CS 4510 Automata and Complexity

Exam 1A: Regular languages

Date: 3:30pm-4:45pm Tuesday, September 20

- Name: ____ GTID#: ____
- **When handing in this test, please go to a TA with your BuzzCard to verify the name on the test.**
- Do not open this exam until you are directed to do so. Read all the instructions first.
- This booklet contains 4 **questions on** 5 **pages**, including this one.
- **Write your name and GTID# (numbers, not letters) on the top of every page.** Your GTID# can be found on your BuzzCard.
- By submitting this exam, you agree that your actions during this exam conform with the Georgia Tech Honor Code.
- Write your solutions in the space provided. If you run out of space, continue your answer on the back of the last page, and make a notation on the front of the sheet.
- There is a page of scratch paper at the end of the exam. We can provide additional scratch paper if you need it. The only outside material you may use on the exam is one (1) double-sided, hand-written, 8.5"x11" page of notes.
- Calculators are NOT permitted.
- You may use any of the theorems/facts/lemmas from the lecture notes, homeworks, or textbook without re-proving them unless explicitly stated otherwise.
- If you have a question, you may ask a TA. You should not communicate with anyone other than teaching staff during the exam.
- Do not spend too much time on any one problem! If you get stuck, move on and come back to that one later.
- Good luck!

Name: ______________________________________ GTID#: ______________________________________

0. **Following Instructions** (1 point extra credit)

   Please write your name and GTID# (numbers, not letters) at the top of EACH page of this exam. Your GTID# can be found on your BuzzCard.

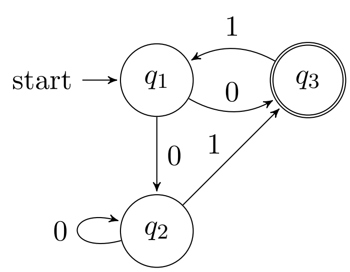
1. **Circle the answers**

   (a) (4 points each) Circle one. No explanation necessary.

   i. Let R be a regular expression. Then $R \cup \emptyset^*$ (ALWAYS / SOMETIMES / NEVER) generates at least one string.

   ii. Suppose L is a language. Then there (ALWAYS / SOMETIMES / NEVER) exists a DFA that recognizes L.

   (b) (4 point) Suppose we wished to convert the following NFA to a DFA as shown in class and in the book:



   Which of the following will be states in the resulting DFA? (Circle all that apply, if any. No explanation necessary.)

   - q_1
   - $\emptyset$
   - $\{q_1, q_3\}$
   - $q_{\textbf{FAIL}}$

   (c) (8 points) You are using the pumping lemma to show that the language $\{w \in \{0,1\}^* \mid w \text{ has an equal number of 0s and 1s}\}$ is not regular. You have already assumed that L is regular, and let p be the pumping length. Which of the following choices for s allow you to successfully complete the proof, where s is the counterexample string which you show cannot be pumped? (Circle all that apply, if any. No explanation necessary.)

   - 0^p1^p
   - 00001111
   - $0^{p+1}1^p$
   - $(01)^p$

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2. **Pumping Lemma** (30 points)
Prove that **one** of the following languages is not regular using the Pumping Lemma.

- $L = \{w \mid w \text{ contains an even number of 0's or every odd position of } w \text{ is a } 1\}$.
- $\Delta = \{1^j 0^k 1^\ell \mid k > 4, j = \ell\}$.
- The language described by the regular expression $(0 \cup \varepsilon)(10)^*(1 \cup \varepsilon)$.

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3. **Regular Expressions** (20 points - 10 each)
   Give a regular expression for **two** of the following binary languages. Use **only** the symbols $\emptyset$, ε, 0, 1, $\circ$, $\cup$, *,), and (. You may leave $\circ$'s implicit. Please box your final answers.

   - $L = \{w \mid w$ contains an even number of 0's or every odd position of w is a $1\}$.
   - $P = \{w \mid w$ contains zero 1's and n^2 0's for some $n \in \mathbb{Z}^{\geq 0}\}$
   - The set of binary strings containing at least three 1's.

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4. **Closure** (30 points)
Here are three operations on languages.

- $\mathsf{MIX}(A, B) = \{a_1 b_1 \cdots a_n b_n \mid a_1 \cdots a_n \in A, b_1 \ldots b_n \in B\}$
- $\mathsf{WEAVE}(A, B, C) = \{a_1 b_1 c_1 \cdots a_n b_n c_n \mid a_1 \cdots a_n \in A, b_1 \ldots b_n \in B, c_1 \ldots c_n \in C\}$
- $\mathsf{2WEAVE}(A, B, C)$ is defined the same as WEAVE except it includes any string where (at least) two of $a_1 \cdots a_n \in A$, $b_1 \ldots b_n \in B$, and $c_1 \ldots c_n \in C$ hold.

Let $A = \{\mathtt{cat}\}$, $B = \{110\}$, and $C = \{\triangle\square\triangle\}$. Then $\mathsf{MIX}(A, B) = \{\mathtt{c1a1t0}\}$ and $\mathsf{WEAVE}(A, B, C) = \{\mathtt{c1\triangle a1\square t0\triangle}\}$. $\mathsf{2WEAVE}(A, B, C)$ includes $\mathtt{c1\square a1\square t0\square}$, $\mathtt{d1\triangle o1\square g0\triangle}$, $\mathtt{c1\triangle a1\square t0\triangle}$, and more. It does not contain are $\mathtt{c1a1t0}$, $\mathtt{1c\square 1a\square 0t\square}$, or $\mathtt{c1\square a1\square t1\square}$. If $A' = \{\mathtt{cats}\}$ then $\mathsf{MIX}(A', B) = \mathsf{WEAVE}(A', B, C) = \emptyset$

(25 points) Prove that if A, B, and C are regular, then so is $\mathsf{WEAVE}(A, B, C)$. (Hint: you might consider trying to prove closure for MIX first and then generalize your idea.)

(5 points) Prove that if A, B, and C are regular, then so is $\mathsf{2WEAVE}(A, B, C)$. (Hint: modify your proof for WEAVE.)

Prof. Jaeger says, "This question was harder than intended. Only around 5% of the students received full credit on this question. We intend not to have any questions that are this hard."

...............SCRATCH PAPER..................