CS 4510 Automata and Complexity

Exam 3: Practice

- Name: ____ GTID: ____
- Any topic covered in lecture after exam 2, in lecture notes 13-21, and/or in homeworks 6-9 are fair game for the exam. (Additionally, although the exam is not cumulative, you are expected to still be familiar with earlier topics.) Absence of a topic from this practice exam does NOT imply an absence of that topic from the exam. Similarly, the actual exam may differ in length, format, and difficulty from this practice exam.
- We will go over this practice exam in class on Monday, April 25. In the meantime, you are encouraged to discuss this exam on Piazza and in office hours!
- Calculators are NOT permitted.
- You may use any of the theorems/facts/lemmas from the lecture notes, homeworks, or textbook without re-proving them unless explicitly stated otherwise.
- Good luck!

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1. **True or False**

   (a) T F If M is a Turing machine that accepts x and y, then M must accept xy.

   (b) T F Let A and B be languages. If $A \cup B$ is decidable, then $\overline{A \cup B}$ must be decidable.

   (c) There are only countably many regular languages.

   (d) T F If $A \leq_m B$, and A is recognizable, then B is never non-recognizable.

   (e) T F If $SAT \in P$, then all NP-hard problems are in P.

   (f) T F Suppose M is a $PSPACE$-decider for L. Then an accepting tableau for M on some string x is always polynomially *wide*. (That is, the length of each row is bounded by a polynomial in the length of x.)

   (g) T F Suppose M is a $PSPACE$-decider for L. Then an accepting tableau for M is always polynomially *tall*. (That is, the number of rows is bounded by a polynomial in the length of x.)

   (h) T F If M decides whether $x \in L$ in super-exponential time (i.e. $O(2^{2^{|x|}})$), it also does so in super-exponential space.

2. **Model Equivalence**

   Let a *match-TM* be a Turing-Machine that doesn't have an accepting state or a rejecting state. Instead it has a single halting state. If a match-TM ever enters the halting state, it immediately stops computing. Then, if the current contents of the tape are exactly equivalent to the starting contents of the tape it accepts, otherwise it rejects.

   Show that match-TM's are equivalent in power to ordinary TM's.

3. **Recognizable Languages**

   For each of the following languages, if that language is recognizable, give a recognizer for it. If its complement is recognizable, give a recognizer for the complement. (For each language below, either it or its complement is recognizable, but not both.)

   - $L_1 = \{M \mid M$ never enters the reject state for any input (it either accepts or loops forever)$\}$
   - $L_2 = \{M, x, y \mid x \in L(M)$ or $y \in L(M)$ or both$\}$

4. **Undecidability Reductions**

   Prove that $\{M \mid M$ accepts "*hello*" or M accepts "*goodbye*" but not both$\}$ is undecidable using a reduction.

5. **Time Complexity**

   Prove that if $L \in P$, then $\overline{L} \in P$.