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Tutorial 13 - Mathematics for CS

1. Suppose that $X \sim U(-3, 8)$. Find:

   (a) $E(X)$

   $$E(X) = \frac{a+b}{2} = \frac{-3+8}{2} = \frac{5}{2} = 2.5$$

   (b) The standard deviation of X

   $$\operatorname{Var}(X) = \frac{(b-a)^2}{12} = \frac{(8-(-3))^2}{12} = \frac{121}{12} = 10.0833$$

   $$\sigma(X) = \sqrt{\operatorname{Var}(X)} \approx 3.17542648$$

   (c) $P(0 \leq X \leq 4)$

   $$P(0 \leq X \leq 4) = \frac{4-0}{8-(-3)} = \frac{4}{11}$$

2. A new battery supposedly with a charge of 1.5 volts actually has a voltage with a uniform distribution between 1.43 and 1.6 volts.

   (a) What is the expectation of the voltage?

   $$E(X) = \frac{a+b}{2} = \frac{1.43+1.6}{2} = 1.515$$

   (b) What is the standard deviation of the voltage?

   $$\operatorname{Var}(X) = \frac{(b-a)^2}{12} = \frac{(1.6-1.43)^2}{12} = \frac{0.17^2}{12} = 0.00240833$$

   $$\sigma(X) = \sqrt{\operatorname{Var}} \approx 0.04907477$$

   (c) What is the cumulative distribution function of the voltage?

   $$F(x) = \begin{cases} 0 & x < 1.43 \\ \dfrac{x-1.43}{0.17} & 1.43 \leq x \leq 1.6 \\ 1 & x > 1.6 \end{cases}$$

   (d) What is the probability that a battery has a voltage less than 1.48 volts?

   $$P(X < 1.48) = F(1.48) = \frac{1.48-1.43}{0.17} = 5/17$$

(e) If a box contains 50 batteries, what are the expectation and variance of the number of batteries in the box with a voltage less than 1.5 volts?
Probability of that the voltage of a battery is less than 1.5 $p = P(X < 1.5) = \frac{1.5-1.43}{0.17} = \frac{0.07}{0.17} = 7/17$
Let N be the number of such batteries. Then $N \sim \mathrm{B}(50, 7/17)$

$$E(N) = np = 50(7/17) = 20.588235294$$

$$\mathrm{Var}(N) = np(1-p) = 50(7/17)(10/17) \approx 12.1107266436$$

3. Use integration by parts to show that if X has an exponential distribution with parameter λ, then

   (a) $E(X) = \frac{1}{\lambda}$
   Since $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$,

$$E(X) = \int_0^\infty x\lambda e^{-\lambda x}\, dx = -xe^{-\lambda x}\Big|_0^\infty - \int_0^\infty -e^{-\lambda x}\, dx = 0 - \left.\frac{e^{-\lambda x}}{\lambda}\right|_0^\infty = \frac{1}{\lambda} = \frac{1}{\lambda}$$

   (b) $E(X^2) = \frac{2}{\lambda^2}$

$$E(X^2) = \int_0^\infty x^2\lambda e^{-\lambda x}\, dx = -x^2e^{-\lambda x}\Big|_0^\infty + 2\int_0^\infty xe^{-\lambda x}\, dx = \frac{2E(X)}{\lambda} = \frac{2}{\lambda^2}$$

4. Suppose that you are waiting for a friend to call you and that the time you wait in minutes has an exponential distribution with parameter $\lambda = 0.1$.

   (a) What is the expectation of your waiting time?

$$E(X) = \frac{1}{\lambda} = \frac{1}{0.1} = 10$$

   (b) What is the probability that you will wait longer than 10 minutes?

$$P(X > 10) = 1 - (1 - e^{-10\lambda}) = e^{-1} = 0.367879$$

   (c) What is the probability that you will wait less than 5 minutes?

$$P(X < 5) = 1 - e^{-5\lambda} = 1 - e^{-0.5} = 1 - 0.6065 = 0.3935$$

   (d) Suppose that after 5 minutes, you are still waiting for the call. What is the distribution of your additional waiting time?

$$P(X > 15 | X > 5) = \frac{P(X > 15)}{P(X > 5)} = \frac{e^{-15\lambda}}{e^{-5\lambda}} = e^{-10\lambda} = 0.367879$$

   (e) Suppose now that the time you wait in minutes for the call has $U(0, 20)$ distribution. What is the expectation of your waiting time? If after 5 minutes you are still waiting for the call, what is the distribution of your additional waiting time?

$$E(X) = \frac{0+20}{2} = 10$$

   The conditional distribution of additional waiting time is $Y = X - 5$ for $5 \leq X \leq 20$ or $Y \sim U(0, 15)$.

Done by Huynh Le An Khanh on November 11, 2025.