CS 4510 Automata and Complexity

Exam 3A: Turing Machines

Date: 2:40pm-5:30pm Tuesday, Dec. 13

Name: _____ GTID: _____

- **When handing in this test, please go to a TA with your BuzzCard to verify the name on the test.**
- Do not open this exam until you are directed to do so. Read all the instructions first.
- **Write your name and GTID# (numbers, not letters) on the top of every page.** Your GTID# can be found on your BuzzCard.
- By submitting this exam, you agree that your actions during this exam conform with the Georgia Tech Honor Code.
- This exam is out of 100 points, but there are **more than 100 points up for grabs**. Any points you earn above 100 are bonus points!
- Write your solutions in the space provided. If you run out of space, continue your answer on the page of scratch paper provided at the end of the exam.
- The only outside material you may use on the exam is one (1) double-sided, hand-written, 8.5"x11" page of notes.
- Calculators are NOT permitted.
- You may use any of the theorems/facts/lemmas from the lecture notes, homeworks, or textbook without re-proving them unless explicitly stated otherwise.
- If you have a question, you may ask a TA. You should not communicate with anyone other than teaching staff during the exam.
- Do not spend too much time on any one problem! If you get stuck, move on and come back to that one later.
- Good luck!

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0. **Following Instructions** (1 bonus point)

   Please write your name and GTID (letters, not numbers) at the top of EACH page of this exam.

1. **Classes of Languages** Consider the following sets of languages over alphabet $\{0, 1\}$.

   (a) $\mathcal{P}(\{0,1\}^*)$

   (b) The set of context free languages

   (c) The set of Turing decidable languages

   (d) The set of Turing recognizable languages

   (e) The set of regular languages

   (f) $\emptyset$

   Part 1: Fill in the blanks below to show their relationship to each other. (Just write the letters in the blanks.)

   $$\_\_\_\_ \subset \_\_\_\_ \subset \_\_\_\_ \subset \_\_\_\_ \subset \_\_\_\_ \subset \_\_\_\_$$

   Part 2: Fill in the blanks (just write the letters) to indicate which of the sets of languages above are described by each line.

   - ___ is the set of languages decided by a DFA.
   - ___ is the set of languages decided by a PDA.
   - ___ is the set of languages decided by a NFA.
   - ___ is the set of languages decided by a TM.
   - ___ is the set of languages generated by a CFG.
   - ___ is the set of languages described by a regular expression.

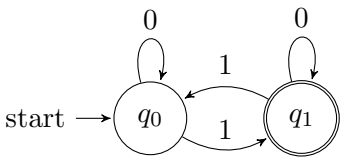
2. **Complexity Classes** Fill in the blanks to show the known relationship between the complexity classes PSPACE, NPSPACE, NP, P, EXPTIME, and TIME(n^2).

   $$\_\_\_\_\_ \subset \_\_\_\_\_ \subseteq \_\_\_\_\_ \subseteq \_\_\_\_\_ = \_\_\_\_\_ \subseteq \_\_\_\_\_$$

3. **Machine understanding**

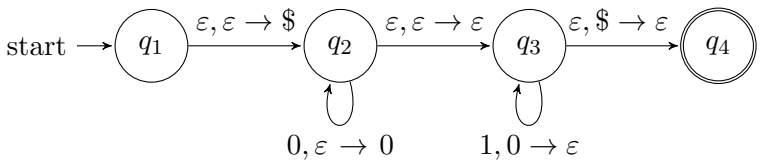
(a) Fill in the blank to describe the language of this DFA.

$$L = \{x \in \{0,1\}^* \mid \underline{\qquad\qquad\qquad\qquad\qquad\qquad}\}$$



(b) Fill in the blank to describe the language of this PDA.

$$L = \{x \in \{0,1\}^* \mid \underline{\qquad\qquad\qquad\qquad\qquad\qquad}\}$$



4. **Difficulty of languages**

(a) Circle the three languages below which are regular.

- $\{w \mid \text{every odd position of } w \text{ is } 1\}$
- $\{0^i 1^j 0^k \mid i, j, k \in \mathbb{Z}^{\geq 0}\}$
- $\{a^i b^j c^k \mid 2i + j = k\}$
- $\{w 1^{n^2} \mid |w| \text{ is even}, n \in \mathbb{Z}^{\geq 0}\}$
- $\{0, \varepsilon, 111, 00110\}$
- $\{0^n 1^n \mid n \in \mathbb{Z}^{\geq 0}\}$

(b) Circle the three languages below which are context free.

- $\{w\#x\#w^{\mathcal{R}}\#1^n \mid |x| = n\}$
- $\{w\#w^{\mathcal{R}}\#x\#1^n \mid |x| = n\}$
- $\{1^i\#0^j\#0^k \mid i + 3k = 5j\}$
- $\{1^m\ 0^{n^3} 1^m \mid n, m \in \mathbb{Z}^{\geq 0}\}$
- $\{ww\#0^n 1^n \mid n \in \mathbb{Z}^{\geq 0}\}$
- $\{0^n 1^n \mid n \in \mathbb{Z}^{\geq 0}\}$

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5. **Multiple-choice** (12 points - 2 points each)

   Circle the correct answer. No justification necessary. Recall that "$A \leq_P B$" means "there is a polynomial-time mapping reduction from A to B."

   (a) Suppose $L = \{00, \varepsilon\}$ and $L' = \{0,1\}^*$. Then $L \leq_m L'$ MUST / MAY / DOESN'T holds.

   (b) If L is decided by some PDA P, then there MUST / MAY / DOESN'T exist a Turing machine that recognizes L.

   (c) Suppose $HALT \leq_m L$. Then there MUST / MAY / DOESN'T exist a TM which decides L.

   (d) Suppose $CLIQUE \in P$, then $P = NP$ MUST / MAY / DOESN'T hold.

   (e) There are COUNTABLY / UNCOUNTABLY many Turing decidable languages over the alphabet $\{0,1\}$.

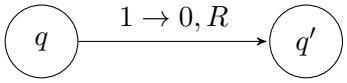
   (f) If TM M is a recognizer for a language L and $w \notin L$, then M MUST / MAY / DOESN'T halt when run on input w.

6. **Language complexity** (12 points - 3 points each)

   For each language, write D in the blank if it is Turing decidable, R in the blank if it is Turing recognizable but not decidable, or C in the blank if its complement is Turing recognizable but not decidable.

   (a) $\{\langle M\rangle, x \mid M \text{ does not accept } x\}$, _____
   (b) $\{\langle M\rangle, \langle N\rangle \mid \text{M and } N \text{ are both TMs}\}$, _____
   (c) $\{\langle M\rangle \mid M \text{ accepts when run on input } \langle M\rangle\}$, _____
   (d) $\{\langle M\rangle, x \mid \text{there is some } y \text{ such that } M \text{ rejects } xy\}$, _____

7. **Tableau** (8 points - 2 points each)
   Suppose M is a TM which includes the following transition.



   Assume there are no other transitions into q'. For each of the following 2×3 windows, circle them if they will not occur in any valid tableau for M.

   (a)
0	0	1
q'	0	0

   (c)
q	1	1
0	q'	1

   (b)
1	q	1
0	1	q'

   (d)
0	1	0
0	1	q'

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8. **Model Conversion** (20 points)

Let a 2-*head, alternating* Turing machine be similar to an ordinary 1-tape Turing machine, except it has 2 different tape heads which take turns. We call them the "even" and the "odd" tape heads. At the beginning of computation both of the heads are pointing at the first character in the input string. During each even step of computation, the "even" tape head reads a value from the tape, write a value to the tape, and moves left or right. During each odd step of computation, the "odd" tape head reads a value from the tape, write a value to the tape, and moves left or right.

Prove that 2-head alternating Turing machines are equivalent in power to ordinary Turing machines.

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9. **Undecidability** (20 points)

   Prove that $L = \{\langle M \rangle, \langle N \rangle \mid L(M) \neq L(N)\}$ is undecidable.

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10. **Recognizability** (10 points)

(a) Give a deterministic Turing machine which recognizes

$$L_1 = \{\langle M \rangle, x \mid \text{there exists a } y \text{ such that } M \text{ accepts } xy\}$$

(b) Give a deterministic Turing machine which recognizes the complement of L_2, where

$$L_2 = \{\langle M \rangle, \langle N \rangle \mid M \text{ does not accept } \langle N \rangle \text{ or } N \text{ does not accept } \langle M \rangle\}$$

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11. **Time Complexity** (23 points)
Consider the $SORT()$ operation, defined as follows: Let $L \subseteq \{0,1\}^*$ be a language. Then $SORT(L) = \{0^n1^m \mid \text{there is some } x \in L \text{ with exactly } n \text{ zeros's and } m \text{ one's}\}$. For example, if $L = \{01, 01000, 1010\}$, then $SORT(L) = \{01, 00001, 0011\}$.

Show that if $SAT \in P$, then P is closed under $SORT$. That is, show that if $L \in P$, then $SORT(L) \in P$. (Hint: First show closure of NP under $SORT$. Then use the *very important* fact we know about SAT.)

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12. **Space Complexity** (20 points)
Suppose L and L' are both in NPSPACE. Prove that $L \cup L' \in$ EXPTIME.

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.....SCRATCH PAPER.....