

Attempt ALL questions. Total mark 100.

Q1. $[10 \times 3 = 30 \text{ marks}]$

Complete ANY TEN of the following questions. If more than ten questions are answered, only the first ten that appear in your answer book will be marked. (No working is required.)

- (1) List all elements of the set $\{x \mid |x - 2|^3 < 8, x \text{ is integer}\}$.
- (2) List all elements in $\{x \mid |x - 1| \leq 1\} \cap \{x \mid |x + 1| \geq 2\}$.
- (3) Solve the inequality $7x - 5 < 3x + 7$.
- (4) Solve to the inequality $\frac{x-7}{x+4} < 0$.
- (5) Find the sum of the first 20 terms in the arithmetic progression 5, 7, 9, \dots .
- (6) Find the sum of the first 13 terms in the geometric progression 3, 6, 12, \dots .
- (7) Find the inverse of $\begin{bmatrix} 0 & -2 \\ 5 & 0 \end{bmatrix}$.
- (8) Is $\begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$ the inverse of $\begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$?
- (9) Let $f(x)$ and $g(x)$ have domains $[0, 3]$ and $[2, 5]$, respectively. If $g(x) \neq 0$ for all $x \in [2, 5]$, what is the domain of $f(x)/g(x)$.
- (10) Find the domain of $f(x) = \frac{x^6 - 1}{x^2 - 7}$.
- (11) Find the domain of $f(x, y) = \frac{x-3}{y-3}$.
- (12) Find the range of $f(x) = \frac{x^2 - 6x + 9}{|x - 3|}$.
- (13) Find the roots of $x^2 + 7x + 12$.
- (14) For a 3×3 matrix, is the operation “replacing Row 3 by Row 2” an elementary row operation?
- (15) For a 3×3 matrix, is the operation “multiplying Row 3 by 0” an elementary row operation?
- (16) Is matrix $\begin{bmatrix} 2 & -2 \\ 0 & 1 \end{bmatrix}$ in reduced row echelon form?
- (17) Which of the following definite integrals represents the area (in the 1st quadrant) under the curve $y = \log(x + 1)$, $0 \leq x \leq 1$?
- (18) What are the solutions to the nonlinear system $x - y^3 = 0$, $x - 9y = 0$?
- (19) Find $\frac{\partial f}{\partial x}$ for $f(x, y) = \sqrt{1 + x^2 + \sin(y)}$.
- (20) Find $\frac{\partial f}{\partial y}$ for $f(x, y) = \sqrt{1 + x^2 + \sin(y)}$.
- (21) The unit vector in the direction of $\mathbf{u} = (-2, 5, -4)$.
- (22) The angle between $\mathbf{u} = (-2, 5, -4)$ and $\mathbf{v} = (3, -2, 6)$.

For the remaining questions (Q2 ~ Q7), full marks may not be awarded unless sufficient working is shown.

Q2. $[6 + (2 + 7 + 3) = 18 \text{ marks}]$ Solve the following questions.

- (a) Solve the nonlinear system $x - y^3 = 0$, $x - 9y = 0$.
- (b) Consider the following linear system:

$$\begin{array}{rrrrrr} x & - & 3y & - & z & = & 2 \\ x & + & 2y & + & z & = & -5 \\ 3x & - & y & + & z & = & 6 \end{array}$$

- i. Write down the augmented matrix of the linear system.
- ii. Reduce the augmented matrix to echelon form.
- iii. Solve the system by ‘back substitution’.

Q3. $[4 \times 3 = 12 \text{ marks}]$

Find each of the following limits.

$$\begin{array}{ll} \text{(a)} \lim_{t \rightarrow \infty} \frac{t^2 + 4t - 2}{t^3 - 4t^2} & \text{(c)} \lim_{x \rightarrow 1} \frac{x^5 + 5x^4 + 3x}{2x^4 - 3x^3} \\ \text{(b)} \lim_{x \rightarrow 2} \frac{x^3 - 4x^2 + 4x}{x - 2} & \text{(d)} \lim_{t \rightarrow \infty} \frac{t^7 + 3t^5 - 8}{t + 100t^6} \end{array}$$

Q4. $[2 + 2 + 3 + 3 = 10 \text{ marks}]$

Find the indicated derivative for each of the following functions.

- (a) For $f(x) = \sin(2x^2 + 1) - 6x^{3/2} - 3x$, find $f'(x)$.
- (b) For $f(x) = e^x \cos(x + 1) - 1$, find $f'(x)$.
- (c) Find $\frac{\partial f}{\partial x}$ for $f(x, y) = \sqrt{1 + x^2 + \sin(y)}$.
- (d) Find $\frac{\partial f}{\partial y}$ for $f(x, y) = \sqrt{1 + x^2 + \sin(y)}$.

Q5. $[4 + 4 = 8 \text{ marks}]$ Let $f(x) = 3x^4 - 24x^2 - 5$.

- (i) Find the critical points of $f(x)$;
- (ii) Classify each critical point found in (i).

Q6. $[4 + 4 + 6 = 14 \text{ marks}]$

Consider

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & -2 & 0 \\ 1 & -1 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$$

- (a) Find $\det(A)$.
- (b) Is $\begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}$ an eigenvector of A ? If yes, what is the corresponding eigenvalue?
- (c) Find the eigenvalues of B .

Q7. $[8 \text{ marks}]$ Perform 3 iterations of Newton's method to find an approximate solution to the equation $x^2 - \log(x) - 15 = 0$, starting from $x_0 = 5$.