Introduction to Probability

Tutorial 8

1. Consider a random variable measuring the following quantities. In each case, state with reasons whether you think it is more appropriate to define the random variable as discrete or as continuous.

   (a) A person's height

   (b) A student's course grade

   (c) The thickness of a metal plate

   (d) The purity of a chemical solution

   (e) The type of personal computer a person owns

   (f) A person's age

2. A random variable X takes values between 4 and 6 with a probability density function

$$f(x) = \frac{1}{x\ \ln(1.5)}$$

   for $4 \leq x \leq 6$ and $f(x) = 0$ elsewhere.

   (a) Make a sketch of the probability density function.

   (b) Check that the total area under the probability density function is equal to 1.

   (c) What is $P(4.5 \leq X \leq 5.5)$?

   (d) Construct and sketch the cumulative distribution function.

   (e) What is the expected value of this random variable?

   (f) What is the median of this random variable?

   (g) What is the variance and the standard deviation of this random variable?

3. A random variable X takes values between 0 and ∞ with a cumulative distribution function

$$F(x) = A + Be^{-x}$$

   for $0 \leq x < \infty$.

   (a) Find the values of A and B and sketch the cumulative distribution function.

   (b) What is $P(2 \leq X \leq 3)$?

   (c) Construct and sketch the probability density function.

4. (Just for fun) Sometimes a random variable is a mix of discrete and continuous components. For example, suppose that the dial-spinning game is modified in the following way. First a fair coin is tossed and if a head is obtained, the player wins \$500 and the dial is not spun. However, if a tail is obtained, the player spins the dial and receives winnings of $\$1000 \times \frac{\theta}{180}$ as before. In this game there is a probability of 0.5 of winning \$500, with all the other possible winnings between \$0 and \$1000 being equally likely. The coin toss provides a discrete element to the winnings, and the dial spin provides a continuous element. The best way to describe the probabilistic properties of mixed random variables such as this is through a cumulative distribution function. The cumulative distribution function of the winnings from this game is given by the figure below.

(a) What is the probability of winning less than \$200?

(b) What is the probability of winning between \$400 and \$700?