Lab Exam Solution (2018 S2)

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Q1

Q1 (a) ¶

In [39]:
```
import QuadGK.quadgk
```

In [40]:
```
# Area under the curve f(x)=sin^2(x) over [0 pi] is the integral of f(x)
 from 0 to pi
function f(x)
    sin(x).^2
end
quadgk(f,0,pi)
```

Out[40]:
```
(1.5707963267948966, 1.3063771620025477e-8)
```

Thus, the area is 1.5707963267948966

Q1 (b)

In [6]:
```
iin=0
N=10000000
for i in 1:N
    x=rand(1,2).*[pi 1]
    if x[2]<=sin(x[1])^2
        iin=iin+1
    end
end
Area=(pi)*iin/N
```

Out[6]:
```
1.5708830347521356
```

So, the estimated area is 1.5708830347521356

Q2

Q2 (a)

In [7]:
```
# Input the Leslie matrix here
A=zeros(7,7)
A[1,:]=[0 1 2 2 2 1 0]
A[2,1]= 0.52
A[3,2]= 0.66
A[4,3]= 0.75
A[5,4]= 0.79
A[6,5]= 0.6
A[7,6]= 0.4
A
```

Out[7]:
```
7×7 Array{Float64,2}:
 0.0   1.0   2.0   2.0   2.0  1.0  0.0
 0.52  0.0   0.0   0.0   0.0  0.0  0.0
 0.0   0.66  0.0   0.0   0.0  0.0  0.0
 0.0   0.0   0.75  0.0   0.0  0.0  0.0
 0.0   0.0   0.0   0.79  0.0  0.0  0.0
 0.0   0.0   0.0   0.0   0.6  0.0  0.0
 0.0   0.0   0.0   0.0   0.0  0.4  0.0
```

Q2 (b)

In [8]:
```
eigvals(A)
```

Out[8]:
```
7-element Array{Complex{Float64},1}:
       0.0+0.0im
   1.27299+0.0im
 0.0996348+0.679909im
 0.0996348-0.679909im
  -0.52263+0.44967im
  -0.52263-0.44967im
 -0.426997+0.0im
```

Largest real eigenvalue is 1.27299>1, the population will survive.

Q2 (c)

In [9]:
```
# Input the Leslie matrix A1 after harvesting
A1=A
A1[2,1]= 0.52
A1[3,2]= 0.46
A1[4,3]= 0.55
A1[5,4]= 0.59
A1[6,5]= 0.4
A1[7,6]= 0.4
A1
```

Out[9]:
```
7×7 Array{Float64,2}:
 0.0   1.0   2.0   2.0   2.0  1.0  0.0
 0.52  0.0   0.0   0.0   0.0  0.0  0.0
 0.0   0.46  0.0   0.0   0.0  0.0  0.0
 0.0   0.0   0.55  0.0   0.0  0.0  0.0
 0.0   0.0   0.0   0.59  0.0  0.0  0.0
 0.0   0.0   0.0   0.0   0.4  0.0  0.0
 0.0   0.0   0.0   0.0   0.0  0.4  0.0
```

In [10]:
```
eigvals(A1)
```

Out[10]:
```
7-element Array{Complex{Float64},1}:
       0.0+0.0im
   1.12975+0.0im
 0.0567917+0.526022im
 0.0567917-0.526022im
 -0.479185+0.338987im
 -0.479185-0.338987im
 -0.284962+0.0im
```

Largest real eigenvalue is 1.12975 > 1, the population is sustanable.

Q3

Q3 (a)

In [15]:
```
# Define functions f(x) and f'(x):
 function ff(x)
    x.^2-2*cos(x).^2+8*x-6
 end
 function dffdx(x)
    2*x+4sin(x)cos(x)+8
 end
```

Out[15]:
```
dffdx (generic function with 1 method)
```

In [16]:
```
x0=3.0
x=x0
for i in 1:10
    x=x-ff(x)/dffdx(x)
end
x
```

Out[16]: 0.7940311687683047

In [17]:
```
ff(x)
```

Out[17]: 0.0

After 10 iterations, we obtain x10=0.7940311687683047

Q3 (b)

Q3b(i)

In [23]:
```
# Critical points of f(x) are roots of f'(x)=0, so we f'(x)=0.
# To use Newton's method, we need functions f'(x) and f''(x).
# Define functions f'(x) and f''(x).
function df(x)
    3*x.^2-0.21*exp(x)+0.5*cos(x)-12.4;
end
function ddf(x)
    6*x-0.21*exp(x)-0.5*sin(x);
end
```

Out[23]: ddf (generic function with 1 method)

In [24]:
```
# solve f'(x)=0 starting from x0=1.0
x0=3
x=x0
for i in 1:10
    x=x-df(x)/ddf(x)
end
x
```

Out[24]: 2.2063679897168815

In [25]:
```
df(x) # checking if x is realy a root for f'(x)=0
```

Out[25]: 3.552713678800501e-15

In [26]:
```
x10=x; # x10=2.2063679897168815 is an approximate critical point.
```

Q3b(ii)

In [28]:
```
# Evaluate f''(x10), then determine if x10 is max or min solution
# or if the 2nd derivative test fails.
ddf(x10)
```

Out[28]: 10.9284814100052

So, f''(x10)>0 and x10=2.2063679897168815 is a (local) minimum point.

Q4

In [32]:
```
using DifferentialEquations
```

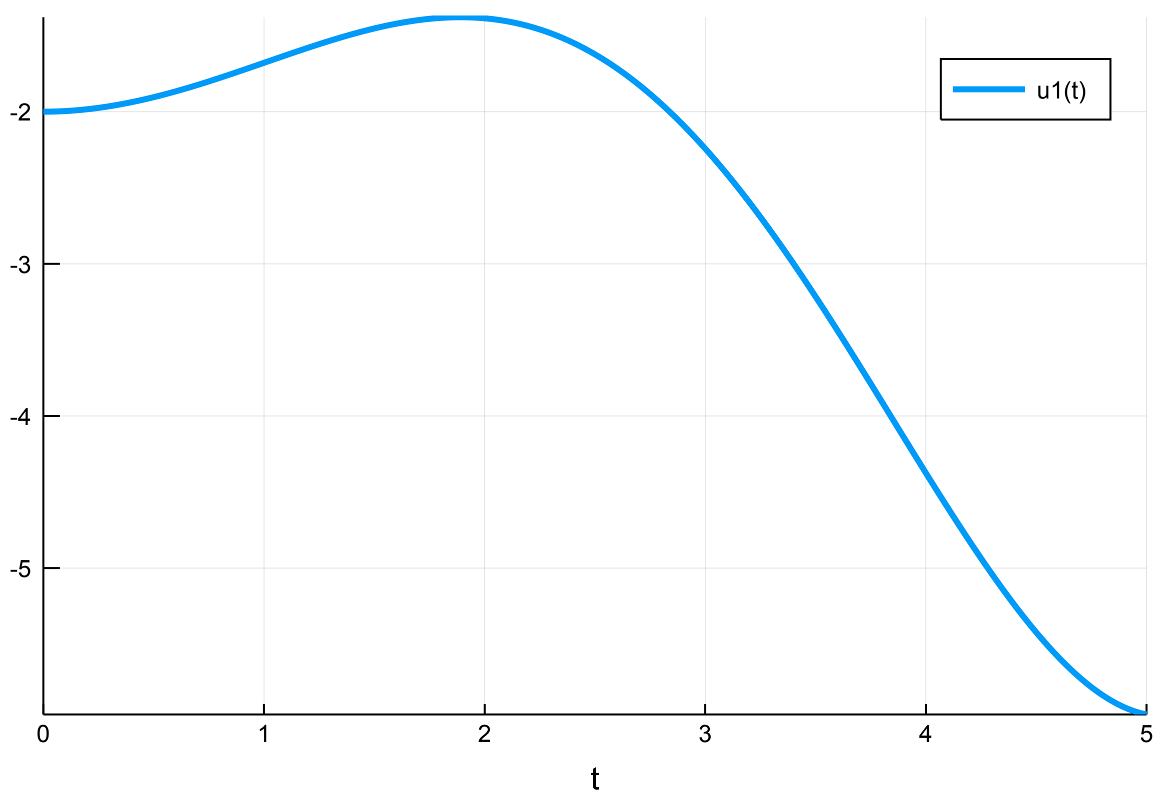
In [33]:
```
using Plots
```

In [34]:
```
function f(x,p,t)
    0.1*t.^2-t.*sin(t.-sqrt(t+1))  #input function expression in t
end
tspan=(0.0,5.0)          #timespan of the solution (input floating number
s for the time span such as (0.0, 2.0))
x0=    -2.0              #Initial values of the states
prob=ODEProblem(f,x0,tspan);
```

In [35]:
```
soln_x=solve(prob,Tsit5(),reltol=1e-8,abstol=1e-8);
```

In [36]:
```
plot(soln_x)
```

Out[36]:



In [37]:
```
soln_x(5)
```

Out[37]:
```
-5.962106741718836
```