数字图像处理及应用 第2次作业

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A

Part I Exercises

Ex.1

(**a**) Give a continuous function for implementing the contrast stretching transformation shown in **Fig. 3.2(a)**. In addition to k, your function must include a parameter, E, for controlling the slope of the function as it transitions from low to high intensity values. Your function should be normalized so that its minimum and maximum values are 0 and 1, respectively.
(**b**) Sketch a family of transformations as a function of parameter E, for a fixed value $m = L/4$ where L is the number of intensity levels in the image.
(**c**) What is the smallest value of E that will make your function effectively perform as the function in **Fig. 3.2(b)**? In other words, your function does not have to be identical to **Fig. 3.2(b)**. It just has to yield the same result of producing a binary image. Assume that you are working with 8-bit images, and let $k = 128$. Let C denote the smallest positive number representable in the computer you are using.

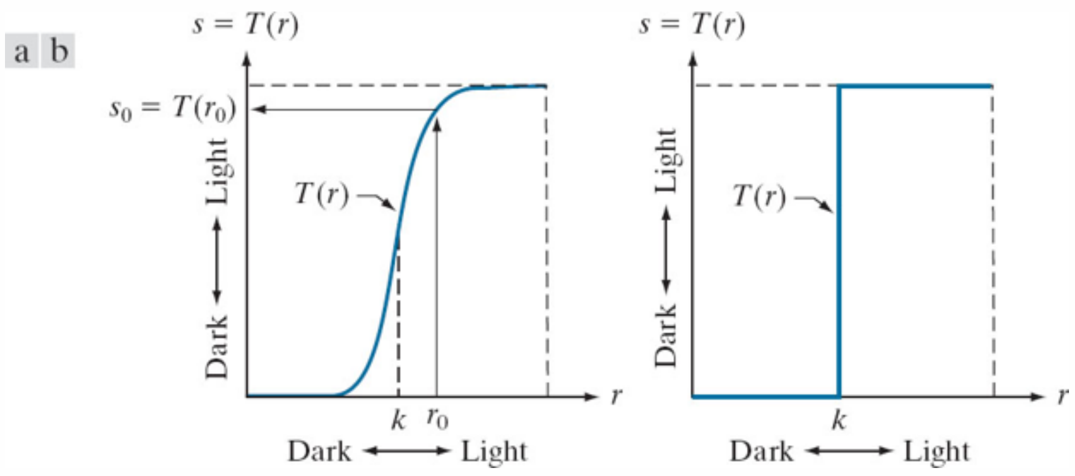


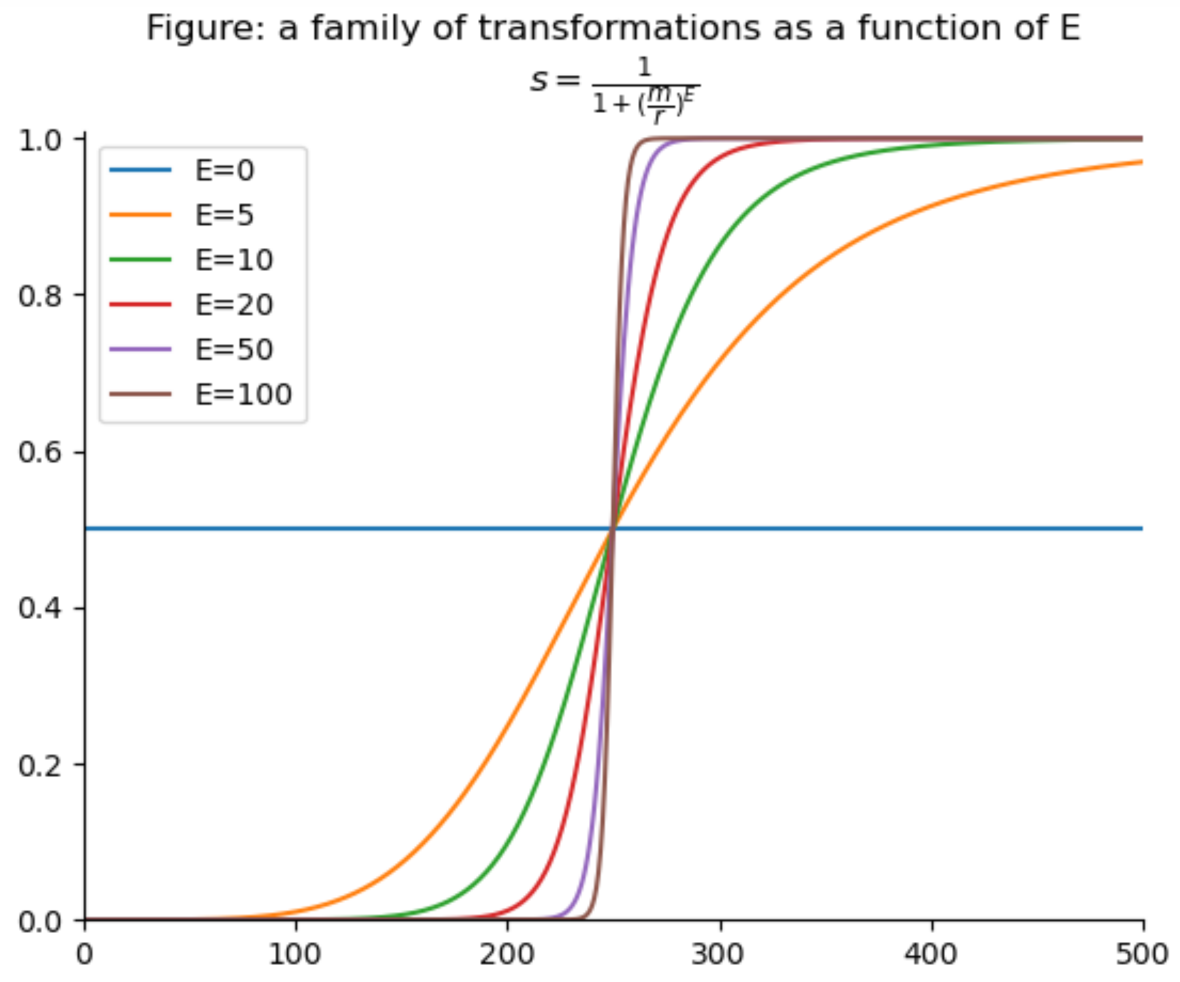
FIGURE 3.2 Intensity transformation functions

Answer:

(a)

$$\begin{aligned} s &= T(r) \\ &= \frac{1}{1+(\frac{m}{r})^E}. \end{aligned}$$

(b)



(c) The contrast stretching function shown in **Fig 3.2(b)** could be described as

$$s = T(r) = \begin{cases} 0, & r \le m-1 \\ 1, & r \ge m+1. \end{cases}$$

In equation (a), if $r = m$, it is evident that $s = T(r) = 0.5$, regardless of any value of parameter E.

As a consequence,

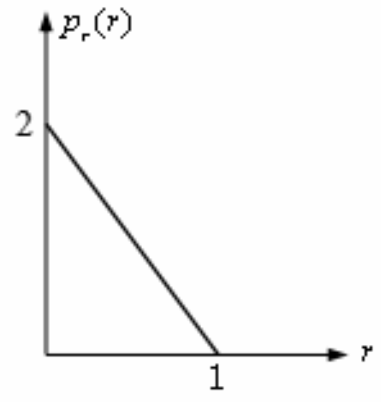
$$s = T(r) = \begin{cases} 0, & r < m-1 \\ 0.5, & r = m \\ 1, & r > m+1. \end{cases}$$

C is the smallest positive number representable in the computer while s is also positive. Finding the smallest value of E is to solve the following equation where $m = 128$.

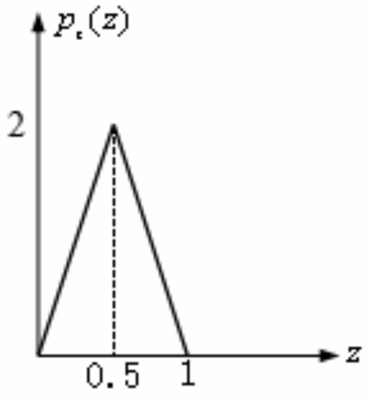
$$\frac{1}{1+\left(\frac{m}{m-1}\right)^E} < \frac{C}{2}$$

Ex.2

An image with intensities in the range [0, 1] has the PDF $p_r(r)$ shown in the following diagram. It is desired to transform the intensity levels of this image so that they will have the specified $p_{z(z)}$ shown. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.



(a)



(b)

Answer: From diagrams above,

$$p_r(r) = -2r + 2$$

$$p_z(z) = \begin{cases} 4z, & z \in [0, 0.5] \\ -4z + 4, & z \in (0.5, 1] \\ 0, & else. \end{cases}$$

Equalizing $p_r(r)$ as follows.

$$\begin{aligned} s &= \int_0^r p_r(w)dw \\ &= \int_0^r (-2w + 2)dw \\ &= -r^2 + 2r \end{aligned}$$

Equalizing $p_z(z)$ as follows.

$$G(z) = \begin{cases} \int_0^z 4vdv = 2z^2, & 0 \leq z < \frac{1}{2} \\ \int_0^{\frac{1}{2}} 4vdv \ + \ \int_{\frac{1}{2}}^z (-4v + 4)dv = -2z^2 + 4z - 1, & \frac{1}{2} \leq z < 1. \\ \int_0^{\frac{1}{2}} 4vdv \ + \ \int_{\frac{1}{2}}^1 (-4v + 4)dv = 1, & 1 \leq z \end{cases}$$

$$G(z) = r = \begin{cases} 2z^2, & 0 \leq z < \frac{1}{2} \\ -2z^2 + 4z - 1, & \frac{1}{2} \leq z < 1 \\ 1, & 1 \leq z \end{cases}$$

$$\Rightarrow z = \begin{cases} \left(\frac{2r - r^2}{2}\right)^{\frac{1}{2}}, & 0 \leq r < 1 - \frac{\sqrt{2}}{2} \\ 1 + \frac{r - 1}{\sqrt{2}}, & 1 - \frac{\sqrt{2}}{2} < r \leq 1 \end{cases}$$

Ex.3 The implementation of linear spatial filters requires moving the center of a mask throughout an image and, at each location, computing the sum of products of the mask coefficients with the corresponding pixels at that location (see **Section 3.4**). A lowpass filter can be implemented by setting all coefficients to 1, allowing use of a so-called box-filter or moving-average algorithm, which consists of updating only the part of the computation that changes from one location to the next.

(**a**) Formulate such an algorithm for an $n \times n$ filter, showing the nature of the computations involved and the scanning sequence used for moving the mask around the image.

(**b**) The ratio of the number of computations performed by a brute-force implementation to the number of computations performed by the box-filter algorithm is called the *computational advantage*. Obtain the computational advantage in this case and plot it as a function of n for $n > 1$. The $1/n^2$ scaling factor is common to both approaches, so you need not consider it in obtaining the computational advantage. Assume that the image has an outer border of zeros that is wide enough to allow you to ignore border effects in your analysis.

Answer:

(a) Consider a 3×3 mask first. Because all the coefficients are 1 (we are ignoring the 1/9 scale factor), the net effect of the lowpass filter operation is to add all the intensity values of pixels under the mask. Initially, it takes 8 additions to produce the response of the mask. However, when the mask moves one pixel location to the right, it picks up only one new column. The new response can be computed as

$$R_{new} = R_{old} - C_1 + C_3$$

where C_1 is the sum of pixels under the first column of the mask before it was moved, and C_3 is the similar sum in the column it picked up after it moved. This is the basic box-filter or moving-average equation. For a 3×3 mask it takes 2 additions to get C_3 (C_1 was already computed). To this, we add one subtraction and one addition to get R_{new}. Thus, a total of 4 arithmetic operations are needed to update the response after one move. This is a recursive procedure for moving from left to right along one row of the image. When we get to the end of a row, we move down one pixel (the nature of the computation is the same) and continue the scan in the opposite direction.
For a mask of size $n \times n$, $(n-1)$ additions are needed to obtain C_3, plus the single subtraction and addition needed to obtain R_{new}, which gives a total of $(n+1)$ arithmetic operations after each move. A brute-force implementation would require n^2 - 1 additions after each move.

(b) The computational advantage is

$$A = \frac{n^2 - 1}{n+1} = \frac{(n+1)(n-1)}{(n+1)} = n - 1$$

The plot of A as a function of n is a simple linear function starting at A=1 for n = 2. (图) ?

Ex.4
(**a**) Suppose that you filter an image, $f(x, y)$, with a spatial filter mask, $w(x, y)$, using convolution, as defined in **Eq. (3.4-2)**, where the mask is smaller than the image in both spatial directions. Show the important property that, if the coefficients of the mask sum to zero, then the sum of all the elements in the resulting convolution array (filtered image) will be zero also (you may ignore computational inaccuracies). Also, you may assume that the border of the image has been padded with the appropriate number of zeros.

$$w(x,y)\bigstar f(x,y)=\sum_{s=-a}^{s=a}\sum_{t=-b}^{t=b}w(x,y)f(x-s,y-t) \tag{3.4-2}$$

here, the symbol ★ stands for convolution operation.

(**b**) Would the result to (a) be the same if the filtering is implemented using correlation, as defined in **Eq. (3.4-1)**?

$$w(x,y)\otimes f(x,y)=\sum_{s=-a}^{s=a}\sum_{t=-b}^{t=b}w(x,y)f(x+s,y+t) \tag{3.4-1}$$

here, the symbol ⊗ stands for correlation operation.

Answer:

(a) Assume that

$$w(x,y)=\begin{bmatrix}1&1&1\\-1&0&1\\-1&-1&-1\end{bmatrix}$$

$$f(x,y)=\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}.$$

And

$$\sum_{i=1}^{3}\sum_{y=1}^{3}w(i,y)=0$$

$$w(x,y)\star f(x,y)=\begin{bmatrix}1&1&1\\-1&0&1\\-1&-1&-1\end{bmatrix}=0.$$

As a conclusion, if the coefficients of the mask sum to zero, then the sum of all the elements in the resulting convolution array (filtered image) will be zero also.

(b) Similarly, correlation is inverting the convolution, thus the same results is expected.

Ex.5 Discuss the limiting effect of repeatedly filtering an image with a 3×3 lowpass filter kernel. You may ignore border effects.

Answer: Given that 3×3 lowpass filter kernel will smooth the image, making the image more blurred. Using repeatedly will deepen this situation and make the image smoother.

Ex.6 You are given the following kernel and image:

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(**a**) By inspection, find two kernels, w_1 and w_2 so that $w = w_1 \star w_2$. ($\star$ stands for convolution operation)

(**b**) Compute $w_1 \star f$ using the minimum zero padding. Show the details of your computation when the kernel is centered at point (2, 3) (2nd row, 3rd col) of f and then show the full convolution.

(**c**) Compute the convolution of w_2 with the result from (**b**). Show the details of your computation when the kernel is centered at point (3, 3) of the result from (b), and then show the full convolution. Compare with the result of $w \star f$.

Answer:

(a)

$$\begin{aligned} w_1 = w &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \\ w_2 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

(b) Minimum zero padding of f:

$$f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus,

$$\begin{aligned} w_1 \star f &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \star \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix}. \end{aligned}$$

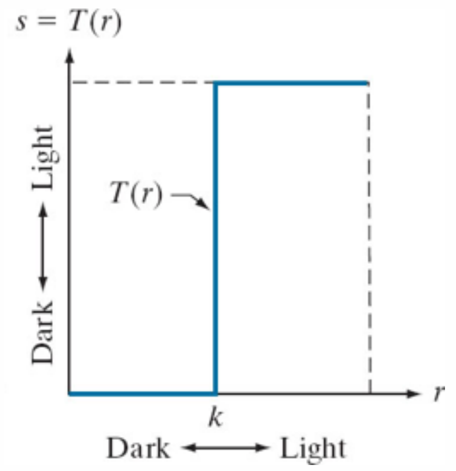
(c)

$$w_2 \star (w_1 \star f) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \star \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 6 & 3 & 0 & 0 \\ 0 & 0 & 4 & 8 & 4 & 0 & 0 \\ 0 & 0 & 3 & 6 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$w \star f = w_1 \star f = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 3 & 6 & 3 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix} = w_2 \star (w_1 \star f).$$

Ex.7 In a character recognition application, text pages are reduced to binary using a thresholding transformation function of the form shown in following figure.



This is followed by a procedure that thins the characters until they become strings of binary 1's on a background of 0's. Due to noise, binarization and thinning result in broken strings of characters with gaps ranging from 1 to 3 pixels. One way to "repair" the gaps is to run a smoothing kernel over the binary image to blur it, and thus create bridges of nonzero pixels between gaps.

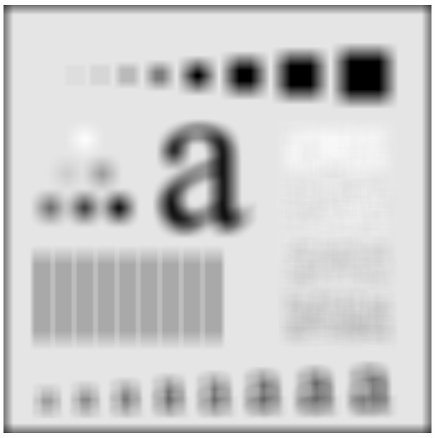
(**a**) Give the (odd) size of the smallest box kernel capable of performing this task.
(**b**) After bridging the gaps, the image is thresholded to convert it back to binary form. For your answer in (a), what is the minimum value of the threshold required to accomplish this, without causing the segments to break up again.

Answer:

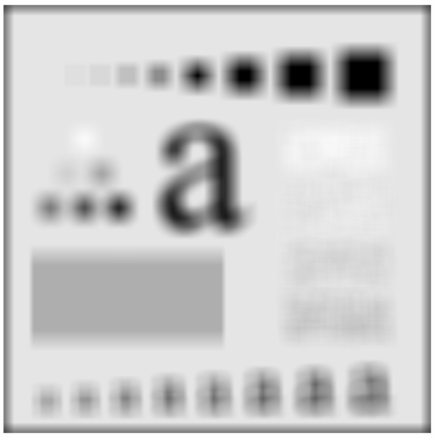
(a) The most extreme case is when the mask is positioned on the centre pixel of a 3-pixel gap, along a thin segment, in which case a 3×3 mask would encompass a completely blank field. Since this is known to be the largest gap, the next (odd) mask size up is guaranteed to encompass some of the pixels in the segment. Thus, the smallest mask that will do the job is a 5×5 averaging mask.

(b) The smallest average value produced by the mask is when it encompasses only two pixels of the segment. This average value is a grey-scale value, not binary, like the rest of the segment pixels. Denote the smallest average value by Amin, and the binary values of pixels in the thin segment by B. Clearly, is less than B. Then, setting the binarizing threshold slightly smaller will create one binary pixel of value B in the centre of the mask.

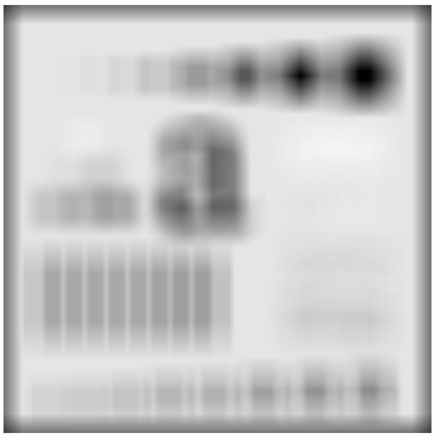
Ex.8 In the original image used to generate the three blurred images shown, the vertical bars are 5 pixels wide, 100 pixels high, and their separation is 20 pixels. The image was blurred using square box kernels of sizes 23, 25, and 45 elements on the side, respectively. The vertical bars on the left, lower part of (**a**) and (**c**) are blurred, but a clear separation exists between them.



(**a**)



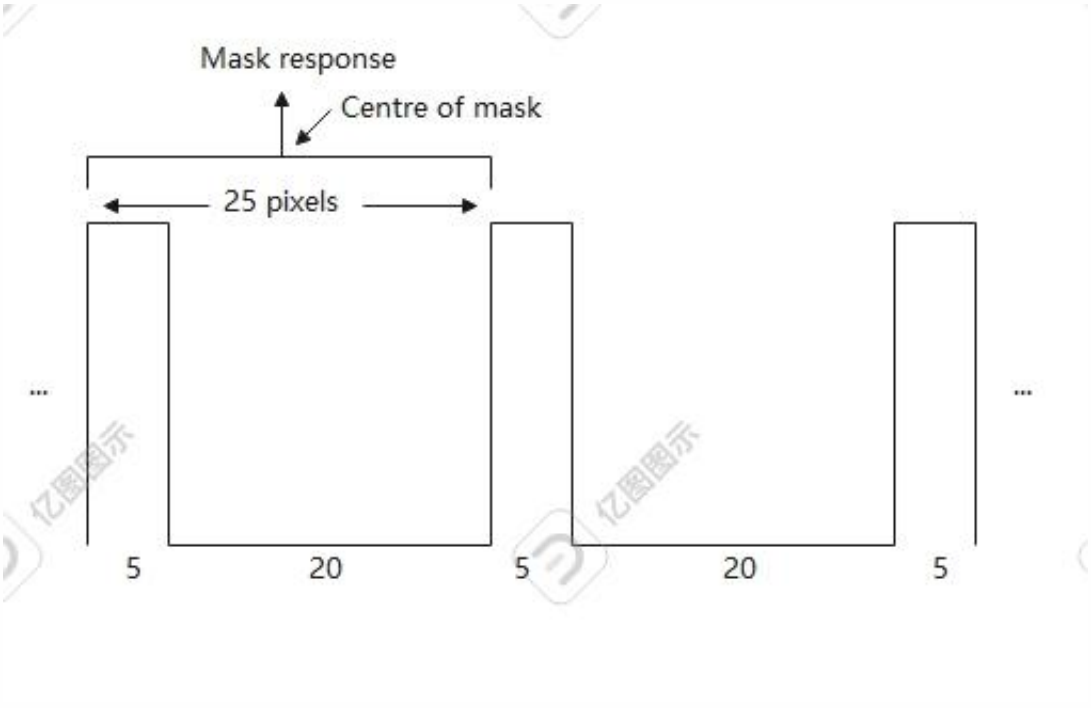
(**b**)


(**c**)

However, the bars have merged in image (**b**), despite the fact that the kernel used to generate this image is much smaller than the kernel that produced image (**c**). Explain the reason for this.

Answer:

From the Fig above, we know that the vertical bars are 5 pixels wide and 100 pixels high and the separation is 20 pixels. The phenomenon in question is related to the horizontal separation between bars, so we can simplify the question by considering a single scan line through the bars in the image. The key to solving this question lies in the fact that the distance (in pixels) between the onset of one bar and the onset of the next one is 25 pixels. Consider the scan line shown in the Fig below. Also shown is a cross-section of a 25×25 mask. The response of the mask is the average of the pixels that it encompasses. Notice that when the mask moves one pixel to the right, it loses one value of the vertical bar on the left, but it picks up an identical one on the right, so the response does not change. In fact, the number of pixels belonging to the vertical bars and contained within the mask does not change, regardless of where the mask is located (as long as it is contained within the bars, and not near the edges of the set of bars). The fact that the number of bar pixels under the mask does not change is due to the peculiar separation between bars and the width of the lines in relation to the 25-pixel width of the mask. This constant response is the reason why no white gaps are seen in the image shown in the problem statement. Note that this constant response does not happen with the 23×23 or the 45×45 masks because they are not "synchronized" with the width of the bars and their separation.



Ex.9 Consider an application such as the one shown in **Fig. 3.34**, in which it is desired to eliminate objects smaller than those enclosed by a square of size $q \times q$ pixels. Suppose that we want to reduce the average intensity of those objects to one-tenth of their original average value. In this way, those objects will be closer to the intensity of the background and they can then be eliminated by thresholding. Give the (odd) size of the smallest averaging mask that will accomplish the desired reduction in average intensity in only one pass of the mask over the image.

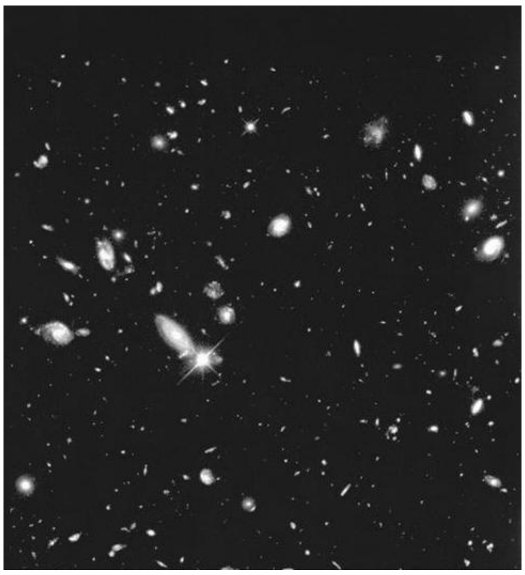


FIGURE 3.34 Image from Hubble Space Telescope

Answer:

In this area, we need up to q^2 points to reduce the average intensity of those objects to one-tenth of their original average value. At the same time I can conclude: the size of mask is larger than the $q \times q$ square neighbourhood. Suppose the size of the filter mask is $n \times n$.The point set outside the object below the mask is called the background.

Suppose a_i, a_j and a_k respectively represent the intensity value of points in the mask, object and background. Thus, the response of the averaging mask at any point on the image is:

$$\begin{aligned} R &= \frac{1}{n^2}\sum_{a_i \in A_1} a_i = \frac{1}{n^2}[\sum_{a_j \in A_2} a_j + \sum_{a_k \in A_3} a_k] \\ &= \frac{1}{n^2}[\frac{q^2}{q^2}\sum_{a_j \in A_2} a_j] + \frac{1}{n^2}\sum_{a_k \in A_3} a_k = \frac{q^2}{n^2}Q + \frac{1}{n^2}(n^2 - q^2) \times S \end{aligned} \tag{1}$$

$A_1 A_2 A_3$ respectively represent the pixel set of mask, object and background. S and Q represent the average intensity value of the background and object points respectively.
Let Equ.(1)$< \dfrac{Q}{10}$, I can get:

$$n > q \times [\frac{10(Q-S)}{Q-10S}]^{0.5}$$

Therefore the size of the smallest averaging mask is $q \times [\dfrac{10(Q-S)}{Q-10S}]^{0.5}$.

Ex.10 A CCD TV camera is used to perform a long-term study by observing the same area 24 hours a day, for 30 days. Digital images are captured and transmitted to a central location every 5 minutes. The illumination of the scene changes from natural daylight to artificial lighting. At no time is the scene without illumination, so it is always possible to obtain an image. Because the range of illumination is such that it is always in the linear operating range of the camera, it is decided not to employ any compensating mechanisms on the camera itself. Rather, it is decided to use image processing techniques to postprocess, and thus normalize, the images to the equivalent of constant illumination. Propose a method to do this. You are at liberty to use any method you wish, but state clearly all the assumptions you made in arriving at your design.

Answer:

It is given that the range of illumination stays in the linear portion of the camera response range, but no values for the range are given. The fact that images stay in the linear range implies that images will not be saturated at the high end or be driven in the low end to such an extent that the camera will not be able to respond, thus losing image information irretrievably. The only way to establish a benchmark value for illumination is when the variable (daylight) illumination is not present. Let $f_o(x, y)$ denote an image taken under artificial illumination only. with no moving objects (e.g., people or vehicles) in the scene. This becomes the standard by which all other images will be normalized. There are numerous ways to solve this problem, but the student must show awareness that areas in the image likely to change due to moving objects should be excluded from the illumination-correction approach.

One way is to select various representative subareas of $f_o(x, y)$ not likely to be obscured by moving objects and compute their average intensities. We then select the minimum and maximum of all the individual average values, denoted by, $\bar{f}_{min}$ and $\bar{f}_{max}$. The objective then is to process any input image, $f(x, y)$, so that its minimum and maximum will be equal to $\bar{f}_{min}$ and $\bar{f}_{max}$, respectively. The easiest way to do this is with a linear transformation function of the form

$$f_{out}(x,y) = af(x,y) + b$$

where f_{out} is the scaled output image. It is easily verified that the output image will have the required minimum and maximum values if we choose

$$a = \frac{\bar{f}_{max} - \bar{f}_{min}}{f_{max} - f_{min}}$$

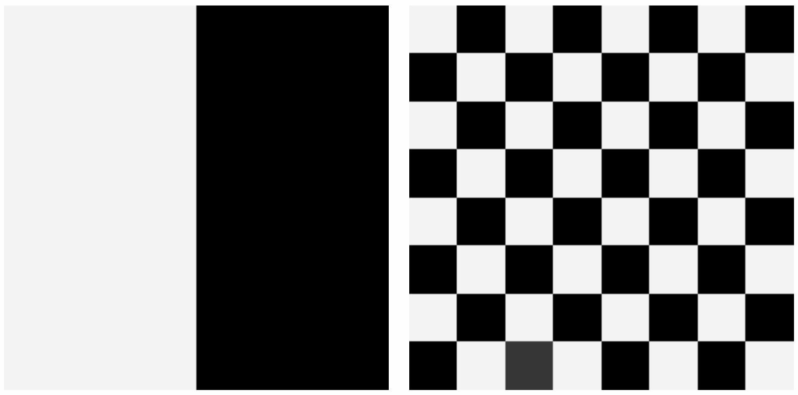
and

$$b = \frac{\bar{f}_{min} f_{max} - f_{min} \bar{f}_{max}}{f_{max} - f_{min}}$$

where f_{max} and $f_{min}$are the maximum and minimum values of the input image.

Part II Programming

1. ***Generate two images with the size of*** 256×256 ***pixels*** which looks like the following images just containing two gray-levels. These two images are quite different, but their histograms are the same. Suppose that each image is blurred with an averaging mask.
(**a**) Would the histograms of the blurred images still be equal? Explain.
(**b**) If your answer is no, sketch the two histograms.



(*followed by* ***Matlab live Scripts*** *or* ***Jupyter Scripts*** *and running results*)

(a) No. The histograms of the blurred images will not be equal as image a possesses fewer boundaries than image b.Or, from another point of view, their local histograms is not equal everywhere. This conclusion could be proved by simulation.

(b) The following programme could Generate two images with the size of 256×256 pixels which looks like the following images just containing two gray-levels. These two images are quite different, but their histograms are the same.

Import dependencies.

```python
import numpy as np
import matplotlib.pyplot as plt
```

Generate image a, using `np.concatenate()` to piece its left part and right part together.

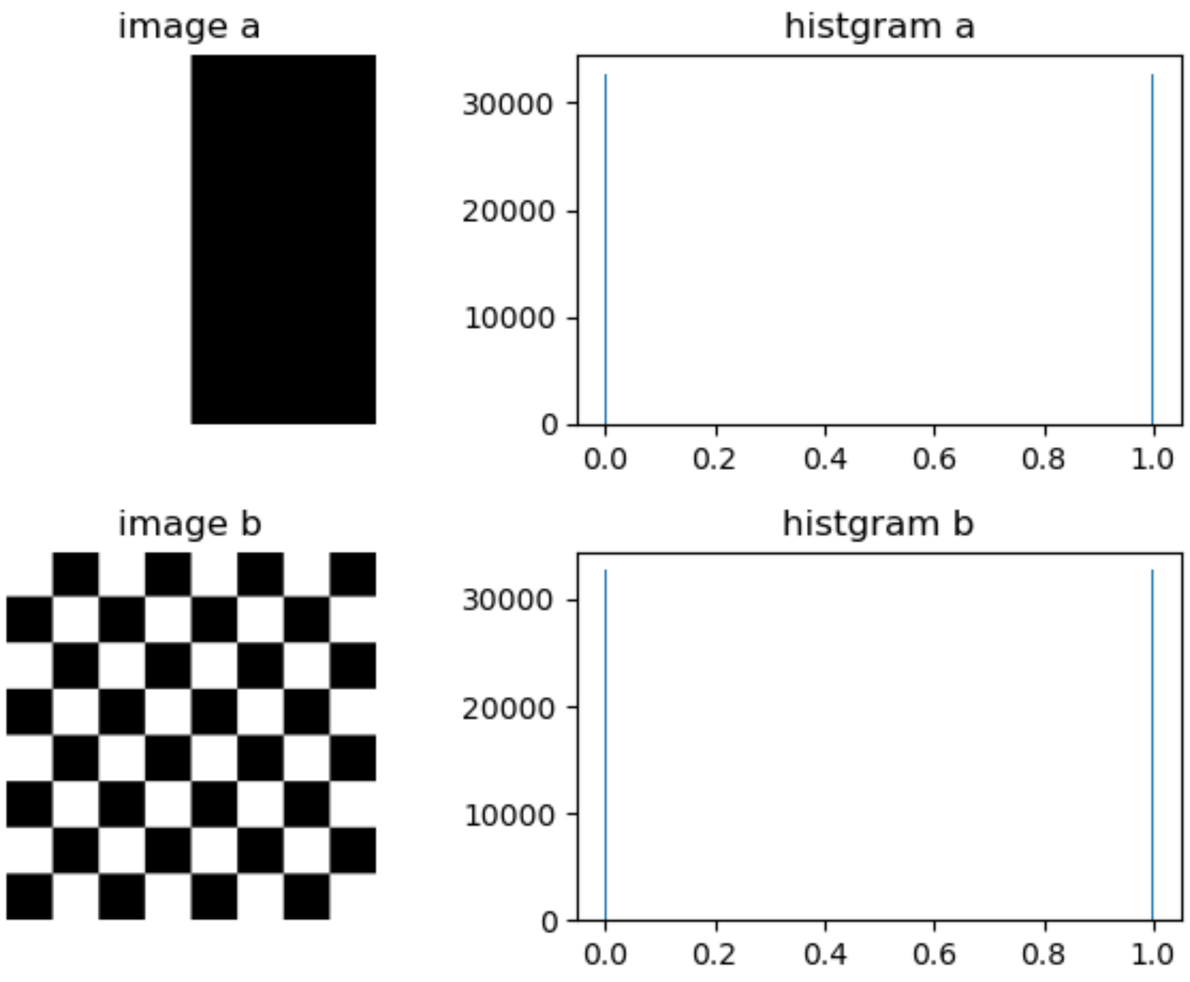
```python
img_a_lt = np.ones((256, 128))
img_a_rt = np.zeros((256, 128))
img_a = np.concatenate((img_a_lt, img_a_rt), axis=1)
img_a = img_a.astype(np.uint8)
```

Generate image b, replicating its piece by `np,tile()`.

```python
img_b_white = np.ones((32, 32))
img_b_black = np.zeros((32, 32))
img_b_piece = np.concatenate(
    (np.concatenate((img_b_white, img_b_black), axis=1),
np.concatenate((img_b_black, img_b_white), axis=1)), axis=0)
img_b = np.tile(img_b_piece, (4, 4))
img_b = img_b.astype(np.uint8)
```

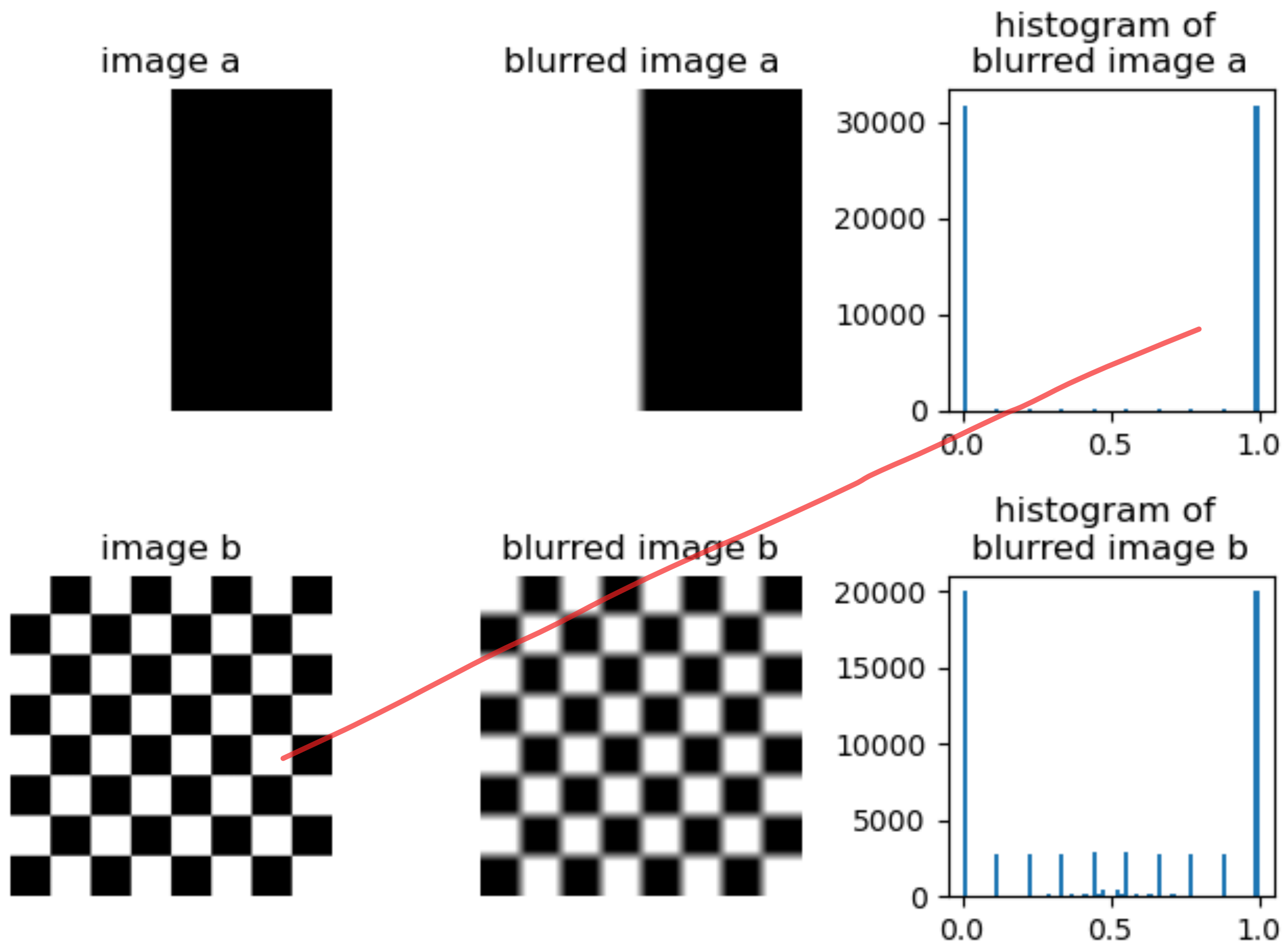
Display the results.

```python
plt.subplot(221), plt.imshow(img_a, cmap='gray'), plt.title('image a'),
plt.axis('off')
plt.subplot(222), plt.hist(img_a.ravel(), bins=256), plt.title('histgram a')
plt.subplot(223), plt.imshow(img_b, cmap='gray'), plt.title('image b'),
plt.axis('off')
plt.subplot(224), plt.hist(img_b.ravel(), bins=256), plt.title('histgram b')
plt.tight_layout()
plt.show()

```



The following programmes implemented averaging filtering using a 9×9 kernel.

```python
import cv2
import numpy as np
import matplotlib.pyplot as plt

img_a_lt = np.ones((256, 128))
img_a_rt = np.zeros((256, 128))
img_a = np.concatenate((img_a_lt, img_a_rt), axis=1)

img_b_white = np.ones((32, 32))
img_b_black = np.zeros((32, 32))
img_b_piece = np.concatenate(
    (np.concatenate((img_b_white, img_b_black), axis=1), np.concatenate((img_b_black, img_b_white), axis=1)), axis=0)
img_b = np.tile(img_b_piece, (4, 4))

# blur images
img_a_blurred = cv2.blur(img_a, ksize=(9, 9))
img_b_blurred = cv2.blur(img_b, ksize=(9, 9))

plt.subplot(231), plt.imshow(img_a, cmap='gray'), plt.title('image a'), plt.axis('off')
plt.subplot(232), plt.imshow(img_a_blurred, cmap='gray'), plt.title('blurred image a'), plt.axis('off')
plt.subplot(233), plt.hist(img_a_blurred.ravel(), bins=64), plt.title('histogram of \nblurred image a')
plt.subplot(234), plt.imshow(img_b, cmap='gray'), plt.title('image b'), plt.axis('off')
plt.subplot(235), plt.imshow(img_b_blurred, cmap='gray'), plt.title('blurred image b'), plt.axis('off')
plt.subplot(236), plt.hist(img_b_blurred.ravel(), bins=64), plt.title('histogram of \nblurred image b')
plt.tight_layout()
plt.savefig('../images/ansfig_A2-Programming.1(b).png')
plt.show()

```



2. Implement the whole procedure described in **section 3.7** "Combining spatial enhancement methods."

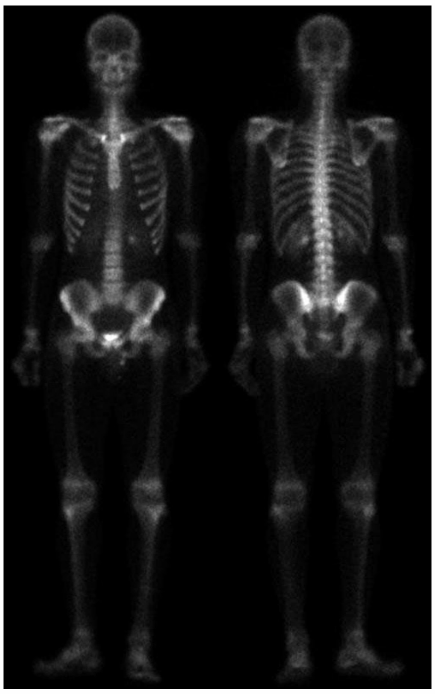
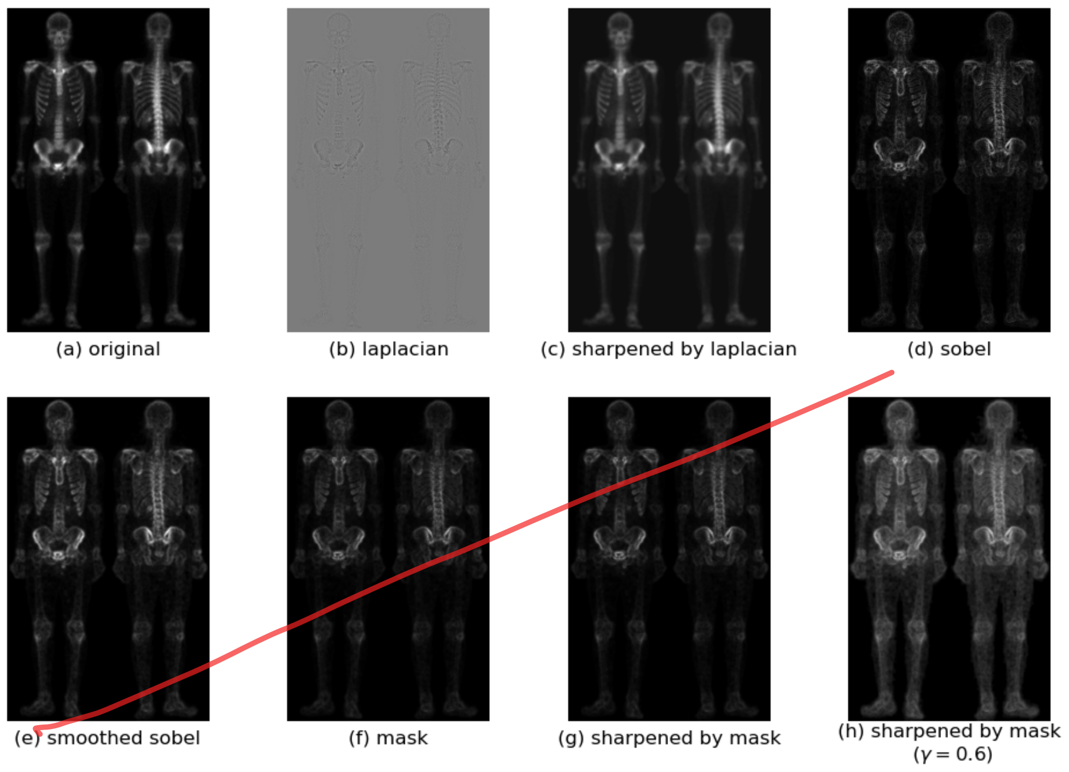


FIGURE 3.43(a) Image of whole body bone scan

(*followed by* ***Matlab live Scripts*** *or* ***Jupyter Scripts*** *and running results*)

```
import cv2
import numpy as np
import matplotlib.pyplot as plt

GAMMA = 0.6

a_original_img = cv2.imread('../images/Fig0343(a)(skeleton_orig).png', 0)
a_original_img = a_original_img.astype(np.float64)

b_laplacian_img = cv2.Laplacian(a_original_img, cv2.CV_64F, None)
norm_b_laplacian_img = cv2.normalize(b_laplacian_img, None, 0, 256, cv2.NORM_MINMAX)

c_sharpened_img = cv2.add(a_original_img, b_laplacian_img)  # c_sharpened_img = a_original_img + b_laplacian_img

d_sobel_img_x = cv2.Sobel(a_original_img, cv2.CV_64F, 1, 0)
d_sobel_img_x = cv2.convertScaleAbs(d_sobel_img_x)
d_sobel_img_y = cv2.Sobel(a_original_img, cv2.CV_64F, 0, 1)
d_sobel_img_y = cv2.convertScaleAbs(d_sobel_img_y)
d_sobel_img = cv2.addWeighted(d_sobel_img_x, 0.5, d_sobel_img_y, 0.5, 0, dtype=cv2.CV_64F)

e_smoothed_sobel_img = cv2.blur(d_sobel_img, (5, 5))

f_mask_img = np.multiply(norm_b_laplacian_img, e_smoothed_sobel_img)

g_sharpened_img = cv2.add(a_original_img, f_mask_img)
g_sharpened_img = cv2.normalize(g_sharpened_img, None, 0, 256, cv2.NORM_MINMAX)
h_gamma_trans_img = (cv2.pow((g_sharpened_img / 255), GAMMA)) * 255  # transformed from g_sharpened_img

plt.figure(figsize=(12, 8))

ls = ['a', 'b', 'c', 'd', 'e', 'f', 'g', 'h']
i = 1
plt.subplot(2, 4, i)
plt.imshow(a_original_img, cmap='gray'), plt.title(f'({ls[i - 1]}) original', y=-0.1), plt.axis('off')
i += 1
plt.subplot(2, 4, i)
plt.imshow(norm_b_laplacian_img, cmap='gray'), plt.title(f'({ls[i - 1]}) laplacian', y=-0.1), plt.axis('off')
i += 1
plt.subplot(2, 4, i)
plt.imshow(c_sharpened_img, cmap='gray'), plt.title(f'({ls[i - 1]}) sharpened by laplacian', y=-0.1), plt.axis('off')
i += 1
plt.subplot(2, 4, i)
plt.imshow(d_sobel_img, cmap='gray'), plt.title(f'({ls[i - 1]}) sobel', y=-0.1), plt.axis('off')
i += 1
plt.subplot(2, 4, i)
plt.imshow(e_smoothed_sobel_img, cmap='gray'), plt.title(f'({ls[i - 1]}) smoothed sobel', y=-0.1), plt.axis('off')
i += 1
plt.subplot(2, 4, i)
plt.imshow(f_mask_img, cmap='gray'), plt.title(f'({ls[i - 1]}) mask', y=-0.1), plt.axis('off')
i += 1
plt.subplot(2, 4, i)
plt.imshow(g_sharpened_img, cmap='gray'), plt.title(f'({ls[i - 1]}) sharpened by mask', y=-0.1), plt.axis('off')
i += 1
plt.subplot(2, 4, i), plt.imshow(h_gamma_trans_img, cmap='gray')
plt.title(f'({ls[i - 1]}) sharpened by mask\n ($\gamma = {GAMMA}$)', y=-0.15), plt.axis('off')

plt.tight_layout()
plt.show()

```



Correction: caption of image (h) shall be 'gamma adjusted ($\gamma = 0.6$)'.