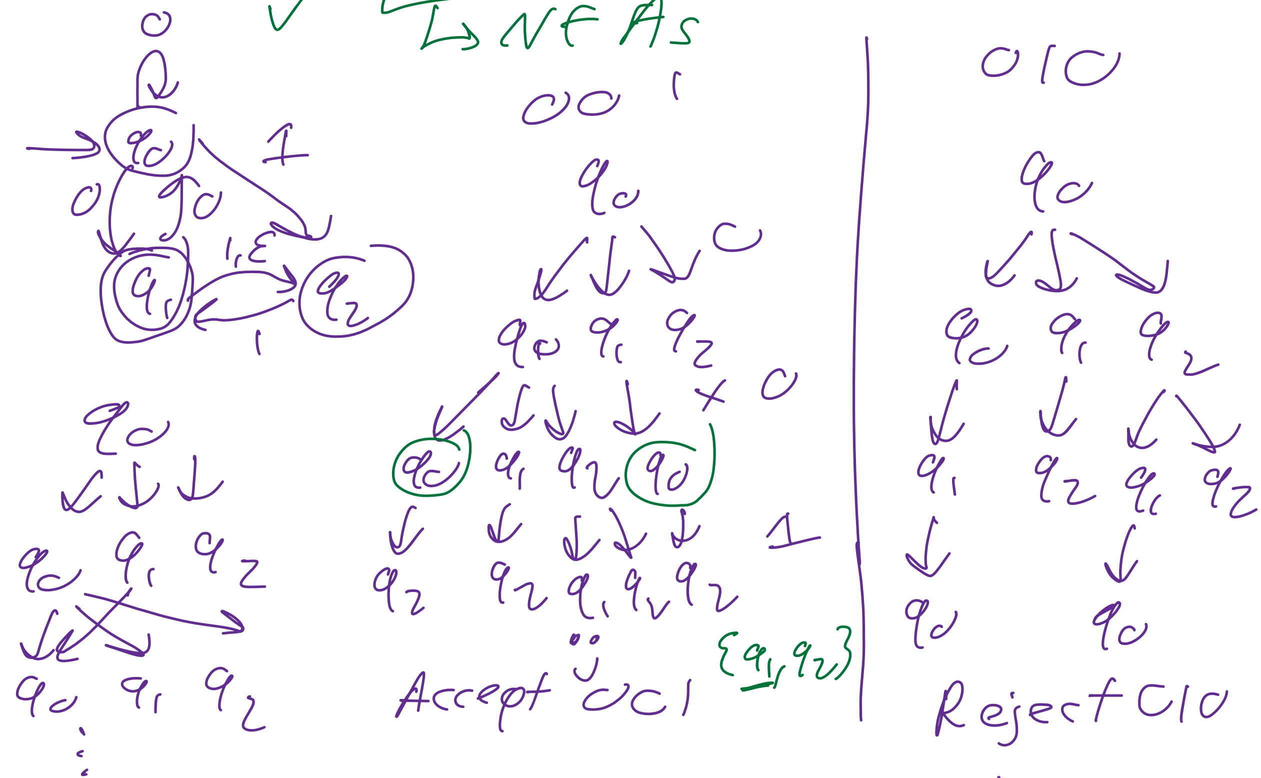
Announcements:
- OH Schedule posted
- HW1 due R at 10pm

- Where are we?

Closure $\cup$, $\circ$, $*$ $\to$ NFAs



Thm: L is regular iff $L = \mathcal{L}(N)$ for some NFA.

Pf. ($\Rightarrow$) is trivial

L regular $\Rightarrow$ $L = \mathcal{L}(M)$ for DFA M

then interpret M as NFA.

($\Leftarrow$) $\boxed{\text{Have our DFA state store sets of NFA states}}$

Let N be given. We build M as follows.

$M.\Sigma = N.\Sigma$

$M.Q = P(N.Q) = \{S \subseteq N.Q\}$

$M.q_0 = \cancel{\{N.q_0\}}$ almost

$= E(\{q_0\})$

$M.F = \cancel{\{\{q \mid q \in N.F\}\}}$

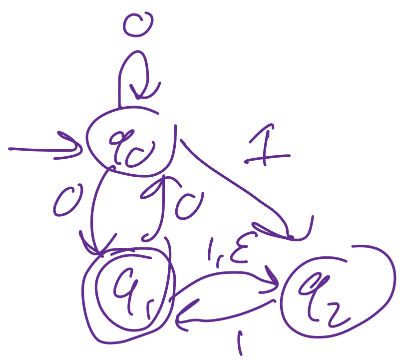
$= \{S \mid S \cap N.F \neq \emptyset\}$

$M.\delta(S, x) = \cancel{\bigcup_{q \in S} \delta(q, x}$

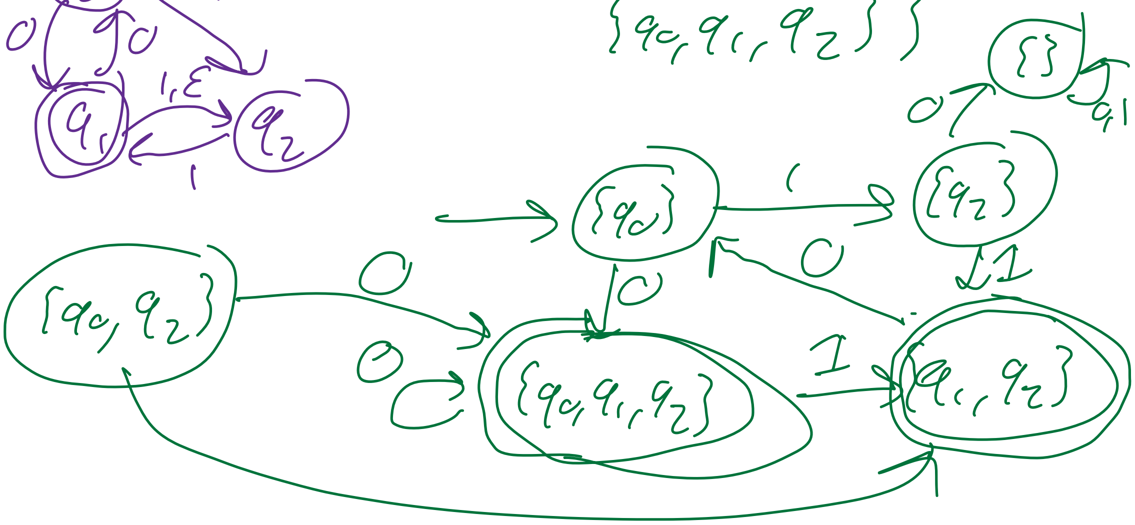
$= \bigcup_{q \in S} E(\underbrace{\delta}_{N.\delta}(q, x))$

Notation

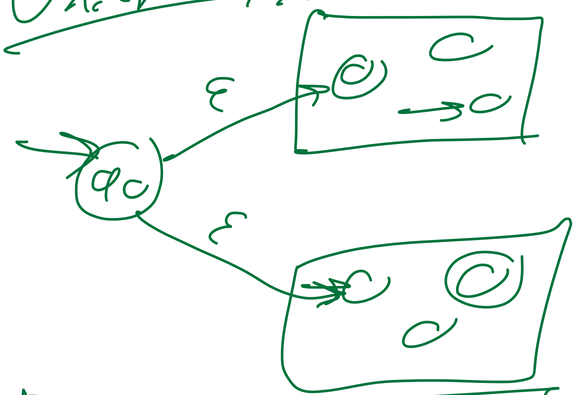
If $S \subseteq N.Q$ (i.e. $S \in P(N.Q)$) then let $E(S) = \{q \in N.Q \mid q$ is reachable from some $q' \in S$ using (zero or more) ε transitions$\}$



$M.Q = \{\{\}, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_2\}, \{q_0, q_1, q_2\}\}$



Union w/ NFAs

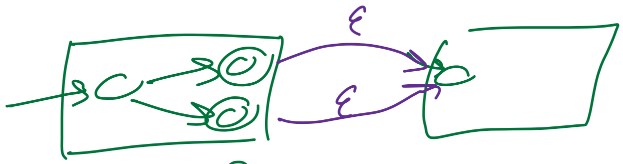


for ∪ add extra q_0 and ε transitions into original start states.

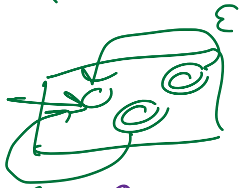
Closure under Kleene Star

$\{cat, doggy\}^* = \{\varepsilon, cat, doggy, catcat, catdoggy, \ldots\}$

Recall Concat



Kleene Star Idea:



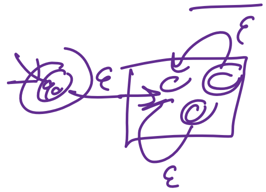
Add ε transition from all accept states start state

$\{cat, catpig, doggy\}^\varepsilon$

May not accept ε! :(

Formally let NFA N be given we build N'.

Add extra q_0



$N'.Q = N.Q \cup \{q_0\}$

$N'.q_0 = q_0$

$N'.F = N.F \cup \{q_0\}$

$N'.\Sigma = N.\Sigma$

$$N'.\delta(q, x) = \begin{cases} \{N.q_0\} & \text{if } q = q_0, x = \varepsilon \\ \emptyset & \text{if } q = q_0, x \neq \varepsilon \\ N.\delta(q, x) & \text{if } q \notin N'.F \cup \{q_0\} \text{ or } x \neq \varepsilon \\ N.\delta(q, \varepsilon) \cup \{N.q_0\} & \text{if } x = \varepsilon \text{ and } q \in N.F \end{cases}$$

A language R is regular iff it can be generated by the following rules (applied a finite number of times)

⓪ $R = \emptyset, \{\varepsilon\}, \{0\}, \{1\}$ → $\{a\}$ for some $a \in \Sigma$

① $R = R_1 \cup R_2$ for regular R_1, R_2

② $R = R_1 \circ R_2$ " "

③ $R = R_1^*$ for regular R_1

Not actually needed. $\{\varepsilon\} = \emptyset^*$

(also $\{\varepsilon\} = \{\varepsilon\}^*$.)

Show how to construct these regular languages

① $\{001\}$ e.g. $\{0\} \circ \{0\} \circ \{1\}$

② all strings ending in 001 $= (\{0\} \cup \{1\})^* \circ \{0\} \circ \{0\} \circ \{1\}$

③ all strings containing 001 as a substring $\underbrace{(\{0\}\cup\{1\})^* \circ \{0\} \circ \{0\} \circ \{1\}}_{\text{that}} \circ (\{0\} \cup \{1\})^*$

④ all strings $(\{0\} \cup \{1\})^*$

⑤ all strings not containing 001 as a substring

⑥ all even length strings

$\Sigma \big(\underbrace{(\{0\} \cup \{\varepsilon\}) \circ \{1\})^*}$ $\circ \{0\}^*$

this part is $\{01, 1\}^*$ which can also be written as

$((\{0\} \circ \{1\}) \cup \{1\})^*$