

# In house high Q Electro-Optic Modulator for Molecular Imaging

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## Abstract

## 1 Introduction

This paper outlines the theory and the method to construct a resonant electro-optic modular (EOM) for molecular imaging.

## 2 Theory

An electro-optic phase modulator is a device which converts an time-varying electric field (typically in the RF frequency) into a modulate of the phase of an optical frequency. This is achieved by sending linearly polarized light along the an axis of a nonlinear crystal while modulating the crystal's index of refraction  $n_e$ .

Applying a voltage  $V$  across the crystal will induce a phase shift,  $\Delta\phi$ , of [1]

$$\Delta\phi = \frac{\pi n_e^3 r_{33}}{\lambda} \frac{Vl}{d} \quad (1)$$

$r_{33}$  is the electro-optic coefficient which is defined by the choice of crystal,  $\lambda$  is the wavelength of the light,  $l$  is the length of the crystal, and  $d$  is the distance between the electrodes.

EOMs are characterised by the half wave voltage which results in a phase shift of  $\Delta\phi = \pi$ . This is given by

$$V_\pi = \frac{\lambda d}{n_e^3 r_{33} l}. \quad (2)$$

Treating the crystal as a distributed capacitor we can design a LCR tank circuit to resonate at the desired frequency. The resonant frequency is given by,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}, \quad (3)$$

where  $C$  is the capacitance of the crystal and  $L$  is the inductance of the tank circuit which we select to define the operational frequency of the EOM. Driving the EOM at the resonant frequency from an RF source will result in the maximum power transfer into the crystal and hence the maximum phase modulation.

Applying a time varying voltage across the crystal induces a time dependence onto the refractive index of the crystal. This results in a phase modulation which produces sidebands around the carrier frequency  $\Omega$  which the separation dictated the RF driving frequency,  $\omega$ . The amplitude of the sidebands are given by,

$$\mathbf{E}(t) = \mathbf{E}_0 \sum_{n=-\infty}^{\infty} J_n(M) e^{it(\Omega+n\omega)}, \quad (4)$$

where  $\mathbf{E}_0$  is the amplitude of the carrier,  $J_n$  is the Bessel function of the  $n$  kind, and  $M$  is the modulation index given by,

$$M = \frac{\pi V}{V_\pi}, \quad (5)$$

where  $V$  is the amplitude of the driving voltage. For typical crystal specifications it is easy to produce the first order sidebands, even with a modulation index of  $M \ll 1$ . To produce higher order sidebands it is evident from equation 5 that a higher driving voltage is required.

It is typical to use a RF amplifier to increase the driving voltage to get the desired modulation index. An important note is in a resonant LCR tank circuit the Q-factor increases the voltage across the crystal by a factor of Q, so it is imperative to design a EOM with a high Quality factor.

In order to minimise the power reflection into the EOM crystal it is important to match the impedance of the driving source, typically  $50 \Omega$ , to the characteristic impedance of the LCR circuit. This duty can be preformed by a transformer whose turns ratio is given by,

$$\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}, \quad (6)$$

where  $N_1$  and  $N_2$  are the number of turns in the primary and secondary and  $Z_1$  and  $Z_2$  are the characteristic impedances of the driving source and the LCR circuit respectively.

### 3 Construction

Our EOM uses a 5% MgO doped LiNbO<sub>3</sub> crystal with a length of 40mm and a width and height of 3mm. The crystal has a refractive index of 2.2 and a electro-optic coefficient of  $r_{33} = 30.9 \text{ pm/V}$ . This was mounted on a electrically isolated acrylic block, and secured with two copper electrodes. The electrodes were sufficiently large enough to act as a heat sink for the crystal in order to minimise thermal induced lensing effects.

## References

- [1] C Mok, M Weel, E Rotberg, and A Kumarakrishnan. Design and construction of an efficient electro-optic modulator for laser spectroscopy (2016). Canadian Journal of Physics. 84(9): 775-786.