Announcements

- HW1 due 10 pm
- HW2 released 10pm
- Practice worksheets on Canvas

Last time

~ "Reg is minimal class of languages containing all finite languages and closed under $\cup, \circ, *$"

Set	Reg. Exp.
$\{0\}\circ\{0\}\circ\{1\}$	001
	$(0\cup1)^*001$

$L(001) = \{0\}\circ\{0\}\circ\{1\}$

Show how to construct these regular languages

1. $\{001\}$ e.g. $\{0\}\circ\{0\}\circ\{1\}$
2. all strings ending in $001 = (\{0\}\cup\{1\})^*\circ\{0\}\circ\{0\}\circ\{1\}$
3. all strings containing 001 as a substring $\underbrace{(\{0\}\cup\{1\})^*\circ\{0\}\circ\{0\}\circ\{1\}}_{\text{that}}\circ(\{0\}\cup\{1\})^*$
4. all strings $(\{0\}\cup\{1\})^*$
5. all strings not containing 001 as a substring
6. all even length strings

$\Sigma\Big(\underbrace{\big((\{0\}\cup\{\varepsilon\})\circ\{1\}\big)^*}\circ\{0\}^*$

this part is $\underline{\{01, 1\}^*}$ which can also be written as

$\big((\{0\}\circ\{1\})\cup\{1\}\big)^*$

$\big((\{0\}\cup\{1\})\circ(\{0\}\cup\{1\})\big)^*$

$(\Sigma\Sigma)^*$

Prop $\{\ \}, \circ$

<u>Defn:</u> A regular expression is

- ε, $\emptyset$, c, or 1
- or $(R) \cup (R)'$, $(R)(R')$, or $(R)^*$ where R, R' are regular expressions

Associative $(A+B)+C = A+(B+C)$

$(A \times B) \times C = A \times (B \times C)$

$A+B+C \qquad A \times B \times C$

$\cup, \circ$ are both associative

$A \cup (B \cup C) \qquad A \cup B \cup C$

$A(BC) \qquad ABC$

$A \times B + C = (A \times B) + C$

$()$ before $*$ before $\circ$ before $\cup$

<u>ab^*</u> vs. $(ab)^*$

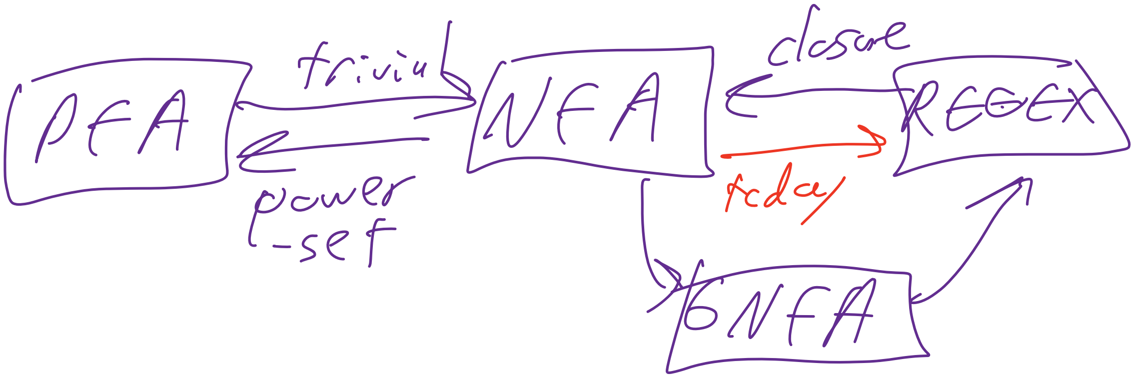
$ab\ldots b \qquad abab\ldots$

- Notation write ε in place of $\emptyset \cup 1$ (or similar)

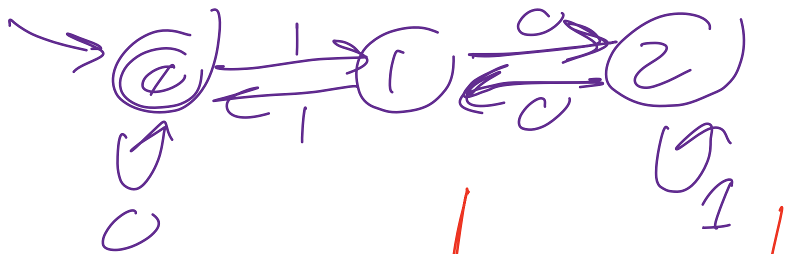
$a^+ = aa^*$

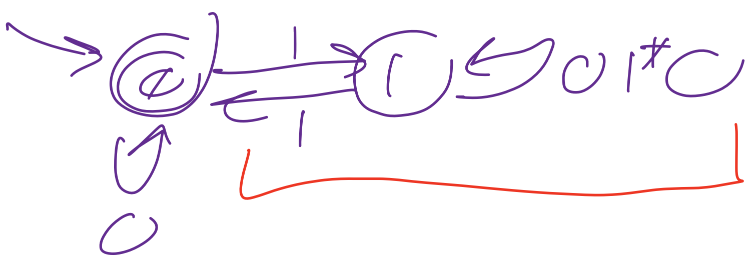
- $\emptyset R = \emptyset$ • $\emptyset^* = \varepsilon$ • $\emptyset \cup R = R$
- $\varepsilon R = R$ • $\varepsilon^* = \varepsilon$ • $\varepsilon \cup R = \varepsilon \cup R$

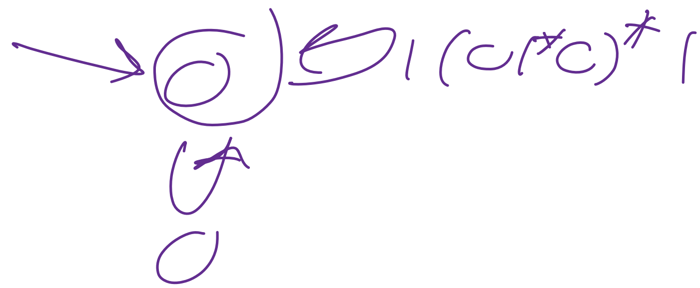
$\underline{\emptyset} \circ \underline{R}$



- Binary numbers divisible by 3.







$$(0 \cup 1(01^*0)^*1)^*$$

Defn: A GNFA is "like" an NFA except transitions can be labeled with any reg. exp.

E.g. $\bigcirc \xrightarrow{\varepsilon} \bigcirc$ are as before

$\bigcirc \xrightarrow{\phi} \bigcirc$ cannot be taken.

DFA/NFA w/ n states $\rightarrow$ GNFA with n states $\rightarrow$ "special" GNFA w/ n+2 states

$\rightarrow$ special GNFA w/ n+1 states $\rightarrow \ldots \rightarrow$ GNFA w/ 2 states

Special Restrictions

- Start state w/ no incoming transitions
- One accepting state w/ no outgoing transitions.

• Otherwise every pair of states has exactly one transition between them.

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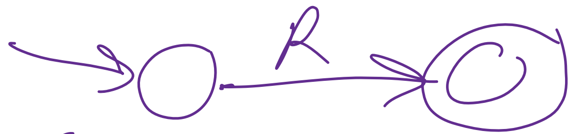
w/2 states



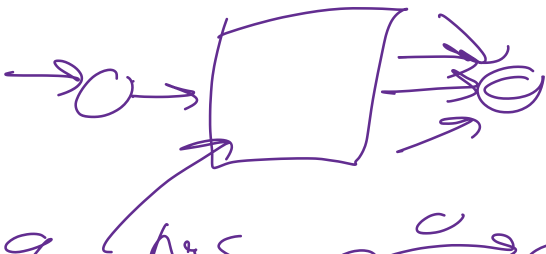
So R is a reg. exp. for this machine.

① Start DFA A. View as GNFA.

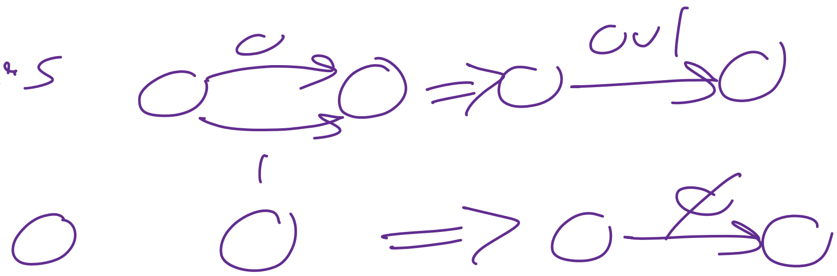
② make it follow rule 1.

③ make it follow rule 2.

④ make it follow rule 3.



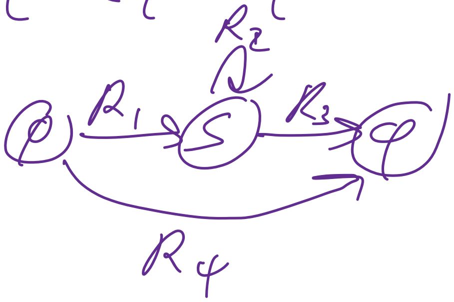
E.g. has



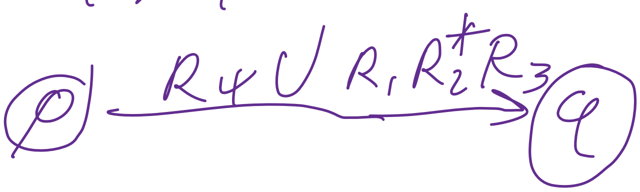
⑤ Loop to remove states.

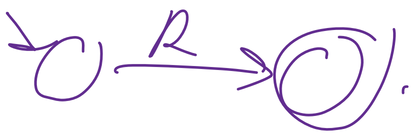
• Pick a non-start/accepting state s we will remove it.

- for every other pair of states p, q ($p = q$ allow) we have



After ripping out s we will make p, q have



Repeat until 

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R is your reg. ex.

Let's show closure properties via reg. ex.

If w is a string, let w^R be w written backwards. (e.g. $(abc)^R = cba$.

If L is a language $L^R = \{w^R : w \in L\}$.

Claim. REG is closed under R.

Pf. We're going to define a function (recursively) $f : \text{REGEX} \to \text{REGEX}$

$L(f(G)) = L(G)^R$.

	reg. ex. G	$f(G)$
Base case	$\emptyset$	$\emptyset$
	ε	ε
	0	0
	1	1
Inductive step	$R_1 \cup R_2$	$f(R_1) \cup f(R_2)$
	$R_1 \circ R_2$	$f(R_2) \circ f(R_1)$
	R^*	$f(R)^*$

$\emptyset^R = \emptyset$

$\{\varepsilon\}^R = \{\varepsilon\}$

$(L_1 \cup L_2)^R = L_1^R \cup L_2^R$

$(01)(ab)$

$ba10$

E.g. Given $(0\cup1)^*001$ $\xrightarrow{\text{answer}}$ $100(0\cup1)^*$

$$f(((0\cup1)^*0)(01)) = f(01)\, f((0\cup1)^*0)$$
$$= f(1)f(0)\ f(0)\ f((0\cup1)^*)$$
$$= 100\ f((0\cup1))^*$$
$$= 100(f(0)\cup f(1))^*$$
$$= 100(0\cup1)^*$$

Let $enc : \Sigma \to \Sigma^*$.

Define $enc(L) = \{enc(w_1)enc(w_2)\ldots \mid w_1 w_2 \ldots \in L\}$

REG is closed under enc.

first proof of non-regularity.

$\{0^n 1^n : n \in \mathbb{Z}\}$ is non-regular.

Observation: Let $x, y, z \in \Sigma^*$ be given.
If $zx \in L$ and $yx \notin L$. Then if D is DFA for L, D must map z and y to different states.

(If z, y went to same state, then zx and yx must go to same state as each other. Contradiction!)

Assume D is a DFA for $\{0^n 1^n \mid n \in \mathbb{Z}\} = L$.
Consider 0^i and 0^j for $i \neq j$.

$$0^i 1^i \in L$$
$$0^j 1^i \notin L$$

so 0^i and 0^j must go to different states.
Then we have infinitely many states, contradicting assume D is DFA.
Hence L is not regular.