Essential Mathematics

Tutorial 5

Semester 2 2018

1. Let

$$\mathbf{a} = (1, 3, -2)$$
$$\mathbf{b} = (4, 5, 1)$$
$$\mathbf{c} = (3, 3, 2)$$

Find:

(a) $\mathbf{a} + \mathbf{b}$

(b) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

(c) $\mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

(d) The angle θ between $\mathbf{a}$ and $\mathbf{c}$

Solution:

$$\mathbf{a} + \mathbf{b} = (5, 8, -1)$$
$$(\mathbf{a} \cdot \mathbf{b})\mathbf{c} = 17(3, 3, 2) = (51, 51, 34)$$
$$\mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} = 17 + 8 = 25$$
$$\theta = \cos^{-1}\left(\frac{\mathbf{a} \cdot \mathbf{c}}{|\mathbf{a}| \times |\mathbf{c}|}\right) \approx 1.0975 \text{ radians} \quad (62.8809^\circ)$$

2. Let

$$\mathbf{a} = \mathbf{i} + 3\mathbf{j}$$
$$\mathbf{b} = \mathbf{i} + x\mathbf{j}$$

Find x such that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to one another.

Solution:

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= 0 \\ 1 \times 1 + 3x &= 0 \\ 1 + 3x &= 0 \\ x &= -\frac{1}{3} \end{aligned}$$

1 Scalar and Vector Projections

3. Consider the following:

$$\begin{aligned} \mathbf{a} &= [2,\ -2,\ 4] \\ \mathbf{b} &= [-1,\ -1,\ 1] \end{aligned}$$

Find:

1. The scalar projection of **a** in **b**;
2. The vector projection of **a** in **b**.

Solution:

1. scalar projection $= \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|} = \dfrac{4}{\sqrt{3}}$;
2. vector projection $= \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2}\mathbf{b} = \dfrac{4}{3}\mathbf{b} = \dfrac{4}{3}[-1,\ -1,\ 1]$.

2 Linear Independence

4. Consider the following vectors:

$$\begin{aligned} \mathbf{a} &= \mathbf{i} + 2\mathbf{j} \\ \mathbf{b} &= 3\mathbf{i} + 4\mathbf{j} \\ \mathbf{c} &= 5\mathbf{i} + 6\mathbf{j} \end{aligned}$$

Express **c** as a linear combination of **a** and **b** (HINT: solve $x\mathbf{a} + y\mathbf{b} = \mathbf{c}$ for x and y)

5. Consider the following system of linear equations:

$$\begin{array}{rrrr} x & +y & +z & = 10 \\ 2x & -3y & 4z & = 1 \\ 4x & -y & +6z & = 21 \end{array}$$

(a) When represented in matrix form ($A\mathbf{x} = \mathbf{b}$), what is the rank of A?

(b) What is the rank of the augmented matrix?

(c) Can you solve for x, y and z? If so, find x, y and z

6. Consider the following system of linear equations:

$$\begin{array}{rrrr} x & +y & +z & = 10 \\ 2x & -3y & +4z & = 1 \\ 6x & +y & +8z & = 25 \end{array}$$

(a) When represented in matrix form ($A\mathbf{x} = \mathbf{b}$), what is the rank of A?

(b) What is the rank of the augmented matrix?

(c) Can you solve for x, y and z? If so, find x, y and z