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Tutorial 4 - Mathematics for CS

1. Two cards are chosen from a pack of cards without replacement. Calculate the probabilities:
   (a) Both are picture cards.
   (b) Both are from red suits.
   (c) One card is from a red suit and one card is from a black suit.

   Repeat the above calculation when the second drawing is made with replacement.
   Compare your answers in the 2 cases.
   Assuming it's a regular 52 cards deck, the probabilities without replacement is:

   (a) $P(\text{Both are picture cards}) = \dfrac{12 \times 11}{52 \times 51} = 11/221.$
   (b) $P(\text{Both are from red suits}) = \dfrac{26 \times 25}{52 \times 51} = 25/102.$
   (c) $P(\text{One card is from a red suit and one card is from a black suit}) = \dfrac{2 \times 26 \times 26}{52 \times 51} = 26/51$

   Assuming it's a regular 52 cards deck, the probabilities with replacement is:

   (a) $P(\text{Both are picture cards}) = \dfrac{12}{52} \times \dfrac{12}{52} = 9/169.$
   (b) $P(\text{Both are from red suits}) = \dfrac{26}{52} \times \dfrac{26}{52} = 1/4.$
   (c) $P(\text{One card is from a red suit and one card is from a black suit}) = 1 \times \dfrac{26}{52} = 1/2$

   The chances of drawing two cards of the same types are greater with replacement, while the probability of drawing two cards of the different types is less.

2. Show that if the events A and B are independent events ($A \perp B$), then so are the events
   (a) A and B'
   $$A \perp B \Leftrightarrow P(A \cap B) = P(A)P(B)$$
   $$\Leftrightarrow P(A \cap B') = P(A) - P(A \cap B) = P(A) - P(A)P(B) = P(A)P(B') \Leftrightarrow A \perp B'$$
   (b) A' and B This is (a) but A and B are swapped.
   (c) A' and B' Since $A \perp B$ then $A \perp B'$. Let C be A and D be B'. Due to (b), $C' \perp D$ or $A' \perp B'$

3. Suppose that an insurance company insures its clients for flood damage to the property.
   Can the company reasonably expect that the claims from its clients will be independent of each other?
   No, if on client has flood damage, it is likely flooding and hence likely for others to have flood damage.

4. (a) If a fair die is rolled 5 times, what is the probability that the numbers obtained are all even numbers?

$$P(\text{even} \times 5) = P(\text{even})^5 = (1/2)^5 = 1/32$$

(b) If a fair die is rolled 3 times, what is the probability that the 3 numbers obtained are all different?
The number of ways to get 3 different numbers from 3 rolls: $A_6^3 = 120$
Since all rolls are equally likely:

$$P(\text{3 numbers obtained are all different}) = \frac{120}{6^3} = 5/9$$

5. Consider the network given in the figure with three switches. Suppose that the switches operate independently of each other and that switch 1 allows a message through with probability 0.88, switch 2 allows a message through with probability 0.92 and switch 3 allows a message through with probability 0.90. What is the probability that a message will find a route through the network?
The probability of the message passing through the upper path: $0.88 \times 0.92 = 0.8096$
The probability of the message not passing through either paths: $(1-0.8096)\times(1-0.9) = 0.01904$
The probability of the message passing through one path (find a route through the network): 0.98096

6. (Monty Hall problem, for fun only) Suppose you are on a game show, and you are given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say Door 1, and the host, who knows what is behind the doors, opens another door, say Door 3, which has a goat. He then says to you, "Do you want to pick Door 2?" Is it to your advantage to switch your choice?

   This is a classic problem. Suppose intially, we pick door 1. Since the host show us what's behind door 2 or door 3 is a goat, we know that if only one of door 2 or 3 is goat, we can garuantee that the behind the other door is a car. And since the chance of there being only one goat behind both door 2 or 3 are 2/3, the chances to pick the door with the car is: $2/3 \times 1 + 1/3 \times 0 = 2/3$.

Submitted by Vo Nguyen Minh Tri on October 8, 2025.