HW3: report

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算法

问题描述

如上两幅图，现我们需要将Figure 3.1中的女孩搬到Figure 3.2的海水中，为使得复制粘贴更加逼真自然，我们需要设计算法来满足我们两幅图像融合的需要。

Poisson Image Editing

Poisson Image Editing算法的基本思想是在尽可能保持原图像内部梯度的前提下，让粘贴后图像的边界值与新的背景图相同，以实现无缝粘贴的效果。从数学上讲，对于原图像$f(x,y)$，新背景$f^*(x,y)$和嵌入新背景后的新图像$v(x,y)$，等价于解最优化问题：

$$\min_f \iint_\Omega |\nabla f - \nabla \boldsymbol{v}|^2 \ \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

利用变分法可转化为具有Dirichlet边界条件的Poisson方程：

$$\Delta f = \Delta \boldsymbol{v} \text{ over } \Omega \ \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

以Figure 3.1和Figure 3.2为例，将Figure 3.1中需要复制的区域设为S，定义N_p为S中的每一个像素p四个方向连接邻域，令$< p, q >$为满足$q \in N_p$的像素对。边界Ω定义为

$$\partial\Omega = p \in S \setminus \Omega : N_p \cap \Omega \neq \emptyset$$

设f_p为p处的像素值f，目标即求解像素值集$f|_\Omega = f_p, p \in \Omega$。

利用Poisson Image Editing算法的基本原理，上述问题转化为求解最优化问题：

$$\min_{f|_\Omega} \sum_{<p,q>\cap\Omega\neq\emptyset} (f_p - f_q - v_{pq})^2, \text{with } f_p = f_p^*, \text{for all} p \in \partial\Omega$$

化为求解线性方程组：

$$\text{for all } p \in \Omega,\ |N_p| f_p - \sum_{q\in N_p\cap\Omega} f_q = \sum_{q\in N_p\cap\partial\Omega} f_p^* + \sum_{q\in N_p} v_{pq}$$

对于梯度场$\boldsymbol{v}(\boldsymbol{x})$的选择，文献给出两种方法，一种是完全使用前景图像的内部梯度，即：

$$\text{for all } < p, q >, v_{pq} = g_p - g_q$$

另一种是使用混合梯度：

$$\text{for all } \boldsymbol{x} \in \Omega,\ \boldsymbol{v}(\boldsymbol{x}) = \begin{cases} \nabla f^*(\boldsymbol{x}) & \text{if } |\nabla f^*(\boldsymbol{x})| > |\nabla g(\boldsymbol{x})|, \\ \nabla g(\boldsymbol{x}) & \text{otherwise} \end{cases}$$

实现

predecomposer

矩阵预分解工具。在选区完成时既完成A的填充和分解。

- 接口

```
class Predecomposer
{
   public:
    Predecomposer(std::shared_ptr<USTC_CG::Image> mask) : mask_(mask){}
    ~Predecomposer() = default;

    void solve();

   public:
    std::shared_ptr<USTC_CG::Image> mask_;

    Eigen::SparseMatrix<double> A_;

    // pos to index map
    std::unordered_map<std::pair<int, int>, int, pair_hash> index_map;
    // index to neighbors pos map
    std::unordered_map<int, std::vector<std::pair<int, int>>> neighbor_map;
    // index to borders pos (exclude, if exists) map
    std::unordered_map<int, std::vector<std::pair<int, int>>> border_map;

    Eigen::SimplicialLLT<Eigen::SparseMatrix<double>> solver;
};
```

- 核心算法`solve()`实现及解释

```cpp
void Predecomposer::solve()
{
    //根据编号索引构造index map
    //初始化border map
    int counter = 0;
    for (int i = 0; i < mask_->width(); ++i)
    {
        for (int j = 0; j < mask_->height(); ++j)
        {
            if (mask_->get_pixel(i, j)[0] > 0)
            {
                index_map.emplace(std::pair<int, int>(i, j), counter);
                border_map.emplace(
                    counter, std::vector<std::pair<int, int>>(0));
                counter++;
            }
        }
    }
    A_.resize(counter, counter);

    //构造A矩阵
    for (int i = 0; i < mask_->width(); ++i)
    {
        for (int j = 0; j < mask_->height(); ++j)
        {
            //对于掩码中的每个像素
            if (mask_->get_pixel(i, j)[0] > 0)
            {
                int idx = index_map[std::pair<int, int>(i, j)];
                int neighbor_count = 0;
                std::vector<std::pair<int, int>> near = {
                    { i - 1, j }, { i + 1, j }, { i, j - 1 }, { i, j + 1 }
                };
                //遍历(i, j)4个邻居节点
                for (const auto& n : near)
                {
                    int near_x = n.first;
                    int near_y = n.second;
                    // 如果没有超出图像范围
                    if (0 <= near_x && near_x < mask_->width() && 0 <= near_y &&
                        near_y < mask_->height())
                    {
                        //将邻居节点加入该节点(i, j)的邻居索引中
                        neighbor_map[idx].push_back(
                            std::pair<int, int>(near_x, near_y));
                        //如果没有超出掩码范围, 调整矩阵A对应条目
                        if (mask_->get_pixel(near_x, near_y)[0] > 0)
                            A_.coeffRef(idx, index_map[{ near_x, near_y }]) =
                                -1;
                        //如果超出范围，将邻居节点加入该节点(i, j)的边界索引中
                        else
                            border_map[idx].push_back(
                                std::pair<int, int>(near_x, near_y));
                        ++neighbor_count;
                    }
                }
                A_.coeffRef(idx, idx) = neighbor_count;
            }
        }
    }
    A_.makeCompressed();
    solver.compute(A_);
}
```

SeamlessCloner

利用预分解信息进行实时求解。

- 接口

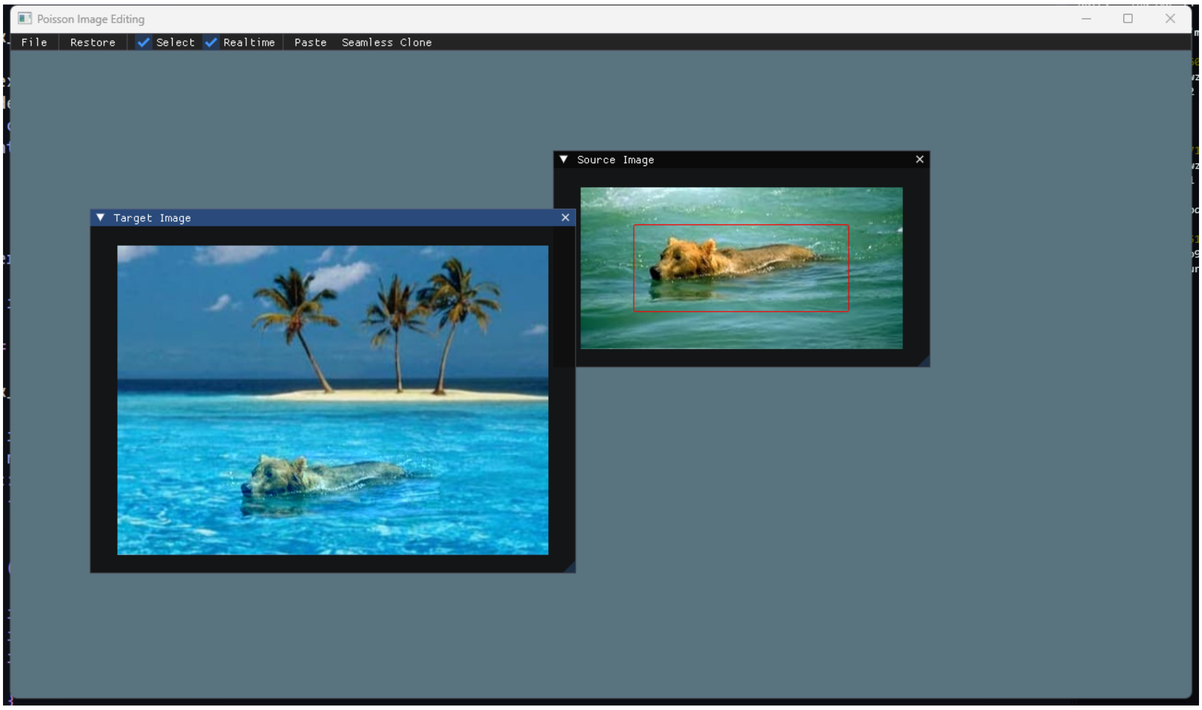
```
#pragma once

#include "comp_source_image.h"
#include "predecomposer.h"

class SeamlessCloner
{
   public:
    SeamlessCloner() = default;
    ~SeamlessCloner() = default;

    void set_decomposer(std::shared_ptr<Predecomposer> decomposer);
    void set_target_image(std::shared_ptr<USTC_CG::Image> dst_image);
    void set_source_image(std::shared_ptr<USTC_CG::Image> src_image);
    void set_offset(int offset_x, int offset_y);

    void solve();

    std::vector<unsigned char> get_pixel(int i, int j, int channels);
```

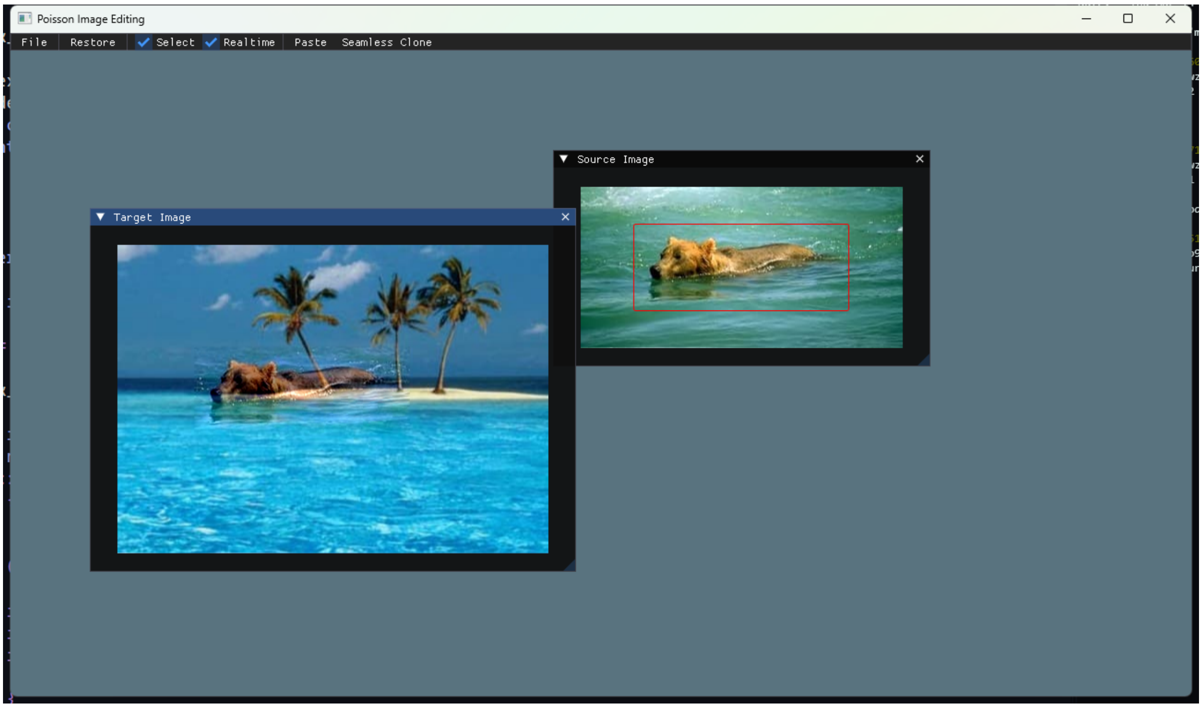
```cpp

   private:
    inline double get_gradient_mix(double f_p, double f_q, double g_p, double g_q)
    {
        double v_pq_f = f_p - f_q;
        double v_pq_g = g_p - g_q;
        return (v_pq_f * v_pq_f) > (v_pq_g * v_pq_g) ? v_pq_f : v_pq_g;
    }

   private:
    inline bool check_valid_range(int i, int j)
    {
        return i >= dst_image_->width() ||
            j >= dst_image_->height();
    }

   private:
    int offset_x_;
    int offset_y_;
    std::shared_ptr<USTC_CG::Image> src_image_;
    std::shared_ptr<USTC_CG::Image> dst_image_;
    std::shared_ptr<Predecomposer> decomposer_;
    std::vector<Eigen::VectorXd> color_vec;
};
```

- 核心算法solve()实现及解释

```cpp
void SeamlessCloner::solve()
{
    color_vec.clear();
    int channels = src_image_->channels();

    // 分channel单列求解
    for (auto channel = 0; channel < channels; ++channel)
    {
        Eigen::VectorXd b = Eigen::VectorXd::Zero(decomposer_->A_.rows());
        //index map存储了所有掩码内的像素及对应索引
        for (const auto& pair : decomposer_->index_map)
        {
            int i = pair.first.first, j = pair.first.second;
            int idx = pair.second;
            if (check_valid_range(i + offset_x_, j + offset_y_)) continue;

            // calculate gradient mix color channel
            double total = 0;
            double d_pivot = dst_image_->get_pixel(i + offset_x_, j + offset_y_)[channel];
            double s_pivot = src_image_->get_pixel(i, j)[channel];
            for (auto &neighbor : decomposer_->neighbor_map[idx])
            {
                if (check_valid_range(neighbor.first + offset_x_, neighbor.second + offset_y_)) continue;

                double d_neighbor = dst_image_->get_pixel(
                    neighbor.first + offset_x_, neighbor.second + offset_y_)[channel];
                double s_neighbor = src_image_->get_pixel(neighbor.first, neighbor.second)[channel];
                total += get_gradient_mix(d_pivot, d_neighbor, s_pivot, s_neighbor);
            }

            b(idx) = total;
            // smooth border
            for (auto &border : decomposer_->border_map[idx])
            {
                if (check_valid_range(border.first + offset_x_, border.second + offset_y_)) continue;

                b(idx) += dst_image_->get_pixel(
                    border.first + offset_x_, border.second + offset_y_)[channel];
            }
        }

        auto x = decomposer_->solver.solve(b);
        //按列添加到最终颜色向量集合中
        color_vec.push_back(x);
    }
}
```

演示


Poisson Image Editing
File
Restore
Select
Realtime
Paste
Seamless Clone
Source Image
Target Image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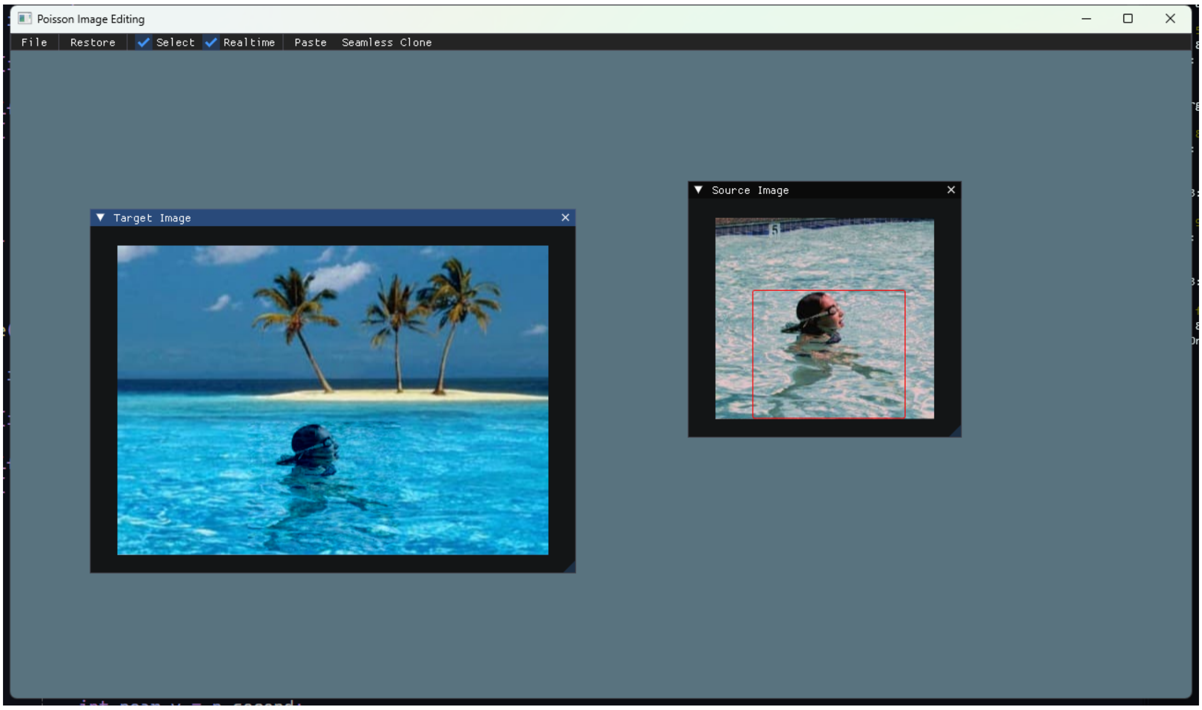
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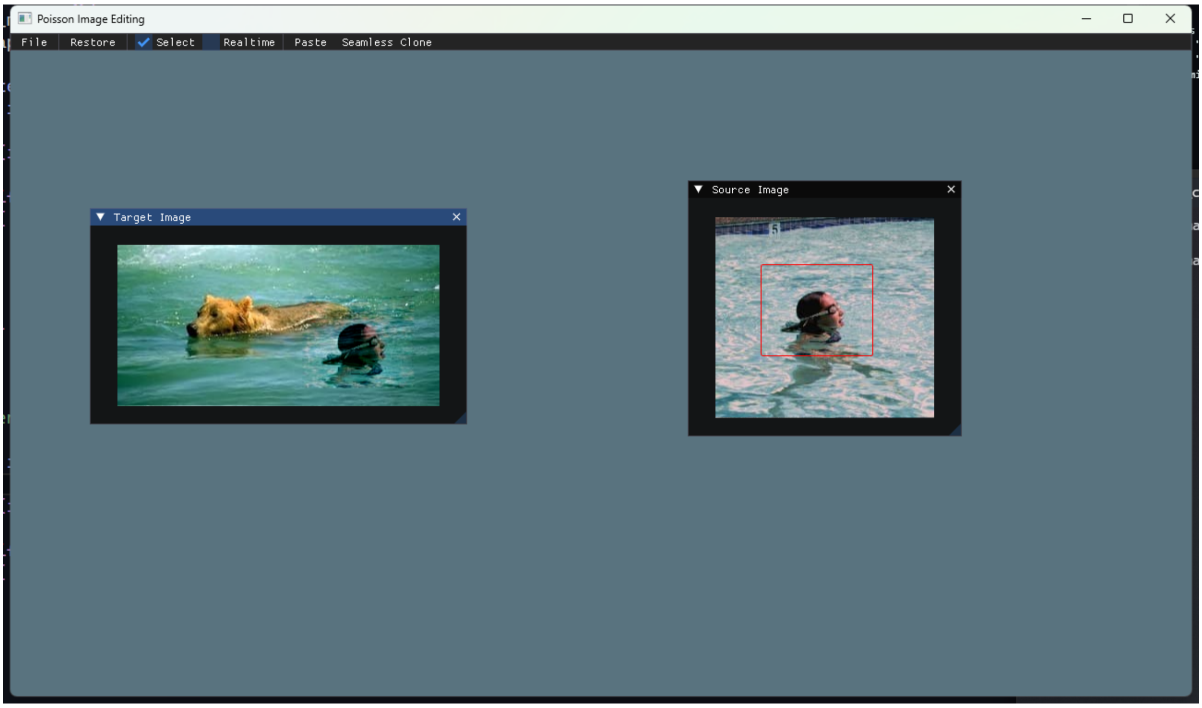
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