RMIT UNIVERSITY

SCHOOL OF SCIENCE

Family Name	
Given Names	
Student No.	

MATH2267 ESSENTIAL MATHEMATICS FOR ANALYTICS

Sample Laboratory Exam

Semester 2, 2018

Instructions

This exam paper **MUST be returned to the teaching staff** at the end of the lab exam, after filling in your names and student number. Photos of the exam paper are not allowed.

A template Julia solution file LabExam2018S2.ipynb should be downloaded from Canvas and used for all your calculations and answers. When completed, print your solution as a PDF file and submit to Canvas (go to Assignments in Canvas, then Lab Exam, and follow the instructions from there).

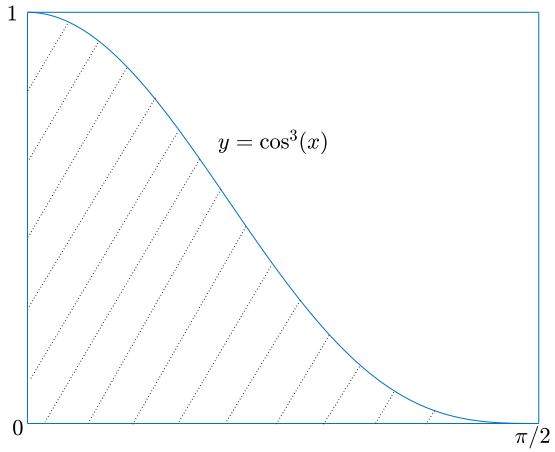
This Lab Exam is open book. You may not communicate with any one other than the teaching team during this Exam. This includes online communication.

(Time 110 minutes including submission time, Total Marks 100)

Attempt all FOUR questions:

1. **(5+25=30 marks)**

   Find the (approximate) area under the curve $y = \cos^3 x$ over $0 \le x \le \pi/2$ (the shaded region in figure), using the numerical methods specified below:



   (a) Numerical integration method (quadgk);

   (b) Using Monte-Carlo method (with no less than $200,000$ random points).

2. **(8+7+10=25 marks)**

A species has a life span of six years. They start to produce offsprings in their second year and each individual of year 2, 3, 4, 5 and 6 produces 2, 3, 3, 2 and 1 babies, respectively. The death rates of all age groups are:

Year	1	2	3	4	5	6
Death rate	38%	24%	15%	21%	30%	100%

(a) Input in IJulia the Leslie matrix for the population model.

(b) Is the population likely to survive?

(c) If 20%, 40%, 40%, and 25% of the year groups 2, 3, 4, and 5, respectively, are sold each year, will the population still be sustainable?

(HINT: Survival rate $= 1-$ death rate;
Survival rate after selling $= 1-$ death rate $-$ percentage sold)

3. **(15+15=30 marks)**

Complete each of the following.

(a) Perform 10 iterations of Newton's method (Newton-Raphson method) to find an approximate solution of the equation

$$\frac{1}{3}x^3 - 2x\sin(x) + 3x - 10 = 0,$$

starting from $x_0 = -1$. Show x_{10}.

(b) Solve the following ODE using the DifferentialEquations package in Julia Box. Plot your solution $x(t)$ on $[0, 5]$.

$$\begin{aligned} \dot{x}(t) &= 2t\sin(t^2 - 1),\ 0 < t < 5 \\ x(0) &= 1.0 \end{aligned}$$

4. **(10+5=15 marks)**

Consider the function $f(x) = x^2 + x\cos(x)$. Its first and second derivatives are

$$f'(x) = 2x - x\sin(x) + \cos(x)$$

and

$$f''(x) = 2 - x\cos(x) - 2\sin(x).$$

(a) Using Julia find a critical point for $f(x)$.

HINT: Solve $f'(x) = 0$ with a initial guess $x_0 = -1.0$. Run 10 iterations using Newton's method to obtain approximate critical point x_{10}.

(b) Calculate $f''(x_{10})$ and determine if x_{10} is a minimum point, maximum point or the 2_{nd} order test fails in this case.