Worksheet Solutions 4: Context-Free Grammars

Spoiler alert: The following content contains spoilers to worksheet 4. If you have not done so already, you are strongly recommended to try out those questions yourself first.

- **Problem 2**

  a. $S \rightarrow A1A$
  $A \rightarrow 0A1 \mid 1A0 \mid AA \mid 1A \mid \varepsilon$

  c. $S \rightarrow 0S0 \mid 1S1 \mid A$
  $A \rightarrow 1B0 \mid 0B1$
  $B \rightarrow 0B \mid 1B \mid \varepsilon$

  e. $S \rightarrow A \mid B$
  $A \rightarrow 0A1 \mid A1 \mid 1$
  $B \rightarrow 00B1 \mid 0B \mid 0$

- **Problem 4**

  a. $S \rightarrow S_1 D \mid AS_2$
  $S_1 \rightarrow aS_1c \mid B$
  $B \rightarrow Bb \mid \varepsilon$
  $D \rightarrow Dd \mid \varepsilon$
  $S_2 \rightarrow bS_2d \mid C$
  $A \rightarrow Aa \mid \varepsilon$
  $C \rightarrow C \mid \varepsilon$

  b. Note that either i ≥ j or k ≥ m.
  If i ≥ j, we can say that we are actually generating $a^x(a^jb^j)c^kd^m$ where $x+k = m$.
  $S_1 \rightarrow aSd \mid BC$
  $B \rightarrow aBb \mid \varepsilon$
  $C \rightarrow cCd \mid \varepsilon$

  The case where k ≥ m implies we are actually generating $a^ib^jc^k(c^md^m)$ where $i + k = m$.
  $S_2 \rightarrow AD$
  $D \rightarrow cDd \mid \varepsilon$
  $A \rightarrow LR$
  $L \rightarrow aLb \mid \varepsilon$
  $R \rightarrow bRc \mid \varepsilon$

- **Problem 6**

  $S \to A\,\text{``0''} \mid A\,\text{``1''} \mid A\,\text{``}\varepsilon\text{''} \mid S\,\text{``}\cup\text{''}\,S \mid \text{``(''}\,S\,\text{``)''} \mid S\,\text{``*''}$

  $A \to S \mid \varepsilon$

- **Problem 7**

  $S \to \varepsilon \mid SS \mid A$

  $S \to \text{<aaa>}S\text{</aaa>} \mid \text{<bbb>}S\text{</bbb>} \mid \text{<ccc>}S\text{</ccc>}$

  $S \to \text{<abc>}S\text{</abc>} \mid \text{<cba>}S\text{</cba>}$

  $S \to \text{<aaa/>} \mid \text{<bbb/>} \mid \text{<ccc/>} \mid \text{<abc/>} \mid \text{<cba/>}$