Probability & Statistics

Tutorial 14

1. Suppose $Z \sim N(0, 1)$. Find

(a) $P(Z \leq 1.34)$ (b) $P(Z \geq -0.22)$ (c) $P(-2.19 \leq Z \leq 0.43)$

(d) $P(0.09 \leq Z \leq 1.76)$ (e) $P(|Z| \leq 0.38)$ (f) $P(|Z| \geq 0.91)$

(g) The value of x for which $P(Z \leq x) = 0.55$

(h) The value of x for which $P(Z \geq x) = 0.72$

(i) The value of x for which $P(|Z| \leq x) = 0.31$

(j) The value of x for which $P(|Z| \geq x) = 0.42$

2. Suppose $X \sim N(10, 2)$. Find

(a) $P(X \leq 10.34)$ (b) $P(X \geq 11.98)$ (c) $P(7.67 \leq X \leq 9.90)$

(d) $P(10.88 \leq X \leq$ (e) $P(|X - 10| \leq 3)$ (f) $P(|X - 10| \geq 8)$
13.22)

(g) The value of x for which $P(X \leq x) = 0.81$

(h) The value of x for which $P(X \geq x) = 0.04$

(i) The value of x for which $P(|X - 10| \geq x) = 0.63$

(j) The value of x for which $P(|X - 10| \leq x) = 0.44$

3. Suppose that $X \sim N(\mu, \sigma^2)$ and that

$$P(X \leq 5) = 0.8 \quad \text{and} \quad P(X \geq 0) = 0.6.$$

What are the values of μ and σ^2?

4. The thicknesses of glass sheets produced by a certain process are normally distributed with a mean of $\mu = 3$ mm and a standard deviation of $\sigma = 0.12$ mm.

(a)   What is the probability that a glass sheet is thicker than 3.2 mm?

(b)   What is the probability that a glass sheet is thinner than 2.7 mm?

(c)   What is the value of c for which there is a 99% probability that a glass sheet has a thickness within the interval $[3 - c, 3 + c]$?

(d)   What is the probability that three glass sheets placed one on top of another have a total thickness greater than 9.5 mm?

(e)   What is the probability that seven glass sheets have an average thickness less than 3.1 mm?

5. The weights of bags filled by a machine are normally distributed with a standard deviation of 0.05 kg and a mean that can be set by the operator. At what level should the mean be set if it is required that only 1% of the bags weigh less than 10 kg?

6. An investment in company A has an expected return of $30,000 with a standard deviation of $4000. What is the probability that the return will be at least $25,000 if it has a normal distribution?