数字图像处理及应用 第4次作业

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B

Part I Exercises

Ex.1 The image shown in FIGURE 1 consists of two infinitesimally thin white lines on a black background, intersecting at some point in the image. The image is input into a linear, position invariant system with the impulse response given as Eq.1.

$$h(x,y) = e^{-[(x-\alpha)^2+(y-\beta)^2]} \tag{1}$$

Assuming continuous variables and negligible noise, find an expression for the output image $g(x,y)$.

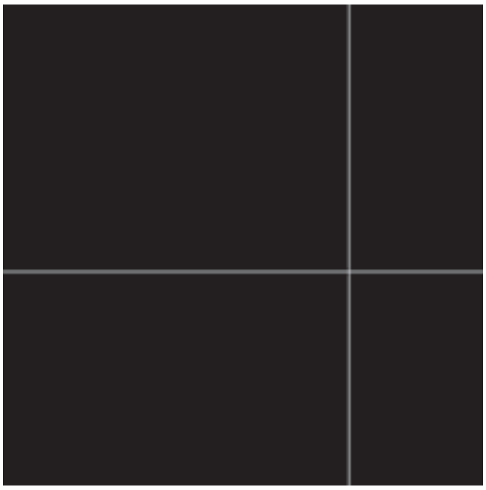


FIGURE 1

Answer:

we know that: this image can be modeled to:

$$f(x,y) = \delta(x-a) + \delta(y-b)$$

so, we can get that:

$$F(u,v) = 2\pi\delta(v)e^{-j2\pi ua} + 2\pi\delta(u)e^{-j2\pi vb}$$

since $h(x,y) = e^{-[(x-\alpha)^2+(y-\beta)^2]}$:

$$H(u,v) \quad = \sqrt{\pi}e^{-\pi^2u^2}e^{-j2\pi u\alpha}\sqrt{\pi}e^{-\pi^2v^2}e^{-j2\pi v\beta}$$

and:

$$\begin{aligned} G(u,v) &= F(u,v)H(u,v) \\ &= \sqrt{\pi}e^{-\pi^2u^2}e^{-j2\pi u(\alpha+a)}\sqrt{\pi}e^{-\pi^2v^2}e^{-j2\pi v\beta}2\pi\delta(v) + \sqrt{\pi}e^{-\pi^2v^2}e^{-j2\pi v(\beta+b)}\sqrt{\pi}e^{-\pi^2u^2}e^{-j2\pi u\alpha}2\pi\delta(u) \end{aligned}$$

for what we want $g(x,y) = \mathscr{F}^{-1}[G(u,v)]$:

$$
\begin{aligned}
g(x,y) &= \int_{-\infty}^{+\infty} 2\pi\delta(v)\sqrt{\pi}e^{-\pi^2v^2}e^{-j2\pi v\beta}e^{j2\pi vy}dv \int_{-\infty}^{+\infty} \sqrt{\pi}e^{-\pi^2u^2}e^{-j2\pi u(\alpha+a)}e^{j2\pi ux}du \\
&+ \int_{-\infty}^{+\infty} 2\pi\delta(u)\sqrt{\pi}e^{-\pi^2u^2}e^{-j2\pi u\alpha}e^{j2\pi ux}du \int_{-\infty}^{+\infty} \sqrt{\pi}e^{-\pi^2v^2}e^{-j2\pi v(\beta+b)}e^{j2\pi vy}dv \\
&= \sqrt{\pi}e^{-[x-(\alpha+a)^2]} + \sqrt{\pi}e^{-[y-(\beta+b)^2]} \\
&= \sqrt{\pi}\{e^{-[x-(\alpha+a)^2]} + e^{-[y-(\beta+b)^2]}\}
\end{aligned}
$$

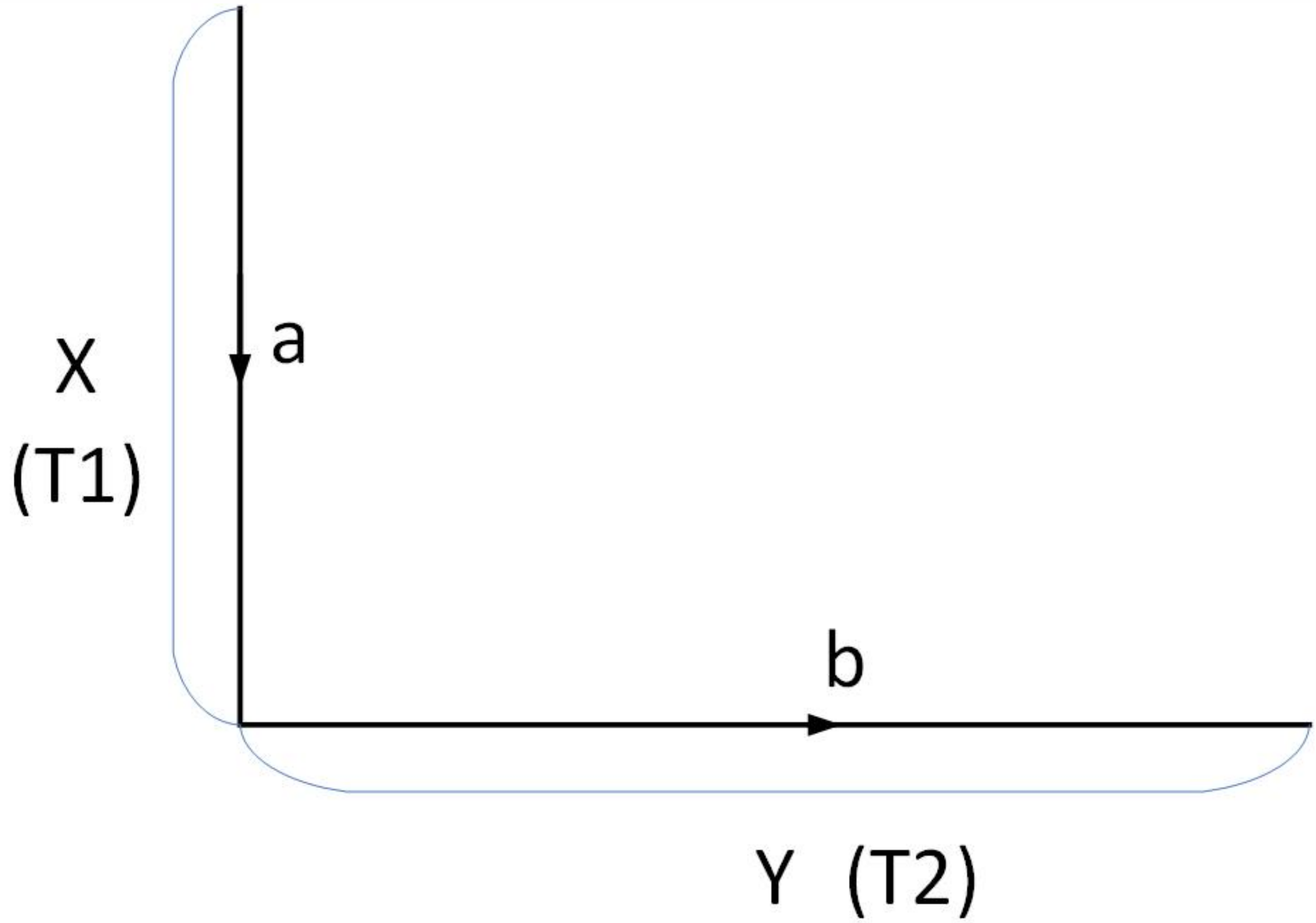
so we can know the finally answer is:

$$
g(x,y) = \sqrt{\pi}\{e^{-[x-(\alpha+a)^2]} + e^{-[y-(\beta+b)^2]}\}
$$

Ex.2 During acquisition, an image undergoes uniform linear motion in the vertical direction for a time T_1. The direction of motion then switches to the horizontal direction for a time interval T_2. Assuming that the time it takes the image to change directions is negligible, and that shutter opening and closing times are negligible also, give an expression for the blurring function, $H(u,v)$.

Answer:

First down, then to the right, and at constant velocity, you get



In vertical direction

$$X(t)=\begin{cases} a/T_1\cdot t, & 0\le t\le T_1\\ a, & t>T_1\end{cases}$$

In horizontal direction

$$Y(t)=\begin{cases} b/T_2\cdot (t-T_1), & T_1<t\le T_2\\ 0, & 0\le t\le T_1\end{cases}$$

With the formula

$$H(u,v)=\int_0^T e^{-j2\pi[ux_o(t)+vy_0(t)]}dt$$

we can get

$$\begin{aligned} H(u,v) &= \int_0^{T_1} e^{-j2\pi u\cdot at/T_1}dt+\int_{T_1}^{T_1+T_2} e^{-j2\pi[ua+v\cdot b(t-T_1)/T_2]}dt\\ &=\frac{1}{-j2\pi ua/T_1}\cdot[e^{-j2\pi uat/T_1}]_0^{T_1}+\frac{1}{-j2\pi bv/T_2}\cdot[e^{-j2\pi vbt/T_2}]_{T_1}^{T_1+T_2}\cdot e^{-j2\pi[ua-bT_1/T_2]}\\ &=\frac{e^{-j2\pi ua}-1}{-j2\pi ua/T_1}+\frac{e^{-j2\pi(ua+vb)}-e^{-j2\pi ua}}{-j2\pi vb/T_2}\\ &=e^{-jua\pi}\sin(\pi ua)\frac{T_1}{\pi ua}+e^{-j2\pi ua}e^{-jvb\pi}\sin(\pi vb)\frac{T_2}{\pi vb}\end{aligned}$$

Ex.3

(**a**) The image in (b) and (c) were obtained by inverse and Wiener-filtering the image in (a), which is a motion blurred image that, in addition, is corrupted by additive Gaussian noise. The blurring itself is corrected in (b) and (c). However, the restored image (b) has a strong streak pattern that is not apparent in (a) [for example, compare the area of constant white in the top right of (b) with the corresponding are in (a).] On the other hand, the streak pattern does not appear in (c). Explain how this pattern originated and why Wiener filter can avoid it.

(**a**) (**b**) (**c**)

FIGURE 2 Inverse and Wiener filtering

Answer:

If you use inverse filtering, there will be:

$$\widehat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

It is easy to see that when H(u,v) is small, it amplifies the effect of noise, so it will have a different effect, if Wiener filter is used, it will have:

$$\begin{aligned}
\widehat{F}(u,v) &= [\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}] \cdot G(u,v) \\
&= \frac{G(u,v)}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \\
&= [F(u,v) + \frac{N(u,v)}{H(u,v)}] \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}
\end{aligned}$$

Therefore, when H(u,v) is very small, the effect of noise amplified by H(u,v) is attenuated, so as to avoid this phenomenon

---

Ex.4 A certain X-ray imaging geometry produces a blurring degradation that can be modeled as the convolution of the sensed image with the spatial, circularly symmetric function

$$h(x,y) = \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{2}$$

Assuming continuous variables, show that the degradation in the frequency domain is given by the expression

$$H(u,v) = -8\pi^3\sigma^2(u^2+v^2)e^{-2\pi^2\sigma^2(u^2+v^2)} \tag{3}$$

Answer:

We can see that from

$$h(x,y) = \frac{x^2+y^2-2\sigma^2}{\sigma^4}\cdot e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

We can get:

$$\begin{aligned}
h(x,y) &= \frac{\partial^2(e^{\frac{-(x^2+y^2)}{\sigma^2}})}{\partial x^2} + \frac{\partial^2(e^{\frac{-(x^2+y^2)}{2\sigma^2}})}{\partial y^2} \\
&= \frac{x^2-\sigma^2}{\sigma^4} + \frac{y^2-\sigma^2}{\sigma^4} \\
&= \frac{x^2+y^2-2\sigma^2}{\sigma^4}
\end{aligned}$$

Let's say

$$f(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

From the knowledge of the previous chapter:

$$\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} \Longleftrightarrow -4\pi^2(u^2+v^2)F(u,v)$$

And then

$$F(u,v) = 2\pi\sigma^2 e^{-2\pi\sigma^2(u^2+v^2)}$$

Therefore

$$\begin{aligned}
h(x,y) \Longleftrightarrow H(u,v) &= -4\pi^2(u^2+v^2)\cdot 2\pi\sigma^2 e^{-2\pi\sigma^2(u^2+v^2)} \\
&= -8\pi^3\sigma^2(u^2+v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}
\end{aligned}$$

H(u,v) as shown in the figure

Ex.5 The image shown is a blurred, 2-D projection of a volumetric rendition of a heart. It is known that each of the cross hairs on the right bottom part of the image was 4 pixels wide, 20 pixels long, and had an intensity value of 255 before blurring. Provide a step-by-step procedure indicating how you would use the information just given to obtain the blurring function $H(u,v)$.

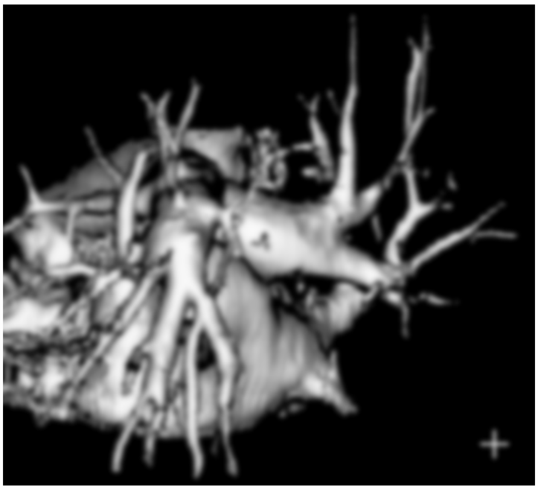


FIGURE 3 Volumetric rendition of a heart

Answer:

Firstly, according to the linear space invariant system is completely represented by its impulse response, we only need to observe a transformation of the whole image, then we can estimate the fuzzy image $H(u,v)$ of the whole image through the fuzzy function $H'(u,v)$.Since the subject has accurate image information before the crosshair blur, set $g(x,y)$, and set $g'(x,y)$ after the blur. The Fourier transform corresponding to the frequency domain gives you $G(u,v)$ and $G'(u,v)$. Then the approximate fuzzy function can be estimated in terms of

$$H(u,v) = \frac{G'(u,v)}{G(u,v)}$$

Ex.6 Explain the reason for the formation of image (d) in FIGURE 4 (refer to Example 4.6 in page 252)，which is acquired by an imaging system with maximum sampling rate of 96×96. The original image of (d) is a checkerboard like image, which each of its square is of 0.4798×0.4798 pixels.

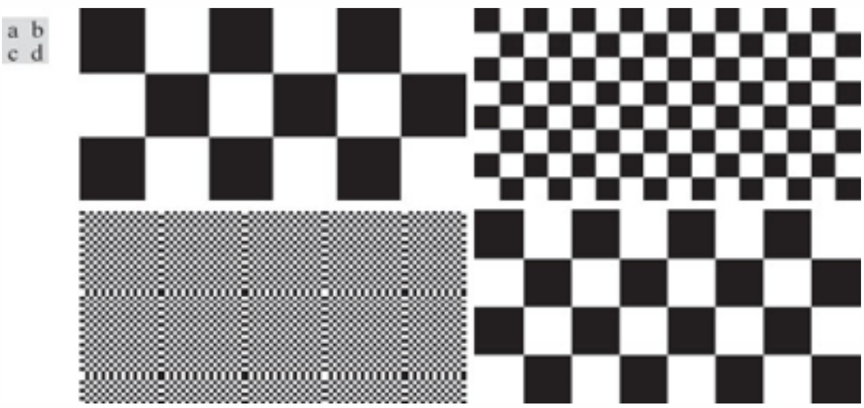


FIGURE 4 Aliasing in image

Answer:

For image of d, we need at least more than 200 sampling points, so aliasing will occur theoretically. In this case, the aliased result looks like a normal checkerboard pattern. In fact, this image would result from sampling a checker- board image whose squares were 12 pixels on the side. Thus, the situation shown in Figure d occurs

Part II Programming

1. The arithmetic mean filter is defined as

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t).$$

The white bars in the test pattern shown are 7 pixels wide and 210 pixels high. The separation between bars is 17 pixels. What would this image look like after application of

(**a**) A 3×3 arithmetic mean filter?
(**b**) A 7×7 arithmetic mean filter?
(**c**) A 9×9 arithmetic mean filter?

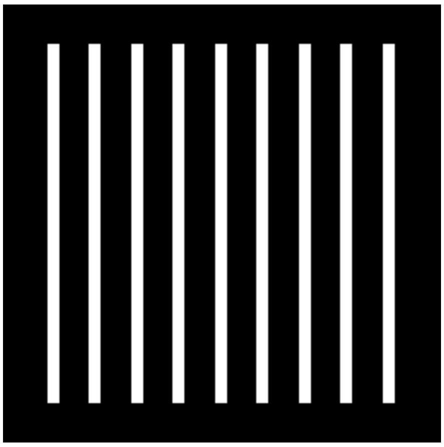
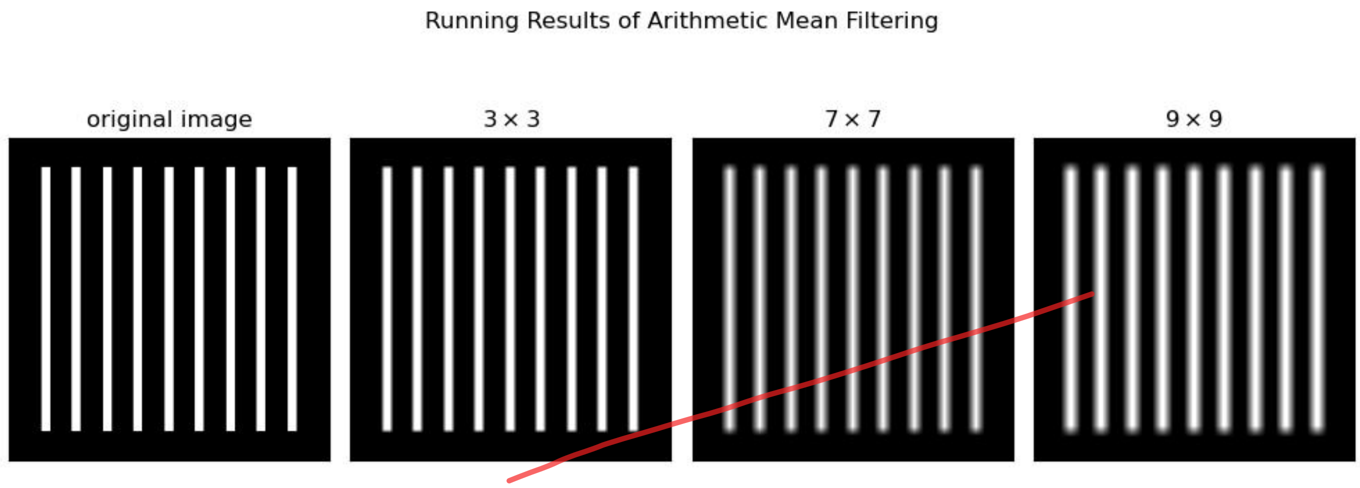


FIGURE 5 Test pattern

(*followed by* ***Matlab live Scripts*** *or* ***Jupyter Scripts*** *and running results*)

```
import cv2
import matplotlib.pyplot as plt

# open
original_image = cv2.imread("../images/FigP0501.png", flags=0)
size1 = (3, 3)
size2 = (7, 7)
size3 = (9, 9)
size = [3, 7, 9]

target_image = []
target_image.append(cv2.blur(original_image, size1))
target_image.append(cv2.blur(original_image, size2))
target_image.append(cv2.blur(original_image, size3))

# show
fig, axs = plt.subplots(nrows=1, ncols=4, figsize=(10, 4))

ax = axs[0]
ax.imshow(original_image, cmap='gray')
ax.set_title(f"original image")
```

```python
ax.set_xticks([])
ax.set_yticks([])

for i in range(3):
    ax = axs[i + 1]
    ax.imshow(target_image[i], cmap='gray')
    ax.set_title(fr"${size[i]}\times${size[i]}")
    ax.set_xticks([])
    ax.set_yticks([])

plt.suptitle("Running Results of Arithmetic Mean Filtering")

plt.tight_layout()

# output = f'../images/Arithmetic Mean Filtering.jpg'
# plt.savefig(output)

plt.show()

```



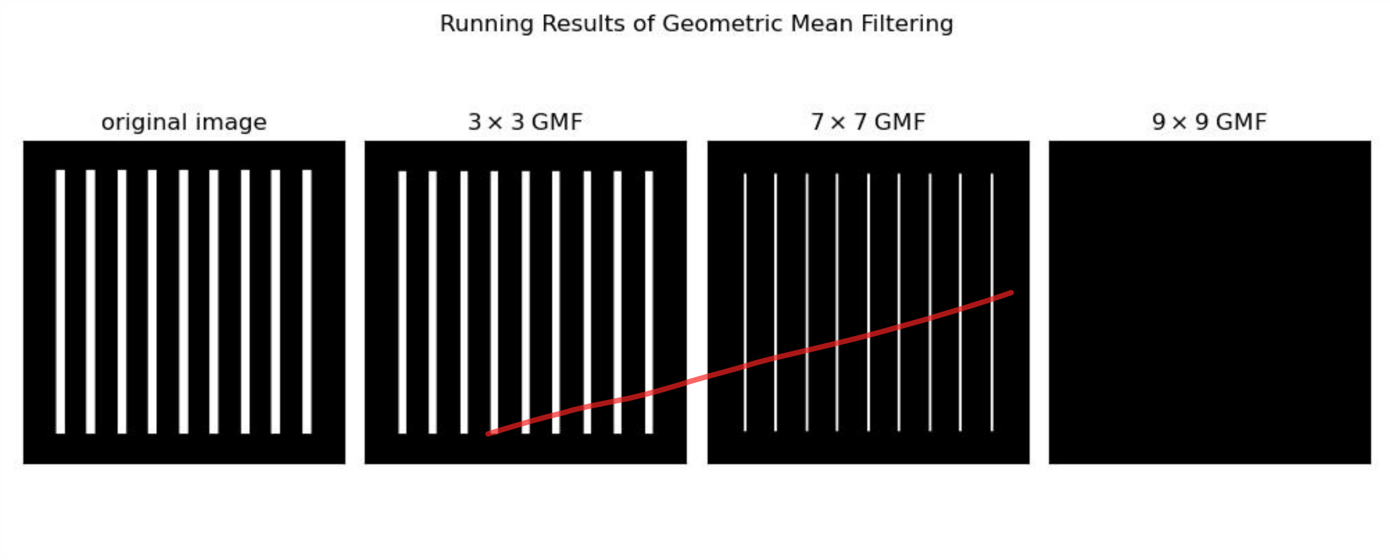
2. Repeat **1** using a geometric mean filter which is defined as

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}.$$

(*followed by **Matlab live Scripts** or **Jupyter Scripts** and running results*)

```python
import cv2
import numpy as np
import matplotlib.pyplot as plt


def geometric_mean_filter(img, ksize):
    h, w = img.shape[:2]
    expo = 1 / (ksize * ksize)
```

```python
    pad = int((ksize - 1) / 2)
    # pad the image using `cv2.copyMakeBorder()` whose effect can also be produced by`np.pad()`
    padded = cv2.copyMakeBorder(img, pad, pad, pad, pad, borderType=cv2.BORDER_REFLECT_101)
    filtered = np.zeros(img.shape)
    for i in range(pad, pad + h):
        for j in range(pad, pad + w):
            prod = np.prod(padded[i - pad:i + pad, j - pad:j + pad])
            filtered[i - pad][j - pad] = np.power(prod, expo)
    return filtered


def GMF(img, ksize):
    return geometric_mean_filter(img, ksize)


kernel_size = [3, 7, 9]

# read the original image
original_image = cv2.imread("../images/FigP0501.png", flags=0)

filtered_img = []

# apply filters with different kernel sizes respectively
for ksize in kernel_size:
    filtered_img.append(GMF(original_image, ksize))

# display the results
fig, axs = plt.subplots(nrows=1, ncols=4, figsize=(10, 4))
ax = axs[0]
ax.imshow(original_image, cmap='gray'), ax.set_title(f"original image"), ax.set_xticks([]), ax.set_yticks([])
for i in range(3):
    ax = axs[i + 1]
    ax.imshow(filtered_img[i], 'gray')
    ax.set_title(fr"${kernel_size[i]}\times${kernel_size[i]} GMF")
    ax.set_xticks([]), ax.set_yticks([])

plt.suptitle("Running Results of Geometric Mean Filtering")

plt.tight_layout()

# output = f'../images/Geometric Mean Filtering.jpg'
# plt.savefig(output)

plt.show()

```



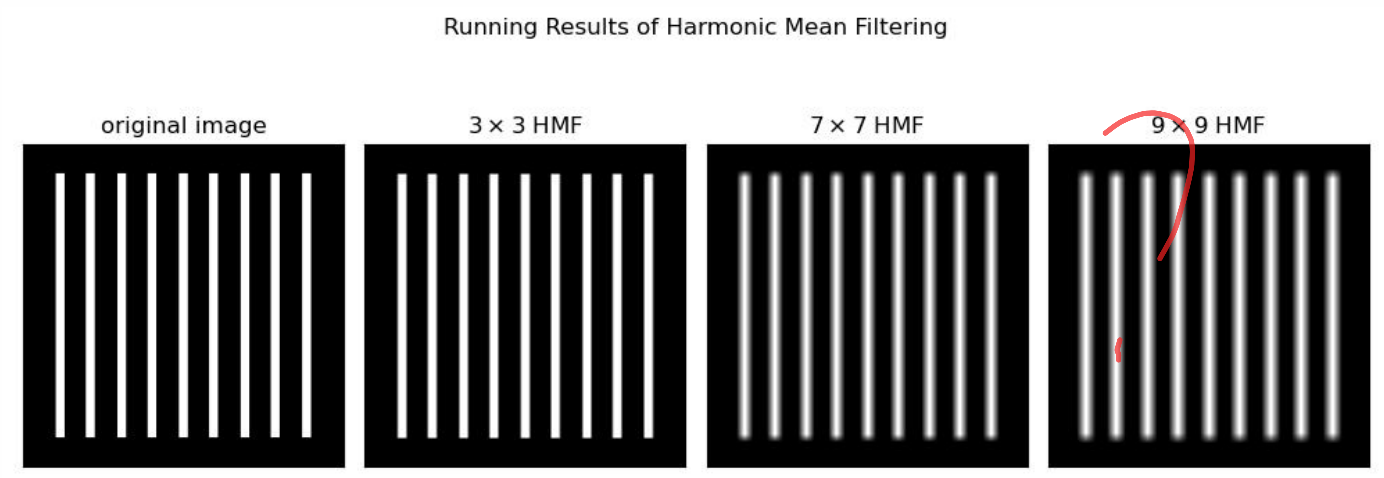
3. Repeat **1** using a harmonic mean filter which is defined as

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}.$$

(*followed by **Matlab live Scripts** or **Jupyter Scripts** and running results*)

```python
import cv2
import matplotlib.pyplot as plt
import numpy as np


def harmonic_mean_filter(img, ksize):
    h, w = img.shape[:2]
    order = ksize * ksize
    pad = int((ksize - 1) / 2)
    # pad the image using `np.pad()` whose effect can also be produced by`cv2.copyMakeBorder()`
    padded = np.pad(img, pad, 'symmetric')
    filtered = np.zeros(img.shape)
    for i in range(pad, pad + h):
        for j in range(pad, pad + w):
            s = np.sum(1 / (1e10 + padded[i - pad:i + pad, j - pad:j + pad]))
            filtered[i - pad][j - pad] = order / s
    return filtered


# read the original image
original_image = cv2.imread("../images/FigP0501.png", flags=0)
ksize = [3, 7, 9]

filtered_image = []
for size in ksize:
    filtered_image.append(harmonic_mean_filter(original_image, size))

# display the results
```

```python
fig, axs = plt.subplots(nrows=1, ncols=4, figsize=(10, 4))

ax = axs[0]
ax.imshow(original_image, cmap='gray')
ax.set_title(f"original image")
ax.set_xticks([])
ax.set_yticks([])

for i in range(3):
    ax = axs[i + 1]
    ax.imshow(filtered_image[i], cmap='gray')
    ax.set_title(fr"${ksize[i]}\times${ksize[i]} HMF")
    ax.set_xticks([])
    ax.set_yticks([])

plt.suptitle("Running Results of Harmonic Mean Filtering")

plt.tight_layout()

# output = f'../images/Harmonic Mean Filtering.jpg'
# plt.savefig(output)

plt.show()

```



4. Sketch what the image in FIGURE 6 would look like if it were blurred using the transfer function

$$H(u,v) = \frac{T}{\pi(ua+vb)} sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

(**a**) With $a = b = 0.1$, and $T = 1$.

(**b**) In addition, add Gaussian noise into the resulting image of (a), with zero mean and variance of 650.

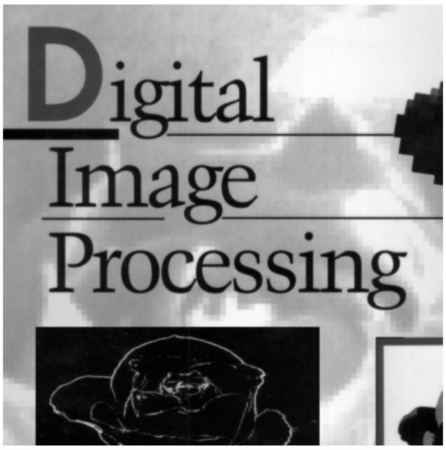


FIGURE 6

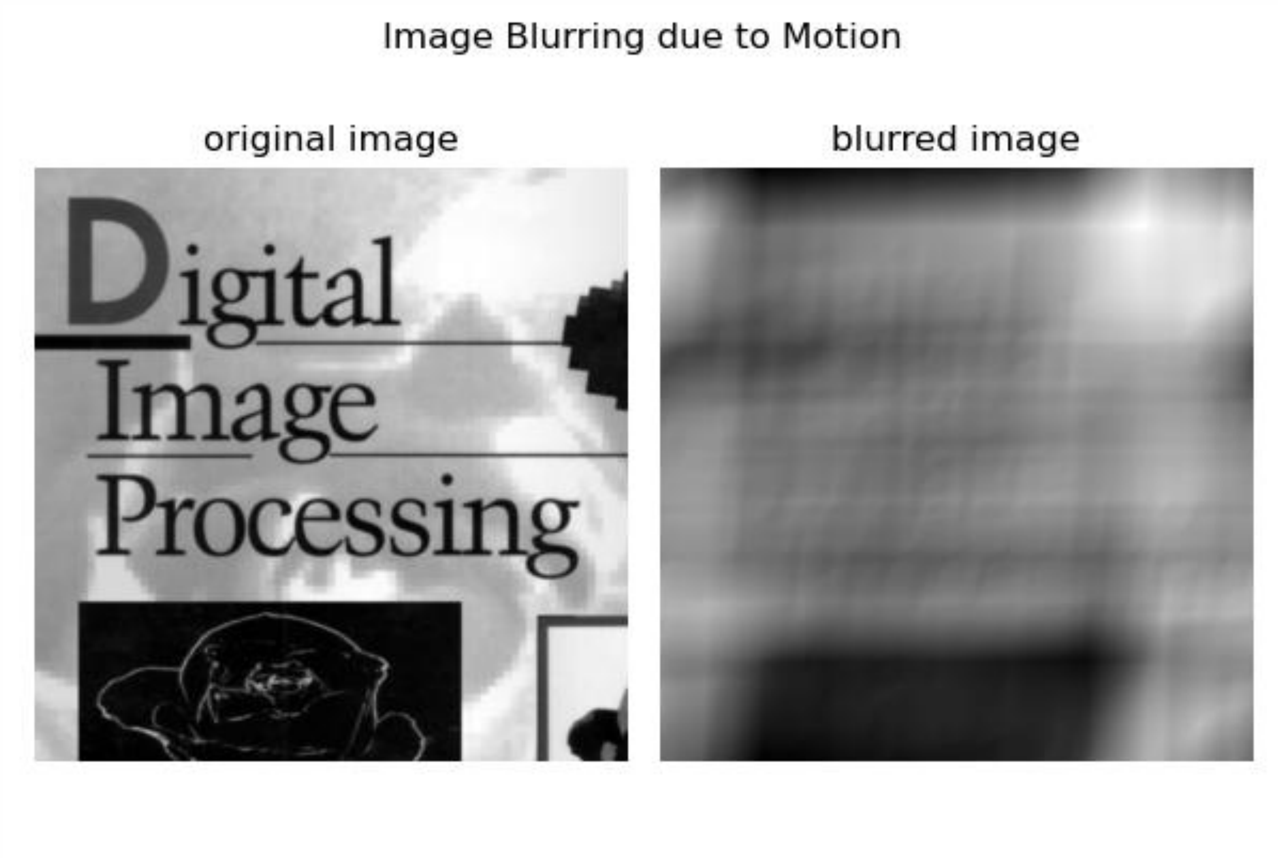
Try to restore the degraded image after procedure (b) using inverse filter, Wiener filter, and constrained least squares filter.

(*followed by* ***Matlab live Scripts*** *or* ***Jupyter Scripts*** *and running results*)

(**a**)

```
import cv2
import numpy as np
import matplotlib.pyplot as plt


def motion_blur(img, a=0.1, b=0.1, T=1):
    M, N = img.shape[:2]
    H = np.empty(img.shape, dtype=complex)
    # calculate the transfer function of motion blur
    for u in range(M):
        for v in range(N):
            s = u * a + v * b
            H[u, v] = (T / (np.pi * s + np.finfo(float).eps)) * np.sin(np.pi * s) * np.exp(-1j * np.pi * s)

    # apply the transfer function of motion blur
    f = img
    F = np.fft.fft2(f)
    G = H * F
    g = np.fft.ifft2(G)
    g = np.real(g)
    return g


original_img = cv2.imread("../images/Fig0526(a).png", 0)

blurred_img = motion_blur(original_img)

plt.figure(figsize=(6, 4))
plt.subplot(121), plt.imshow(original_img, 'gray'), plt.title("original image"), plt.axis('off')
```

```python
plt.subplot(122), plt.imshow(blurred_img, 'gray'), plt.title("blurred image"), plt.axis('off')
plt.suptitle("Image Blurring due to Motion")
plt.tight_layout()

# output = f'../images/Image Blurring due to Motion.jpg'
# plt.savefig(output)

plt.show()

```



(**b**)

```python
import cv2
import numpy as np
import matplotlib.pyplot as plt


def motion_degrade_function(img, a=0.1, b=0.1, T=1):
    M, N = img.shape[:2]
    H = np.empty(img.shape, dtype=complex)
    # calculate the transfer function of motion blur
    for u in range(M):
        for v in range(N):
            s = u * a + v * b
            H[u, v] = (T / (np.pi * s + np.finfo(complex).eps)) * np.sin(np.pi * s) * np.exp(-1j * np.pi * s)
    return H
```

```python


def motion_blur(img, a=0.1, b=0.1, T=1):
    H = motion_degrade_function(img, a, b, T)
    # apply the transfer function of motion blur
    f = img.copy()
    F = np.fft.fft2(f)
    G = H * F
    g = np.fft.ifft2(G)
    g = np.real(g)
    return g


def inverse_motion_blur(img, a=0.1, b=0.1, T=1):
    H = motion_degrade_function(img, a, b, T)
    # apply the transfer function of motion blur
    g = img.copy()
    G = np.fft.fft2(g)
    F = G / (H + np.finfo(complex).eps)
    f = np.fft.ifft2(F)
    f = np.real(f)
    return f


def gauss_blur(img, mean, stand_deviation):
    n = np.random.normal(mean, stand_deviation, img.shape)
    f = img.copy()
    g = f + n
    return g


def wiener_filter(img, H, K):
    G = np.fft.fft2(img)
    H_square = np.power(H, 2)
    H_abs = np.abs(H)
    F = ((1 / (H_abs + np.finfo(complex).eps)) * (H_square / (H_square + K))) * G
    f = np.real(np.fft.ifft2(F))
    return f


original_img = cv2.imread("../images/Fig0526(a).png", 0)
trans_func = motion_degrade_function(original_img)
motion_blurred_img = motion_blur(original_img)

gauss_blurred_img = gauss_blur(motion_blurred_img, 0, np.sqrt(650))

inverse_filtered_img = inverse_motion_blur(gauss_blurred_img)
wiener_filtered_img = wiener_filter(gauss_blurred_img, trans_func, K=0.1)
CLS_filtered_img = 0

# display the results
```

```python
img_ls = [original_img, motion_blurred_img, gauss_blurred_img, inverse_filtered_img, wiener_filtered_img,
          CLS_filtered_img]
title_ls = ['original image', 'motion blurred', 'motion blurred with Gaussian noise', 'inverse filtered',
            'wiener filtered', 'constrained least squares filtered']
fig, axs = plt.subplots(2, 3, figsize=(10, 8))
for i in range(5):
    ax = axs.flat[i]
    ax.imshow(img_ls[i], cmap='gray')
    ax.set_title(title_ls[i])
    ax.set_xticks([])
    ax.set_yticks([])
axs[1, 2].set_visible(False)
plt.tight_layout()

# output = f'../images/blur&noise.jpg'
# plt.savefig(output)

plt.show()

```