CS663 Assignment 1 Question 3

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This document describes the findings regarding question 3 of assignment 1. The mathematical derivations for Bi-histogram equalization is shown and examples on its application is also shown.

1 Question 3a

Split histogram $h(I)$ into two histograms $h_1(I)$ and $h_2(I)$ over the domain [0,a] and [a,1] respectively for some arbitrary aϵ(0,1)

Assume mass of histogram $h_1(I)$ in [0,a] is $\alpha \Rightarrow$ mass of histogram $h_2(I)$ in [a,1] is (1-α)

Given that masses of both $h_1(I)$ and $h_2(I)$ are preserved after performing equalization

In the domain [0,a], after equalization of $h_1(I)$, the resulting histogram will be of uniform distribution in the region [0,a], i.e. $h_{1E}(I) = \frac{\alpha}{a}$.

In the domain (a,1], after equalization of $h_2(I)$, the resulting histogram will be of uniform distribution in the region [a,1], i.e $h_{2E}(I)=\frac{1-\alpha}{1-a}$.

In both the region, it is ensured that the mass is conserved. Hence the mass α is conserved in [0,a], and mass (1-α) is conserved in [a,1].

Now, we will calculate the mean of the final distribution that we have achieved:

$$E[I] = \int_0^1 If(I)dI = \frac{\alpha}{a}\frac{I^2}{2}\Big|_0^a + \frac{1-\alpha}{1-a}\frac{I^2}{2}\Big|_a^1$$

$$\text{E[I]} = \frac{\alpha a}{2} + \frac{(1-a)(1-\alpha)}{2}$$

$$\text{E[I]}=\frac{1-\alpha+a}{2}$$

2 Question 3b

It is given a is the median intensity, this means α=0.5

From above expression obtained in a it can be found that

$$E[I] = \frac{0.5 + a}{2}$$

3 Question 3c

Scenario:
Bi-histogram equalization method can be used in the conditions where we want to conserve our image brightness to some extent while bringing out details and using full range of intensities.

Reasons:
In simple histogram equalization the final mean become close to 0.5, which distorts original brightness.In bi-histogram equalization final mean is close to intial mean a.This preserves the overall brightness at some extent

4 Question 3d

Figure 1: Histogram and bi-histogram equalized image for ans3d.jpg

The original image has a mean value of 0.2597
The histogram equalized image has a mean value of 0.5050 , and bi-histogram equalized image has a mean value of 0.4745(more towards the mean of original image)

Thus, in the particular example shown above as the original brightness is preserved more in the bi-histogram equalization as compared to the global histogram equalization.