- HW due Thurs → Come with questions.
- Review session Thurs
- Exam on Tuesday

**PDA**

( ) [ }

$\{0^i 1^j 2^h 3^\ell : i=j, h=()\}$

$\{$ " " " $| i=h, j=13\}$

$\{$ " " " $| i=\ell, j=h\}$

→ ① Push 0's on stack
② Pop for each 1
③ If get perfectly to empty stack, repeat w/ 2's 3's.

$\varepsilon, \$ \to \varepsilon$
$\varepsilon, \$ \to \$$
$\varepsilon, \varepsilon \to \$$
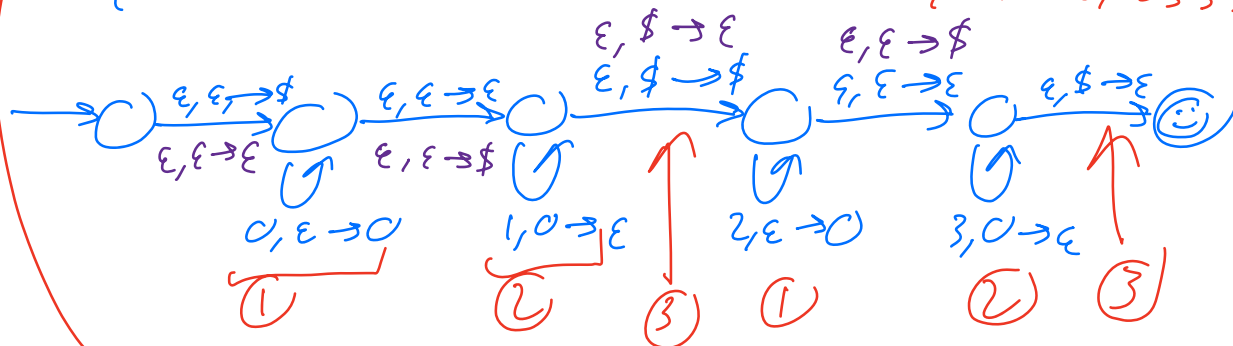


$\varepsilon, \varepsilon \to \$$     $\varepsilon, \varepsilon \to \varepsilon$     $\varepsilon, \varepsilon \to \$$     $\varepsilon, \varepsilon \to \varepsilon$     $\varepsilon, \$ \to \varepsilon$
$\varepsilon, \varepsilon \to \varepsilon$     $\varepsilon, \varepsilon \to \$$

$0, \varepsilon \to 0$     $1, 0 \to \varepsilon$     $2, \varepsilon \to 0$     $3, 0 \to \varepsilon$
① ② ③ ① ② ③

① Add 0's to que
② Deque for each 1
③ If perfect amount continue ($\$$ at end of que)

( [ ) ]

$\{0^i 1^j 2^i 3^j : i, j \geq 0\}$ → Impossible with stack
↳ Recall in opposite order you remembered.
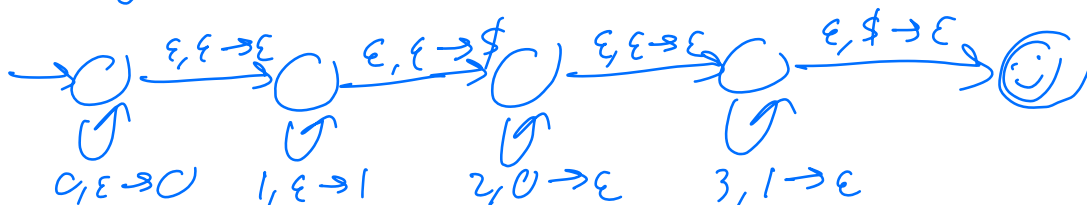


$\varepsilon, \varepsilon \to \varepsilon$     $\varepsilon, \varepsilon \to \$$     $\varepsilon, \varepsilon \to \varepsilon$     $\varepsilon, \$ \to \varepsilon$

$0, \varepsilon \to 0$     $1, \varepsilon \to 1$     $2, 0 \to \varepsilon$     $3, 1 \to \varepsilon$

$\{0^i 1^j 2^j 3^i | i, j \geq 0\}$

for PDA (similar to this GA.)

([])     with Q A

(1) Setup → $ OOC...# ||| ...△
   Queue
     front                ← back

(2) Cycle → # ||| ...△ $ OO...O
   to
   reorder

(3) Straightforward
    Comparison.         OO...#..((



Transitions:
$\varepsilon, \varepsilon \to \#$ ; $\varepsilon, \varepsilon \to \$$ ; $\varepsilon, \# \to \#$ ; $\varepsilon, \# \to \varepsilon$

self-loops: $C, \varepsilon \to O$ ; $|, \varepsilon \to |$ ; $\varepsilon, O \to O$ ; $\$, O \to \varepsilon$ ; $\varepsilon, | \to \varepsilon$

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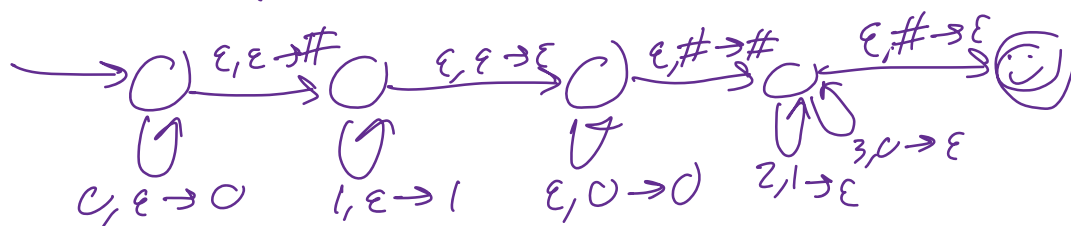
$\{ w \$ w^R \mid w \in \{C,|\}^* \}$
   ↳ Straight forward w/ PDA
     ↳ Many extra cycles w/ Q.A.

↑ Exam 1 ends here.

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Identifying Reg. Exp.
$\Sigma = \{ (, ), \emptyset, C, |, \cup, * \}$
L is strings generated by

- $\emptyset, C, | \in L$

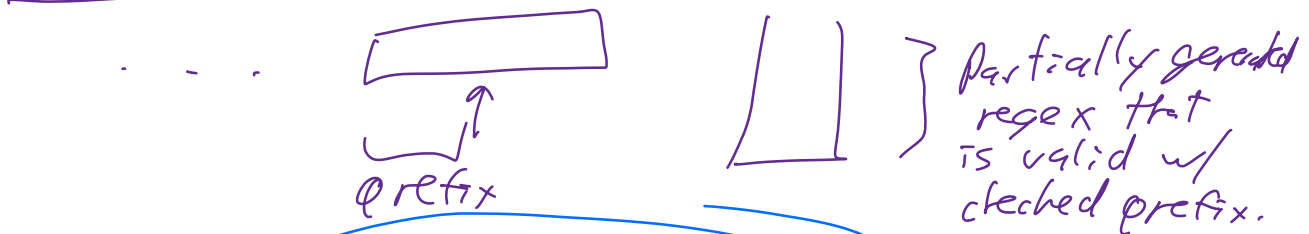- $(R_1 R_2), (R_1 \cup R_2), (R_1^*) \in L$ whenever $R_1, R_2 \in L$.

$(\emptyset \cup |) \in L$
$)( \notin L$

<u>Goal</u>: Recognize L w/ PDA.

┌─────────┐
│ High Level │ ( • Apply rules a bunch of times.
│ Idea │ ( • Have something in L on the stack
└─────────┘

Actually interweave this.

• Compare this to input, accept if match.

$(\emptyset \cup 1)$

$\begin{bmatrix} C \\ R \\ \cup \\ R \\ ) \end{bmatrix} \Rightarrow \begin{bmatrix} C \\ \emptyset \\ \cup \\ R \\ ) \end{bmatrix} \Rightarrow \begin{bmatrix} C \\ \emptyset \\ \cup \\ 1 \\ ) \end{bmatrix}$

Input
$[ ( \emptyset \cup 1 ) ]$
↑

Stack

$\begin{bmatrix} C \\ R \\ \cup \\ R \\ ) \end{bmatrix} \Rightarrow$

$[ ( \emptyset \cup 1 ) ]$
↑

$\begin{bmatrix} R \\ \cup \\ R \\ ) \end{bmatrix}$

$\Rightarrow$ $[ ( \emptyset \cup 1 ) ]$
↑

$\begin{bmatrix} \emptyset \\ \cup \\ R \\ ) \end{bmatrix}$

. . .

⬜
↑
prefix

$\begin{bmatrix} \\ \end{bmatrix}$ } Partially generated regex that is valid w/ checked prefix.

$c, 0 \to [ 1, 1 \to \varepsilon \quad \leftarrow$ also $(, ), \cup, *$
$q, c \to \varepsilon$

$\varepsilon, \varepsilon \to R\$ \qquad \varepsilon, \$ \to \varepsilon$

$\begin{bmatrix} R \\ \$ \end{bmatrix}$

$\varepsilon, R \to \emptyset$
$\varepsilon, R \to e$
$\varepsilon, R \to 1$
$\varepsilon, R \to (R R)$
$\varepsilon, R \to (R \cup R)$
$\varepsilon, R \to (R *)$

$q \quad a, b \to w_1 w_2 \dots w_n \quad r$

$q \quad a, b \to v_n \quad \varepsilon, \varepsilon \to w_{8-1} \varepsilon, \varepsilon \to w_{8-2} \to \dots @$

wlog, allow "multiples"

$1, 1 \to \varepsilon$
$c, 0 \to c$

$$\downarrow \quad \begin{array}{l} \emptyset,\emptyset \to \varepsilon \\ ),) \to \varepsilon \\ (,( \to \varepsilon \quad \#,\# \to \varepsilon \\ \cup,\cup \to \varepsilon \end{array}$$

$$\Sigma = \{ \overset{1}{(}, \overset{r_q}{)}, \overset{q}{\emptyset}, \overset{p}{\cup}, [ , \cup, \# \}$$
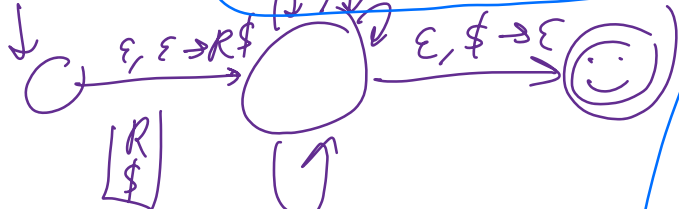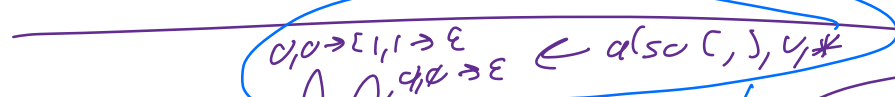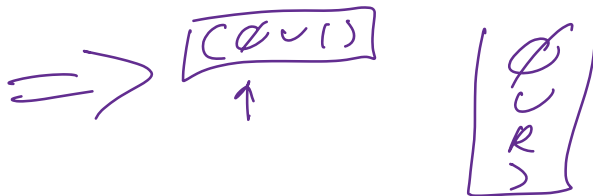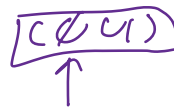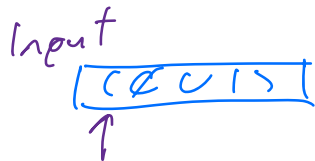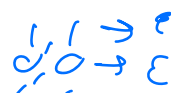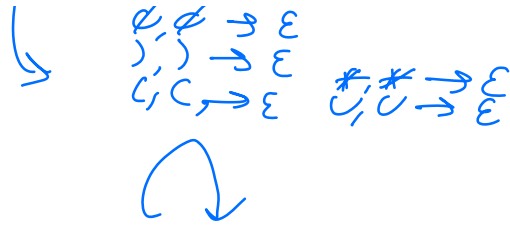
formally, a PDA is a 6-tuple

$$M = (Q, \Sigma, \Gamma, q_0, \delta, f)$$

$\quad \quad \quad \llcorner$ stack alphabet
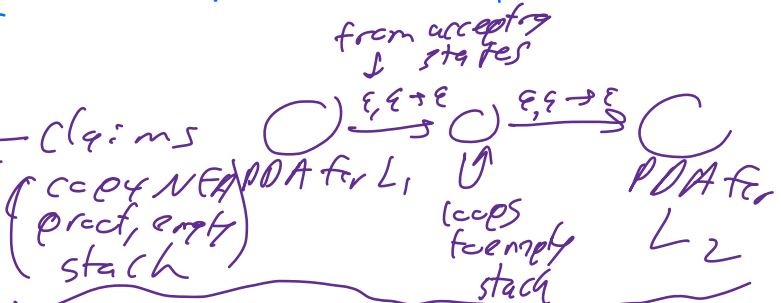$\quad \quad \quad \quad$ finite set of symbols

$\cdot \delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \longrightarrow P(Q \times (\Gamma \cup \{\varepsilon\}))$
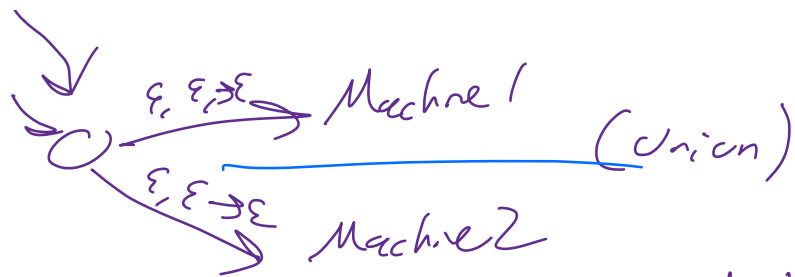


state diagram: states $q$, $r$, $s$ with transitions $a,b \to c$; $a,b \to d$; $a,b \to c$
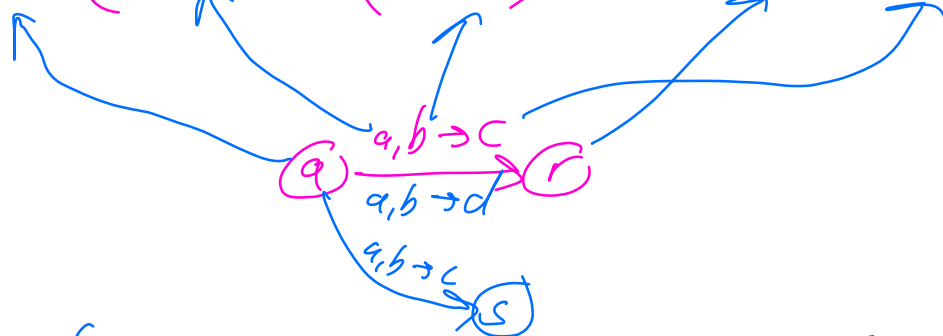
Is the class of languages recognized by PDAs closed under

$\checkmark \cdot$ Union
$\checkmark \cdot$ Concat $\longleftarrow$ claims
$\checkmark \cdot$ * $\longleftarrow$ $\checkmark$ (copy NFA⇒PDA for $L_1$
$\quad \quad \quad \quad \quad \quad \quad \quad$ (copy, empty
$\quad \quad \quad \quad \quad \quad \quad \quad$ stack)
$\times \cdot$ complement
$\times \cdot$ intersection
$\checkmark \cdot$ INSERT

from accepting
$\downarrow$ states

$\bigcirc \overset{\varepsilon,\varepsilon \to \varepsilon}{\longrightarrow} \bigcirc \overset{\varepsilon,\varepsilon \to \varepsilon}{\longrightarrow} \bigcirc$ PDA for $L_2$

$\cup$ loops
to empty
stack

$\boxed{A \wedge B = C}$

$\varepsilon, \varepsilon \to \varepsilon$ Machine 1

$\varepsilon, \varepsilon \to \varepsilon$ Machine 2

(Union)

High level idea works, but needs refinement.
(Expanded in concatenation addendum.)