UE22MA242A: Mathematics for Computer Science Engineers

Session: Aug\_Dec-2024

Tutorial Session-1 Questions

| SI.No | Questions |
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| 1. 3PES1202100541v | A physical education professor wants to study the physical fitness levels of students at her university. There are 20,000 students enrolled at the university, and she wants to draw a sample of size 100 to take a physical fitness test. She obtains a list of all 20,000 students, numbered from 1 to 20,000. She uses a computer random number generator to generate 100 random integers between 1 and 20,000 and then invites the 100 students corresponding to those numbers to participate in the study. Is this a simple random sample? |
|  | Each of the following processes involves sampling from a population. Define the population, and state whether it is tangible or conceptual.  a. A shipment of bolts is received from a vendor. To check whether the shipment is acceptable with regard to shear strength, an engineer reaches into the container and selects 10 bolts, one by one, to test.  b. The resistance of a certain resistor is measured five times with the same ohmmeter.  c. A graduate student majoring in environmental science is part of a study team that is assessing the risk posed to human health of a certain contaminant present in the tap water in their town. Part of the assessment process involves estimating the amount of time that people who live in that town are in contact with tap water. The student recruits residents of the town to keep diaries for a month, detailing day by day the amount of time they were in contact with tap water.  d. Eight welds are made with the same process, and the strength of each is measured. e. A quality engineer needs to estimate the percentage of parts manufactured on a certain day that are defective. At 2:30 in the afternoon he samples the last 100 parts to be manufactured. |
| 3. | The length of a rivet manufactured by a certain process has mean μX = 50 mm and standard deviation σX = 0.45 mm. What is the largest possible value for the probability that the length of the rivet is outside the interval 49.1–50.9 mm? |
| 4. | Assume that the probability density function for X, the length of a rivet in the above Example is    It can be verified that μX = 50 and σX = 0.45. Compute the probability that the length of the rivet is outside the interval 49.1–50.9 mm. How close is this probability to the Chebyshev bound of 1/4? |
| 5. | As part of a quality-control study aimed at improving a production line, the weights (in ounces) of 50 bars of soap are measured. The results are as follows, sorted from smallest to largest. 11.6 12.6 12.7 12.8 13.1 13.3 13.6 13.7 13.8 14.1 14.3 14.3 14.6 14.8 15.1 15.2 15.6 15.6 15.7 15.8 15.8 15.9 15.9 16.1 16.2 16.2 16.3 16.4 16.5 16.5 16.5 16.6 17.0 17.1 17.3 17.3 17.4 17.4 17.4 17.6 17.7 18.1 18.3 18.3 18.3 18.5 18.5 18.8 19.2 20.3 Construct a normal probability plot for these data. Do these data appear to come from an approximately normal distribution? |
| 6. | Let X denote the number of flaws in a 1 in. length of copper wire. The probability mass function of X is presented in the following table.    One hundred wires are sampled from this population. What is the probability that the average number of flaws per wire in this sample is less than 0.5? |
| 7. | At a large university, the mean age of the students is 22.3 years, and the standard deviation is 4 years. A random sample of 64 students is drawn. What is the probability that the average age of these students is greater than 23 years? |
| 8. | Let X1,..., Xn be a random sample from a N(μ, σ2) population. The sample variance is s2 = n i=1(Xi − X)2/(n − 1). It can be shown that s2 has mean μs2 = σ2 and variance σ2 s2 = 2σ4/(n − 1). Consider the estimator σ2 = n i=1(Xi − X)2/n, in which the sum of the squared deviations is divided by n rather than n − 1. Compute the bias, variance, and mean squared error of both s2 and σ2. Show that σ2 has smaller mean squared error than s2. |
| 9. | Let X1,..., Xn be a random sample from an Exp(λ) population, where λ is unknown. Find the MLE of λ. |
| 10. | Let X1,..., Xn be a random sample from a N(μ, σ2 ) population. Find the MLEs of μ and of σ. (Hint: The likelihood function is a function of two parameters, μ and σ. Compute partial derivatives with respect to μ and σ and set them equal to 0 to find the values μ and σ that maximize the likelihood function.) |
| 11. | Let X1,..., Xn be a random sample from a population with the Poisson(λ) distribution. Find the MLE of λ. |
| 12. | Lifetimes of batteries in a certain application are normally distributed with mean 50 hours and standard deviation 5 hours. Find the probability that a randomly chosen battery lasts between 42 and 52 hours. |
| 13. | A process manufactures ball bearings whose diameters are normally distributed with mean 2.505 cm and standard deviation 0.008 cm. Specifications call for the diameter to be in the interval 2.5 ± 0.01 cm. What proportion of the ball bearings will meet the specification? |
| 14. | Grandma bakes chocolate chip cookies in batches of 100. She puts 300 chips into the dough. When the cookies are done, she gives you one. What is the probability that your cookie contains no chocolate chips? |
| 15. | Grandma’s grandchildren have been complaining that Grandma is too stingy with the chocolate chips. Grandma agrees to add enough chips to the dough so that only 1% of the cookies will contain no chips. How many chips must she include in a batch of 100 cookies to achieve this? |
| 16. | Find the probability mass function of the random variable X if X ∼ Bin(10, 0.4). Find P(X = 5). |
| 17. | A fair die is rolled eight times. Find the probability that no more than 2 sixes come up. |