Observation: If $\# \notin M.\Gamma$, then M "cannot distinguish" between empty stack and stack w/ $\#$ on top.

Let M, N be PDAs. We want PDA for $L(M) \circ L(N)$.

① Build O with copies of M and N. Extra start state $O.q_0$ with $\to (O.q_0) \xrightarrow{\varepsilon, \varepsilon \to \#} (M.q_0)$ where $\# \notin M.\Gamma$. Then extra intermediate state r with transitions

- $(q) \xrightarrow{\varepsilon, \varepsilon \to \varepsilon} (r)$ for all $q \in M.F$
- $(r) \circlearrowleft \varepsilon, a \to \varepsilon$ for all $a \in M.\Gamma$
- $(r) \xrightarrow{\varepsilon, \# \to \varepsilon} (N.q_0)$.

Idea: We empty out the stack at r, checking $\#$ to know it is empty.

Accept states are those of N.

② Alternatively, with $\# \notin N.\Gamma$ we can have $O.q_0 = M.q_0$ and $(q) \xrightarrow{\varepsilon, \varepsilon \to \#} (N.q_0)$ for each $q \in M.F$.
Accept states are those of N.

Idea: putting $\#$ on top makes it equivalent to an empty stack from N's perspective