# Tutorial 1 - Solutions

2018

## 1 Sets

1. If $A = \{a, b, c, d, e, f\}$ and $B = \{e, g, u, d, w\}$:
   (a) What is $A \cup B$?

   **Solution:** $A \cup B = \{a, b, c, d, e, , g, u, w\}$

   (b) What is $A \cap B$?

   **Solution:** $A \cap B = \{d, e\}$

   (c) What is $A \setminus B$?

   **Solution:** $A \setminus B = \{a, b, c, f\}$

   (d) What is $B \setminus A$?

   **Solution:** $B \setminus A = \{g, u, w\}$

2. If $A = \{x \in \mathbb{Z} | x^2 \leq 4\}$ and $B = [-5, 2)$
   (a) What is $A \cap B$?

   **Solution:** $A$ is the set of integers ($\mathbb{Z}$) such that $x^2 \leq 4$. Listing them, that would be $\{-2, -1, 0, 1, 2\}$.
   $B$ is the set of real numbers such that $-5 \leq x < 2$. All of the numbers from $A$ satisfy the condition for $B$, except for 2 (which is not less than 2). Therefore, the answer is:

   $$\{-2, -1, 0, 1\}$$

   (b) Is $-2 \in A \setminus B$?

   **Solution:** $-2 \in A$ , but it is also an element of $B$. Therefore, no, $-2 \notin A \setminus B$.

## 2 Functions

For each of the following functions, plot it in Julia. State its domain and range. Note any thing interesting about it. Are there any points where it reaches some maximum or minimum value?

1. $f(x) = x^2$

**Solution:** Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Interesting Features: The function reaches a minimum value when $x = 0$

2. $g(x) = x^3$

**Solution:** Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Interesting Features: The function looks like it's about to reach a maximum/minimum when $x = 0$, but then keeps going.

3. $h(x) = e^{-x^2}$

**Solution:** Domain: $(-\infty, \infty)$
Range: $(0, 1]$
Interesting Features: The function reaches a maximum value when $x = 0$

4. $l(x) = \sqrt{9 - x^2}$

**Solution:** Domain: $[-3, 3]$
Range: $[0, 3]$
Interesting Features: The function reaches a maximum value when $x = 0$. It's also a semicircle

# 3 Inequalities

Solve the following inequalities for $x$

1. $2x + 4 > 2$

**Solution:**

$$\begin{aligned} 2x + 4 &> 2 \\ 2x &> -2 \\ x &> -1 \end{aligned}$$

2. $x^2 < 9$

**Solution:**

$$
\begin{aligned}
x^2 &< 9 \\
x < \sqrt{9} \text{ and } &-x < \sqrt{9} \\
x < 3 \text{ and } &x > -3 \\
-3 < x < 3 \text{ or } &x \in (-3, 3)
\end{aligned}
$$

3. $\sqrt{x} > x^2$ (Hint: Use Julia to plot this)

**Solution:**

$$
\begin{aligned}
\sqrt{x} &> x^2 \\
x &> x^4 \\
0 &> x^4 - x \\
x^4 - x &< 0
\end{aligned}
$$

Plotting in Julia shows that $x^4 - x < 0$ while $x \in (0, 1)$