Huynh Le An Khanh
Vietnamese-German University
Binh Duong, Vietnam

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Tutorial 12 - Mathematics for CS

1. Suppose that $X \sim B(10, 0.12)$. Calculate

   (a) $P(X = 3)$

   $$P(X = 3) = \binom{10}{3}(0.12)^3(0.88)^7 = 120(0.001728)(0.39304) \approx 0.0815$$

   (b) $P(X = 6)$

   $$P(X = 6) = \binom{10}{6}(0.12)^6(0.88)^4 = 210(2.985984 \times 10^{-6})(0.59969536) \approx 0.00038$$

   (c) $P(X \leq 2)$

   $$\begin{aligned} P(X \leq 2) &= \sum_{x=0}^{2} \binom{10}{x}(0.12)^x(0.88)^{10-x} \\ &= \binom{10}{0}(0.88)^{10} + \binom{10}{1}(0.12)(0.88)^9 + \binom{10}{2}(0.12)^2(0.88)^8 \approx 0.891318206278 \end{aligned}$$

   (d) $P(X \geq 7)$

   $$P(X \geq 7) = 1 - P(X \leq 6) = 1 - \sum_{x=0}^{6} \binom{10}{x}(0.12)^x(0.88)^{10-x} \approx 0.00003084647$$

   (e) $E(X)$

   $$E(X) = np = 10(0.12) = 1.2$$

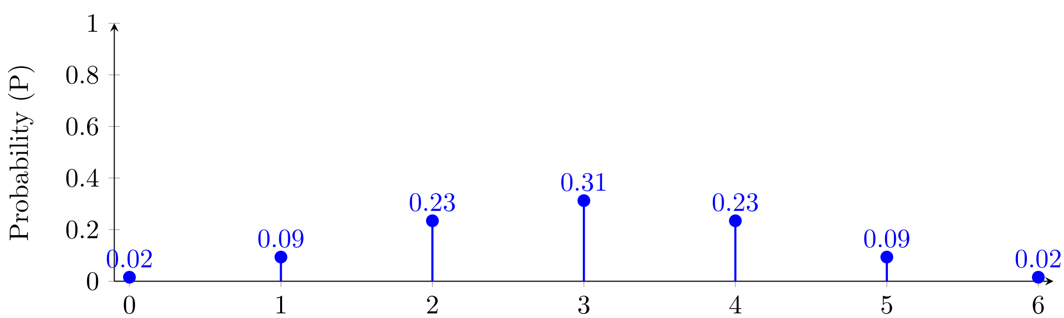
   (f) $\mathrm{Var}(X)$

   $$\mathrm{Var}(X) = np(1-p) = 10(0.12)(0.88) = 1.056$$

2. Draw line graphs of the probability mass functions of a $B(6, 0.5)$ distribution and a $B(6, 0.7)$ distribution. Mark the expected values of the distributions on the line graphs and calculate the standard deviations of the two distributions.

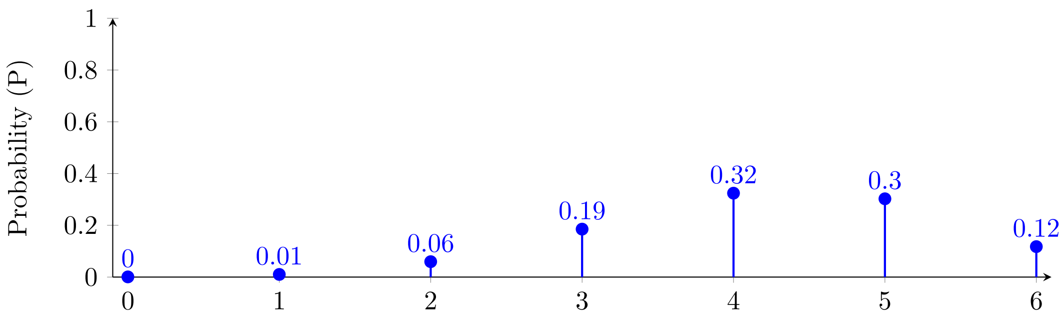
   For $B(6, 0.5)$:

   $$E(X) = 6(0.5) = 3, \quad \sigma = \sqrt{6(0.5)(0.5)} = \sqrt{1.5} \approx 1.225$$



For $B(6, 0.7)$:

$$E(X) = 6(0.7) = 4.2, \quad \sigma = \sqrt{6(0.7)(0.3)} = \sqrt{1.26} \approx 1.122$$



3. A fair die is rolled 8 times. Calculate the probability that there are:

   (a) Exactly 5 even numbers

   $$P(X = 5) = \binom{8}{5}(0.5)^5(0.5)^3 = \binom{8}{5}(0.5)^8 = 56(0.00390625) = 0.21875$$

   (b) Exactly one 6

   $$P(X = 1) = \binom{8}{1}(1/6)(5/6)^7 = 8(1/6)(0.27908) \approx 0.3721$$

   (c) No 4s

   $$P(X = 0) = (5/6)^8 \approx 0.2326$$

4. Consider two independent binomial random variables $X_1 \sim B(n_1, p)$ and $X_2 \sim B(n_2, p)$. If $Y = X_1 + X_2$, explain why $Y \sim B(n_1 + n_2, p)$.
   Since $X_1 \sim B(n_1, p) \Rightarrow X_1 = A_1 + A_2 + \cdots + A_{n_1}$ and $X_2 \sim B(n_2, p) \Rightarrow X_1 = A_{n_1+1} + A_{n_1+2} + \cdots + B_{n_2}$ where A_i is pair wise independent with parameter p. Hence

   $$Y = X_1 + X_2 = \sum_{i=1}^{n_2} A_i \sim B(n_1 + n_2, p)$$

5. If X has a geometric distribution with parameter $p = 0.7$, calculate

   (a) $P(X = 4)$

   $$P(X = 4) = (1 - p)^3 p = (0.3)^3(0.7) = 0.0189$$

   (b) $P(X = 1)$

   $$P(X = 1) = p = 0.7$$

   (c) $P(X \leq 5)$

   $$P(X \leq 5) = 1 - (1 - p)^5 = 1 - (0.3)^5 = 1 - 0.00243 = 0.99757$$

   (d) $P(X \geq 8)$

   $$P(X \geq 8) = (1 - p)^7 = (0.3)^7 = 0.0002187$$

6. Suppose that $X_1, \ldots, X_r$ are independent random variables, each with a geometric distribution with parameter p. Explain why
$$Y = X_1 + \cdots + X_r$$
has a negative binomial distribution with parameters p and r. Use this relationship to establish the mean and variance of a negative binomial distribution.
Negative binomial distribution counts the number of trials until the r-th success. Another way to think of this is the trials until the 1st success, and 2nd success, $\ldots$ rth success with parameter p. Hence,
$$E(Y) = E(X_1 + \cdots + X_r) = \sum_{i=1}^{r} E(X_i) = \sum_{i=1}^{r} \frac{1}{p}$$
$$\text{Var}(Y) = \text{Var}(X_1 + \cdots + X_r) = \sum_{i=1}^{r} \text{Var}(X_i) = \sum_{i=1}^{r} \frac{(1-p)}{p^2} = \frac{r(1-p)}{p^2}$$

7. If X has a negative binomial distribution with parameters $p = 0.6$ and $r = 3$, calculate:
   (a) $P(X = 5)$
   $$P(X = 5) = \binom{5-1}{3-1}(0.6)^3(0.4)^2 = \binom{4}{2}(0.216)(0.16) = 6(0.03456) = 0.20736$$
   (b) $P(X = 8)$
   $$P(X = 8) = \binom{8-1}{3-1}(0.6)^3(0.4)^5 = \binom{7}{2}(0.216)(0.01024) = 21(0.00221184) = 0.04645$$
   (c) $P(X \leq 7)$
   $$P(X \leq 7) = \sum_{x=3}^{7} \binom{x-1}{2}(0.6)^3(0.4)^{x-3} \approx 0.952$$
   (d) $P(X \geq 7)$
   $$P(X \geq 7) = 1 - P(X \leq 6) \approx 1 - 0.901 = 0.099$$

8. An archer hits a bull's-eye with probability 0.09 and results of different attempts are independent.
   (a) Probability that the first bull's-eye is scored with the fourth arrow:
   $$P(X = 4) = (1 - 0.09)^3(0.09) = 0.91^3(0.09) = 0.0684$$
   (b) Probability that the third bull's-eye is scored with the tenth arrow:
   $$P(X = 10) = \binom{9}{2}(0.09)^3(0.91)^7 = 36(0.000729)(0.501) = 0.0132$$
   (c) Expected number of arrows before first bull's-eye:
   $$E(X) = \frac{1}{p} = \frac{1}{0.09} \approx 11.11$$
   (d) Expected number of arrows before third bull's-eye:
   $$E(X) = \frac{r}{p} = \frac{3}{0.09} = 33.33$$

Done by Huynh Le An Khanh on November 7, 2025.