CS 4510 Automata and Complexity

Exam 3: Turing Machines

Date: 2:40pm-5:30pm Friday, April 29

- Name: ____ GTID: ____
- **When handing in this test, please go to a TA with your GTID to verify the name on the test.**
- Do not open this exam until you are directed to do so. Read all the instructions first.
- **Write your name and user id (letters, not numbers) on the top of every page.**
- By submitting this exam, you agree that your actions during this exam conform with the Georgia Tech Honor Code.
- Write your solutions in the space provided. If you run out of space, continue your answer on the page of scratch paper provided at the end of the exam. Do NOT write on the backs of the pages, only the front of each page will be scanned.
- If you have any work on detached pages that you want graded, staple them to your exam before handing it in.
- The only outside material you may use on the exam is one (1) single-sided, hand-written, 8.5"x11" page of notes.
- Calculators are NOT permitted.
- You may use any of the theorems/facts/lemmas from the lecture notes, homeworks, or textbook without re-proving them unless explicitly stated otherwise.
- If you have a question, you may ask a TA. You should not communicate with anyone other than teaching staff during the exam.
- Do not spend too much time on any one problem! If you get stuck, move on and come back to that one later.
- Good luck!

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0. **Following Instructions** (1 bonus point)

   Please write your name and GTID (letters, not numbers) at the top of EACH page of this exam.

1. **Short answer** (24 points)

   Circle TRUE or FALSE. No justification necessary. Recall that "$A \leq_P B$" means "there is a polynomial-time mapping reduction from A to B."

   (a) (TRUE / FALSE) If M never halts on any input, then $L(M) = \emptyset$.

   (b) (TRUE / FALSE) If $\overline{HALT} \leq_m A$, then A must be non-recognizable.

   (c) (TRUE / FALSE) There is no A such that both A and $\overline{A}$ are recognizable but not decidable.

   (d) (TRUE / FALSE) If $A \leq_P B$, and A is NP-complete, then B must be NP-complete.

   (e) (TRUE / FALSE) $HALT$ is a countable set.

   (f) (TRUE / FALSE) If A is NP-complete, and B is NP-complete, then it is definitely the case that $A \leq_P B$ *and* $B \leq_P A$.

   (g) (TRUE / FALSE) Let M be a Turing machine that never uses any tape cells except for the ones originally containing the input (which it may overwrite as normal). Then the number of rows in an accepting tableau for M on any string x is bounded by a polynomial function of the length of x.

   (h) (TRUE / FALSE) If P = PSPACE, then there must be some polynomial time algorithm for SAT.

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2. **Model Conversion** (22 points)

   Let an *eager-write* Turing machine be similar to an ordinary Turing machine except that, whenever it writes a character to a square, it also writes that same character onto the cells to the immediate left and right of the tape head. Prove that eager-write Turing machines are equal in power to ordinary Turing machines. Note that, like ordinary Turing machines, eager-write Turing machines *must* write during each step.

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3. **Undecidability** (22 points)

   Prove that $L = \{\langle M \rangle, \langle N \rangle, x \mid M$ accepts x or N accepts x but *not* both$\}$ is undecidable.

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4. **Recognizability** (12 points)

For each of the following languages, if that language is recognizable, give a recognizer for it. If its complement is recognizable, give a recognizer for the complement. (For each language below, either it or its complement is recognizable, but not both.) **Clearly indicate whether you are recognizing the given language or its complement.**

(a) $L_1 = \{\langle M \rangle, x \mid \text{ there is some } y \text{ such that } M \text{ accepts } xy\}$

(b) $L_2 = \{\langle M \rangle \mid M \text{ accepts at most one of ``hello'' and ``goodbye''}\}$

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5. **Time Complexity** (20 points)

   Prove that if L is in NP, then L^* is in NP.

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6. **Extra Credit** (6 points)

Recall that, in the proof of the Cook-Levin theorem, the sub-formula φ_{start} forces the first row of the tableau to be the starting configuration of M on w. Recall also that $x_{i,j,a}$ is a boolean variable which is true if and only if the cell at row i and column j in the tableau contains symbol a.

Let M be a machine with start state q_s that always uses exactly $2n$ tape cells on inputs of size n (including the input cells), and let $w =$ "*hello*". Give the formula for φ_{start}.

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7. **Extra Credit 2** (6 points)

   Prove that $DIF = \{\langle M\rangle, \langle N\rangle \mid L(M) \neq L(N)\}$ is **non-recognizable**.

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