第六章形态学图像处理

简介

- 形态学是生物学的一个分支，主要研究动植物的形态和结构
- 形态学图像处理是用来提取图像中的特定分量，用于图像的表示和描述，如表示区域的形状，包括边界、骨架和凸壳等。
- 同时，也广泛用于图像的预处理和后处理，如形态学过滤、细化和修剪。

前言 (1)

- 集合反射

集合B的反射，记为$\hat{B}$，定义为:

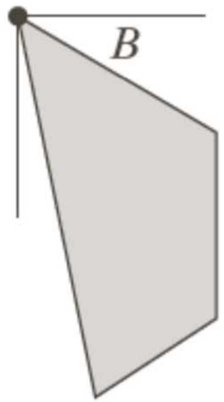
$$\hat{B} = \{w \mid w = -b,\ b \in B\}$$

- 集合平移

集合B关于点 $z = (z_1, z_2)$ 的平移，记作$(B)_Z$，定义为：

$$(B)_Z = \{c \mid c = b + z,\ b \in B\}$$

示例:反射和平移



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

前言(2)

- **结构元（Structure elements, SE）**

小集合或子图像，用于探测图像以获得感兴趣的性质。

示例：结构元(1)

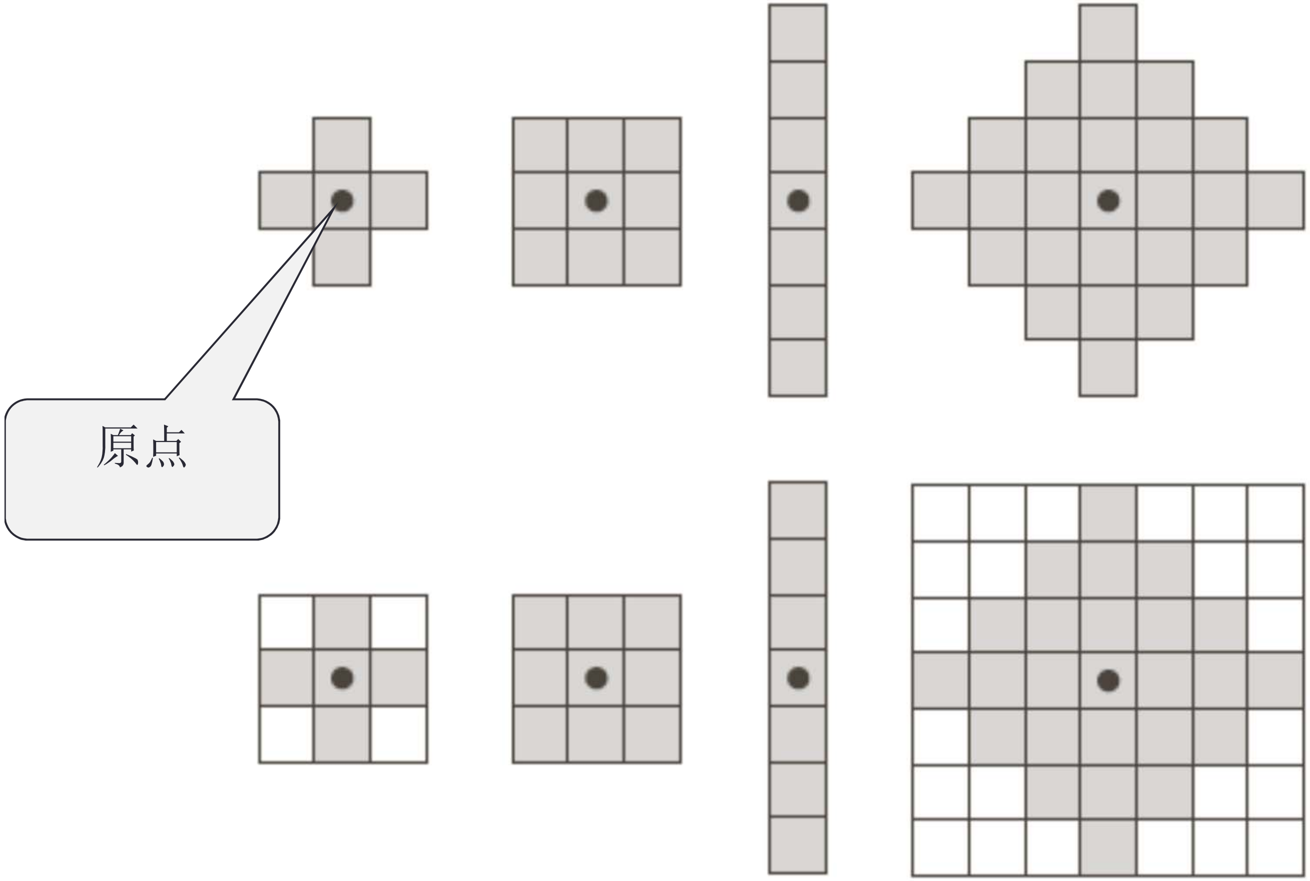
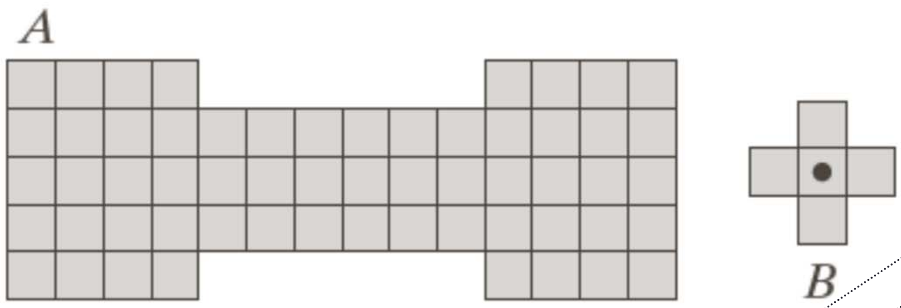


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

示例：结构元(2)

当其原点位于原始集合A的边界时，容纳所有结构元



B的原点访问A的每个元素

在B的原点的每个位置，如果B完全包含在A中，则该位置是新集合的成员，否则它不是新集合的成员。

a b
c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

腐蚀

作为 Z^2中的集合A 和 B ，用B 对A 进行腐蚀，

表示为 $A \ominus B$，定义为

$$A \ominus B = \left\{ z \mid (B)_Z \subseteq A \right\}$$

是所有点z 组成集合，B 用点z 平移后应包含在A 内。

$$A \ominus B = \left\{ z \mid (B)_Z \cap A^c = \varnothing \right\}$$

腐蚀实例(1)

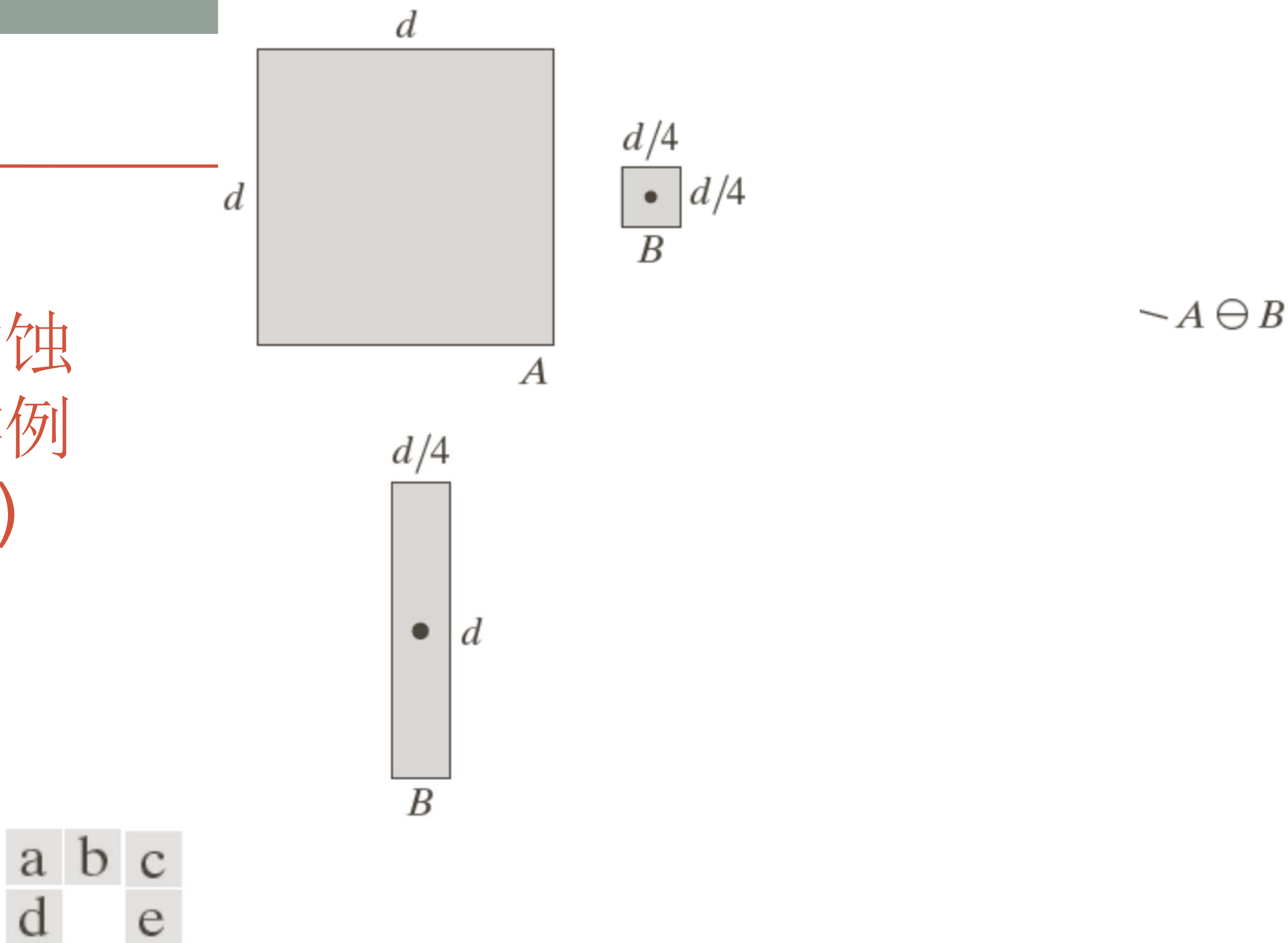
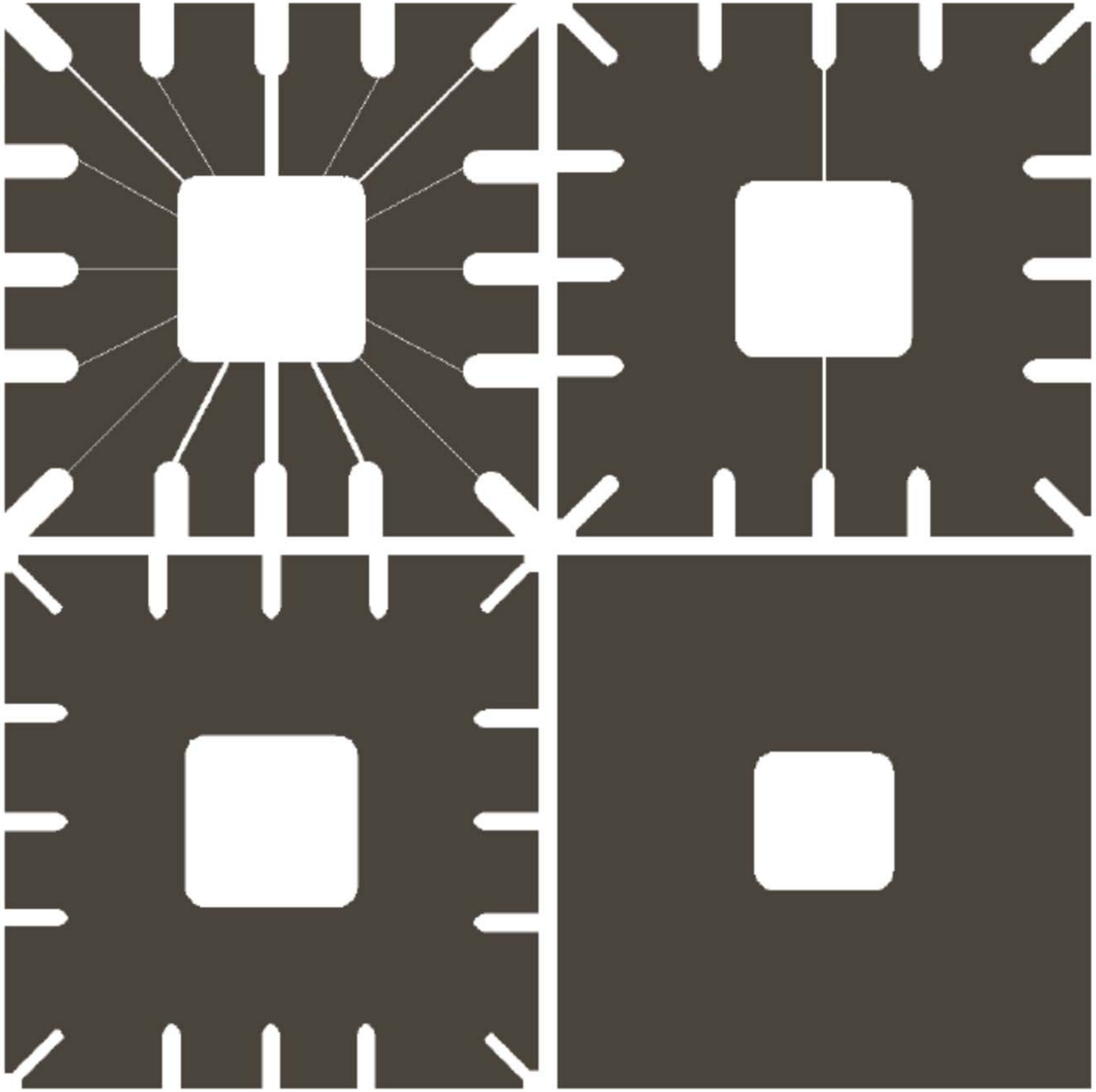


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

腐蚀实例(2)



a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

膨胀

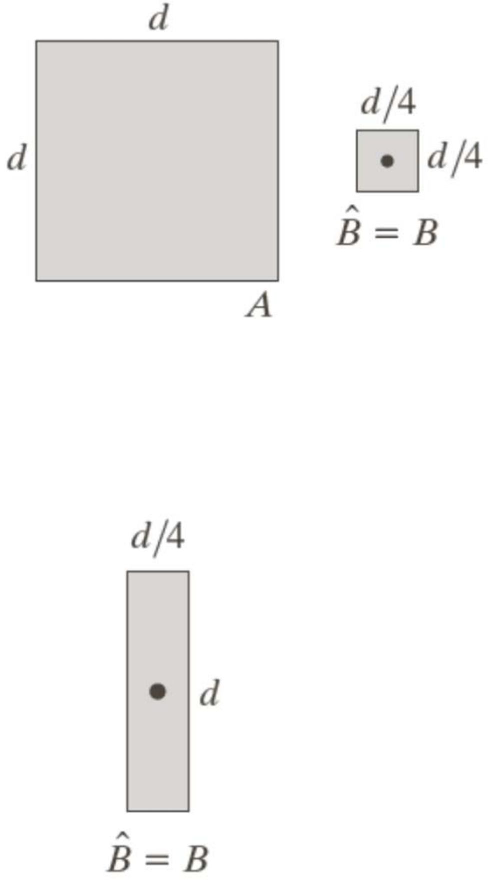
作为 Z^2中的集合 A 和 B，用 B 对A 进行膨胀，表示为 $A \oplus B$，定义为

$$\mathrm{A} \oplus \mathrm{B} = \left\{ z \mid \left(\hat{B} \right)_z \cap A \neq \varnothing \right\}$$

所有位移 z 应使平移的 $\hat{B}$ 和 A 的集合至少有一个元素重叠。

$$A \oplus B = \left\{ z \mid \left[\left(\hat{B} \right)_z \cap A \right] \subseteq A \right\}$$

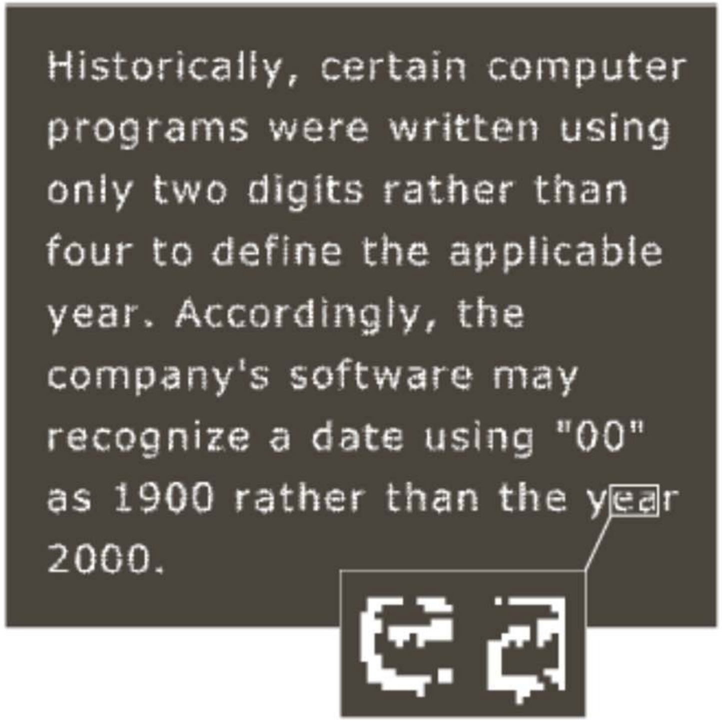
膨胀的例子(1)



a b c
d e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

膨胀的例子 (2)



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

开操作和闭操作

用结构元B 对集合A 执行开操作，记作$A \circ B$

定义为：

$$A \circ B = (A \ominus B) \oplus B$$

用结构元B 对集合A 执行闭操作，记作$A \bullet B$

定义为：

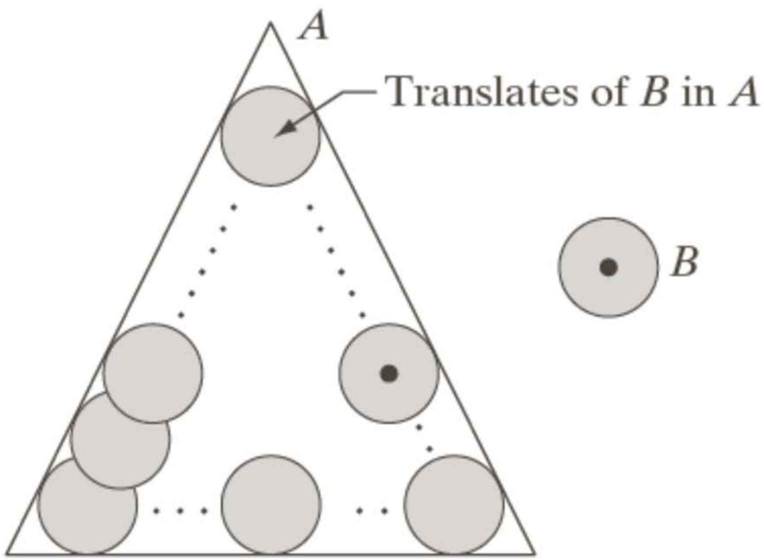
$$A \bullet B = (A \oplus B) \ominus B$$

开操作和闭操作

开操作通常使物体轮廓平滑，断开狭窄的峡口，消除细小的突出物。

闭操作同样趋向于平滑物体的轮廓，但它会弥合狭窄的间断和细长的沟壑，消除小的孔洞，填补轮廓线中的断裂

示例:开操作



a b c d

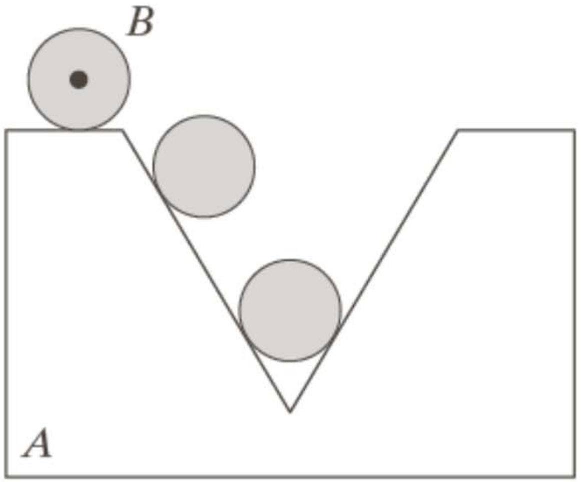
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

开操作

- 等效于使用结构元B去拟合集合A

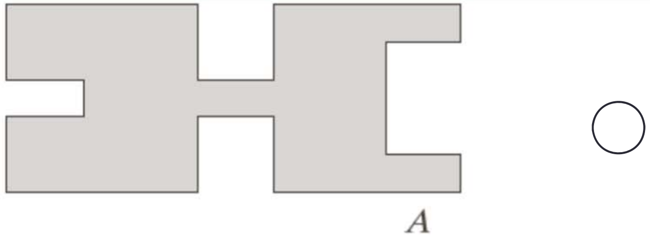
$$A \circ B = \bigcup \left\{ (B)_z \mid (B)_z \subseteq A \right\}$$

示例: 闭操作



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.



a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

开操作和闭操作的性质

- 开操作的性质

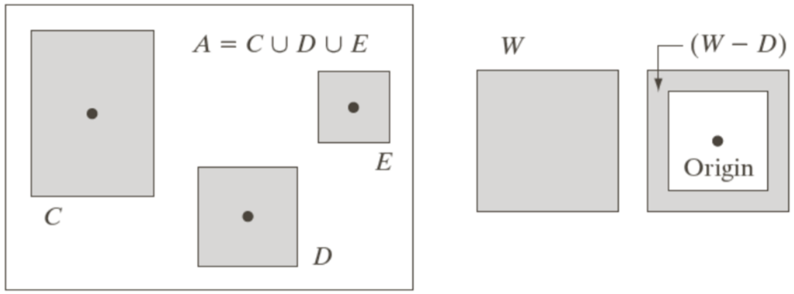
  (a) $A \circ B$ 是 A 的一个子集（子图像）

- 闭操作的性质

击中-击不中变换

如果 B 表示由D 及背景组成的集合, 则A 中 B 的匹配项（或匹配集）表示为$A \circledast B$,

$$A \circledast B = \left(A \ominus D\right) \cap \left[A^c \ominus \left(W - D\right)\right]$$



a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

击中-击不中变换

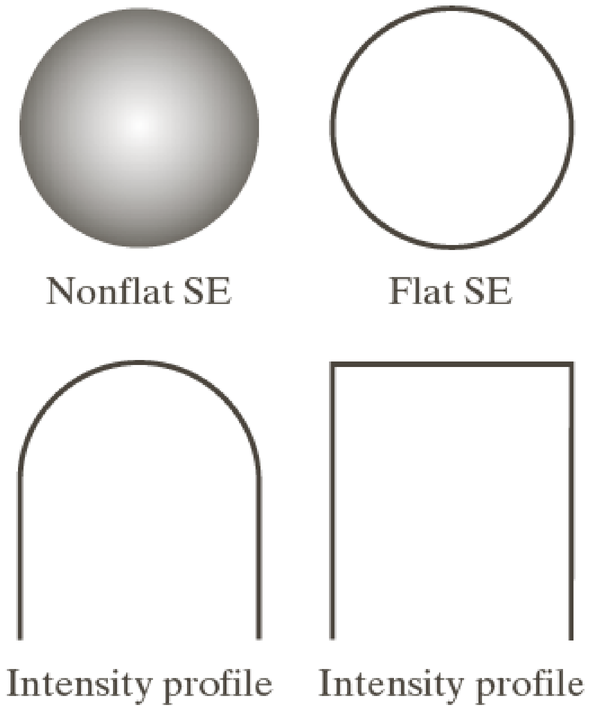
如果$B=\left(B_1,B_2\right)$，B_1 表示物体，B_2 表示背景

$$A\circledast B=\left(A\ominus B_1\right)\cap\left(A^c\ominus B_2\right)$$

灰度形态学

$f(x, y)$： 灰度图像

$b(x, y)$： 结构元



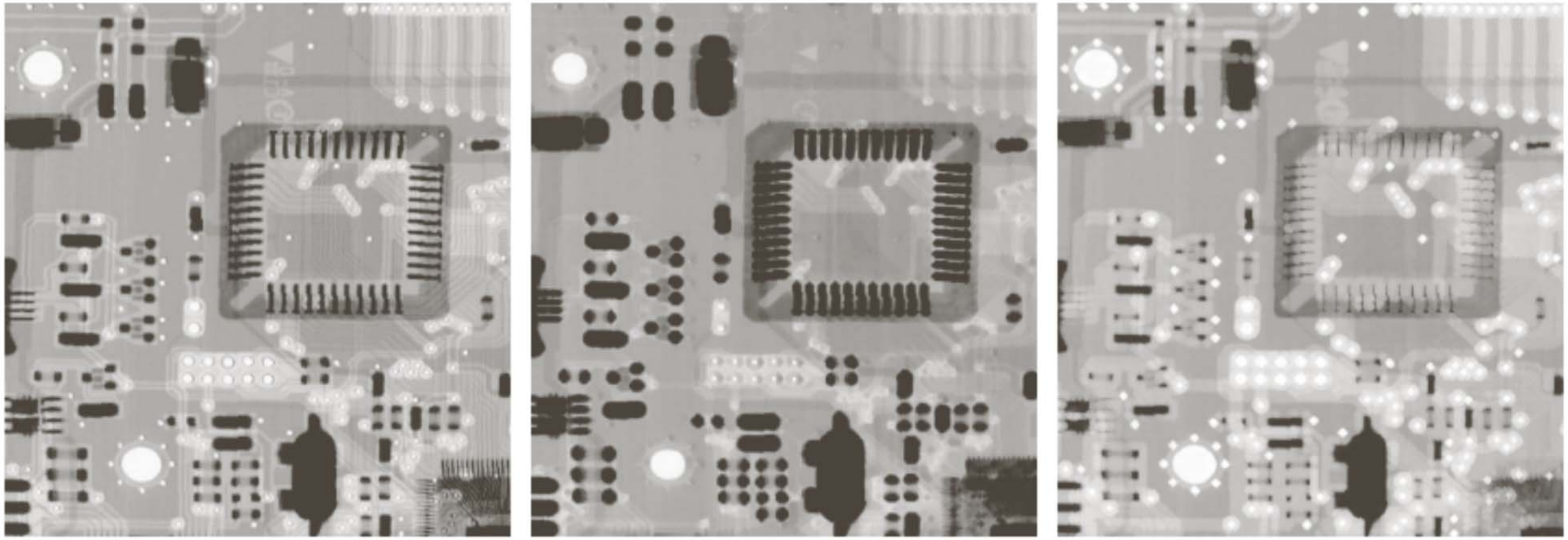
a b
c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

灰度形态学:腐蚀和膨胀

$$[f \ominus b](x,y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$$

$$[f \oplus b](x,y) = \max_{(s,t)\in b}\{f(x-s, y-t)\}$$



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 $\times$ 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

灰度形态学:使用非平坦结构元的腐蚀和膨胀

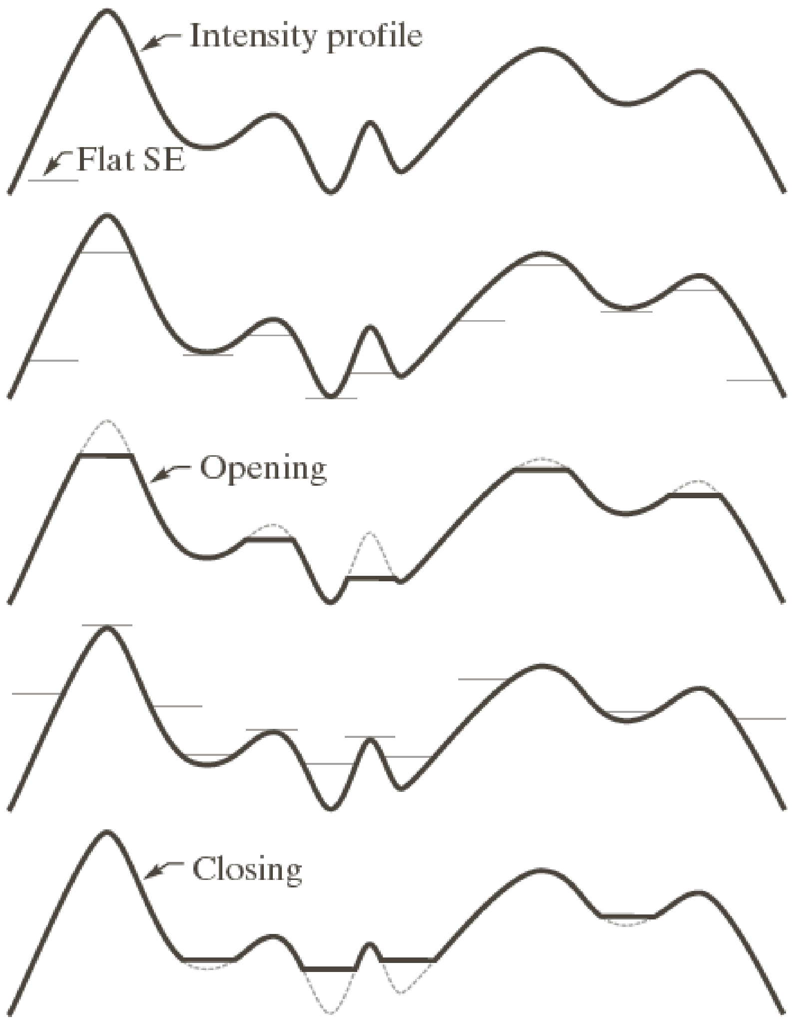
$$[f \ominus b_N](x,y) = \min_{(s,t)\in b}\{f(x+s,y+t)-b_N(s,t)\}$$

$$[f \oplus b_N](x,y) = \max_{(s,t)\in b}\{f(x-s,y-t)+b_N(s,t)\}$$

开操作和闭操作

$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$



a
b
c
d
e

FIGURE 9.36
Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal.
(c) Opening.
(d) Flat structuring element pushed down along the top of the signal.
(e) Closing.

灰度图像开操作的性质

$$(a) \quad f \circ b \dashv f$$

$$(b) \quad \text{if } f_1 \dashv f_2, \text{ then } (f_1 \circ b) \dashv (f_2 \circ b)$$

$$(c) \quad (f \circ b) \circ b = f \circ b$$

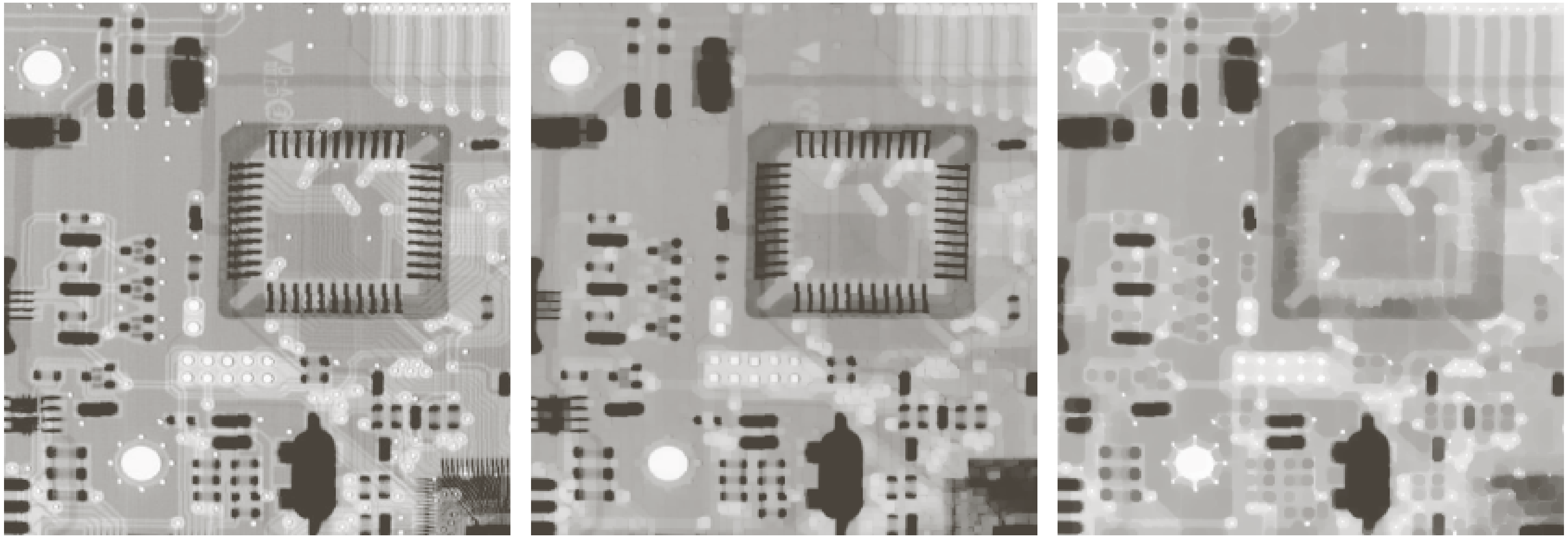
此式中 $e \dashv r$ 表示 e 的值域包含在 r 的值域中，且 $e(x, y) \le r(x, y)$.

灰度图像闭操作的性质

$$(a)\quad f \dashv f \bullet b$$

$$(b)\quad \text{if } f_1 \dashv f_2,\ \text{then } \left(f_1 \bullet b\right) \dashv \left(f_2 \bullet b\right)$$

$$(c)\quad \left(f \bullet b\right) \bullet b = f \bullet b$$

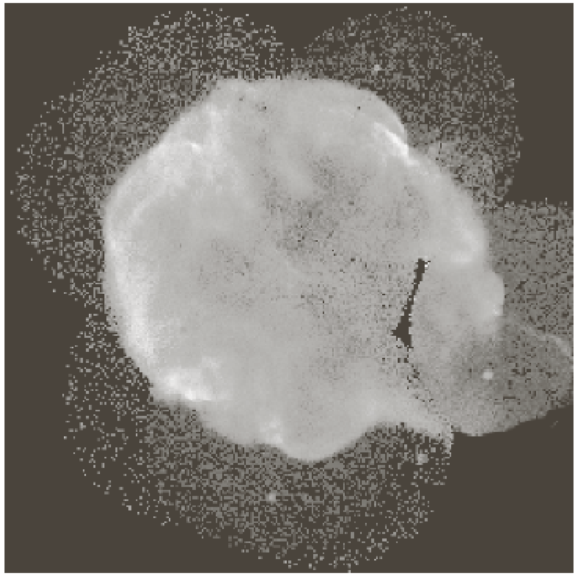


a b c

形态学平滑

- 开操作会抑制小于指定结构元的明亮细节，而闭操作会抑制黑暗的细节。
- 通常可以将开操作和闭操作结合起来使用来进行形态滤波，用于图像平滑和噪声消除。

形态学平滑



a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

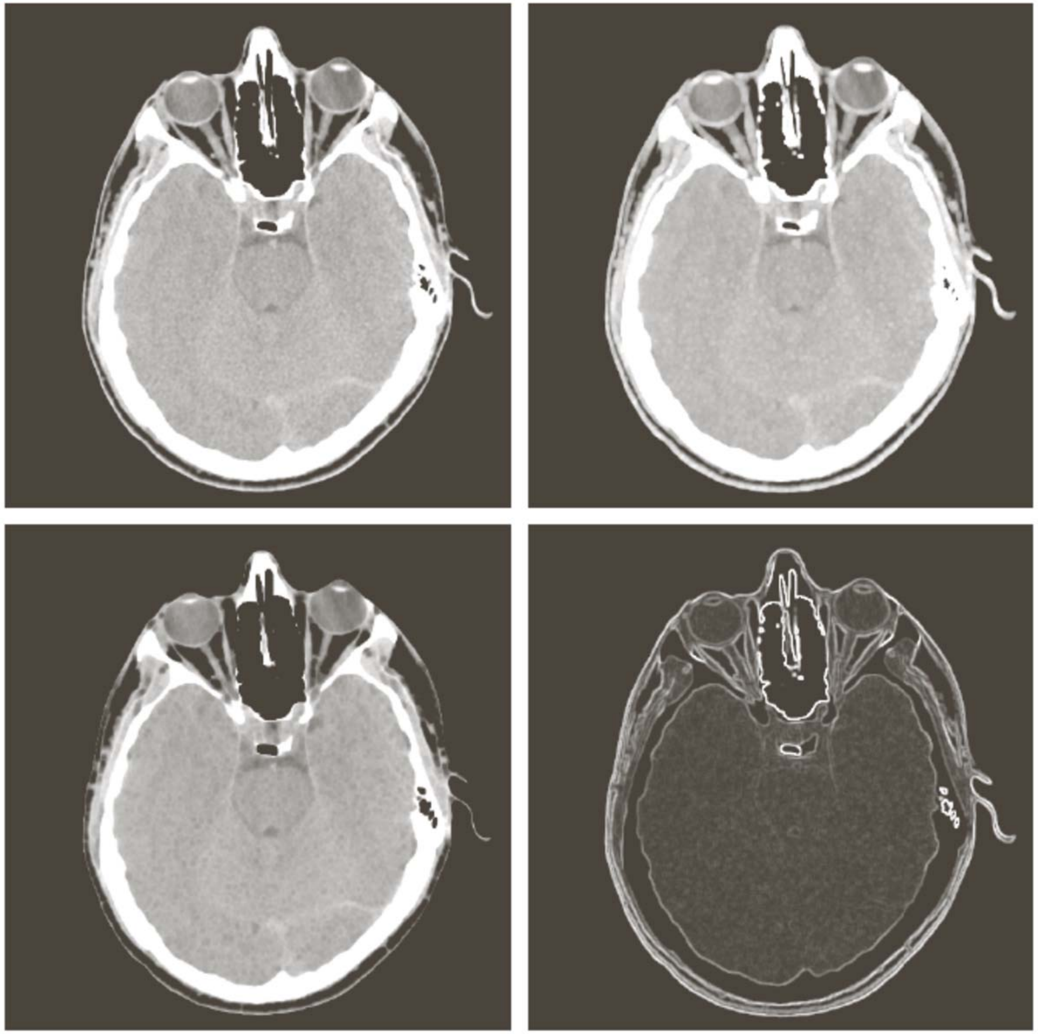
形态学梯度

- 膨胀和腐蚀结合图像减法可以得到图像的形态学梯度，表示为g：

$$g = (f \oplus b) - (f \ominus b)$$

- 作用：边缘得到增强，同质区域的贡献得到抑制，从而产生一种“类导数”（梯度）效应。

形态学梯度



a	b
c	d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

顶帽变换和底帽变换

- 灰度图像 f 的顶帽变换定义为 f 减去其开操作的值：

$$T_{hat}(f) = f - (f \circ b)$$

- 灰度图像 f 的底帽变换定义为其闭操作的值减去 f：

$$B_{hat}(f) = (f \bullet b) - f$$

将顶帽变换用于分割

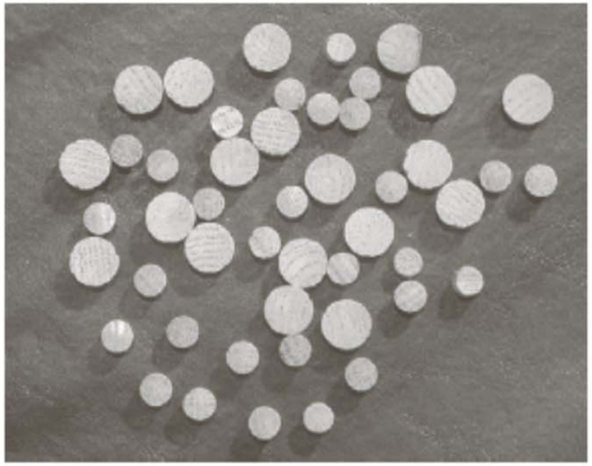
a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600×600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

粒度测定

- 粒度测定用于确定图像中颗粒大小的分布
- 特定大小的开操作对包含与结构元大小相似颗粒图像区域影响最大
- 对于每次开操作，计算像素值的总和（**表面积**）

示例



a	b	c
d	e	f

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

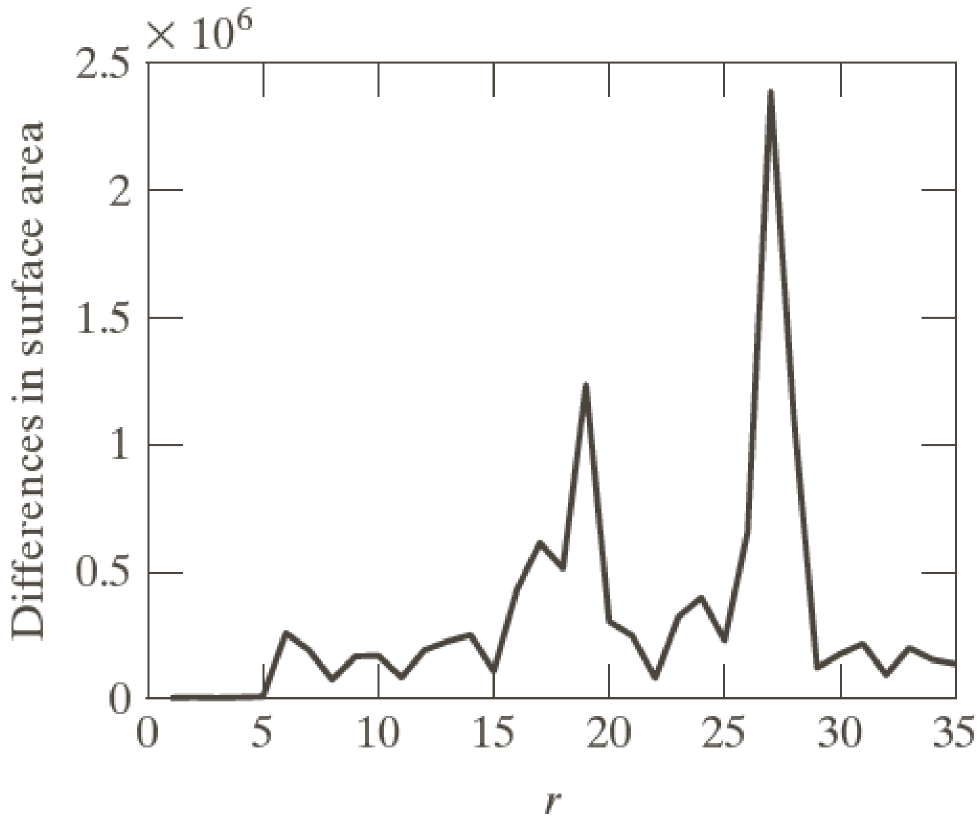
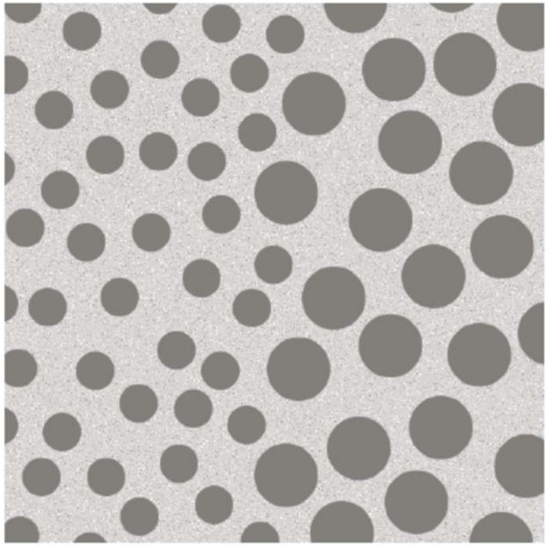


FIGURE 9.42 Differences in surface area as a function of SE disk radius, *r*. The two peaks are indicative of two dominant particle sizes in the image.

纹理分割

- 分割：将图像细分为区域的过程。



a	b
c	d

FIGURE 9.43 Textural segmentation. (a) A 600×600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

算法实践

- 编程实现习题9.17的算法过程。