CS 4510 Automata and Complexity

Homework Worksheet 3: Select Solutions

The answers to select questions are provided here. For the rest of questions, however, you are encouraged to discuss on Piazza and try to solve them collaboratively. You are also welcomed to discuss these problem during office hours.

Select Solutions

Problem 1 Solution

Let's pick the string $0^p1^{p^2} \in L_1$. This string can be split into $s = xyz$ such that y is not the empty string and $|xy| \leq p$. By this definition, xy consists of only 0s. Let's say y consists of k 0s such that $1 \leq k \leq p$. Pumping on xyz once to get $xyyz$ gives us the string $0^{p+k}1^{p^2}$. $(p+k)^2 = p^2+2k+k^2 > p^2$. Thus $xyyz \notin L_1$ and L_1 is not regular by the pumping lemma.

Problem 2 Solution

Suppose L_2 is regular. Let p be as in the Pumping Lemma. Select $s = (10)^p1^p \in L_2$ such that $|s| \geq p$. Splitting s into xyz gives 2 options for xy depending on whether p is even or odd. If p is even, y is a non-empty substring of $(10)^k$ for $1 \leq k \leq \frac{p}{2}$. If p is odd, y is a non-empty substring of $(10)^k1$ for $1 \leq k \leq \frac{p}{2}$. This gives us 3 cases to consider. The first case is that y starts with and ends with a 0. If we down pump on such a string, xz will contain 110 as a substring which cannot occur for any string in L_2. The second case is the y starts with and ends with a 1. Down pumping on such a string will result in a string that contains 00 as a substring which also cannot occur for any string in L_2. The last case is more tricky where y starts and ends with different symbols. Here, y is either $(10)^i$ or $(01)^i$ where $0 < i < \frac{p}{2}$. Down pumping on this string will gives $xz = (10)^{p-i}1^p \notin L_2$ as $p - i \neq p$. As we reach a contradiction for all cases regarding y, we conclude that L_2 is not regular in violation of the pumping lemma.

Problem 3 Solution

Assume L_3 is regular. Now, from pumping lemma, let's pick string 1^{2^p} for some p. Since $1^{2^p} \in L_3$. we can split this string into $s = xyz$ such that y is non-empty and $|xy| \leq p$. Therefore, $y = 1^m$ for some $1 \leq m \leq p$. Now, pumping up once, we get $xyyz = 1^{2^p+m}$. Let's check if $xyyz$ belongs to L_3 or not. Note that, $1^{2^p} \in L_3$ and $1^{2^{p+1}} \in L_3$. As $2^p < 2^p + m \leq 2^p + p < 2^p + 2^p = 2 * (2^p) = 2^{p+1}$, $xyyz \notin L_3$ and hence L_3 is not regular.

Problem 7 Solution

Assume that L_7 is regular. This would mean that the Pumping Lemma applies to L_7. n can be defined as the pumping length and p can be defined as an arbitrary prime larger than n, which exists since there are infinitely many primes. Let s be a string of 0^p such that the $|s| > n$. Because $s \in L$ and $|s| > n$, the Pumping Lemma guarantees that s can be split into $s = xyz$ where for any $i \geq 0$, $xy^iz \in L_7$. The string xy^iz will have a length $p + (i-1)|y|$. Since i can be any non negative integer, i can be $p+1$ meaning that the length of xy^iz is $p + p|y| = p(1+|y|)$ after pumping. $p(1+|y|)$ is a composite number since it can be obtained by multiplying 2 numbers other than itself and 1, in this case p and $(1+|y|)$. If $s = xy^{p+1}z$ is of composite length, $s \notin L_7$ violating the Pumping Lemma. This contradiction means that L_7 is not regular.

Problem 8 Solution

Yes. It can be written as a regex: $uuuuv^*u^*vvvvvvv$

Problem 9 Solution

This proof assumes that s should be pump-able. But this would require that s is longer than the pumping length of L. So let the pumping length of L be larger than the length of s. Now there is no contradiction.

Problem 11 Solution

Let p be a prime number larger than the pumping length. Then the string $s = a^p b^{(p-1)!}$ is in the language where xy consists exclusively of a's. Pick a $y = a^k$ where $1 \leq k \leq p$. consider $xz = a^{p-k}b^{(p-1)!}$. $p - k$ is a factor of $(p-1)!$ meaning that they are not realtively prime. This string cannot be pumped down, which is a contradiction and the language is irregular by the Pumping Lemma.