CS4510 - Automata & Complexity

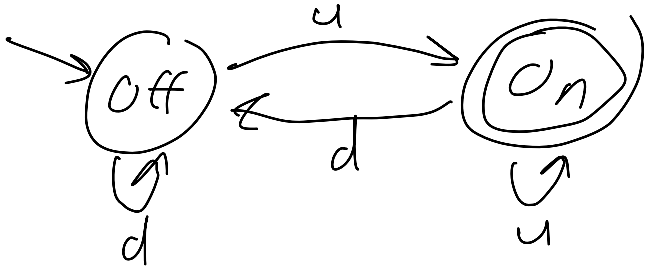
Prof: Jaeger

Understanding Computation ↗ Computer ↗ People → Thinking

① Computability: What problems are solvable?
(How strong does computer need to be?)

② Complexity: How "efficiently" can we compute?

→ formal, mathematic ⟹ Automata



uuddd u

Defn: A deterministic finite automat (DFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$:

① Q is a finite set of sets   e.g. $Q = \{On, Off\}$

② Σ is a finite alphabet (set of symbols) $\Sigma = \{u, d\}$

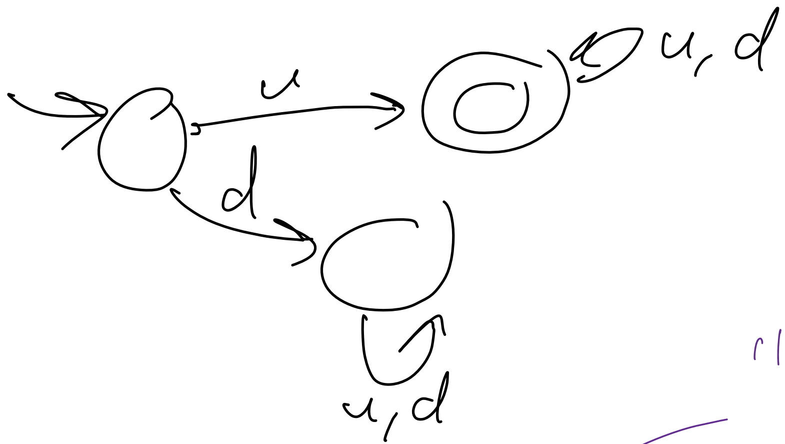
③ δ is our transition function   e.g. $\delta(\cdot, u) = On$

$\delta : Q \times \Sigma \to Q$ $\delta(\cdot, d) = Off$

④ $q_0 \in Q$ is start state $q_0 = Off$

⑤ $F \subseteq Q$ is set of accepting states $F = \{On\}$

E.g.: Everything that starts with u.



"dot notation"

Kleen star

Defn: If M is a DFA and $w \in M.\Sigma^*$ then M accepts w if there exists a sequence of states $q_1, \dots, q_{|w|}$ s.t. $\forall i \; \delta(q_i, w_i) = q_{i+1}$ and $q_{|w|} \in F$.

set of sequences of characters from $M.\Sigma^*$

$i = 0, \dots, |w|-1$

ith character in w

$(w = w_0 w_1 w_2 \dots w_{|w|-1})$

Defn: $L(M) = \{ w : w \in M.\Sigma^*, \; M \text{ accepts } w \}$

e.g. If $M.\Sigma = \{u, d\}$

Then $M.\Sigma^*$ is all finite strings consisting of u's and d's

$M.\Sigma^* = \{u, d, uu, ud, du, dd, uuu, \ldots\}$

$|u| = 1$ $|uuu| = 3$ $|du| = 2$

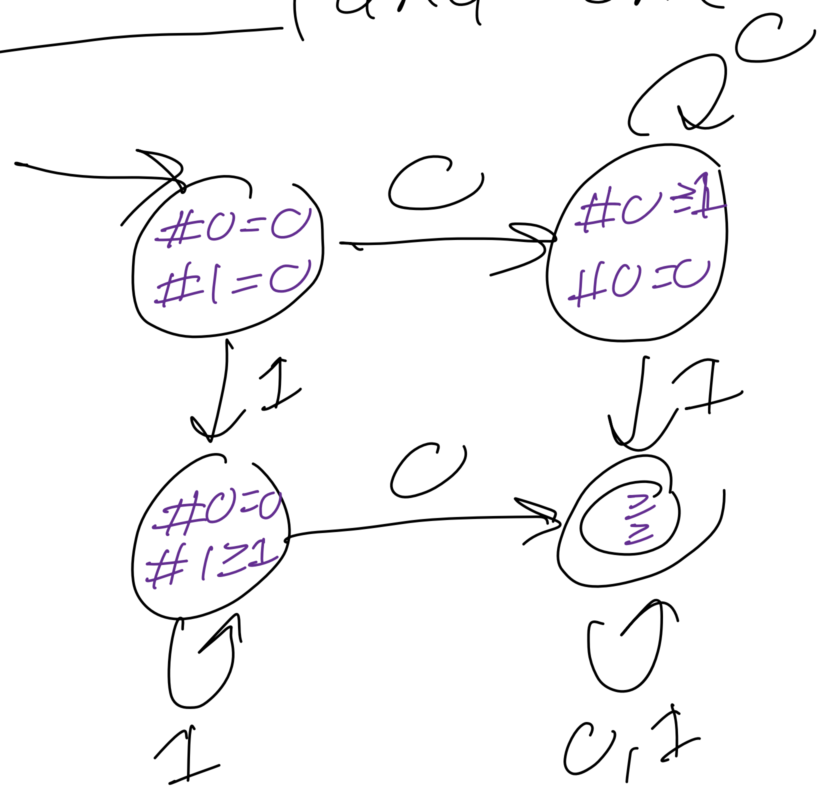
$M.\Sigma^*$ also contains the empty string.

We call this string ε.

\varepsilon

E.g. Build M s.t. $L(M) = \{w \in \{0,1\}^* : w$ contains

$\boxed{M.\ \Sigma=\{0,1\}}$ $\{$at least one 1 and one 0.$\}$



To prove minimality, show that ε, 0, 1, and 01 all must go to different states.

<u>Thm.</u> If L is accepted by some DFA, then its complement $\overline{L}$ is accepted by some DFA.

Pf. Let M accepts L. $(L(M)=L)$

Then build N which is the exact same except

$$\underline{N.F = M.G - M.F}.$$

set subtraction.

Clearly $L(N) = \bar{L}$.

Defn. If $L = L(M)$ for DFA M, then we say L is regular.

Above Thm. says that the set of regular languages is closed under complement.

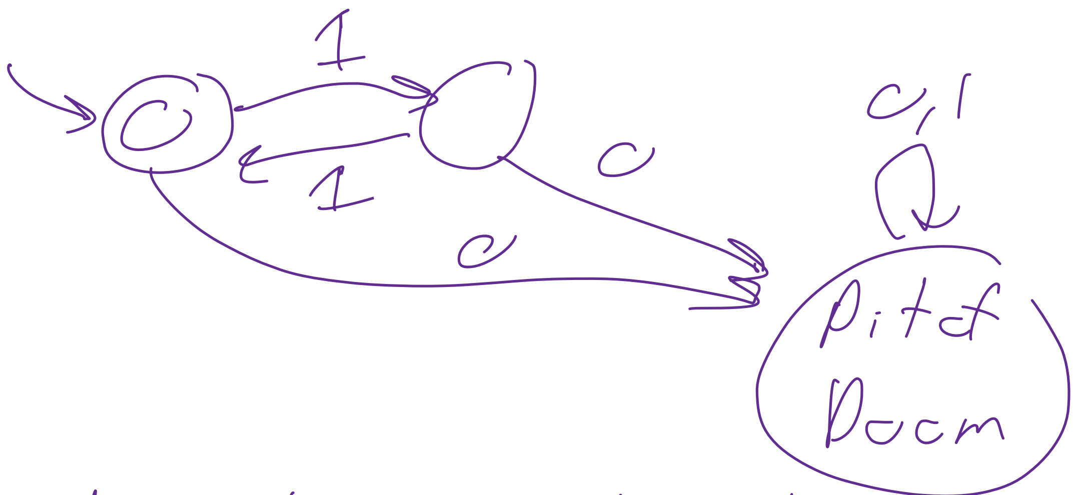
These are "languages" which are sets of strings.

(i.e. subsets of Σ^*.)

<u>Define</u> $REG = \{L : L = L(M)\}$ for some DFA M.

REG closed under complement means $\bar{L} \in REG$ whenever $L \in REG$.

<u>E.g.</u> $L = \{w : w = 1^n$ and n is even.$\}$



$L = \{w : w = 1^n$ n is divisible by $7\}$

$Q = \{0, \dots, 6\}$

$q_0 = 0$

$\Sigma = \{1\}$

$F = \{0\}$

$\delta(q, x) = q + 1 \bmod 7$

$L = \{ w \in \underbrace{\{0,1\}^*}_{} : \text{int}(w) = 3 \bmod 7 \}$

$\{0, \dots, 9\}$ is easy extension

1011 is eleven

$\underbrace{1011}_{},\underbrace{0}_{}$

$1011 \to 10110$, $1011 \to 10111$

① multiply by 10

② Add 0 or 1

$Q = \{0, \dots, 6\}$

$q_0 = 0$

$F = \{3\}$

$\Sigma = \{0, 1\}$

$\delta(q, x) = \underbrace{10q + x \bmod 111}_{2q + x \bmod 7}$

E.g. If L, L' are regular

is $L \cap L' = \{w : w \in L \text{ and } w \in L'\}$

also regular?

(Is REG closed under intersection?)

Pf. Yes. Let $L(M) = L$ and $L(N) = L'$.

Build O.

$$O.\Sigma = M.\Sigma = N.\Sigma$$

$$O.Q = M.Q \times N.Q = \{(q, q') : q \in M.Q,\ q' \in N.Q\}$$

$$q_0 = (M.q_0, N.q_0)$$

$$F = M.F \times N.F$$

$$\delta((q, q'), x) = (M.\delta(q, x), N.\delta(q', x))$$

$$L = \{w : w = 0^n 1^n\}$$