GOVERNMENT COLLEGE OF ENGINEERING & TEXTILE TECHNOLOGY,SERAMPORE

Topic: Improper integral, Types of improper integral and illustration.

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INTRODUCTION TO IMPROPER INTEGRALS

Improper integrals are a type of integral where something happens that makes the regular rules of integration not work. This can happen in two main cases: 1) When the limits tend to infinity 2) When the function becomes discontinuous after putting the value of limits.

The rules of improper integrals act as a saviour where the rules of basic integration fail.

Regular integration fails in improper integrals because the standard rules assume:

1.Finite Limits: In regular integration, the interval of integration must be finite, like $\int_5^{10} x$. If the limits are infinite, the process doesn't work directly because infinity isn't a number we can use in calculations.

2.Well-Defined Functions: The function being integrated must be continuous throughout the interval. If the function has a discontinuity eg :1/X at X=0 the integral doesn't follow the usual rules because the function " becomes undefined.

<u>FOUNDATION ON IMPROPER INTEGRALS</u>

The definite integral $\int_a^b f(x)$ is called improper integral, if either range of integration is infinite or integrand f(x) is unbounded on [a,b] or both . Some of the Examples are:

$$1.\ \int_0^1 \frac{dx}{x} \quad 2.\ \int_0^3 \frac{dx}{\sqrt{3-x}} \quad 3.\ \int_0^1 \frac{1}{1-x} \quad 4.\ \int_0^{\frac{\pi}{2}} \sec x$$

Here, in the first sum on putting

1. Here in the first integral , placing 0 in place of x in the denominator would give infinite as an output of the integral. Even while solving the integral we might get as follows:

$$\int_0^1 \frac{dx}{x} = \lim_{\alpha\to 0+} \int_\alpha^1 \frac{dx}{x}$$
$$= \lim_{\alpha\to 0+} \left[\ln|x|\right]_\alpha^1$$
$$= \lim_{\alpha\to 0+} (\ln 1 - \ln\alpha) = \infty.$$

2. Here in the integral , placing 3 in place of x in the $\sqrt{3-x}$ would give infinite as an output of the integral. Even while solving the integral we might get as follows:

$$\int_0^3 \frac{1}{\sqrt{3-x}}dx = \lim_{c\to 3} \int_0^c \frac{1}{\sqrt{3-x}}dx$$
$$= \lim_{c\to 3} | -2\sqrt{3-x}|$$
$$= \lim_{c\to 3}(-2\sqrt{3-c}+2\sqrt{3})$$
$$= 2\sqrt{3}$$

3. Here in the integral , placing 1 in place of x in the $1-x$ would give infinite as an output of the integral. Even while solving the integral we might get as follows:

$$\int_0^1 \frac{1}{1-x}dx.$$

$$\lim_{b\to 1^-} \int_0^b \frac{1}{1-x}dx = \lim_{b\to 1^-} \left[-\ln|1-x|\right]_0^b$$
$$= \lim_{b\to 1^-} [-\ln(1-b)+0] = \infty.$$

4. Here in the integral , placing pi/2 in place of in the x in $secx$ would give infinite as an output of the integral. Even while solving the integral we might get as follows

$$\int_0^{\frac{\pi}{2}} \sec(x)\,dx = \int_0^{\frac{\pi}{2}} \frac{1}{\cos(x)}\,dx \qquad \overline{\cos\tfrac{\pi}{2}} = 0$$
$$= \lim_{t\to\frac{\pi}{2}} \int_0^t \sec(x)dx = \lim_{t\to\frac{\pi}{2}} \ln|\sec x+\tan x|\Big|_0^t$$
$$= \lim_{t\to\frac{\pi}{2}} \ln|\sec t+\tan t| - 0 = \cdots = \infty$$

CONCEPT OF CONVERGENCE AND DIVERGENCE

An Integral is said to be converging if the integral exists and gives a finite number and divergent if it becomes discontinuous at some points within the given bounds or the integral tends to infinity.

Types of Convergence:

1. Integral $\int_{a}^{\infty} f(x)dx$

The improper integral is convergent if value of Integral $\int_{a}^{\infty} f(x) = \lim x -> \infty \int_{a}^{x} f(X)$ is finite and if the value tends to - ∞ or ∞ , it is said to be divergent.

2. Integral: $\int_{-\infty}^{b} f(x)dx$

The improper integral is convergent if value of Integral $\int_{-\infty}^{b} f(x) = \lim x -> -\infty \int_{x}^{b} f(X)$ is finite and if the value tends to - ∞ or ∞ , it is said to be divergent.

3. Integral: $\int_{-\infty}^{\infty} f(x)dx$

The improper integral is convergent if value of Integral $\int_{-\infty}^{\infty} f(x) = \lim x -> -\infty \int_{x}^{c} f(X) + \lim x -> \infty \int_{c}^{x} f(X)$ is finite and if the value tends to - ∞ or ∞ , it is said to be divergent.

STEPS FOR SOLVING IMPROPER INTEGRALS

1. Identify the Type of Improper Integrals

   . Infinite Limits: If the integral's limits are infinite

   . Discontinuous Integrand: If the function is undefined at some point in the integration range

2. Rewriting the Integral with Limits

   . For infinite limit replace the infinite point with a variable

   . For an undefined point break the integration limit (if required) then change the point of discontinuity by adding or subtracting with a number very close to zero.

3. Evaluating the given integral
4. Putting the values of limits in their place and checking whether the integral is convergent or not
5. If the integral is convergent, we are going to get a finite answer. If not then the integral is divergent and we will be getting an infinite answer.

Some solved examples on improper integrals

1.
$$\int_0^1 \frac{dx}{x} = \lim_{\alpha\to 0+} \int_\alpha^1 \frac{dx}{x} = \lim_{\alpha\to 0+} \Big[\ln|x|\Big]_\alpha^1 = \lim_{\alpha\to 0+} \left(\ln 1 - \ln\alpha\right) = \infty.$$

2.
$$\begin{aligned}\int_0^\infty xe^{-2x^2}\,dx &= \lim_{N\to\infty}\int_0^N xe^{-2x^2}\,dx \\ &= \lim_{N\to\infty} -\frac{1}{4}e^{-2x^2}\Big|_{x=0}^{x=N} \\ &= \lim_{N\to\infty} -\frac{1}{4}\left(e^{-2N^2} - e^0\right) = \frac{1}{4}\end{aligned}$$

3.
$$\int_0^1 \frac{1}{1-x}\,dx.$$

$$\begin{aligned}\lim_{b\to 1^-}\int_0^b \frac{1}{1-x}\,dx &= \lim_{b\to 1^-}\Big[-\ln|1-x|\Big]_0^b \\ &= \lim_{b\to 1^-}\left[-\ln(1-b)+0\right] = \infty.\end{aligned}$$

APPLICATIONS OF IMPROPER INTEGRALS

Improper Integrals have a lot of applications such as:

1. PHYSICS:

   . For calculating gravitational and electric field intensities over infinite regions

   . Modeling wave function in quantum mechanics

2. PROBABILITY

   . For defining probability distributions where integrals extend over infinite ranges

3. MATHEMATICS

   . For solving those integrals where the bound of the integrals ranges to infinite or the integral becomes discontinuous at some points within the given bounds, thus, helping in place where rules of normal integrals fail.

4. ENGINEERING

   . Helps in analyzing signal processing problems, such as Fourier transforms

CONCLUSION

In conclusion, this report shows improper integrals, focusing on how to handle cases with infinite limits or discontinuities. Several methods are shown for determining whether these integrals converge and how to calculate their values when they do. The report show the importance of these integrals in real-world applications, such as physics and engineering, where they help shape infinite or discontinuous behaviors.

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