Essential Mathematics

Tutorial 3

Semester 2 2017

1 Systems of Linear Equations

1. Find x and y such that:

   (a)

$$x + y = 3 \tag{1}$$
$$x - y = 4 \tag{2}$$

Solution: (1) + (2):

$$\begin{aligned} (x + y) + (x - y) &= 3 + 4 \\ 2x &= 7 \\ x &= 7/2 \end{aligned}$$

Substituting into (1):

$$\begin{aligned} 7/2 + y &= 3 \\ y &= 6/2 - 7/2 \\ y &= -1/2 \end{aligned}$$

   (b)

$$1.5x + 3y = 9.75 \tag{3}$$
$$2x - 0.5y = 9.5 \tag{4}$$

Solution: $(3) + 6 \times (4)$:

$$\begin{aligned} (1.5x + 3y) + 6(2x - 0.5y) &= 9.75 + 9.5 \times 6 \\ 1.5x + 3y + 12x - 3y &= 9.75 + 57 \\ 13.5x &= 66.75 \\ x &= \frac{89}{18} \end{aligned}$$

Substituting into (4):

$$\begin{aligned} 2 \times \frac{89}{18} - 0.5y &= 9.5 \\ -0.5y &= 9.5 - \frac{89}{9} \\ -0.5y &= -\frac{7}{18} \\ y &= \frac{14}{18} \end{aligned}$$

2. For each of the following systems, is there one solution, no solution, or an infinite number of solutions?

(a)

$$\begin{aligned} 19x + 14y &= 8 \\ 133x + 98y &= 56 \end{aligned}$$

Solution: Observe that $133/19 = 98/14 = 56/8 = 7$, which means that one of the equations is just a multiple of the other. Thus, the two straight lines determined by the two equations coincide. There are an infinite number of solutions.

Arguing based on mathematical formula, we have:

$$\begin{array}{rrcr} 19x & +14y & = & 8 \\ 133x & +98y & = & 56 \end{array} \Rightarrow \begin{array}{rrcr} 19x & +14y & = & 8 \\ & 0 & = & 0 \end{array} \qquad (\text{Eq2} - (133/19)\text{Eq1})$$

For all real value of y, say $y = t$, $x = (8 - 14t)/19$. So, there are infinitely many solutions $\{(t, (8 - 14t)/19) \mid t \in \mathbb{R}\}$.

(b)

$$\begin{aligned} 19x + 14y &= 8 \\ 133x + 98y &= 57 \end{aligned}$$

Solution: No solution: $133/19 = 98/14 = 7 \neq 57/8 = 7.125$. Left hand sides of equations are just a multiples, but right hand side doesn't match.

Arguing based on mathematical formula, we have:

$$\begin{array}{rrcr} 19x & +14y & = & 8 \\ 133x & +98y & = & 57 \end{array} \Rightarrow \begin{array}{rrcr} 19x & +14y & = & 8 \\ & 0 & = & 1 \end{array} \qquad (\text{Eq2} - (133/19)\text{Eq1})$$

$0 = 1$ is impossible that means that the two equations do not hold true at the same time. Thus, there is no solution.

(c)

$$\begin{aligned} 20x + 53y &= 3066 \\ 7x - 12y &= 798 \end{aligned}$$

Solution: One solution: $20/7 = 2.857 \neq 53/12 = 4.417$. Left hand sides of equations are not multiples of each other.

2 Systems of nonlinear equations

1. For each of the following systems, find all values for x and y

   (a)

$$\begin{aligned} x - y^2 &= 0 && (5) \\ x - 3y &= 10 && (6) \end{aligned}$$

Solution: $x - y^2 = 0 \Rightarrow x = y^2$. Substituting into (6)

$$\begin{aligned} y^2 - 3y &= 10 \\ y^2 - 3y - 10 &= 0 \\ y^2 - 3y - 10 &= 0 \\ y^2 + (2-5)y - 10 &= 0 \\ (y+2)(y-5) &= 0 \end{aligned}$$

So $x = -2$ or 5. We have two solutions to check. Substituting $y = -2$ into (6)

$$\begin{aligned} x - 3 \times -2 &= 10 \\ x + 6 &= 10 \\ x &= 4 \end{aligned}$$

Now substituting $y = 5$:

$$\begin{aligned} x - 3 \times 5 &= 10 \\ x - 15 &= 10 \\ x &= 25 \end{aligned}$$

So, $(x, y) = (4, -2)$ or $(25, 5)$

   (b)

$$\begin{aligned} -3x - y &= -2 && (7) \\ x^2 + 2y &= -4 && (8) \end{aligned}$$

Solution: $-3x - y = -2 \Rightarrow y = -3x + 2$. Substituting into (8)

$$
\begin{aligned}
x^2 + 2(-3x+2) &= -4 \\
x^2 - 6x + 4 &= -4 \\
x^2 - 6x + 8 &= 0 \\
x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times 8}}{2} \\
x &= \frac{6 \pm \sqrt{36-32}}{2} \\
x &= \frac{6 \pm \sqrt{4}}{2} \\
x &= \frac{6 \pm 2}{2} \\
x &= 3 \pm 1
\end{aligned}
$$

So $x = 2$ or 4. We have two solutions to check: Substituting $x = 2$ into (7):

$$
\begin{aligned}
-3 \times 2 - y &= -2 \\
-6 - y &= -2 \\
-y &= 4 \\
y &= -4
\end{aligned}
$$

Substituting $x = 4$ into (7):

$$
\begin{aligned}
-3 \times 4 - y &= -2 \\
-12 - y &= -2 \\
-y &= 10 \\
y &= -10
\end{aligned}
$$

So $(x, y) = (2, -4)$ or $(4, -10)$