CS 4510 Automata and Complexity 04/06/2022

Worksheet 6: Undecidability Reductions

Worksheets are provided for your study purposes only. They are not graded. The answers to some of the questions will be uploaded to Canvas soon. For the rest of questions, however, you are encouraged to discuss on Canvas and try to solve them collaboratively. You are also welcomed to discuss the problem during office hours.

In this worksheet, you will be asked to provide regular expressions for regular languages.

1. **Problem 1**

   $L_1 = \{\langle M_1\rangle, \langle M_2\rangle, x | x \notin L(M_1) \text{ and } x \in L(M_2)\}$. Show that L_1 is undecidable.

2. **Problem 2**

   $L_2 = \{\langle M\rangle | L(M) \text{ contains the string "GT"}\}$. Show that L_2 is undecidable.

3. **Problem 3**

   a. Show that $L_3 = \{\langle M\rangle | M \text{ accepts } \langle M\rangle\}$ is undecidable.

   b. Show that $L_4 = \{\langle M\rangle, x, y | M \text{ accepts x or } M \text{ accepts y but not both}\}$ is undecidable.

   c. Show that $L_5 = \{\langle M\rangle, \langle N\rangle | \ |L(M)| < |L(N)|\}$ is undecidable.

   ***d.** Show that $L_6 = \{\langle M\rangle, x | M \text{ never moves its head to the left on input } x\}$ is **decidable**.

4. **Problem 4**

   a. Show that $L_7 = \{\langle M\rangle | L(M) = \Phi\}$ is undecidable.

   b. Show that $L_8 = \{\langle M\rangle, \langle N\rangle | L(M) = L(N)\}$ is undecidable.

5. **Problem 5**

   Show that $Finite_{TM} = \{\langle M\rangle | M \text{ is a TM and } L(M) \text{ is finite}\}$ is undecidable.

6. **Problem 6**

   A useless state in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

7. **Problem 7**

   EVEN-HAL$T_{TM} = \{\langle M\rangle, x | M \text{ is a TM and M(x) halts after an even number of steps}\}$. Show that EVEN-HAL$T_{TM}$ is undecidable.