Announcements: • HW2 due today
• HW3 released today
• Exam review next Thursday
↳ "Student Led": I come without a plan. You ask me about what you want. (Practice Exams? Worksheets? ...)

Non-regular Languages
• $\{ww \mid w \in \{0,1\}^*\}$
• $\{0^m 1^n \mid m > n\}$ ↳ Pump down $i = 0$
• $\{wv \mid v \text{ is } w \text{ with bits flipped}\} = A$

Pf. We use PL (Contrapositive).
Let p be given. Pick $s = 0^p 1^p$.
Let $xyz = s$ be given with $|xy| \leq p$ and $|y| \neq 0$. Then y is all zeros.
Picking $i = 2$, xy^2z will have more 0's than 1's so is not in the language. ☒

Pf (Closure Properties)
General strategy: Assume A is regular. Use closure properties to show B is regular.
Prove B is not regular. Contradiction → A is nonregular.
Pf: Proposal 1 $A \circ A^R$ ← doesn't work.
Assume A is reg. Then $A \cap 0^*1^* = \{0^n 1^n \mid n \geq 0\}$

0*1* is regular and reg. lang. are closed under $\cap$, so B is reg. Contrad.

(Be careful about direction. We construct B from A. Not vice versa.)

Ex. $\{a^n b^n \mid n \geq 0\} = A$

Recall) If $enc : \Sigma \to \Sigma^*$ then enc(A) is regular if A is.

$B = \{0^n 1^n : n \geq 0\}$ define $enc(a) = 0$, $enc(b) = 1$

then $enc(A) = B$ so A is nonregular.

Ex. Define $\Sigma = \{0, 1, (,), \phi, \cup, *\}$

A is minimal language such that

- $\phi, 0, 1 \in A$
- $(R_1 \cup R_2), (R_1 R_2), (R_1^*) \in A$ if $R_1, R_2 \in A$.

Define $A' = \{$strings w/ balanced parens$\}$

↳ $() \in A'$ $)(\notin A'$

$(()()) \in A'$ $()) \notin A'$

A' is non-regular. $A' \cap (*)^* =$
(Closure & contradiction) $\{(^n)^n \mid n \geq 0\}$

Pf. Assume A is regular.

Define $enc(x) = \begin{cases} (& \text{if } x = (\\ ) & \text{if } x =) \\ \varepsilon & \text{o.w.} \end{cases}$

Now $enc(A) = A'$. So A' "is regular"
Contradiction. So A is non regular.

Why couldn't DFAs recognize these languages? finite memory

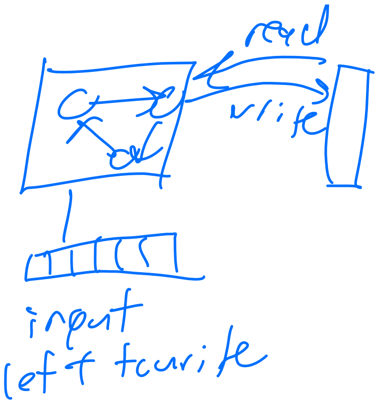
what happens with more memory?

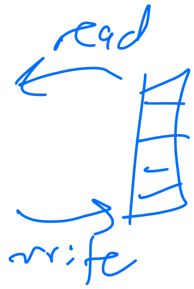
Let's DFAs w/ stacks and queues!

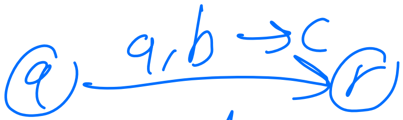
Pushdown Automata (PDA)

Queue Automata (QA)

(Always non-deterministic)







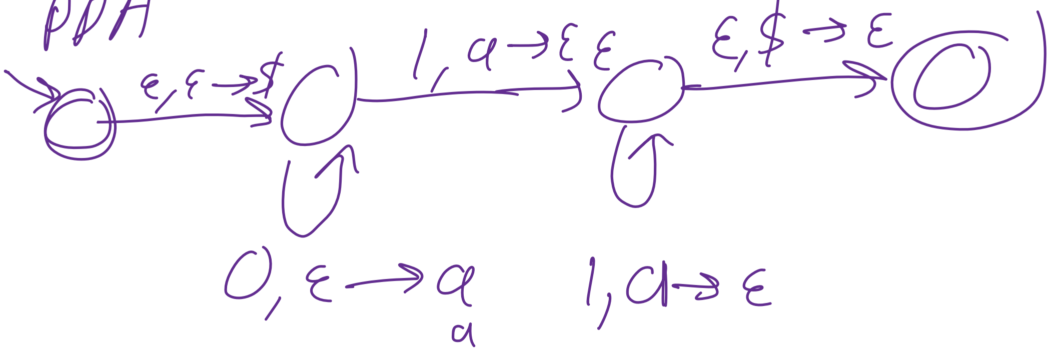
- read a off of input
- read b off external memory
- write c to external memory

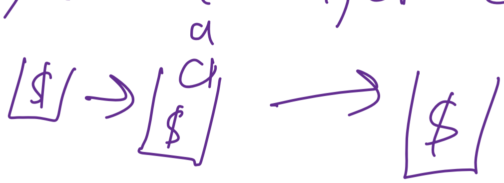
pop/dequeue → (read b off external memory)

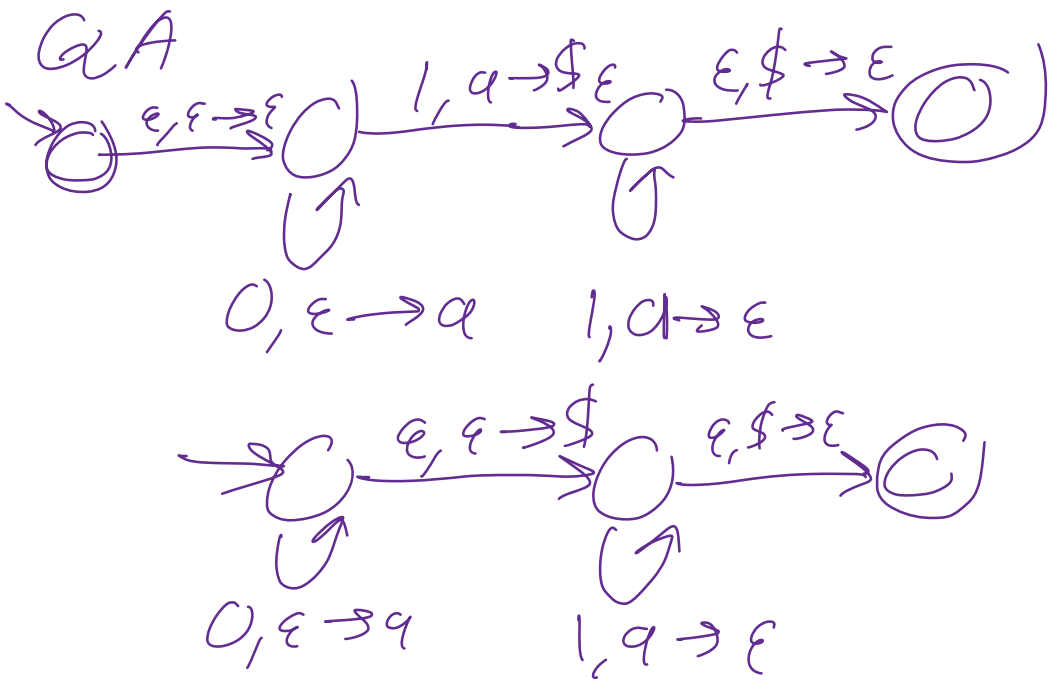
ε allowed for each

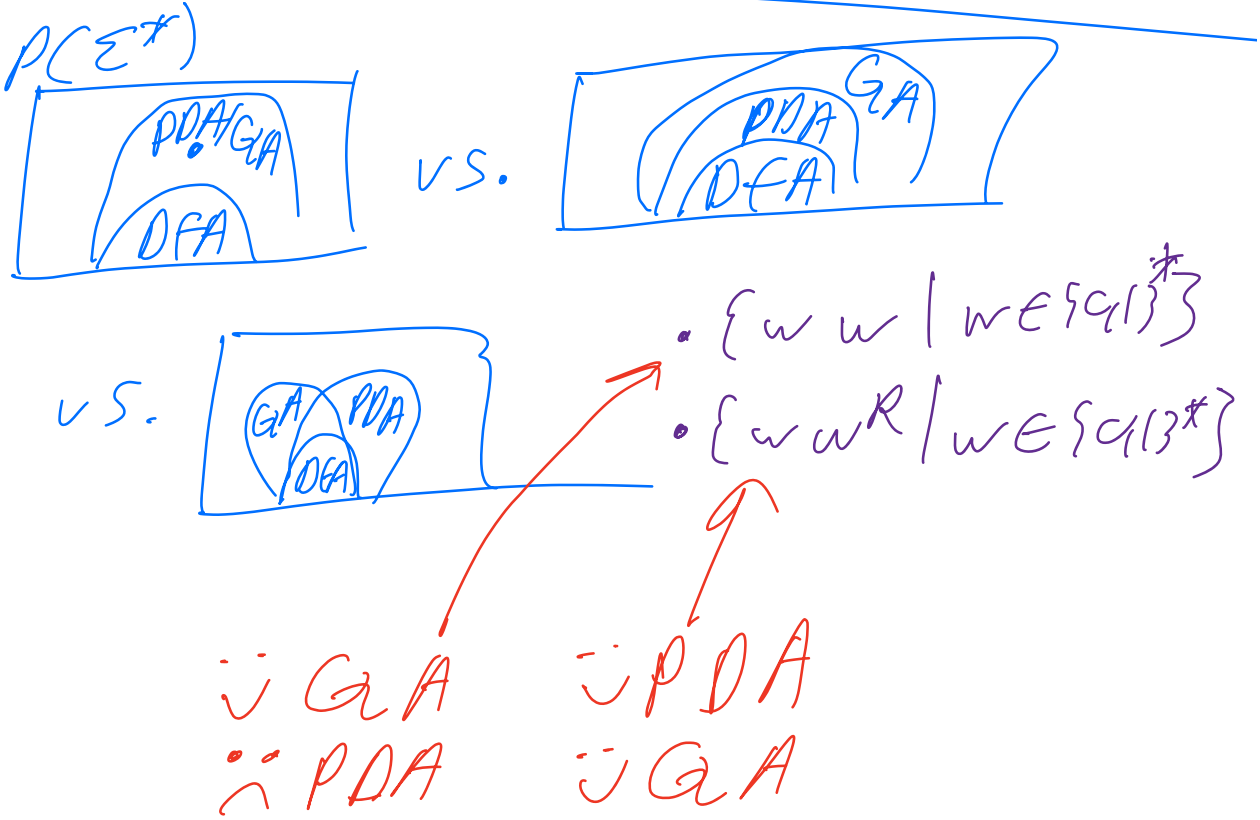
$\{0^n 1^n \mid n \geq 0\}$

PDA






GA
ε, ε → ε
1, a → $ε
ε, $ → ε
0, ε → a
1, a → ε
ε, ε → $
ε, $ → ε
0, ε → a
1, a → ε


P(Σ*)
PDA/GA
DFA
vs.
GA
PDA
DFA
vs.
GA
PDA
DFA
• {w w | w ∈ {a,b}*}
• {w wR | w ∈ {a,b}*}
GA
PDA
PDA
GA

E.g. • $\{0^i 1^j 2^k 3^\ell \mid i=k,\ j=\ell\}$

• $\{\quad\quad\quad\quad \mid i=j,\ k=\ell\}$

Possible w/ GA
Not with PDA

Easy with PDA
Easy with GA.