

## CS 4510 Automata and Complexity

### Exam 2: Context-Free Languages

*Date: 3:30pm-4:45pm Monday, March 14*

- Name: \_\_\_\_\_ GTID: \_\_\_\_\_
- **When handing in this test, please go to a TA with your GTID to verify the name on the test.**
- Do not open this exam until you are directed to do so. Read all the instructions first.
- **Write your name and user id (letters, not numbers) on the top of every page.**
- By submitting this exam, you agree that your actions during this exam conform with the Georgia Tech Honor Code.
- Write your solutions in the space provided. If you run out of space, continue your answer on the page of scratch paper provided at the end of the exam. Do NOT write on the backs of the pages, only the front of each page will be scanned.
- The only outside material you may use on the exam is one (1) single-sided, hand-written, 8.5"x11" page of notes.
- Calculators are NOT permitted.
- You may use any of the theorems/facts/lemmas from the lecture notes, homeworks, or textbook without re-proving them unless explicitly stated otherwise.
- If you have a question, you may ask a TA. You should not communicate with anyone other than teaching staff during the exam.
- Do not spend too much time on any one problem! If you get stuck, move on and come back to that one later.
- Good luck!

	Grade	Grading TA
Name on EACH page		
Short Answer		
Grammars		
Pushdown Automata		
Pumping Lemma		
Bonus		
Total		

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**0. Following Instructions** (1 bonus point)

Please write your name and GTID (letters, not numbers) at the top of EACH page of this exam.

**1. Short answer**

- (a) (5 points) Give an unambiguous grammar which is equivalent to the following grammar:

$$S \rightarrow SS \mid a$$

- (b) (5 points) Aibek is practicing the context-free pumping lemma, and he is currently trying to prove that  $L = \{x \mid x \text{ has exactly the same number of 0's, 1's and 2's}\}$  is not context-free. He has already assumed that  $L$  is context-free and let  $p$  be the pumping length. The string he has chosen as the counterexample string which cannot be pumped is  $(012)^p$ . Explain to Aibek why his proof will fail in two sentences or less.

- (c) (5 points) Give a Chomsky Normal Form grammar which is equivalent to the following grammar:

$$\begin{aligned} S &\rightarrow BA \\ A &\rightarrow a \mid \varepsilon \\ B &\rightarrow BB \mid b \end{aligned}$$

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2. **Context-Free Grammars** (30 points - 15 each)

Give context-free grammars for each of the following languages:

(a)  $L_1 = \{a^i b^{2i-2} \mid i \in \mathbb{Z}^+\}$

(b)  $L_2 = \{a^i b^j c^k d^m \mid i, j, k, m \in \mathbb{Z}^{\geq 0}, i + j + m = k\}$

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3. **Pushdown Automata** (30 points - 15 each)

Give a pushdown automata for each of the following languages.

(a)  $L_1$  as defined in problem 2a.

(b)  $L_3 = \{w \mid w \text{ is a binary palindrome of odd length, whose first and middle characters are the same}\}$

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4. **Pumping Lemma** (25 points)

Prove that the following language is not context-free:

$$L = \{w_1\#w_2 \mid w_1 \text{ and } w_2 \text{ are binary strings of the same length and contain the same number of 0's}\}$$

Some example strings in this language are 1#1 or 0101#1001.

.....SCRATCH PAPER.....