POWER SYSTEM OPERATION AND CONTROL (15 / 17EE81)

MODULE – 02

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MODULE – 02:

- **HYDRO-THERMAL SCHEDULING**
- **AUTOMATIC GENERATION CONTROL (AGC)**

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HYDRO-THERMAL SCHEDULING:

INTRODUCTION:

- **Hydro power developed is dependent on water head 'h' and discharge of water 'q'**
- **Water head 'h'**
  - **Vertical distance in the water reservoir.**
  - **Starting point is where the water begins to impact the pressure and ending point is where it ceases to exert pressure.**
  - **A low head site – 10 m or below, high head site – >20 m**

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INTRODUCTION:

- **Hydraulic power is**

  P_h **= head * water flow rate * Acceleration due to gravity**
- **The hydraulic system efficiency can be expected around 50% to 60%.**
- **Electric power developed in hydro systems is,**

  $$P_{GH} = \eta \,.\, \rho \,.\, q \,.\, g \,.\, h \text{ Watts}$$
- **Two types or kinds of Hydro-thermal scheduling:**
  - **Long-range or Long-time or Long-term scheduling**
  - **Short-range or Short-time or Short-term scheduling**

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INTRODUCTION:
LONG-RANGE SCHEDULING:

- Involves Long-range forecasting of water availability and scheduling of water releases from reservoir.
- This scheduling can be 1 week to 1 year or several years.
- While scheduling, the following uncertainties are to be accounted.
    - Load
    - Water inflow
    - Growth in thermal generation
- Hydro-thermal solution of this scheduling can be obtained with some assumptions, such as,
    - Constant thermal power or constant hydro power

INTRODUCTION:
SHORT-TERM SCHEDULING:

- Involves hour-by-hour scheduling of all generators so as to optimize total production cost.
- There are three scheduling categories.
    - Systems with only hydro plants. Only constraint is water inflow.
    - Hydro-thermal systems with major hydro generations. Scheduling is done to minimize thermal generation cost.
    - Hydro-thermal systems with equality in thermal and hydro plants or hydro plants are minor portion. Scheduling is done to minimize thermal generation cost keeping hydro generation constraints in account.

INTRODUCTION:
NOMENCLATURE USED FOR HYDRO-THERMAL SCHEDULING:

- ${P_{G_T}}^K$- Thermal power generation in k^{th} time interval
- ${P_{G_H}}^K$- Hydro power generation in k^{th} time interval
- ${P_D}^K$- Load demand in k^{th} time interval
- ${P_{Loss}}^K$- Real Power Loss in k^{th} time interval
- N – Total No. of time intervals.
- k – A particular time interval.
- N_T - No. of intervals the thermal plant is ON.
- h^k - No. of hours in k^{th} interval.

INTRODUCTION:
NOMENCLATURE USED FOR HYDRO-THERMAL SCHEDULING:

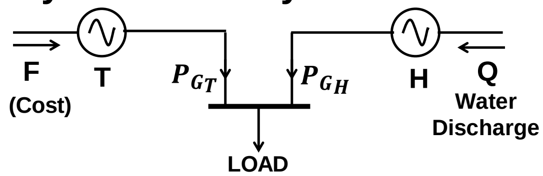
- ${Q_{in}}^K$- Water inflow rate in k^{th} time interval in m^3/s or m^3/hr.
- ${Q_o}^K$- Water discharge rate in k^{th} time interval in m^3/s or m^3/hr.
- T_{max} - Total time interval / period of operation.
- T_T - Total time for which the thermal plant is ON.

ALGORITHMS IN HYDRO-THERMAL SCHEDULING:

- Hydro-thermal scheduling based on energy
- Hydro-thermal scheduling – Discrete time interval method
- Short-term hydro thermal scheduling using $\gamma - \lambda$ iterations
- Short-term hydro thermal scheduling using penalty factors

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

- Fundamental hydrothermal system



- Objective of scheduling is
  - to determine the water discharge at various intervals, to minimize the cost of thermal power generation and
  - to meet the operating constraints of hydro plant.

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

MATHEMATICAL FORMULATION OF OBJECTIVE FUNCTION:

- Let period of operation of fundamental hydro-thermal system be T_{max}.
- Following assumptions are made.
  - Water storages of reservoir at the beginning and end of the period are specified (known/given).
  - Water inflow rate is known for the complete period of operation.
  - Load demand to be satisfied is known for the complete period of operation.

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

MATHEMATICAL FORMULATION OF OBJECTIVE FUNCTION:

- The objective of this scheduling is,
  - To determine the water discharge rate so as to minimize the cost of thermal power generation.
- Mathematically,

$$\text{Minimize } F_T = \int_0^{T_{max}} F'\left(P_{GT}(t)\right) dt \quad \text{-------------------} \quad (1)$$

where F_T - Total cost.

- For this problem, the operational constraints to be considered are,

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

MATHEMATICAL FORMULATION OF OBJECTIVE FUNCTION:

- **Power Balance constraints:**

$$P_{GT}(t) + P_{GH}(t) - P_{Loss}(t) - P_D(t) = 0 \text{ ------------------ } (2)$$

- **Water availability constraints:**

$$S(T_{max}) - S(0) - \int_0^{T_{max}} Q_{in}(t)dt + \int_0^{T_{max}} Q_o(t)dt = 0 \text{ --- } (3)$$

Where, S is water storage in m^3 and

Q is water rate in m^3/s.

- **Hydropower generation constraints:**

Hydropower generation $P_{GH}(t) = f\big(S(t), Q_o(t)\big)$ ------ (4)

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

MATHEMATICAL FORMULATION OF OBJECTIVE FUNCTION:

Discretization:

- Let period of operation from 0 to T_{max} be divided into N no. of sub-intervals each of time ΔT.
- Let all the variables remains fixed within each sub-interval.
- The above defined mathematical problem (Eqns. (1) to (4)) can be discretized with the above discretization parameters.

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GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

MATHEMATICAL FORMULATION OF OBJECTIVE FUNCTION:

- Minimize total cost of thermal generation,

$$F_T = \sum_{k=1}^{N} F\big(P_{GT}{}^k\big) \text{ -- } (5)$$

under the constraints,

1) $P_{GT}{}^k + P_{GH}{}^k - P_{Loss}{}^k - P_D{}^k = 0$ ------------------------ (6)
2) $S^k - S^{k-1} - Q_{in}{}^k\,\Delta T + Q_o{}^k\,\Delta T = 0$

(Equation in quantity of water S in m^3)

Dividing by ΔT, $(S')^k - (S')^{k-1} - Q_{in}{}^k + Q_o{}^k = 0$ --- (7)

(Equation in water rate Q in m^3/s)

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GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

MATHEMATICAL FORMULATION OF OBJECTIVE FUNCTION:

3) Hydropower generated can be expressed as,

$$P_{GH}{}^k = 9.81 * 10^{-3} * h_{av} * \big(Q_o{}^k - \rho\big) \text{ MW}$$

Where, h_{av} is the Average water head.

$$h_{av} = h_o * \Big[1 + 0.5\, e\, \Big((S')^k + (S')^{k-1}\Big)\Big]$$

Water head correction factor $e = \frac{\Delta T}{A\, h_o}$

$$\therefore\ P_{GH}{}^k = 9.81 * 10^{-3} * h_o * \Big[1 + 0.5\, e\, \Big((S')^k + (S')^{k-1}\Big)\Big] * \big(Q_o{}^k - \rho\big)$$

$$P_{GH}{}^k = h_o{}' \Big[1 + 0.5\, e\, \Big((S')^k + (S')^{k-1}\Big)\Big] * \big(Q_o{}^k - \rho\big) \text{ ----- } (8)$$

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GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

DEPENDENT VARIABLES FOR SOLUTION:

- Let us add water availability eqn (7) for all N sub-intervals.

$$(S')^N - (S')^0 - \sum_{k=1}^{N} Q_{in}{}^k + \sum_{k=1}^{N} Q_o{}^k = 0 \text{ ------------ } (9)$$

- In this eqn. out of N values of $Q_o{}^k$, (N-1) values can be specified (independent variable) and one value of $Q_o{}^k$ will be unspecified (dependent variable).
- Let $Q_o{}^k$ for k = 2, 3, ….. N be specified (independent variable) and then $Q_o{}^k$ for k = 1, i.e. $Q_o{}^1$ will become unspecified (dependent variable).
- From eqn. (9), $Q_o{}^1 = (S')^0 - (S')^N + \sum_{k=1}^{N} Q_{in}{}^k - \sum_{k=2}^{N} Q_o{}^k$ --(10)

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

DEPENDENT VARIABLES FOR SOLUTION:

- The other dependent variables are,

$$P_{GT}{}^k,\ P_{GH}{}^k \text{ and } (S')^k \ (k \neq 0 \neq N)$$

- Finally, the dependent variables for solving the hydro-thermal problem are listed as,
  - Thermal power generation at all sub-intervals k, $P_{GT}{}^k$
  - Hydro power generation at all sub-intervals k, $P_{GH}{}^k$
  - Water storages in all sub-intervals except at k=0 (initial) and k=N (final), $(S')^k$
  - Water discharge rate at first interval (k=1), $Q_o{}^1$

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

- The Lagrangian function is framed by augmenting the cost function (eqn. 5) with the constraints (eqns. 6, 7 and 8) using Lagrangian multipliers.
- $$\mathcal{L} = \sum_k \Big\{ F(P_{GT}{}^k) - \lambda_1{}^k (P_{GT}{}^k + P_{GH}{}^k - P_{Loss}{}^k - P_D{}^k) + \lambda_2{}^k \left((S')^k - (S')^{k-1} - Q_{in}{}^k + Q_o{}^k \right) + \lambda_3{}^k \left(P_{GH}{}^k - h_o{}' \left[1 + 0.5\, e \left((S')^k + (S')^{k-1} \right) \right] * (Q_o{}^k - \rho) \right) \Big\} \text{ ------------------------ } (11)$$
- To compute Lagrangian multipliers $\lambda_1{}^k$, $\lambda_2{}^k$ and $\lambda_3{}^k$:
- Equate the partial derivatives of Lagrangian function $\mathcal{L}$ w.r.t. dependent variables ($P_{GT}{}^k$, $P_{GH}{}^k$, $(S')^k$ k $\neq 0 \neq N$ and $Q_o{}^1$) to zero.

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

1. Equate the partial derivative of Lagrangian function $\mathcal{L}$ w.r.t. dependent variables $P_{GT}{}^k$ to zero.

$$\frac{\partial \mathcal{L}}{\partial P_{GT}{}^k} = 0$$

$$\frac{dF(P_{GT}{}^k)}{dP_{GT}{}^k} - \lambda_1{}^k \left[1 - \frac{\partial P_{Loss}{}^k}{\partial P_{GT}{}^k} \right] = 0 \text{ --------------------- } (12)$$

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

2. Equate the partial derivative of Lagrangian function $\mathcal{L}$ w.r.t. dependent variables $P_{GH}{}^{k}$ to zero.

$$\frac{\partial \mathcal{L}}{\partial P_{GH}{}^{k}} = 0$$

$$-\lambda_1{}^{k}\left[1 - \frac{\partial P_{Loss}{}^{k}}{\partial P_{GH}{}^{k}}\right] + \lambda_3{}^{k} = 0$$

$$\lambda_3{}^{k} - \lambda_1{}^{k}\left[1 - \frac{\partial P_{Loss}{}^{k}}{\partial P_{GH}{}^{k}}\right] = 0 \text{ ---------------------------------- } (13)$$

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

3. Equate the partial derivative of Lagrangian function $\mathcal{L}$ w.r.t. dependent variables $(S')^{k}$ to zero.

- **Let us modify the Lagrangian function $\mathcal{L}$ (eqn. 11) to incorporate $(S')^{k}$.**
- **In eqn. 11, let us consider only intervals k and k+1.**
- $\mathcal{L} = F(P_{GT}{}^{k}) - \lambda_1{}^{k}(P_{GT}{}^{k} + P_{GH}{}^{k} - P_{Loss}{}^{k} - P_D{}^{k}) + \lambda_2{}^{k}\left((S')^{k} - (S')^{k-1} - Q_{in}{}^{k} + Q_o{}^{k}\right) + \lambda_2{}^{k+1}\left((S')^{k+1} - (S')^{k} - Q_{in}{}^{k+1} + Q_o{}^{k+1}\right) + \lambda_3{}^{k}\left(P_{GH}{}^{k} - h_o{}'\left[1 + 0.5\,e\,\left((S')^{k} + (S')^{k-1}\right)\right] * (Q_o{}^{k} - \rho)\right) + \lambda_3{}^{k+1}\left(P_{GH}{}^{k+1} - h_o{}'\left[1 + 0.5\,e\,\left((S')^{k+1} + (S')^{k}\right)\right] * (Q_o{}^{k+1} - \rho)\right)$ -- (14)

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

Now, $\dfrac{\partial \mathcal{L}}{\partial (S')^{k}} = 0$

$$\lambda_2{}^{k} - \lambda_2{}^{k+1} + \lambda_3{}^{k}\left(-h_o{}'[0.5\,e] * (Q_o{}^{k} - \rho)\right) + \lambda_3{}^{k+1}\left(-h_o{}'[0.5\,e\,] * (Q_o{}^{k+1} - \rho)\right) = 0$$

$$\lambda_2{}^{k} - \lambda_2{}^{k+1} - \lambda_3{}^{k}\,h_o{}'\,0.5\,e\,(Q_o{}^{k} - \rho) - \lambda_3{}^{k+1}\,h_o{}'\,0.5\,e\,(Q_o{}^{k+1} - \rho) = 0 \text{ --- } (15)$$

4. Equate the partial derivative of Lagrangian function $\mathcal{L}$ w.r.t. dependent variable $Q_o{}^{1}$ to zero.

- **To achieve this, Let us modify the Lagrangian function $\mathcal{L}$ (eqn. 11) to incorporate $Q_o{}^{1}$.**

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

- **Let k = 1 in eqn. (7) gives,**

$$(S')^{1} - (S')^{0} - Q_{in}{}^{1} + Q_o{}^{1} = 0$$

$$\Longrightarrow (S')^{1} = (S')^{0} + Q_{in}{}^{1} - Q_o{}^{1} \text{------------- } (16)$$

- **Let k = 1 in Lagrangian function $\mathcal{L}$ (eqn. 11) gives,**
- $\mathcal{L} = F(P_{GT}{}^{1}) - \lambda_1{}^{1}(P_{GT}{}^{1} + P_{GH}{}^{1} - P_{Loss}{}^{1} - P_D{}^{1}) + \lambda_2{}^{1}\left((S')^{1} - (S')^{0} - Q_{in}{}^{1} + Q_o{}^{1}\right) + \lambda_3{}^{1}\left(P_{GH}{}^{1} - h_o{}'\left[1 + 0.5\,e\,\left((S')^{1} + (S')^{0}\right)\right] * (Q_o{}^{1} - \rho)\right)$
- **Substituting eqn. (16) here, gives, (only in red coloured)**
- $\mathcal{L} = F(P_{GT}{}^{1}) - \lambda_1{}^{1}(P_{GT}{}^{1} + P_{GH}{}^{1} - P_{Loss}{}^{1} - P_D{}^{1}) + \lambda_2{}^{1}\left((S')^{1} - (S')^{0} - Q_{in}{}^{1} + Q_o{}^{1}\right) + \lambda_3{}^{1}\left(P_{GH}{}^{1} - h_o{}'[1 + 0.5\,e\,(2(S')^{0} + Q_{in}{}^{1} - Q_o{}^{1})] * (Q_o{}^{1} - \rho)\right)$--------$(17)$

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

Now, $\frac{\partial \mathcal{L}}{\partial Q_o^{1}} = 0$

From eqn. (17),

$$\lambda_2^{1} + \lambda_3^{1}(-h_o'[0.5\,e\,(-1)] * (Q_o^{1} - \rho) - h_o'[1 + 0.5\,e\,(2\,(S')^{0} + Q_{in}^{1} - Q_o^{1})]) = 0$$

$$\lambda_2^{1} - \lambda_3^{1}\,h_o'\left(-0.5\,e * (Q_o^{1} - \rho) + 1 + 0.5\,e\left(2\,(S')^{0} + Q_{in}^{1} - Q_o^{1}\right)\right) = 0$$

$$\lambda_2^{1} - \lambda_3^{1}\,h_o'\left(1 + 0.5\,e * \left(2\,(S')^{0} + Q_{in}^{1} - Q_o^{1} - Q_o^{1} + \rho\right)\right) = 0$$

$$\lambda_2^{1} - \lambda_3^{1}\,h_o'\left(1 + 0.5\,e * \left(2\,(S')^{0} + Q_{in}^{1} - 2\,Q_o^{1} + \rho\right)\right) = 0 \text{ ----- } (18)$$

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

- Now the gradient vector is required to end the iterative process.
- It is the partial derivative of the Lagrangian function $\mathcal{L}$ (eqn. 11) w.r.t. independent variables $Q_o^{k}(\mathrm{k} \neq 1)$
- To achieve this, Lagrangian function $\mathcal{L}$ (eqn. 11) is to be modified to incorporate Q_o^{k}
- From eqn. (7), $(S')^{k} = (S')^{k-1} + Q_{in}^{k} - Q_o^{k}$
- Substituting $(S')^{k}$ in eqn. 11 gives, (at λ_3^{k} term only)
- $\mathcal{L} = \sum_k \left\{F(P_{GT}^{k}) - \lambda_1^{k}(P_{GT}^{k} + P_{GH}^{k} - P_{Loss}^{k} - P_D^{k}) + \lambda_2^{k}\left((S')^{k} - (S')^{k-1} - Q_{in}^{k} + Q_o^{k}\right) + \lambda_3^{k}\left(P_{GH}^{k} - h_o'[1 + 0.5\,e\left(2\,(S')^{k-1} + Q_{in}^{k} - Q_o^{k}\right)] * (Q_o^{k} - \rho)\right)\right\}$ ---(19)

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

SOLUTION TECHNIQUES:

- Now, gradient vector is found from eqn. (19) as follows.

$$\left[\frac{\partial \mathcal{L}}{\partial Q_0^{k}}\right]_{k\neq 1} = \lambda_2^{k} + \lambda_3^{k}\begin{pmatrix} -h_o'\left[1 + 0.5\,e\left(2\,(S')^{k-1} + Q_{in}^{k} - Q_o^{k}\right)\right] \\ -h_o'\,[0.5\,e\,(-1)] * (Q_o^{k} - \rho) \end{pmatrix}$$

$$\left[\frac{\partial \mathcal{L}}{\partial Q_0^{k}}\right]_{k\neq 1} = \lambda_2^{k} - \lambda_3^{k}\,h_o'\left[1 + 0.5\,e\left(2\,(S')^{k-1} + Q_{in}^{k} - Q_o^{k}\right) - 0.5\,e\,(Q_o^{k} - \rho)\right]$$

$$\left[\frac{\partial \mathcal{L}}{\partial Q_0^{k}}\right]_{k\neq 1} = \lambda_2^{k} - \lambda_3^{k}\,h_o'\left[1 + 0.5\,e\left(2\,(S')^{k-1} + Q_{in}^{k} - 2\,Q_o^{k} + \rho\right)\right] \text{- } (20)$$

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

ALGORITHM:

Step-1: Assume the water discharges Q_o^{k} for intervals k = 2, 3, ……N as initial values. (independent variables)

Step-2: Calculate water discharge Q_o^{1} at interval k = 1, water storages $(S')^{k}$, hydro power generation P_{GH}^{k} and thermal power generation P_{GT}^{k} using equations (10), (7), (8) and (6) respectively.

Step-3: Obtain Lagrangian multipliers λ_1^{k}, λ_3^{k}, λ_2^{k} $(k \neq 1)$ and λ_2^{1} using equations (12), (13), (15) and (18) respectively.

GENERAL ALGORITHM FOR HYDRO-THERMAL SCHEDULING – DISCRETE TIME INTERVAL METHOD

ALGORITHM:

Step-4: Obtain gradient vector $\left[\frac{\partial \mathcal{L}}{\partial {Q_0}^k}\right]_{k \neq 1}$ using equation (20). If it is less than prescribed tolerance limit ε, then stop the iteration and give the optimum scheduling of hydro and thermal plants, otherwise go to next step.

Step-5: Now, compute ${Q_o}^k(new) = {Q_o}^k\,(old) - \alpha * \left[\frac{\partial \mathcal{L}}{\partial {Q_0}^k}\right]_{k \neq 1}$ for intervals k = 2, 3,..N where α is a positive scalar.

Now, with the new values of ${Q_o}^k$ (k = 2, 3, … N) go to step-2.

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

- A Mathematical model for short-term hydro-thermal scheduling can also be built with more attention on water discharge rates Q_o.
- Let us redefine the mathematical problem formulated.
- Minimize total cost of thermal generation,

$$F_T = \sum_{k=1}^{N} h^k\, F\left({P_{GT}}^k\right) \text{ ------ } (1)$$

Where, h^k - No. of hours in k^{th} interval and $\sum_{k=1}^{N} h^k = T_{max}$

- Under the constraints,

1) Power balance equation: ${P_{GT}}^k + {P_{GH}}^k - {P_{Loss}}^k - {P_D}^k = 0$

$${P_D}^k + {P_{Loss}}^k - {P_{GT}}^k - {P_{GH}}^k = 0 \text{ ------------- } (2)$$

2) Water availability equation: $\sum_{k=1}^{N} h^k\, {Q_o}^k = (Q_o)_{Total}$ -------- (3)

Where, $(Q_o)_{Total}$ - Total volume of water available for discharge in whole period of operation in m^3.

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

- Let us assume a constant head operation.
- Water discharge rate in k^{th} interval, ${Q_o}^k$ depends on hydropower generation, ${P_{GH}}^k$. Hence, ${Q_o}^k = {Q_o}^k\left({P_{GH}}^k\right)$
- For solving the mathematical problem defined in eqns. (1), (2) and (3), let us form the Lagrangian function as follows.

$$\mathcal{L} = \sum_{k=1}^{N} h^k\, F\left({P_{GT}}^k\right) + \lambda^k\left({P_D}^k + {P_{Loss}}^k - {P_{GT}}^k - {P_{GH}}^k\right) + \gamma\left(\sum_{k=1}^{N} h^k\, {Q_o}^k - (Q_o)_{Total}\right) \text{ ----------------- } (4)$$

Where, λ^k and γ are Lagrangian multipliers.

- To compute Lagrangian multipliers, let us equate the partial derivatives of Lagrangian function $\mathcal{L}$ (eqn. 4) w.r.t dependent variables, ${P_{GT}}^k$ and ${P_{GH}}^k$ to zero.

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

1) let us equate the partial derivatives of Lagrangian function $\mathcal{L}$ (eqn. 4) w.r.t dependent variables, ${P_{GT}}^k$ to zero.

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial {P_{GT}}^k} = 0 \qquad \Rightarrow h^k \frac{dF\left({P_{GT}}^k\right)}{d{P_{GT}}^k} + \lambda^k\left[\frac{\partial {P_{Loss}}^k}{\partial {P_{GT}}^k} - 1\right] = 0$$

$$\Rightarrow h^k \frac{dF\left({P_{GT}}^k\right)}{d{P_{GT}}^k} + \lambda^k \frac{\partial {P_{Loss}}^k}{\partial {P_{GT}}^k} = \lambda^k \text{ ----------- } (5)$$

2) let us equate the partial derivatives of Lagrangian function $\mathcal{L}$ (eqn. 4) w.r.t dependent variables, ${P_{GH}}^k$ to zero.

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial {P_{GH}}^k} = 0 \qquad \Rightarrow \lambda^k\left[\frac{\partial {P_{Loss}}^k}{\partial {P_{GH}}^k} - 1\right] + \gamma\, h^k \frac{d{Q_o}^k\left({P_{GH}}^k\right)}{d{P_{GH}}^k} = 0$$

$$\Rightarrow \gamma\, h^k \frac{d{Q_o}^k\left({P_{GH}}^k\right)}{d{P_{GH}}^k} + \lambda^k \frac{\partial {P_{Loss}}^k}{\partial {P_{GH}}^k} = \lambda^k \text{ --- } (6)$$

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

- **The above equations (5) and (6) are called as coordination equations.**
- **NOTE: If losses are neglected, the coordination equations becomes,**

$$h^k \frac{dF(P_{GT}{}^k)}{dP_{GT}{}^k} = \lambda^k$$

$$\gamma\, h^k \frac{dQ_o{}^k(P_{GH}{}^k)}{dP_{GH}{}^k} = \lambda^k$$

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

PRE-REQUISITES FOR $\gamma - \lambda$ ITERATIVE ALGORITHM:

- **General fuel cost function of thermal generation is,**

$$F(P_{GT}{}^k) = a_T + b_T\, P_{GT}{}^k + c_T\, (P_{GT}{}^k)^2\ Rs./hr$$

- **The incremental fuel cost function is,**

$$\frac{dF(P_{GT}{}^k)}{dP_{GT}{}^k} = b_T + 2\, c_T\, P_{GT}{}^k \quad \text{--------------------------------} \ (7)$$

- **Water discharge rate is a function of $P_{GH}{}^k$ and is given by,**

$Q_o{}^k = a_H + b_H\, P_{GH}{}^k$ **(for generation below the max. limit) ------ (8)**

$Q_o{}^k = a_H + b_H\, P_{GH}{}^k + c_H\, (P_{GH}{}^k)^2$ **(for generation above the max. limit)**

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

PRE-REQUISITES FOR $\gamma - \lambda$ ITERATIVE ALGORITHM:

- **Incremental water discharge rate is given by,**

$\frac{dQ_o{}^k(P_{GH}{}^k)}{dP_{GH}{}^k} = b_H$ **(for generation below the max. limit) ------- (9)**

$\frac{dQ_o{}^k(P_{GH}{}^k)}{dP_{GH}{}^k} = b_H + 2\, c_H\, P_{GH}{}^k$ **(for generation above the max. limit)**

- **The Power loss is given by,**

$$P_{Loss}{}^k = B_{TT}(P_{GT}{}^k)^2 + 2\, B_{TH}\, P_{GT}{}^k\, P_{GH}{}^k + B_{HH}(P_{GH}{}^k)^2 \text{--------} \ (10)$$

- **The incremental transmission losses are,**

$$\frac{\partial P_{Loss}{}^k}{\partial P_{GT}{}^k} = 2B_{TT}\, P_{GT}{}^k + 2\, B_{TH}\, P_{GH}{}^k \text{---------------------------} \ (11)$$

$$\frac{\partial P_{Loss}{}^k}{\partial P_{GH}{}^k} = 2B_{TH}\, P_{GT}{}^k + 2\, B_{HH}\, P_{GH}{}^k \text{--------------------------} \ (12)$$

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

PRE-REQUISITES FOR $\gamma - \lambda$ ITERATIVE ALGORITHM:

- **Substituting eqns. (7) and (11) in eqn. (5) gives,**

$$h^k \frac{dF(P_{GT}{}^k)}{dP_{GT}{}^k} + \lambda^k \frac{\partial P_{Loss}{}^k}{\partial P_{GT}{}^k} = \lambda^k$$

$$h^k(b_T + 2\, c_T\, P_{GT}{}^k) + \lambda^k(2B_{TT}\, P_{GT}{}^k + 2\, B_{TH}\, P_{GH}{}^k) = \lambda^k$$

$$h^k b_T + P_{GT}{}^k\, (2\, h^k\, c_T + 2\, \lambda^k\, B_{TT}) + 2\, \lambda^k\, B_{TH}\, P_{GH}{}^k = \lambda^k$$

$$\Longrightarrow \qquad P_{GT}{}^k = \frac{\lambda^k - h^k b_T - 2\, \lambda^k\, B_{TH}\, P_{GH}{}^k}{2\, h^k\, c_T + 2\, \lambda^k\, B_{TT}} \text{------------} \ (13)$$

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

PRE-REQUISITES FOR $\gamma - \lambda$ ITERATIVE ALGORITHM:

- **Substituting eqns. (9) and (12) in eqn. (6) gives,**

$$\gamma\, h^k \frac{dQ_o{}^k(P_{GH}{}^k)}{dP_{GH}{}^k} + \lambda^k \frac{\partial P_{Loss}{}^k}{\partial P_{GH}{}^k} = \lambda^k$$

$$\gamma\, h^k\, b_H + \lambda^k\left(2B_{TH}\, P_{GT}{}^k + 2\, B_{HH}\, P_{GH}{}^k\right) = \lambda^k$$

$$\gamma\, h^k\, b_H + 2\,\lambda^k\, B_{TH}\, P_{GT}{}^k + 2\,\lambda^k\, B_{HH}\, P_{GH}{}^k = \lambda^k$$

$$\Longrightarrow \quad P_{GH}{}^k = \frac{\lambda^k - \gamma\, h^k\, b_H - 2\,\lambda^k\, B_{TH}\, P_{GT}{}^k}{2\,\lambda^k\, B_{HH}} \text{ ------------ } (14)$$

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

$\gamma - \lambda$ ITERATIVE ALGORITHM:

Step-1: Choose a tolerance limits for power balance as ε_1 and for water discharge as ε_2 .

Step-2: Assume initial values of Lagrangian multipliers γ and λ^k. Also assume all power generations $P_{GT}{}^k$ and $P_{GH}{}^k$ are zero or some arbitrary value.

Step-3: Set k = 1 for the first time interval.

Step-4: Using equations (13) and (14), compute the power generations, $P_{GT}{}^k$ and $P_{GH}{}^k$.

Step-5: Now compute the transmission power loss $P_{Loss}{}^k$ using eqn. (10).

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

$\gamma - \lambda$ ITERATIVE ALGORITHM:

Step-6: Then determine the power mismatch ΔP^k.

$$\Delta P^k = P_D{}^k + P_{Loss}{}^k - P_{GT}{}^k - P_{GH}{}^k$$

Step-7: If $\left|\Delta P^k\right| \le \varepsilon_1$, then calculate water discharge $Q_o{}^k$ using eqn. (8) and go to step-9, otherwise go to next step.

Step-8: If $\Delta P^k < 0$, then decrement λ^k by $\Delta\lambda^k$, i.e., $\lambda^k = \lambda^k - \Delta\lambda^k$ or If $\Delta P^k > 0$, then increment λ^k by $\Delta\lambda^k$, i.e., $\lambda^k = \lambda^k + \Delta\lambda^k$, then go to step-4.

Step-9: Now increment the time interval k as, k = k+1 and continue from step-4 till the last interval k = N.

SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

$\gamma - \lambda$ ITERATIVE ALGORITHM:

Step-10: With the help of $Q_o{}^k$ computed in each interval k, determine discharge mismatch or change in water discharge ΔQ.

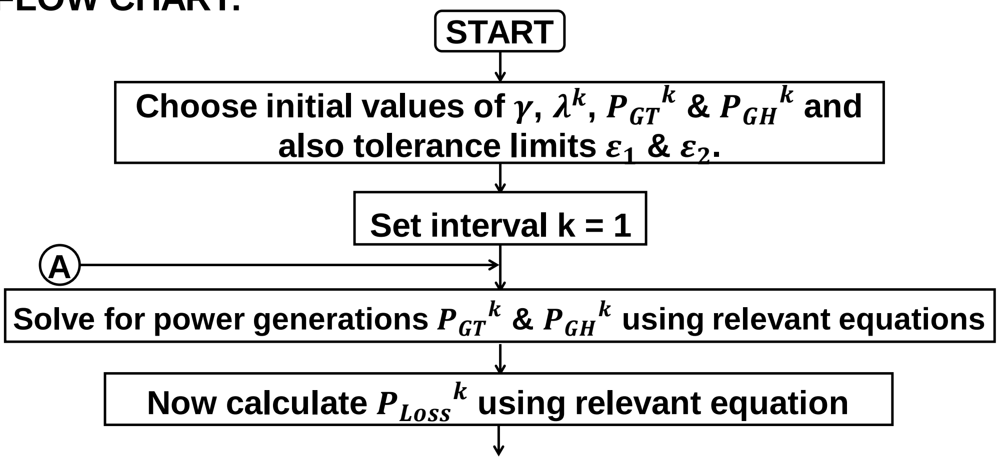
$$\Delta Q = \sum_{k=1}^{N} h^k\, Q_o{}^k - (Q_o)_{Total}$$

Step-11: If $|\Delta Q| \le \varepsilon_2$, then stop the iteration, otherwise go to next step.

Step-12: If $\Delta Q > 0$, then increment γ by $\Delta\gamma$, i.e., $\gamma = \gamma + \Delta\gamma$, otherwise decrement γ by $\Delta\gamma$, i.e., $\gamma = \gamma - \Delta\gamma$. Then go to step-4.

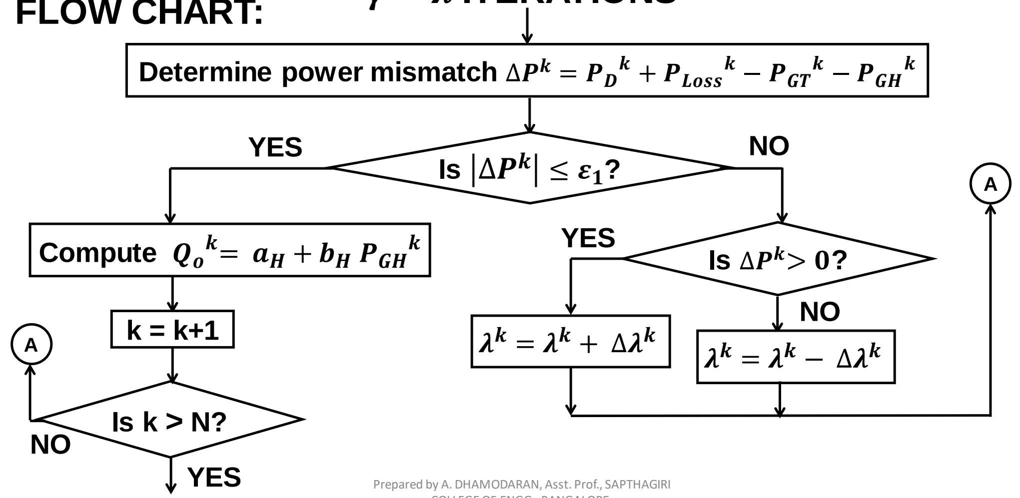
SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

FLOW CHART:



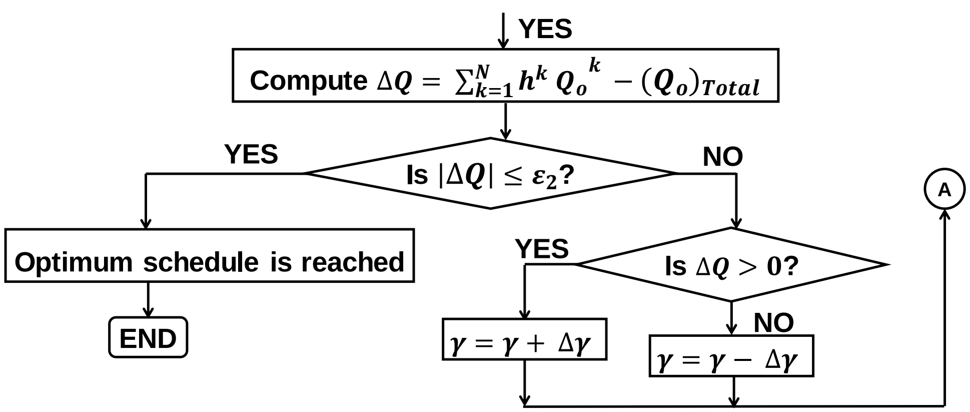
SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

FLOW CHART:



SHORT TERM HYDRO-THERMAL SCHEDULING USING $\gamma - \lambda$ ITERATIONS

FLOW CHART:



SHORT TERM HYDRO-THERMAL SCHEDULING USING PENALTY FACTORS

- **Penalty factors are used in thermal power plant scheduling and are defined as,**

  Penalty factor of a thermal plant, $L_i = \dfrac{1}{1 - \frac{\partial P_{Loss}}{\partial P_{GT}}}$

- **Let us extend the penalty factor concept in hydrothermal scheduling.**
- **Let us consider a fundamental hydrothermal system.**
- **Let the total volume of water available for discharge in hydro plant be $(Q_o)_{Total}$ m³.**
- **Let the total load demand P_D exist as constant for complete period of operation T.**

SHORT TERM HYDRO-THERMAL SCHEDULING USING PENALTY FACTORS

- The objective function can be defined as,

Minimize $C = \int_0^T F_T(P_{GT})\,dt + \gamma \int_0^T Q_o(P_{GH})\,dt$ ------------ (1)

Under the constraints,

1) Power balance constraints: $P_D + P_{Loss} - P_{GH} - P_{GT} = 0$ -- (2)

2) Water availability equation: $\int_0^T Q_o(P_{GH})\,dt = (Q_o)_{Total}$ ---- (3)

- Let in eqn.(2) be an incremental change in parameters.

$$\frac{\partial P_{Loss}}{\partial P_{GH}}\,\Delta P_{GH} + \frac{\partial P_{Loss}}{\partial P_{GT}}\,\Delta P_{GT} - \Delta P_{GH} - \Delta P_{GT} = 0$$

$$-\Delta P_{GT}\left(1 - \frac{\partial P_{Loss}}{\partial P_{GT}}\right) - \Delta P_{GH}\left(1 - \frac{\partial P_{Loss}}{\partial P_{GH}}\right) = 0$$

$$\Delta P_{GH}\left(1 - \frac{\partial P_{Loss}}{\partial P_{GH}}\right) = -\Delta P_{GT}\left(1 - \frac{\partial P_{Loss}}{\partial P_{GT}}\right) \text{ ------------- } (4)$$

SHORT TERM HYDRO-THERMAL SCHEDULING USING PENALTY FACTORS

- For incremental change in generations, the minimization function eqn. (1) can be modified as,

$$\frac{dF_T}{dP_{GT}}\Delta P_{GT} + \gamma\,\frac{dQ_o}{dP_{GH}}\Delta P_{GH} = 0$$

- Multiplying both sides by $-\left(1 - \frac{\partial P_{Loss}}{\partial P_{GH}}\right)$ gives,

$$-\left(1 - \frac{\partial P_{Loss}}{\partial P_{GH}}\right)\frac{dF_T}{dP_{GT}}\Delta P_{GT} - \gamma\left(1 - \frac{\partial P_{Loss}}{\partial P_{GH}}\right)\frac{dQ_o}{dP_{GH}}\Delta P_{GH} = 0$$

- Substituting eqn. (4) gives,

$$-\left(1 - \frac{\partial P_{Loss}}{\partial P_{GH}}\right)\frac{dF_T}{dP_{GT}}\Delta P_{GT} + \gamma\left(1 - \frac{\partial P_{Loss}}{\partial P_{GT}}\right)\frac{dQ_o}{dP_{GH}}\Delta P_{GT} = 0$$

$$\gamma\frac{dQ_o}{dP_{GH}}\left(1 - \frac{\partial P_{Loss}}{\partial P_{GT}}\right) = \frac{dF_T}{dP_{GT}}\left(1 - \frac{\partial P_{Loss}}{\partial P_{GH}}\right)$$

SHORT TERM HYDRO-THERMAL SCHEDULING USING PENALTY FACTORS

$$\gamma\frac{dQ_o}{dP_{GH}}\left[\frac{1}{1-\frac{\partial P_{Loss}}{\partial P_{GH}}}\right] = \frac{dF_T}{dP_{GT}}\left[\frac{1}{1-\frac{\partial P_{Loss}}{\partial P_{GT}}}\right] = \lambda \text{ (say) ----------------------- } (5)$$

From eqn. (5), $\gamma\frac{dQ_o}{dP_{GH}}\left[\frac{1}{1-\frac{\partial P_{Loss}}{\partial P_{GH}}}\right] = \lambda$

$$\Longrightarrow \gamma\ IW_H\ L_H = \lambda \text{ ----------------------- } (6)$$

Where, $IW_H = \frac{dQ_o}{dP_{GH}}$ is the incremental water discharge rate of hydro plant.

$L_H = \left[\frac{1}{1-\frac{\partial P_{Loss}}{\partial P_{GH}}}\right]$ is the penalty factor of hydro plant.

SHORT TERM HYDRO-THERMAL SCHEDULING USING PENALTY FACTORS

From eqn. (5), $\frac{dF_T}{dP_{GT}}\left[\frac{1}{1-\frac{\partial P_{Loss}}{\partial P_{GT}}}\right] = \lambda$

$$\Longrightarrow IC_T\ L_T = \lambda \text{ ------------------------------------ } (7)$$

Where, $IC_T = \frac{dF_T}{dP_{GT}}$ is the incremental fuel cost of thermal plant.

$L_T = \left[\frac{1}{1-\frac{\partial P_{Loss}}{\partial P_{GT}}}\right]$ is the penalty factor of thermal plant.

- Equations (6) and (7) are called coordination equations and they are required to be solved to obtain the scheduling of fundamental hydrothermal plant.

SHORT TERM HYDRO-THERMAL SCHEDULING USING PENALTY FACTORS

NOTE: If losses are neglected in fundamental hydrothermal system, then the coordination equations are modified as,

$$\gamma\ IW_H = \lambda \quad \Longrightarrow \quad \gamma \frac{dQ_o}{dP_{GH}} = \lambda \quad \text{and}$$

$$IC_T = \lambda \quad \Longrightarrow \quad \frac{dF_T}{dP_{GT}} = \lambda$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

1) Find the electrical power generated by a plant with a water head of 150 m, density of water = 1000 kg/m³, water flow rate = 0.095 m³/s, generator efficiency = 89% and turbine efficiency = 85%.

Sol.: Given, water head h = 150 m, Water density ρ = 1000 kg/m³, water flow rate q = 0.095 m³/s, generator efficiency η_G = 0.89

turbine efficiency η_T = 0.85

Overall efficiency η = η_G * η_T = 0.89 * 0.85 = 0.7565

The electrical power developed $P_{GH} = \eta\ \rho\ q\ g\ h$ Watts

= 0.7565 * 1000 * 0.095 * 9.81 * 150

= 105753.03 W

The electrical power developed P_{GH} = 105.753 kW

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

2) A hydro plant and a thermal plant supply a common load of 100 MW for 1 week continuously. The characteristics of the plants are as follows.

Hydro plant: $Q_o = 110 + 5.1\ P_{GH}$ m³/s; $0 \le P_{GH} \le 100$ MW

Thermal plant: $F_T(P_{GT}) = 55 + 11.5\ P_{GT} + 0.022\ P_{GT}^2$ Rs./hr.; $10 \le P_{GT} \le 60$ MW. Determine the generation scheduling of both plants for the given load, if (i) the hydro energy is limited to 12000 MWh and (ii) the volume of water drawn is limited to 3.2 * 10⁸ m³.

Sol.: (i) Constraint on hydro energy:

Given, Hydro plant energy = 12000 MWh, Load P_D = 100 MW

Period of operation = 1 week

Total time period of operation T_{max} = 24 * 7 hrs = 168 hr.

Total energy demand $E_D = P_D * T_{max}$ = 100 * 168 = 16800 MWh

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

Energy balance equation is,

Thermal plant energy E_T + hydro plant energy E_H = Load energy E_D

∴ Thermal plant energy E_T = Load energy E_D – hydro plant energy E_H

Thermal plant energy E_T = 16800 – 12000 = 4800 MWh = $T_T * P_{GT}'$

The thermal power generation at which the thermal unit must be operated to minimize the total cost is, $P_{GT}' = \sqrt{\frac{a}{c}}$

where 'a' and 'c' are cost coefficients of thermal plant and cost equation: $F_T(P_{GT}) = a_T + b_T\ P_{GT} + c_T\ P_{GT}^2$

$$P_{GT}' = \sqrt{\frac{a}{c}} = \sqrt{\frac{55}{0.022}} = 50\ MW$$

Period of thermal plant operation $T_T = \frac{E_T}{P_{GT}'} = \frac{4800}{50}$ = 96 hr.

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

Hence, the generation schedule for the given load demand of 100 MW for 1 week period is given as follows.

(1) For 96 hrs, the thermal power generation is $P_{GT}{}'$ = 50 MW and hydro power generation is $P_{GH} = P_D - P_{GT}{}' = 100 - 50 = 50$ MW to meet the load demand of 100 MW.

(2) For the remaining period of $T_{max} - T_T = 168 - 96 = 72\ hrs$, only the hydro plant will run for full load hydro power generation of 100 MW to meet the load demand of 100 MW.

(ii) Constraint on volume of water:

Given, Total volume of water (limited) $V_T = 3.2 * 10^8\ m^3$

As per (i), optimum thermal power generation $P_{GT}{}' = 50$ MW.

Let it be supplied for T_T hrs.

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

Therefore, the hydro plant should supply P_{GH} = 50 MW for T_T hrs. and P_{GH} = 100 MW for remaining $(168 - T_T)$ hrs.

Given that, rate of water discharge $Q_o = 110 + 5.1\ P_{GH}$ m³/s

$\Rightarrow \quad Q_o = (110 + 5.1\ P_{GH}) * 3600$ m³/hr

∴ For hydro plant supplying P_{GH} = 50 MW during the period of T_T hrs,

Vol. of water required $V_1 = Q_o * T_T = (110 + 5.1\ * 50) * 3600 * T_T\ \ m^3$

For hydro plant supplying 100 MW during the period of $(168 - T_T)$ hrs,

Volume of water required $V_2 = Q_o * (168 - T_T)\ m^3$

$V_2 = (110 + 5.1\ * 100) * 3600 * (168 - T_T)\ m^3$

∴ Total volume of water required for complete period of operation of 168 hr is,

$V_T = V_1 + V_2 = 3.2 * 10^8\ m^3$ (Given)

$\Rightarrow (110 + 5.1\ * 50) * 3600 * T_T\ + (110 + 5.1\ * 100) * 3600 * (168 - T_T) = 3.2 * 10^8\ \ m^3$

Solving, $T_T = 59.89\ hr$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

Hence, the generation schedule for meeting the given load demand of 100 MW is as follows.

(i) The thermal plant supplies P_{GT} =50 MW & hydro plant supplies P_{GH} =50 MW for a period of 59.89 hr.

(ii) The hydro plant alone supplies P_{GH} = 100 MW for the remaining period of $(168 - T_T) = (168 - 59.89) = 108.11\ hrs.$

NOTE: For complete minimization of cost of thermal power generation, the hydro plant alone may generate P_{GH} = 100 MW for complete period of operation of T = 168 hrs.

Total volume of water required $V_T = Q_o * T = Q_o * 168\ \ m^3$

$V_T = (110 + 5.1\ * 100) * 3600 * 168 = 374976000\ m^3$

$= 3.74976 * 10^8\ m^3$

With this volume of water available, the thermal plant can be completely shut down.

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

3) A two plant system with a hydral plant and a thermal plant has the following characteristics. The fuel cost characteristics of the thermal plant is $F_T = 20\ P_{GT} + 0.04\ P_{GT}{}^2$ Rs/hr. The water discharge characteristics of hydral plant is $Q_o = 7.5\ P_{GH} + 0.004\ P_{GH}{}^2$ m³/s. The constant which converts incremental water discharge to incremental plant cost γ is $4.1 * 10^{-4}\ Rs./m^3$ and λ = 70 Rs./MWhr. Take $B_{HH} = 0.0025\ MW^{-1}$. Determine the generation of each plant, the load on the system and the power transmission losses.

Sol.: Given, $F_T = 20\ P_{GT} + 0.04\ P_{GT}{}^2\ Rs./hr$

$Q_o = 7.5\ P_{GH} + 0.004\ P_{GH}{}^2\ m^3/s = \left(7.5\ P_{GH} + 0.004\ P_{GH}{}^2\right) * 3600\ m^3/hr$

$\Rightarrow \quad Q_o = \left(27000\ P_{GH} + 14.4\ P_{GH}{}^2\right) m^3/hr$

$\gamma = 4.1 * 10^{-4}\ Rs./m^3 \quad \lambda = 70\ Rs./MWhr$ and $B_{HH} = 0.0025\ MW^{-1}$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

The transmission power loss is given by,

$$P_{Loss} = B_{TT} * P_{GT}{}^{2} + B_{HH} * P_{GH}{}^{2} + 2 * B_{TH} * P_{GT} * P_{GH}$$

Given that, $B_{HH} = 0.0025\ MW^{-1}$ and let $B_{TT} = B_{TH} = 0$

$$\therefore P_{Loss} = 0.0025 * P_{GH}{}^{2}$$

Hence, $\frac{\partial P_{Loss}}{\partial P_{GT}} = 0$ and $\frac{\partial P_{Loss}}{\partial P_{GH}} = 0.005\ P_{GH}$

Since, Fuel cost of thermal plant is given as,

$$F_T = 20\ P_{GT} + 0.04\ P_{GT}{}^{2}\ Rs./hr$$

then, $\frac{dF_T}{dP_{GT}} = 20 + 0.08\ P_{GT}\ \ Rs./MWhr$

Water discharge rate of hydro plant,

$$Q_o = \left(27000\ P_{GH} + 14.4\ P_{GH}{}^{2}\right) m^3/hr$$

then, $\frac{dQ_0}{dP_{GH}} = 27000 + 28.8\ P_{GH}\ \ m^3/MWhr$

Let the period of operation be, h = 1 hr.

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

The coordination equations for $\gamma - \lambda$ iterative algorithm are,

$$h * \frac{dF_T}{dP_{GT}} + \lambda * \frac{\partial P_{Loss}}{\partial P_{GT}} = \lambda \quad \text{and} \quad \gamma * h * \frac{dQ_0}{dP_{GH}} + \lambda * \frac{\partial P_{Loss}}{\partial P_{GH}} = \lambda$$

Substituting all the parameters in coordination equations gives,

$$h * \frac{dF_T}{dP_{GT}} + \lambda * \frac{\partial P_{Loss}}{\partial P_{GT}} = \lambda \Longrightarrow \quad 1 * (20 + 0.08\ P_{GT}) + 70 * 0 = 70$$

$$\Longrightarrow \quad (20 + 0.08\ P_{GT}) = 70 \qquad \Longrightarrow \quad P_{GT} = 625\ \text{MW}$$

$$\gamma * h * \frac{dQ_0}{dP_{GH}} + \lambda * \frac{\partial P_{Loss}}{\partial P_{GH}} = \lambda$$

$$\Longrightarrow \quad 4.1 * 10^{-4} * 1 * (27000 + 28.8\ P_{GH}) + 70 * 0.005\ P_{GH} = 70$$

Solving, $P_{GH} = 162.88\ \text{MW}$

$$\therefore P_{Loss} = 0.0025 * P_{GH}{}^{2} = 0.0025 * 162.88^2 = 66.33\ \text{MW}$$

Power balance eqn.; $P_D + P_{Loss} - P_{GH} - P_{GT} = 0$

$$\therefore P_D = P_{GT} + P_{GH} - P_{Loss} = 625 + 162.88 - 66.33 = 721.55\ \text{MW}$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

4) Obtain the schedule for a hydrothermal system for a 1 hr. period with the following data: Load = 800 MW, $F_T = 1200 + 8\ P_{GT} + 0.002\ P_{GT}{}^{2}$, $Q_o = 10^5 + 6 * 10^3\ P_{GH}$, total volume of water available $(Q_0)_{Total} = 32 * 10^6\ m^3$. The loss coefficients are, $B_{TT} = B_{TH} = 0$, $B_{HH} = 0.45 * 10^{-5}\ MW^{-1}$. Take initial values as, λ = 9.6 and γ = 1.59385973*10^{-3}. Make the computations for one iteration.

Sol.: Given, $T_{max} = 1$ hr. No. of intervals N = 1, i.e., k = 1.

$P_D = 800$ MW, λ = 9.6 Rs./MWhr, and γ = 1.59385973*10^{-3} Rs./m³

The loss coefficients are, $B_{TT} = B_{TH} = 0$, $B_{HH} = 0.45 * 10^{-5}\ MW^{-1}$

$$\therefore P_{Loss} = B_{TT}\ P_{GT}{}^{2} + 2 * B_{TH}\ P_{GT}\ P_{GH} + B_{HH}\ P_{GH}{}^{2} = B_{HH}\ P_{GH}{}^{2}$$

$$\Longrightarrow \quad P_{Loss} = 0.45 * 10^{-5}\ P_{GH}{}^{2}$$

$$\therefore \frac{\partial P_{Loss}}{\partial P_{GT}} = 0 \qquad \text{and} \qquad \frac{\partial P_{Loss}}{\partial P_{GH}} = 0.9 * 10^{-5}\ P_{GH}$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

Also given, $F_T = 1200 + 8\ P_{GT} + 0.002\ P_{GT}{}^{2}\ Rs./hr$

$$\Longrightarrow \quad \frac{dF_T}{dP_{GT}} = 8 + 0.004\ P_{GT}$$

and $Q_o = 10^5 + 6 * 10^3\ P_{GH}\ \ m^3/hr$ (Say) $\Longrightarrow \frac{dQ_0}{dP_{GH}} = 6 * 10^3$

Now, let us calculate the thermal and hydro generations using coordination equations as follows.

$$h * \frac{dF_T}{dP_{GT}} + \lambda * \frac{\partial P_{Loss}}{\partial P_{GT}} = \lambda \qquad \Longrightarrow 1 * (8 + 0.004\ P_{GT}) + 9.6 * 0 = 9.6$$

$$\Longrightarrow P_{GT} = 400\ \text{MW}$$

and $\gamma * h * \frac{dQ_0}{dP_{GH}} + \lambda * \frac{\partial P_{Loss}}{\partial P_{GH}} = \lambda$

$$\Longrightarrow \quad 1.59385973 * 10^{-3} * 1 * 6 * 10^3 + 9.6 * 0.9 * 10^{-5}\ P_{GH} = 9.6$$

$$\Longrightarrow P_{GH} = 426.39\ \text{MW}$$

$$\therefore P_{Loss} = 0.45 * 10^{-5}\ P_{GH}{}^{2} = 0.45 * 10^{-5} * 426.39^2$$

$$P_{Loss} = 0.8181\ \text{MW}$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

$\therefore$ Change in power $\Delta P = P_D + P_{Loss} - P_{GT} - P_{GH}$

$$\Longrightarrow \Delta P = 800 + 0.8181 - 400 - 426.39$$

$$\Delta P = -25.5719 \text{ MW}$$

Now, volume of water discharged in 1 hr. is,

$$Q_o = 10^5 + 6 * 10^3 \, P_{GH} \; m^3/hr$$

$$\Longrightarrow Q_o = 10^5 + 6 * 10^3 * 426.39 = 2658340 \, m^3$$

$$Q_o = 2.65834 * 10^6 \, m^3$$

$\therefore$ Change in volume of water discharged

$$\Delta Q_0 = \sum_{k=1}^{N} Q_0{}^k * h^k - (Q_0)_{Total}$$

$$\Longrightarrow \Delta Q_o = Q_o * h - (Q_0)_{Total} = 2.65834 * 10^6 - 32 * 10^6$$

$$\Longrightarrow \Delta Q_o = -29.34166 * 10^6 \, m^3$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

5) A steam plant and a hydro plant supply a load of 500 MW for first 12 hrs. and 300 MW for next 12 hrs. in a day. The thermal plant characteristics are given by, $F(P_{GT}{}^k) = 0.06 * (P_{GT}{}^k)^2 + 40 * (P_{GT}{}^k) + 100 \; Rs./hr$. The hydro plant characteristics are given by, $Q_0{}^k = 0.003 * (P_{GH}{}^k)^2 + 0.5 * (P_{GH}{}^k) \; m^3/s$. The power loss is given by, $P_{Loss}{}^k = 0.001 * (P_{GH}{}^k)^2$. Take $\gamma = 80 \; Rs./m^3$. Find the schedule of power and total water discharge. Also determine the daily operating cost of the thermal plant and daily water discharged by the hydro plant. Obtain the schedule (i) neglecting losses and (ii) considering losses. Use penalty factor method.

Sol.: Given, N = 2, $\therefore$ k = 1,2 and ΔT = 12 hrs.

Load demand $P_D{}^k$ are, $P_D{}^1 = 500$ MW and $P_D{}^2 = 300$ MW

and $\gamma = 80 \; Rs./m^3$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

Also, given that, $F(P_{GT}{}^k) = 0.06 * (P_{GT}{}^k)^2 + 40 * (P_{GT}{}^k) + 100 \; Rs./hr$.

$$\Longrightarrow \quad IC_T = \frac{dF(P_{GT}{}^k)}{dP_{GT}{}^k} = 0.12 * P_{GT}{}^k + 40$$

and, $Q_0{}^k = 0.003 * (P_{GH}{}^k)^2 + 0.5 * (P_{GH}{}^k) \; m^3/s$.

$$\Longrightarrow \quad IW_H = \frac{dQ_0{}^k}{dP_{GH}{}^k} = 0.006 * P_{GH}{}^k + 0.5$$

The transmission loss is given by, $P_{Loss}{}^k = 0.001 * (P_{GH}{}^k)^2$

$$\therefore \quad \frac{\partial P_{Loss}{}^k}{\partial P_{GT}{}^k} = 0 \quad \text{and} \quad \frac{\partial P_{Loss}{}^k}{\partial P_{GH}{}^k} = 0.002 \; P_{GH}{}^k$$

The penalty factors are, $L_T = \frac{1}{\left[1 - \frac{\partial P_{Loss}{}^k}{\partial P_{GT}{}^k}\right]} = 1$ and $L_H = \frac{1}{\left[1 - \frac{\partial P_{Loss}{}^k}{\partial P_{GH}{}^k}\right]} = \frac{1}{[1 - 0.002 \; P_{GH}{}^k]}$

Case (i) Neglecting losses:-

If the losses are neglected, the coordination equations are,

$$\gamma * \frac{dQ_0{}^k}{dP_{GH}{}^k} = \lambda \quad \text{and} \quad \frac{dF(P_{GT}{}^k)}{dP_{GT}{}^k} = \lambda \quad \therefore \gamma * \frac{dQ_0{}^k}{dP_{GH}{}^k} = \frac{dF(P_{GT}{}^k)}{dP_{GT}{}^k} = \lambda$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

$$\Longrightarrow \quad 80 * (0.006 * P_{GH}{}^k + 0.5) = 0.12 * P_{GT}{}^k + 40 = \lambda$$

$$\Longrightarrow \quad 80 * (0.006 * P_{GH}{}^k + 0.5) = 0.12 * P_{GT}{}^k + 40$$

$$\Longrightarrow \quad 0.12 \; P_{GT}{}^k - 0.48 \; P_{GH}{}^k = 0 \text{ ------------ } (1)$$

(a)In the first interval k = 1:

The coordination eqn (1) at k = 1 is, $0.12 \; P_{GT}{}^1 - 0.48 \; P_{GH}{}^1 = 0$ -- $(1a)$

In k = 1, $P_D{}^1 = 500$ MW for 12 hrs.

$\therefore$ The power balance equations is, $P_{GT}{}^1 + P_{GH}{}^1 = P_D{}^1 = 500$ ---- (2)

Solving $(1a)$ and (2), $P_{GT}{}^1 = 400$ MW and $P_{GH}{}^1 = 100$ MW

(b) In the first interval k = 2:

The coordination eqn (1) at k = 2 is, $0.12 \; P_{GT}{}^2 - 0.48 \; P_{GH}{}^2 = 0$ -- $(1b)$

In k = 2, $P_D{}^2 = 300$ MW for 12 hrs.

$\therefore$ The power balance equations is, $P_{GT}{}^2 + P_{GH}{}^2 = P_D{}^2 = 300$ ---- (3)

Solving $(1b)$ and (3), $P_{GT}{}^2 = 240$ MW and $P_{GH}{}^2 = 60$ MW

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

The daily operating cost of thermal plant is,

$$F_T(P_{GT}{}^k) = \left(F(P_{GT}{}^1) * \Delta T + F(P_{GT}{}^2) * \Delta T\right) Rs. = \left(F(400) * 12 + F(240) * 12\right) Rs.$$

$$= \left(\left(0.06 * (400)^2 + 40 * (400) + 100\right) * 12 + \left(0.06 * (240)^2 + 40 * (240) + 100\right) * 12\right) Rs.$$

$$F_T(P_{GT}{}^k) = Rs. 4,66,272$$

The daily water discharged by the hydro plant is,

$$(Q_0)_{Total} = \left(Q_0{}^1(P_{GH}{}^1) * 3600 * \Delta T + Q_0{}^2(P_{GH}{}^2) * 3600 * \Delta T\right) \ m^3$$

$$= \left(Q_0{}^1(100) * 3600 * 12 + Q_0{}^2(60) * 3600 * 12\right) \ m^3$$

$$= \left(\left(0.003 * (100)^2 + 0.5 * 100\right) * 3600 * 12 + \left(0.003 * (60)^2 + 0.5 * 60\right) * 3600 * 12\right) \ m^3$$

$$(Q_0)_{Total} = 5218560 \ m^3 = 5.21856 * 10^6 \ m^3$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

Case (ii) Considering losses:-

If the losses are considered, the coordination equations are,

$$\gamma * IW_H * L_H = \gamma * \frac{dQ_0{}^k}{dP_{GH}{}^k} * \frac{1}{\left[1 - \frac{\partial P_{Loss}{}^k}{\partial P_{GH}{}^k}\right]} = \lambda \text{ and } IC_T * L_T = \frac{dF(P_{GT}{}^k)}{dP_{GT}{}^k} * \frac{1}{\left[1 - \frac{\partial P_{Loss}{}^k}{\partial P_{GT}{}^k}\right]} = \lambda$$

$$\therefore \gamma * IW_H * L_H = IC_T * L_T = \lambda$$

$$\Rightarrow 80 * \left(0.006 * P_{GH}{}^k + 0.5\right) * \frac{1}{\left[1 - 0.002 \ P_{GH}{}^k\right]} = \left(0.12 * P_{GT}{}^k + 40\right) * 1 = \lambda$$

$$\Rightarrow 80 * \left(0.006 * P_{GH}{}^k + 0.5\right) * \frac{1}{\left[1 - 0.002 \ P_{GH}{}^k\right]} = \left(0.12 * P_{GT}{}^k + 40\right)$$

$$\Rightarrow \left(0.12 * P_{GT}{}^k + 40\right) * \left(1 - 0.002 \ P_{GH}{}^k\right) = 80 * \left(0.006 * P_{GH}{}^k + 0.5\right)$$

$$\Rightarrow 0.12 * P_{GT}{}^k - 0.00024 * P_{GT}{}^k * P_{GH}{}^k - 0.56 * P_{GH}{}^k = 0 \text{ ----- } (4)$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

(a)In the first interval k = 1:

The coordination equation (4) **at k = 1 is,**

$$0.12 \ P_{GT}{}^1 - 0.00024 \ P_{GT}{}^1 \ P_{GH}{}^1 - 0.56 \ P_{GH}{}^1 = 0 \text{ --------- } (4a)$$

In k = 1, $P_D{}^1 = 500$ **MW for 12 hrs.**

$\therefore$ **The power balance equations is,** $P_{GT}{}^1 + P_{GH}{}^1 - P_D{}^1 - P_{Loss}{}^1 = 0$

$$\Rightarrow \ P_{GT}{}^1 + P_{GH}{}^1 - 500 - 0.001 * \left(P_{GH}{}^1\right)^2 = 0$$

$$\Rightarrow \ P_{GT}{}^1 = 0.001 * \left(P_{GH}{}^1\right)^2 - P_{GH}{}^1 + 500 \text{ --------- } (5)$$

Substituting (5) **in** $(4a)$,

$$0.12 \left(0.001 * \left(P_{GH}{}^1\right)^2 - P_{GH}{}^1 + 500\right) - 0.00024 \left(0.001 * \left(P_{GH}{}^1\right)^2 - P_{GH}{}^1 + 500\right) P_{GH}{}^1 - 0.56 \ P_{GH}{}^1 = 0$$

Simplifying, $-2.4 * 10^{-7}\left(P_{GH}{}^1\right)^3 + 3.6 * 10^{-4}\left(P_{GH}{}^1\right)^2 - 0.8 \ P_{GH}{}^1 + 60 = 0$

Solving, $P_{GH}{}^1 = 77.5675 \text{ MW}, \ 711.216 \text{ MW and } 711.216 \text{ MW}$

Select $P_{GH}{}^1 = 77.5675 \text{ MW}$

$$\therefore \ P_{Loss}{}^1 = 0.001 * \left(P_{GH}{}^1\right)^2 = 0.001 * (77.5675)^2 = 6.017 \text{ MW}$$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

$$\therefore \ P_{GT}{}^1 = 0.001 * \left(P_{GH}{}^1\right)^2 - P_{GH}{}^1 + 500 = 0.001 * (77.5675)^2 - 77.5675 + 500$$

$$P_{GT}{}^1 = 428.4492 \text{ MW}$$

(b) In the first interval k = 2:

The coordination equation (4) **at k = 2 is,**

$$0.12 \ P_{GT}{}^2 - 0.00024 \ P_{GT}{}^2 \ P_{GH}{}^2 - 0.56 \ P_{GH}{}^2 = 0 \text{ --------- } (4b)$$

In k = 2, $P_D{}^2 = 300$ **MW for 12 hrs.**

Hence, the power balance equations is, $P_{GT}{}^2 + P_{GH}{}^2 - P_D{}^2 - P_{Loss}{}^2 = 0$

$$\Rightarrow \ P_{GT}{}^2 + P_{GH}{}^2 - 300 - 0.001 * \left(P_{GH}{}^2\right)^2 = 0$$

$$\Rightarrow \ P_{GT}{}^2 = 0.001 * \left(P_{GH}{}^2\right)^2 - P_{GH}{}^2 + 300 \text{ --------- } (6)$$

Substituting (6) **in** $(4b)$,

$$0.12 \left(0.001 * \left(P_{GH}{}^2\right)^2 - P_{GH}{}^2 + 300\right) - 0.00024 \left(0.001 * \left(P_{GH}{}^2\right)^2 - P_{GH}{}^2 + 300\right) P_{GH}{}^2 - 0.56 \ P_{GH}{}^2 = 0$$

Simplifying, $-2.4 * 10^{-7}\left(P_{GH}{}^2\right)^3 + 3.6 * 10^{-4}\left(P_{GH}{}^2\right)^2 - 0.752 \ P_{GH}{}^2 + 36 = 0$

Solving, $P_{GH}{}^2 = 48.9835 \text{ MW}, \ 725.5083 \text{ MW and } 725.5083 \text{ MW}$

Select $P_{GH}{}^2 = 48.9835 \text{ MW}$

NUMERICAL PROBLEMS – HYDROTHERMAL SCHEDULING:

$$\therefore \quad {P_{Loss}}^{2} = 0.001 * \left({P_{GH}}^{2}\right)^{2} = 0.001 * (48.9835)^{2}$$

$${P_{Loss}}^{2} = 2.3946 \text{ MW}$$

$$\therefore \; {P_{GT}}^{2} = 0.001 * \left({P_{GH}}^{2}\right)^{2} - {P_{GH}}^{2} + 300 = 0.001 * (48.9835)^{2} - 48.9835 + 300$$

$${P_{GT}}^{2} = 253.4159 \text{ MW}$$

The daily operating cost of thermal plant is,

$$F_T\left({P_{GT}}^{k}\right) = \left(F\left({P_{GT}}^{1}\right) * \Delta T + F\left({P_{GT}}^{2}\right) * \Delta T\right) Rs. = Rs.\left(F(428.4492) * 12 + F(253.4159) * 12\right)$$

$$F_T\left({P_{GT}}^{k}\right) = Rs.\begin{pmatrix} \left(0.06 * (428.4492)^2 + 40 * (428.4492) + 100\right) * 12 \\ +\left(0.06 * (235.4159)^2 + 40 * (235.4159) + 100\right) * 12 \end{pmatrix}$$

$$F_T\left({P_{GT}}^{k}\right) = Rs. 4,93,128$$

The daily water discharged by the hydro plant is,

$$(Q_0)_{Total} = \left({Q_0}^{1}\left({P_{GH}}^{1}\right) * 3600 * \Delta T + {Q_0}^{2}\left({P_{GH}}^{2}\right) * 3600 * \Delta T\right) \; m^3$$

$$= \left({Q_0}^{1}(77.5675) * 3600 * 12 + {Q_0}^{2}(48.9835) * 3600 * 12\right) \; m^3$$

$$= \begin{pmatrix} \left(0.003 * (77.5675)^2 + 0.5 * (77.5675)\right) * 3600 * 12 + \\ \left(0.003 \, (48.9835)^2 + 0.5 * (48.9835)\right) * 3600 * 12 \end{pmatrix} m^3$$

$$(Q_0)_{Total} = 3824228.2025736 \; m^3 = 3.824 * 10^6 \; m^3$$

THANK YOU

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