Huynh Le An Khanh
Vietnamese-German University
Binh Duong, Vietnam

October 30, 2025

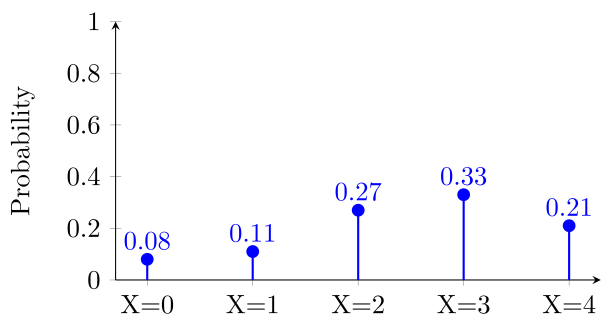
Tutorial 7 - Mathematics for CS

1. An office has 4 copying machine, and the random variable X measures how many of them are in use at a particular moment in time. Suppose that $P(X = 0) = 0.08$, $P(X = 1) = 0.11$, $P(X = 2) = 0.27$ and $P(X = 3) = 0.33$.

   (a) What is $P(X = 4)$?

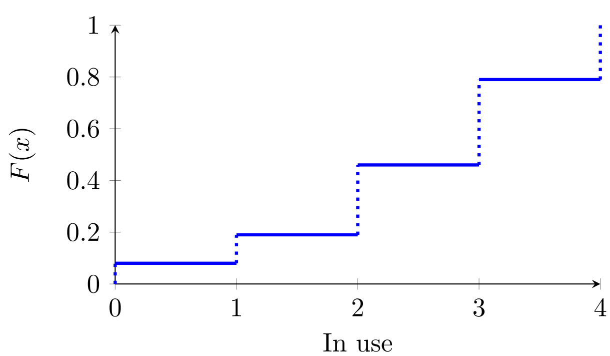
$$\begin{aligned} P(X = 4) = 1 - P(X = 3) - P(X = 2) - P(X = 1) - P(X = 0) \\ = 1 - 0.33 - 0.27 - 0.11 - 0.08 = 0.21 \end{aligned}$$

   (b) Draw a line graph of the probability mass function.



   (c) Construct and plot the cumulative distribution function.

The cumulative distribution function: $F(x) = \begin{cases} 0.08 & 0 \leq x < 1 \\ 0.19 & 1 \leq x < 2 \\ 0.46 & 2 \leq x < 3 \\ 0.79 & 3 \leq x < 4 \\ 1 & x = 4 \end{cases}$ Plot of the cumulative distribution function:



(d) What is the expected number of copying machines at a particular moment in time?

$$\begin{aligned} E(X) &= 0 + P(X=1) + 2P(X=2) + 3P(X=3) + 4P(X=4) \\ &= 0.11 + 2\times 0.27 + 3\times 0.33 + 4\times 0.21 = 2.48(\text{copying machine}) \end{aligned}$$

(e) Calculate the variance and standard deviation of the number of copying machines in use at a particular moment.

$$\text{Var}(X) = \sum_{i=0}^{4} x(x-E(x))^2$$
$$= 0.08(0-2.48)^2 + 0.11(1-2.48)^2 + 0.27(2-2.48)^2 + 0.33(3-2.48)^2 + 0.21(4-2.48)^2 = 1.3696$$

$$\sigma = \sqrt{1.3696} \approx 1.1702991071$$

2. fair coin is tossed 3 times. A player wins \$1 if the first toss is a head, but loses \$1 if the first toss is a tail. Similarly, the player wins \$2 if the second toss is a head, but loses \$2 if the second toss is a tail, and wins or loses \$3 according to the result of the third toss.
Let the random variable X be the total winnings after the 3 tosses (possibly a negative value if losses are incurred).

(a) Construct the probability mass function.
Let (x, y, z) be the result of the throws where x, y, z are 1 for win, and 0 for lost of the 1st, 2nd, 3rd throw respectively.
Let $W(x, y, z)$ be the winnings (\$) of result (x, y, z).
Calculate W:

$$\begin{array}{cccc} W(0,0,0) = -6 & W(1,0,0) = -4 & W(0,1,0) = -2 & W(1,1,0) = 0 \\ W(1,1,1) = 6 & W(0,1,1) = 4 & W(1,0,1) = 2 & W(0,0,1) = 0 \end{array}$$

Since the events are equally likely,

$$P(x) = \begin{cases} 1/8 & x = -6 \\ 1/8 & x = -4 \\ 1/8 & x = -2 \\ 1/4 & x = 0 \\ 1/8 & x = 2 \\ 1/8 & x = 4 \\ 1/8 & x = 6 \end{cases}$$

(b) Construct the cumulative distribution function.

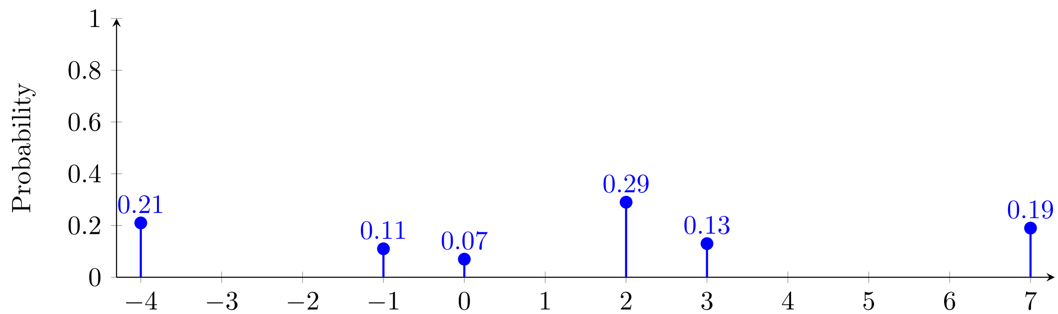
$$F(x) = \begin{cases} 1/8 & -6 \le x < -4 \\ 1/4 & -4 \le x < -2 \\ 3/8 & -2 \le x < 0 \\ 5/8 & 0 \le x < 2 \\ 3/4 & 2 \le x < 4 \\ 7/8 & 4 \le x < 6 \\ 1 & x = 6 \end{cases}$$

(c) What is the most likely value of the random variable X? The most likely value of X is 0.

3. The figure presents the cumulative distribution function of a random variable. Make a table and line graph of its probability mass function.
Probability mass function:

	P(X)
-4	0.21
-1	0.11
0	0.07
2	0.29
3	0.13
7	0.19
otherwise	0

Line graph of the probability mass function:



Done by Huynh Le An Khanh on October 30, 2025.