geq x) = 1 - P(X-10 \leq x) + P(X-10 \leq -x) = 1 - \Phi(x/2) + \Phi(-x/2) = 2\Phi(-x/2)$$

   $$\Rightarrow \Phi(-x/2) = 0.315 \Rightarrow x = -2\Phi^{-1}(0.315) \approx 0.963453699169$$

(j) The value of x for which $P(|X-10| \leq x) = 0.44$ Since $P(|X-10| \leq x) > 0 \Rightarrow x \geq 0$

$$P(|X-10| \leq x) = P(X-10 \leq x) - P(X-10 \leq -x) = \Phi(x/2) - \Phi(-x/2) = 2\Phi(x/2) - 1$$

$$\Rightarrow 2\Phi(x/2) - 1 = 0.44 \Rightarrow x = 2\Phi^{-1}(1+0.44) \approx 1.16568301454$$

3. Suppose that $X \sim N(\mu, \sigma^2)$ and that

$$P(X \leq 5) = 0.8 \text{ and } P(X \geq 0) = 0.6.$$

What are the values of μ and σ^2?

$$P(X \leq 0) = 0.4 \Rightarrow \frac{0-\mu}{\sigma} = \Phi^{-1}(0.4) \qquad \Rightarrow \mu + \Phi^{-1}(0.4)\sigma = 0$$
$$P(X \leq 5) = 0.8 \Rightarrow \frac{5-\mu}{\sigma} = \Phi^{-1}(0.8) \qquad \Rightarrow \mu + \Phi^{-1}(0.8)\sigma = 5$$

Solving the equation gives $\mu \approx 1.15687, \sigma \approx 4.56634$
Therfore $\sigma^2 = 20.88$

4. The thicknesses of glass sheets produced by a certain process are normally distributed with a mean of $\mu = 3$ mm and a standard deviation of $\sigma = 0.12$ mm.

(a) What is the probability that a glass sheet is thicker than 3.2 mm?

$$P(X > 3.2) = 1 - P(X < 3.2) = 1 - \Phi(\frac{3.2-3}{0.12}) \approx 0.0477903522728$$

(b) What is the probability that a glass sheet is thinner than 2.7 mm?

$$P(X < 2.7) = \Phi(\frac{2.7-3}{0.12}) \approx 0.00620966532578$$

(c) What is the value of c for which there is a 99% probability that a glass sheet has a thickness within the interval $[3-c, 3+c]$?

$$P(3-c \leq X \leq 3+c) = 1 - 2P(X \leq 3-c) = 0.99 \Rightarrow P(X \leq 3-c) = 0.005$$
$$\Rightarrow c = -0.12\Phi^{-1}(0.005) \approx 0.309099516426$$

(d) What is the probability that three glass sheets placed one on top of another have a total thickness greater than 9.5 mm?

$$\bar{X} \sim N(3, \frac{0.12^2}{3}) \Rightarrow P(3\bar{X} > 9.5) = 1 - P(\bar{X} < 9.5/3)$$
$$= 1 - \Phi(\frac{9.5/3-3}{\sqrt{0.12^2/3}}) \approx 0.00807238904718$$

(e) What is the probability that seven glass sheets have an average thickness less than 3.1 mm?

$$\bar{X} \sim N(3, \frac{0.12^2}{7}) \Rightarrow P(\bar{X} < 3.1) = \Phi(\frac{3.1-3}{\sqrt{0.12^2/7}}) \approx 0.986265679807$$

5. The weights of bags filled by a machine are normally distributed with a standard deviation of 0.05 kg and a mean that can be set by the operator. At what level should the mean be set if it is required that only 1% of the bags weigh less than 10 kg?

$$P(X < 10) = 0.01 \Rightarrow \frac{10 - \mu}{0.05} = \Phi(0.01) \Rightarrow \mu = 10 - 0.05\Phi^{-1}(0.01) = 10.1163173937$$

6. An investment in company A has an expected return of \$30,000 with a standard deviation of \$4000. What is the probability that the return will be at least \$25,000 if it has a normal distribution?

$$P(X \geq 25000) = 1 - \Phi(\frac{25000 - 30000}{4000}) \approx 0.894350226333$$

Done by Huynh Le An Khanh on November 11, 2025.