## MATH2267 ESSENTIAL MATHEMATICS FOR ANALYTICS
## **Laboratory Exam Revision Exercises**

## Instructions

You may create an ipynb-file (e,g, revision.ipynb) for the solution of this revision exercises. When completed print it as a PDF file and save in your computer.

## Attempt all questions:

1. A species has a life span of eight years. They start to produce offsprings in their second year and each individual of year 2, 3, 4, 5, 6,7 and 8 produces 1, 2, 3, 4, 2, 2 and 0 babies, respectively. The survival rates of all age groups are:

   | Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
   |------|-----|-----|-----|-----|-----|-----|-----|---|
   | Survival rate | 60% | 70% | 75% | 75% | 80% | 65% | 60% | 0 |

   (a) Input in Julia the Leslie matrix for the population model.

   (b) Is the population likely to survive?

   (c) If 25% of the year 1 group, 30% of the year 2 group, 30% of year 3 group, and all of year 8 group are sold each year, will the population be sustainable?

2. Consider the function $f(x) = x^4 - x(\sin^2(x) + 6)$.

   (a) Find a critical point of the function by solving the equation $f'(x) = 0$ using Newton's method. It required to perform at least 10 iterations and start from $x_0 = -2$.

   (b) Determine using second order test check if the critical point found in (a) is a maximum point, minimum point, or otherwise.

3. Complete each of the following.

   (a) Find, by numerical integration, the area under the curve $y = \sqrt{x}\tan(x), \ 0 \le x \le \pi/4$.

   (b) Using Monte-Carlo method (with no less than $100,000$ random points) estimate the area under the curve $y = \sqrt{x}\tan x$ over $0 \le x \le \frac{\pi}{4}$. This is the area of the shaded region in the figure below.

4. Solve the following ODE using DifferentialEquations package in Julia Box. Plot the solution $y(t)$ on the interval $[0, 8]$.

$$y'(t) = e^{\sin(t)}(\sqrt{t} - 1), \ 0 < t < 8,$$
$$y(0) = 2.$$