

Solution to Week 2 Exercises

The following are solutions to Exercises in Week 2 Notes.

Factorization

Exercise 2.1.1 Factorize each of the given polynomials using given method.

(Questions do not need software packages) Factorize each of the given polynomials using given method.

1. $x^2 - 18x + 80$

Solution. $x^2 - 18x + 80 = (x - 8)(x - 10)$

2. $x^3 + 8$

Solution. $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

3. $3x^4 - 3x^3 - 36x^2$

Solution.

$$\begin{aligned} 3x^4 - 3x^3 - 36x^2 &= 3x^2(x^2 - x - 12) \\ &= 3x^2(x + 3)(x - 4) \end{aligned}$$

4. $x^4 - 25$

Solution. $x^4 - 25 = (x^2)^2 - (5)^2 = (x^2 - 5)(x^2 + 5) = (x - \sqrt{5})(x + \sqrt{5})(x^2 + 5)$

5. $x^4 + x^2 - 20$

Solution. (think of x^2 as a variable y)

$$\begin{aligned} x^4 + x^2 - 20 &= (x^2 - 4)(x^2 + 5) \\ &= (x - 2)(x + 2)(x^2 + 5) \end{aligned}$$

Sequences and Series

Exercise 2.2.4

Find the sum of the first 30 terms of the sequence 2,4,6,...

Solution. This is a AP with $a = 2$ and $d = 2$. The sum of its first 30 terms is

$$\frac{30}{2}(2 \times 2 + (30 - 1) \times 2) = 15 \times 62 = 930$$

Exercises 2.2.6

1. Write down the first few terms of a GP which starts with 5 and has a common ratio of 2.

Solution. 5, 10, 20, 40, ...

2. Is the sequence $2, -6, 18, -54, \dots$ a GP and, if so, what is the common ratio?

Solution. yes, $r = -3$ as $-6/2 = -3$, $18/(-6) = -3$, $-54/18 = -3$, ...

3. A GP has first term 4 and common ratio 2. What is the 20th term?

Solution. $a + 20 = 4 \times 2^{19} = 2097152$

Exercises 2.3.1

1. How much will an investment of \$1000 at 5% annual interest rate pay out after 10 years?

Solution. Amount pays out after 10 years = $1000 \times (1 + 0.05)^{10} = \1628.89

2. A bank is offering an annual interest rate of 5% compounded monthly. What will it pay out for an investment of \$1000 after 10 years?

Solution. Monthly interest rate $0.05/12$. Amount it pays out after 10 years (or 120 months) = $1000 \times (1 + 0.05/12)^{120} = \1647.01

3. Ruby will receive a gift of \$1000 in five years time. What is the PV of the gift given that the current cost of living is increasing at 3% per annum?

Solution. $PV \times (1 + 0.03)^5 = 1000 \Rightarrow PV = \frac{1000}{(1.03)^5} = \862.61

4. Jim is saving up to buy a car. He puts \$300 per month into a savings account that pays 6% annual interest compounded monthly. How long will it take to reach his target of \$15000?

Solution. Monthly interest rate $0.06/12 = 0.005$. Suppose that it takes n months to reach \$15000. Then.

$$300(1 + 1.005 + 1.005^2 + \cdots + 1.005^n) = 15000$$

$$300 \times \frac{1 - 1.005^{n+1}}{1 - 1.005} = 15000$$

$$\frac{1 - 1.005^{n+1}}{1 - 1.005} = \frac{15000}{300} = 50$$

$$1.005^{n+1} - 1 = 50 * 0.005 = 0.25$$

$$1.005^{n+1} = 1.25$$

$$(n + 1) \log 1.005 = \log 1.25$$

$$n + 1 = \frac{\log 1.25}{\log 1.005}$$

$$n = \frac{\log 1.25}{\log 1.005} - 1$$

$$n = 43.72$$

It takes Jim 44 months for his account to reach \$15000.

(Note: 43 months is not enough, 44 months reach \$15097.25.)