CS 4510 Automata and Complexity 1/29/2022

Worksheet Solutions 1: Regular Language Closure

Spoiler alert: The following content contains spoilers to worksheet 1. If you have not done so already, you are strongly recommended to try out those questions yourself first.

2. **Set operations**

   a. By De Morgan's Law, $L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$. From lecture, we know (1) the complement of a regular language is regular and (2) union of two regular languages is also regular. Since L_1 and L_2 are regular languages:
   By (1), $\overline{L_1}$ and $\overline{L_2}$ are regular.
   By (2), $\overline{L_1} \cup \overline{L_2}$ is regular.
   By (1), $\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2$ is regular.

   b. We know $L_1 - L_2 = L_1 \cup (L_1 \cap L_2)$.
   By Part a, $L_1 \cap L_2$ is regular,
   By (2), $L_1 \cup (L_1 \cap L_2) = L_1 - L_2$ is regular.

   c. By definition, the symmetric difference between two sets L_1 and L_2 is the union of $L_1 - L_2$ and $L_2 - L_1$.
   By Part b, $L_1 - L_2$ and $L_2 - L_1$ are regular.
   By (2), $(L_1 - L_2) \cup (L_2 - L_1)$ is regular, so is the symmetric difference.

3. **SUBSTRING**

   Since L is regular, there exists a DFA D that recognizes L.

   Convert D to an NFA. Create a new state q. Make q the new starting state. Add epsilon transitions from q to all other states. Then, mark all of the states that have a path to the previously accepting states also accepting. This NFA should accept **SUBSTRING**(L). Therefore, regular languages are closed under **SUBSTRING**.

5. **DOUBLE**

   Since L is regular, there exists a DFA M that recognizes L.

   We build an NFA N by replacing each transition in M from a state q_i to a state q_j with two transitions: one from q_i to a new state q' and another from q' to q_j. All paths from the start state q_0 to a final state in M become twice as long. Therefore, N will accept all strings that have twice the length of some string accepted by M. Notice that since M is a DFA and the alphabet Σ is unary, each state in M has exactly one transition going to another state. Thus the number of additional states required is the number of states in M.

   Since this NFA accepts **DOUBLE**(L), regular languages are closed under **DOUBLE**.

7. **SLICE**

   Since L is regular, there exists a DFA D that recognizes L.

   Convert D to an NFA N. Create a new state q. Make q the new starting state. Add epsilon transitions from q to all states that can be reached using one step from the original starting state. The original final states of D are not necessarily final states of N. Instead, only make states that can reach any of the original accepting states using one step accepting.

   Since this NFA accepts **SLICE**(L), regular languages are closed under **SLICE**.