

Worksheet Solutions 2: Regular Expressions

Spoiler alert: The following content contains spoilers to worksheet 2. If you have not done so already, you are strongly recommended to try out those questions yourself first.

2. Binary Strings

- a. $00^* \cup (0 \cup 1)^*000$ (that is, either all 0s or ends with three 0s)
- b. $(01)^*(0 \cup \varepsilon) \cup (10)^*(1 \cup \varepsilon)$
- c. $(\Sigma 0(((\Sigma \Sigma)^*\Sigma 0) \cup \varepsilon)) \cup (\Sigma 1(((\Sigma \Sigma)^*\Sigma 1) \cup \varepsilon)) \cup (\Sigma 1(\Sigma \Sigma)^*0)$

3. Substring

- a. $0^*(1(1 \cup \varepsilon)0^*0)^*(11 \cup 1 \cup \varepsilon)$
- b. $(0 \cup \varepsilon)(10)^*(1 \cup \varepsilon)$
- c. $0^* \cup 1^*$

5. Binary Comparison Operators

- a. 0^*1110
- b. $0^*(11(0 \cup 1)(0 \cup 1) \cup 101(0 \cup 1) \cup 1(0 \cup 1)(0 \cup 1)(0 \cup 1)(0 \cup 1)(0 \cup 1)^*)$
- c. $0^*(1 \cup 10 \cup 11 \cup 100 \cup 101 \cup 110 \cup 111 \cup 1000 \cup 1001)$
- d. $0^*(11(0 \cup 1)(0 \cup 1) \cup 101(0 \cup 1) \cup 1(0 \cup 1)(0 \cup 1)(0 \cup 1)(0 \cup 1)^* \cup 1 \cup 10 \cup 11 \cup 100 \cup 101)$

7. Alternative Regular Expressions

- a. One direction: L^* can be expressed as $\{\varepsilon \cup L^+\}$.
For reverse direction, L^+ can be expressed as LL^*
- b. One direction: $A \cup B$ can be expressed as $(A \circ A) \circ (B \circ B)$.
For reverse direction(possibly tricky): If we have $A \circ B$, we can convert A and B to their DFAs, invert states, and convert back to regular expressions to get A^c and B^c . Now, $A \circ B = A^cUB^c$