

Essential Mathematics

Additional Exercises for Week 3

1. Use Gaussian elimination to solve the following system of linear equations:

(a)

$$\begin{array}{rrcr} x & - & 2y & + & 3z & = & 2 \\ 2x & + & 3y & - & z & = & 4 \\ x & + & 10y & - & 11z & = & 6 \end{array} \quad \left(\begin{array}{l} \text{Solution: } x = 4 \\ y = -2 \\ z = -2 \end{array} \right)$$

(b)

$$\begin{array}{rrcr} 4x_1 & - & 2x_2 & + & x_3 & = & 1 \\ 2x_1 & + & x_2 & - & x_3 & = & 0 \\ x_1 & + & 3x_2 & - & 2x_3 & = & 2. \end{array} \quad \left(\begin{array}{l} \text{Solution: } x_1 = 1 \\ x_2 = 5 \\ x_3 = 7 \end{array} \right)$$

(c)

$$\begin{array}{rrcr} 2x_1 & - & x_2 & + & 3x_3 & = & 18 \\ -x_1 & + & 2x_2 & - & x_3 & = & -12 \\ 4x_1 & + & x_2 & + & 7x_3 & = & 30. \end{array} \quad \left(\begin{array}{l} \text{Solution: } x = 8 \\ y = -2 \\ z = 0 \end{array} \right)$$

(d)

$$\begin{array}{rrcr} x & + & y & + & z & = & 5 \\ x & - & 2y & + & 3z & = & 25 \\ x_1 & + & 3x_2 & - & 2x_3 & = & 10 \end{array} \quad \left(\begin{array}{l} \text{Solution: } x = 30 \\ y = -14 \\ z = -11 \end{array} \right)$$

(e)

$$\begin{array}{rrcr} x_1 & - & x_2 & + & 3x_3 & = & 1 \\ 2x_1 & + & 3x_2 & - & 2x_3 & = & 3 \\ 4x_1 & + & 2x_2 & + & 2x_3 & = & 2 \end{array} \quad \left(\begin{array}{l} \text{Solution: } x = -10 \\ y = 13 \\ z = 8 \end{array} \right)$$

(f)

$$\begin{array}{rrcr} 2x & + & 3y & + & 4z & = & 3 \\ -x & - & 2y & + & 2z & = & -1 \\ x & + & 2y & - & z & = & 1 \end{array} \quad \left(\begin{array}{l} \text{Solution: } x = 3 \\ y = -1 \\ z = -0 \end{array} \right)$$

(g)

$$\begin{array}{rrcr} 2x_1 & - & x_2 & - & 2x_3 & = & -1 \\ -2x_1 & + & 2x_2 & + & 2x_3 & = & 2 \\ x_1 & + & 2x_2 & - & 2x_3 & = & 1 \end{array} \quad \left(\begin{array}{l} \text{Solution: } x = 1 \\ y = 1 \\ z = 1 \end{array} \right)$$

2. For each of the following systems, find all solution pairs (x, y) .

(a)

$$\begin{array}{rrcr} x^2 & - & y & = & 0 \\ x^2 & + & y & = & 8 \end{array} \quad (\text{Solution: } (2, 4), (-2, 4))$$

(b)

$$\begin{array}{rrcr} x^2 & + & 3x & - & y & = & -2 \\ 2x & - & y & = & -3 \end{array} \quad (\text{Solution: } (1, 2), (1, 5))$$

(c)

$$\begin{array}{rrcr} x^2 & + & y^2 & = & 34 \\ x^2 & - & 2y^2 & = & 7 \end{array} \quad (\text{Solution: } (5, 3), (5, -3), (-5, 3), (-5, -3))$$