

### Tutorial 1 - Mathematics for CS

1. (a) What is the sample space when a coin is tossed 3 times?

$$\mathcal{S} = \{(x, y, z) | x, y, z \in \{\text{head}, \text{tail}\}\}$$

- (b) What is the sample space for counting the number of females in a group of  $n$  people?

$$\mathcal{S} = \{x | 0 \leq x \leq n\}$$

- (c) What is the sample space for the number of aces in a hand of 13 playing cards?

$$\mathcal{S} = \{x | 0 \leq x \leq 4\}$$

- (d) What is the sample space for a person's birthday?

$$\mathcal{S} = \begin{array}{ccc} \text{Jan 1st}, & \dots, & \text{Jan 31st} \\ \text{Feb 1st}, & \dots, & \text{Feb 29th} \\ \text{Mar 1st}, & \dots, & \text{Mar 31st} \\ \vdots & \vdots & \vdots \\ \text{Dec 1st}, & \dots, & \text{Dec 31st} \end{array}$$

- (e) A car repair is performed either on time or late and either satisfactorily or unsatisfactorily. What is the sample space for a car repair?

$$\mathcal{S} = \{(x, y) | x \in \{\text{on time}, \text{late}\}, y \in \{\text{satisfactorily}, \text{unsatisfactorily}\}\}$$

- (f) A bag contains balls that are either red or blue and either dull or shiny. What is the sample space when a ball is chosen from the bag?

$$\mathcal{S} = \{(x, y) | x \in \{\text{red}, \text{blue}\}, y \in \{\text{dull}, \text{shiny}\}\}$$

2. A probability value  $p$  is often reported as an odds ratio, which is  $\frac{p}{1-p}$ . This is the ratio of the probability that the event happens to the probability that an event does not happen.

- (a) If the odds ratio is 1, what is  $p$ ?

$$\frac{p}{1-p} = 1 \Leftrightarrow p = 1-p \Leftrightarrow p = 0.5$$

- (b) If the odds ratio is 2, what is  $p$ ?

$$\frac{p}{1-p} = 2 \Leftrightarrow p = 2-2p \Leftrightarrow p = 2/3$$

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(c) If  $p = 0.25$ , what is the odds ratio?

$$\frac{p}{1-p} = \frac{0.25}{1-0.25} = 1/3$$

(d) What are the possible values for the odds ratio?

Since as  $p \rightarrow 1^-$  :

$$\lim_{p \rightarrow 1^-} \frac{p}{1-p} = +\infty$$

and with  $p = 0$  :  $\frac{p}{1-p} = 0$

and  $\frac{p}{1-p}$  is continuous on  $(0, 1)$

Hence  $\frac{p}{1-p}$  takes all value in  $[0, \infty]$

3. (a) An experiment has 5 outcomes: I, II, III, IV and V. If  $P(I) = 0.13, P(II) = 0.24, P(III) = 0.07$  and  $P(IV) = 0.38$ , what is  $P(V)$ ?

$$P(V) = 1 - P(I) - P(II) - P(III) - P(IV) = 1 - 0.13 - 0.24 - 0.07 - 0.38 = 0.18$$

- (b) An experiment has 5 outcomes: I, II, III, IV and V. If  $P(I) = 0.08, P(II) = 0.2, P(III) = 0.33$ , what are the possible values for the probability of outcome V? If outcomes IV and V are equally likely, what are their probabilities values?

$$2P(V) = P(V) + P(IV) = 1 - P(III) - P(II) - P(I) = 1 - 0.08 - 0.2 - 0.33 = 0.39 \Rightarrow P(V) = 0.195$$

4. An experiment has 3 outcomes: I, II and III. If outcome I is twice likely as outcome II and outcome II is 3 times as likely as outcome III, what are the probability values of the 3 outcomes?

$$\begin{cases} P(I) = 2P(II) \\ P(II) = 3P(III) \\ P(I) + P(II) + P(III) = 1 \end{cases} \Rightarrow \begin{cases} P(I) = 0.6 \\ P(II) = 0.3 \\ P(III) = 0.1 \end{cases}$$

5. A company's advertising expenditure is either low with probability 0.28, average with probability 0.55, or high with probability  $p$ . What is  $p$ ?

$$p = 1 - P(\text{low}) - P(\text{average}) = 1 - 0.28 - 0.55 = 0.17$$

*Submitted by Huynh Le An Khanh on September 18, 2025.*