CS 4510 Automata and Complexity 2/2/2022

Homework Worksheet 3: The Pupming Lemma

Worksheets are provided for your study purposes only. They are not graded. The answers to select questions will be uploaded to Canvas. For the rest of questions, however, you are encouraged to discuss on Piazza and try to solve them collaboratively. You are also welcomed to discuss theses problem during office hours.

In this worksheet, you will be utilizing the Pumping Lemma to show that languages are not regular. The pumping lemma has been defined below for your convenience.

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Figure 1: Pumping Lemma from Sipser.

Problems

Problem 1

Define a language L_1 which consists of binary strings $L_1 = \{0^n 1^{n^2} \mid n > 0\}$. Show that L_1 is not regular using the Pumping Lemma.

Problem 2

Let $L_2 = \{(10)^i 1^j \mid i \geq j,\ i, j \in \mathbb{Z}^{\geq 0}\}$. Prove that L_2 is not regular using the Pumping Lemma.

Problem 3

$L_3 = \{1^z \mid z = 2^k,\ k \in \mathbb{Z}^{\geq 0}\}$. Prove that L_3 is not regular using the Pumping Lemma.

Problem 4

Show that $\{0^i 1^j \mid i < j \text{ or } i > 2j\}$ is not regular.

Problem 5

$L_5 = \{0^i 1^j 2^{i*j} \mid i, j \geq 0\}$. Prove that L_5 is not regular using the Pumping Lemma.

Problem 6

$L_6 = \{0^i 1^j 2^k \mid i, j, k \geq 0, i = j \text{ or } i = k\}$. Show that L is not regular.

More Problems

Problem 7

Let $L_7 = \{0^p \mid p \text{is a prime}\}$. Is it regular? Prove your answer.

Problem 8

Is L_8 regular? $L_8 = \{u^4 v^i u^j v^7 \mid i, j \geq 0\}$

Problem 9

What is wrong with the following proof that all finite languages are regular?

Let L be a finite language. Then it has some longest (or tied for longest) string s. But s cannot be pumped up, since then it would be longer than the longest string in L. This is a contradiction to the fact that all long strings in a regular language can be pumped.

Trickier Problems

Problem 10

Let L_0 be the set of all valid regular expressions (as learned in class) over the alphabet $\Sigma = \{0, 1\}$. For example, $(01)^*, \varepsilon \cup 1 \in L$, but $0\cup, (101 \notin L$. *Note: The Regular Expression operators are part of the input string.*

a) What is the alphabet of L_0?

b) Using the pumping lemma, show L_0 is not a regular language.

Problem 11

Show that $\{a^i b^j \mid i, j$ are relatively prime$\}$ is not regular. *Recall two numbers are relatively prime if they share no factors other than 1.*

Problem 12

Argue that, for any fixed k, the DFA that recognizes the language 1^k has at least $k + 2$ states.