

## 数字图像处理及应用 第4次作业

组号: XX (两位数字) 小组成员: (列出所有小组成员, 成员姓名间用1个空格间隔)

### Part I Exercises

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**Ex.1** The image shown in FIGURE 1 consists of two infinitesimally thin white lines on a black background, intersecting at some point in the image. The image is input into a linear, position invariant system with the impulse response given as Eq.1.

$$h(x, y) = e^{-[(x-\alpha)^2 + (y-\beta)^2]} \quad (1)$$

Assuming continuous variables and negligible noise, find an expression for the output image  $g(x, y)$ .



FIGURE 1

**Answer:**

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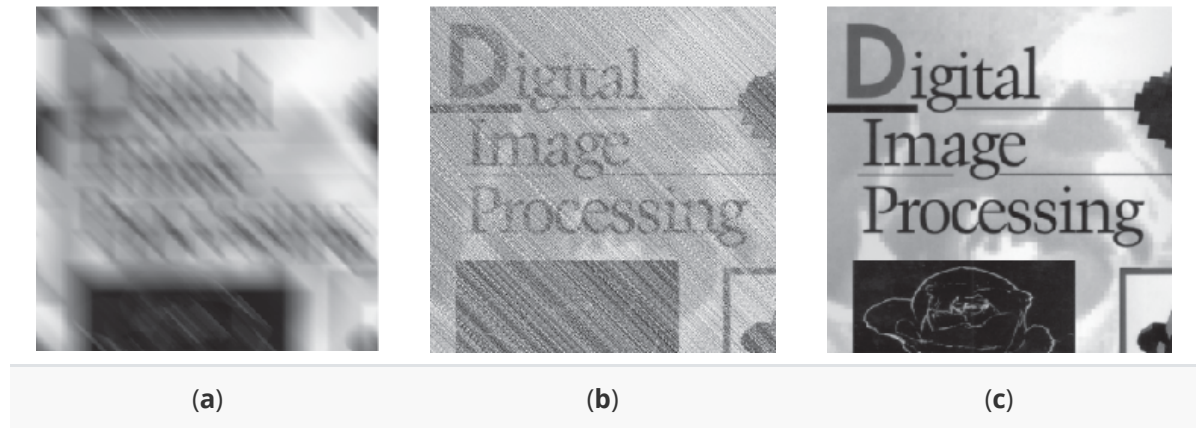
**Ex.2** During acquisition, an image undergoes uniform linear motion in the vertical direction for a time  $T_1$ . The direction of motion then switches to the horizontal direction for a time interval  $T_2$ . Assuming that the time it takes the image to change directions is negligible, and that shutter opening and closing times are negligible also, give an expression for the blurring function,  $H(u, v)$ .

**Answer:**

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**Ex.3**

(a) The image in (b) and (c) were obtained by inverse and Wiener-filtering the image in (a), which is a motion blurred image that, in addition, is corrupted by additive Gaussian noise. The blurring itself is corrected in (b) and (c). However, the restored image (b) has a strong streak pattern that is not apparent in (a) [for example, compare the area of constant white in the top right of (b) with the corresponding area in (a).] On the other hand, the streak pattern does not appear in (c). Explain how this pattern originated and why Wiener filter can avoid it.



**FIGURE 2 Inverse and Wiener filtering**

**Answer:**

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**Ex.4** A certain X-ray imaging geometry produces a blurring degradation that can be modeled as the convolution of the sensed image with the spatial, circularly symmetric function

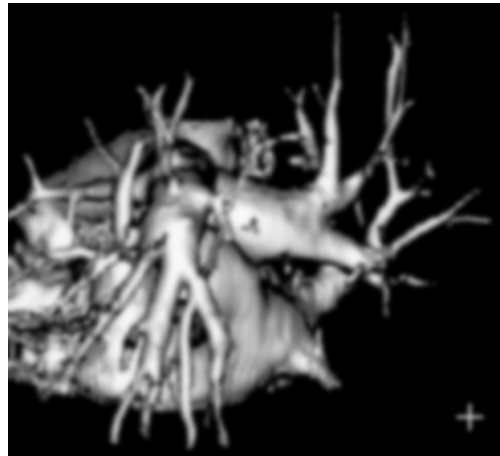
$$h(x, y) = \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} e^{-\frac{x^2 + y^2}{2\sigma^2}} \quad (2)$$

Assuming continuous variables, show that the degradation in the frequency domain is given by the expression

$$H(u, v) = -8\pi^3 \sigma^2 (u^2 + v^2) e^{-2\pi^2 \sigma^2 (u^2 + v^2)} \quad (3)$$

**Answer:**

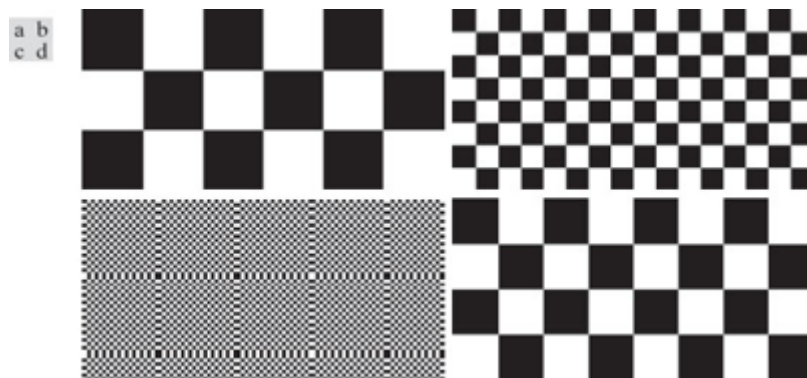
**Ex.5** The image shown is a blurred, 2-D projection of a volumetric rendition of a heart. It is known that each of the cross hairs on the right bottom part of the image was 4 pixels wide, 20 pixels long, and had an intensity value of 255 before blurring. Provide a step-by-step procedure indicating how you would use the information just given to obtain the blurring function  $H(u, v)$ .



**FIGURE 3** Volumetric rendition of a heart

**Answer:**

**Ex.6** Explain the reason for the formation of image (d) in FIGURE 4 (refer to Example 4.6 in page 252), which is acquired by an imaging system with maximum sampling rate of  $96 \times 96$ . The original image of (d) is a checkerboard like image, which each of its square is of  $0.4798 \times 0.4798$  pixels.



**FIGURE 4** Aliasing in image



## Part II Programming

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1. The arithmetic mean filter is defined as

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t). \quad (1)$$

The white bars in the test pattern shown are 7 pixels wide and 210 pixels high. The separation between bars is 17 pixels. What would this image look like after application of

- (a) A  $3 \times 3$  arithmetic mean filter?
- (b) A  $7 \times 7$  arithmetic mean filter?
- (c) A  $9 \times 9$  arithmetic mean filter?

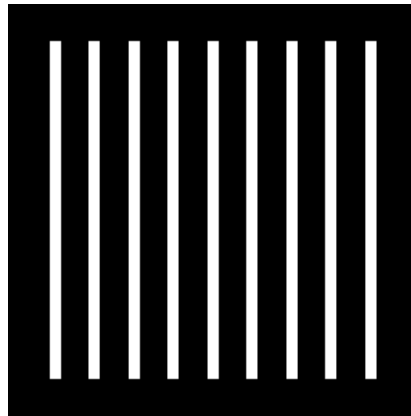


FIGURE 5 Test pattern

(followed by **Matlab live Scripts** or **Jupyter Scripts** and running results)

2. Repeat 1 using a geometric mean filter which is defined as

$$\hat{f}(x, y) = \left[ \prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}. \quad (2)$$

(followed by **Matlab live Scripts** or **Jupyter Scripts** and running results)

3. Repeat 1 using a harmonic mean filter which is defined as

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s, t)}}. \quad (3)$$

(followed by **Matlab live Scripts** or **Jupyter Scripts** and running results)

4. Sketch what the image in FIGURE 6 would look like if it were blurred using the transfer function

$$H(u, v) = \frac{T}{\pi(ua + vb)} \sin[\pi(ua + vb)] e^{-j\pi(ua + vb)} \quad (4)$$

(a) With  $a = b = 0.1$ , and  $T = 1$ .

(b) In addition, add Gaussian noise into the resulting image of (a), with zero mean and variance of 650.

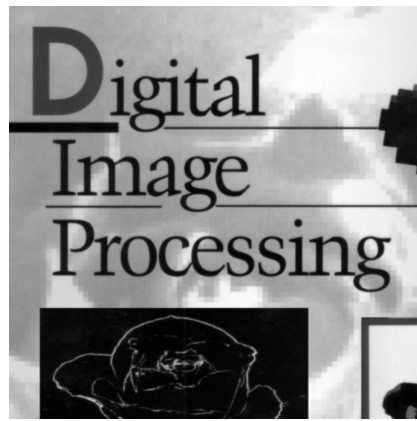


FIGURE 6

Try to restore the degraded image after procedure (b) using inverse filter, Wiener filter, and constrained least squares filter.

(followed by **Matlab live Scripts** or **Jupyter Scripts** and running results)