CS 4510 Automata and Complexity

Exam 2A: Context-Free Languages

Date: 3:30pm-4:45pm Tuesday, October 25

- Name: _____ GTID#: _____
- **When handing in this test, please go to a TA with your BuzzCard to verify the name on the test.**
- Do not open this exam until you are directed to do so. Read all the instructions first.
- **Write your name and GTID# (numbers, not letters) on the top of every page.** Your GTID# can be found on your BuzzCard.
- By submitting this exam, you agree that your actions during this exam conform with the Georgia Tech Honor Code.
- This exam is out of 100 points, but there are **126 points up for grabs**. Any points you earn above 100 are bonus points!
- Write your solutions in the space provided. If you run out of space, continue your answer on the page of scratch paper provided at the end of the exam.
- The only outside material you may use on the exam is one (1) double-sided, hand-written, 8.5"x11" page of notes.
- Calculators are NOT permitted.
- You may use any of the theorems/facts/lemmas from the lecture notes, homeworks, or textbook without re-proving them unless explicitly stated otherwise.
- If you have a question, you may ask a TA. You should not communicate with anyone other than teaching staff during the exam.
- Do not spend too much time on any one problem! If you get stuck, move on and come back to that one later.
- Good luck!

Name: _____ GTID#: _____

0. **Following Instructions** (1 bonus point)

   Write your name and GTID# (numbers, not letters) on the top of every page. Your GTID# can be found on your BuzzCard.

1. **Circle the answers** (10 points total - 2 points each)

   Circle one. No explanation necessary.

   (a) Let L be a language that *is not* generated by any Chomsky normal form grammar. There MUST / MAY / DOESN'T exist a PDA which recognizes L.

   (b) Let G be the grammar which consists only of the rules $S \to SS \mid a$. Then G IS / ISN'T ambiguous.

   (c) Let G be the grammar which consists only of the rules $S \to SS \mid a$. Then G IS / ISN'T in Chomsky normal form.

   (d) Let L be a context-free language, let G be an ambiguous grammar for L, and let $x \in L$. Then G MUST / MAY / DOESN'T have at least two leftmost derivations of x.

   (e) Let M be a PDA that never pops from the stack. Then there MUST / MAY / DOESN'T exist an NFA which recognizes the same language as M.

Name: _____ GTID: _____

2. **Short answer**

(a) (5 points) Give a Chomsky normal form grammar for the language described by the regular expression 1^*.

(b) (5 points) Aibek is practicing the context-free pumping lemma, and he is currently trying to prove that $L = \{ww \mid w \in \{0,1\}^*\}$ is not context-free. He has already assumed that L is context-free and let p be the pumping length. The string he has chosen as the counterexample string which cannot be pumped is $0^p1^p0^p$. Explain to Aibek why his proof will fail in two sentences or less.

(c) (5 points) Give a Chomsky Normal Form grammar which is equivalent to the following grammar:

$$S \to YX$$
$$X \to Y \mid YYX \mid a$$
$$Y \to YY \mid b$$

Name: _____ GTID#: _____

3. **Context-Free Grammars** (30 points - 15 each)
Give context-free grammars for **two** of the following languages over the alphabet $\{0, 1, \#\}$:

   (a) $\Delta_1 = \{1^{n^3} \mid n \in \mathbb{Z}^{\geq 0}\}$

   (b) $\Delta_2 = \{1^n \# w w^{\mathcal{R}} \# 1^n \mid n \in \mathbb{Z}^{\geq 0}, w \in \{0,1\}^*\}$ (Recall $w^{\mathcal{R}}$ is the reverse of w.)

   (c) $\Delta_3 = \{0^i 1^j \mid i, j \in \mathbb{Z}^{\geq 0}, i + 1 = 3j\}$

   (d) $\Delta_4 = \{w \# 0^i \# 1^j \mid w \in \{0,1\}^*, w \text{ has } i \text{ 0's and } j \text{ 1's}\}$

Name: _____ GTID#: _____

4. **Pushdown Automata** (30 points - 15 each)
Give a pushdown automata for **two** of the following languages over the alphabet $\{0, 1, \#\}$. All arrows should be labelled with transitions of the form $x, y \to z$ where each of x, y, z is a single character or the empty string. (In particular, you *may not* push/pop multiple characters at once.)

   (a) $\Delta_1 = \{1^{n^3} \mid n \in \mathbb{Z}^{\geq 0}\}$

   (b) $\Delta_2 = \{1^n \# w w^{\mathcal{R}} \# 1^n \mid n \in \mathbb{Z}^{\geq 0}, w \in \{0,1\}^*\}$ (Recall $w^{\mathcal{R}}$ is the reverse of w.)

   (c) $\Delta_3 = \{0^i 1^j \mid i, j \in \mathbb{Z}^{\geq 0}, i + 1 = 3j\}$

   (d) $\Delta_4 = \{w \# 0^i \# 1^j \mid w \in \{0,1\}^*, w \text{ has } i \text{ 0's and } j \text{ 1's}\}$

Name: _____ GTID#: _____

5. **Pumping Lemma** (40 points - 20 each)
   Prove that **two** of the following languages over the alphabet $\{0, 1, \#\}$ are not context-free:

   (a) $\Delta_1 = \{1^{n^3} \mid n \in \mathbb{Z}^{\geq 0}\}$

   (b) $\Delta_2 = \{1^n \# w w^{\mathcal{R}} \# 1^n \mid n \in \mathbb{Z}^{\geq 0}, w \in \{0,1\}^*\}$ (Recall $w^{\mathcal{R}}$ is the reverse of w.)

   (c) $\Delta_3 = \{0^i 1^j \mid i, j \in \mathbb{Z}^{\geq 0}, i + 1 = 3j\}$

   (d) $\Delta_4 = \{w \# 0^i \# 1^j \mid w \in \{0,1\}^*, w \text{ has } i \text{ 0's and } j \text{ 1's}\}$

.....SCRATCH PAPER.....