Linearized Model for Active and Reactive LMP Considering Bus Voltage Constraints

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***Abstract*—with the increasing integration of large scale renewable energy into power system and the variation of bus voltage, the pricing of reactive power is more necessary than ever before. This paper proposes a novel model to calculate active and reactive Locational Marginal Price (LMP) simultaneously based on a linear optimal power flow (LOPF). First, an augmented Generation Shift Distribution Factors (GSDF) is introduced based on the linear power flow model with considerations of bus voltage and reactive power. Second, the linear optimal power flow model is constructed using GSDF to jointly calculate active LMP and reactive LMP. Third, the components of LMP such as the energy component, line congestion component, and voltage constraint component are derived from LOPF model. Finally, the case study on IEEE 30-bus case illustrates that the model outperforms the DCOPF on the active LMP, and presents acceptable results on reactive LMP compared with ACOPF.**

***Index Terms*—LMP, LMP decomposition, reactive power pricing, Linear Power Flow, Linear Optimal Power Flow.**

Notation

$\mathbf{Y}=\mathbf{G}+j\mathbf{B}$	Admittance matrix of the power system
$\mathbf{Y}'=\mathbf{G}'+j\mathbf{B}'$	Admittance matrix without shunt elements
$Y_{ij}=G_{ij}+jB_{ij}$	Element in the i th row and j th column of $\mathbf{Y}$
$Y'=G'_{ij}+jB'_{ij}$	Element in the i th row and j th column of $\mathbf{Y}'$
$\mathbf{V},\boldsymbol{\theta}$	Vectors of Magnitudes and phase angles of bus voltages
V_i,θ_i	Voltage magnitude and phase angle at bus i
$\mathbf{P},\mathbf{Q}$	Vectors of bus injected active and reactive power
P_i,Q_i	Injected active and reactive power at bus i
P_{ij},Q_{ij}	Power flow from bus i to bus j
$y_{ij}=g_{ij}+jb_{ij}$	Admittance of line (i,j)
$y_{ii}=g_{ii}+jb_{ii}$	Shunt admittance at bus i
M,N	The number of lines and buses
P_i^G,P_i^D	The active power generation and active load at bus i
Q_i^G,Q_i^D	The reactive power generation and reactive load at bus i

I. Introduction

With the integration of large scale renewable energy like solar energy and wind energy to the grid, the variation range of bus voltage increases, which makes the pricing of reactive power more necessary than ever before. The Locational Marginal Price (LMP) is important concept of determining the price of electricity in the electricity market. It has been widely used by ISO (independent system system), such as PJM, New York ISO, ISO-New England, California ISO and Midwest ISO [1]. In practical power system, the LMP is usually calculated by the DC optimal power flow (DCOPF) model because of its high efficiency and robustness. However, the DCOPF approach has the following shortcomings: (1) It introduces errors in the LMP because it ignores the power loss and the variation of bus voltage. (2) It cannot calculate reactive LMP due to the ignorance of the reactive power flow and the reactive power balance. (3) It is not applicable to distribution network because of the neglecting of the line resistance. Comparatively, ACOPF can simultaneously calculate the active LMP (ALMP) and reactive LMP (RLMP) accurately, and is suitable for both transmission grid and distribution network as well [2]. However, it is hard to be applied in the practical market due to its heavy computational burden and its lack of calculation transparency [3]. Thus, it is necessary to develop a new model with both high calculation accuracy and efficiency.

Previous work has been conducted on improving the accuracy of LMP. They mainly focus on the revising the DCOPF model and finding new loss allocation method. Reference [1] improves the accuracy of LMP at different load levels by iterating DCOPF with network loss and distributing the network loss to the buses at both ends of the line. Reference [4] pre-solves the AC optimal power flow (ACOPF) to calculate the loss factor and then use the loss factor for the iteration of DCOPF. Such method is able to consider the impact of loss on LMP while maintaining the independence of choice of the reference bus. Recent work has studied the DCOPF considering bus voltage and reactive power. Reference [5] proposes a linearized OPF model for distribution network considering voltage constraints to calculate both ALMP and RLMP. Reference [6] utilizes Karush–Kuhn–Tucker (KKT) conditions to calculate the ALMP with voltage consideration. Reference [7] presents a linearized and convergence- guaranteed optimal power flow model considering reactive power and voltage magnitude to derive the expression of ALMP and RLMP.

LMP can be decomposed into different components [14, 15]. Reference [1] proposes an iterative DCOPF model to decompose the LMP into marginal energy price, marginal congestion price, and marginal loss price. Reference [8] similarly decomposes the LMP into reference price, loss components, and congestion components for LMP evaluation. Reference [9] proposes a novel risk component for LMP to reflect the system's overall security level. The decomposition of LMP helps the understanding of the power system state, such as the congestion price for congestion management.

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In this paper, we propose a novel model to jointly calculate ALMP and RLMP. An augmented Generation Shift Distribution Factors (GSDF) is first derived using the linearized power flow model considering bus voltage and reactive power. The linear optimal power flow (LOPF) model is then constructed using GSDF to jointly calculate ALMP and RLMP. Various components of ALMP and RLMP are derived from the proposed LOPF model. The accuracy and effectiveness of the model are validated on IEEE 30-bus case.

The contributions of this paper are twofold: 1) a novel model that can jointly calculate ALMP and RLMP considering bus voltage constraint and reactive power balance; 2) the energy component, congestion component, and voltage component of ALMP and RLMP are derived from the model to further facilitate congestion management and reactive power dispatch.

The rest of this paper is organized as follows: Section II introduces the linear power flow (LPF) model, derives the expressions of GSDF from LPF, proposes the linear LMP model, and further derives the components of ALMP and of RLMP. Section III validates the effectiveness of the linear joint model through the IEEE 30-bus case on MATPOWER. Finally, the conclusions are drawn in Section IV.

II. LMP Model and LMP Decomposition

A. LPF Model for LMP Calculation

This paper derives the LOPF model based on the LPF proposed by reference [10]. According to reference [10], the AC power flow equations can be linearized as:

$$\begin{aligned} P_i &= \sum_{j=1}^{N} G_{ij} V_j - \sum_{j=1}^{N} B'_{ij}\theta_j \qquad i = 1,2,\cdots,N \\ Q_i &= -\sum_{j=1}^{N} B_{ij} V_j - \sum_{j=1}^{N} G'_{ij}\theta_j \quad i = 1,2,\cdots,N \end{aligned} \tag{1}$$

The matrix form of linear power flow equation is:

$$\begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} = -\begin{bmatrix} \mathbf{B}' & \mathbf{-G} \\ \mathbf{G}' & \mathbf{B} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{V} \end{bmatrix} \tag{2}$$

The line flow equations are linearized as well:

$$\begin{aligned} P_{ij} &\approx \left(V_i - V_j\right) g_{ij} - b_{ij}\left(\theta_i - \theta_j\right) \\ Q_{ij} &\approx -b_{ij}\left(V_i - V_j\right) - g_{ij}\left(\theta_i - \theta_j\right) \end{aligned} \tag{3}$$

The power flow model above is linear with respect to voltage magnitude and phase angle, and it is distinguished by its high accuracy in voltage magnitude and reactive power flow[10].

B. Active Power and Reactive Power Balance Equations

The purpose of this section is to derive the active power and reactive power balance equations for optimal power flow. According to the Eq. (1), the sum of the injected active power is derived:

$$\begin{aligned} \sum_{i=1}^{N} P_i &= \sum_{i=1}^{N}\sum_{j=1}^{N} G_{ij} V_j - \sum_{i=1}^{N}\sum_{j=1}^{N} B'_{ij}\theta_j = \sum_{j=1}^{N}\left(\sum_{i=1}^{N} G_{ij}\right) V_j - \sum_{j=1}^{N}\left(\sum_{i=1}^{N} B'_{ij}\right)\theta_j \\ &= \sum_{j=1}^{N}\left(\sum_{i=1}^{N} G_{ij}\right) V_j = \sum_{j=1}^{N} g_{jj} V_j \end{aligned} \tag{4}$$

Because $V_j \approx 1$, then:

$$\sum_{i=1}^{N} P_i = \sum_{j=1}^{N} g_{jj} V_j \approx \sum_{j=1}^{N} g_{jj} V_j^2 \approx \sum_{j=1}^{N} g_{jj} \tag{5}$$

Due to the conductance of the grounded branches in real power system is extremely small, the sum of the injected active power is close to zeros. Thus, the active power equation is:

$$\sum_{i=1}^{N} P_i \approx 0 \tag{6}$$

Similarly, the sum of the injected reactive power and the reactive power balance equation is:

$$\sum_{i=1}^{N} Q_i = -\sum_{j=1}^{N} b_{jj} V_j \approx -\sum_{j=1}^{N} b_{jj} V_j^2 \approx -\sum_{j=1}^{N} b_{jj} \tag{7}$$

Actually, the $g_{jj} V_j^2$ and $-\sum_{j=1}^{N} b_{jj} V_j^2$ are the active power flow and reactive power flow of grounded branch at bus j respectively. This means the linear optimal power flow equations consider the influence of grounded branch as well, besides considering the bus voltage and reactive power flow.

C. Augmented GSDF

To obtain LMP model, we next derive the GSDF from the LPF.

Let:

$$\mathbf{C} = \begin{bmatrix} \mathbf{B}' & \mathbf{-G} \\ \mathbf{G}' & \mathbf{B} \end{bmatrix} \tag{8}$$

According to the Eq. (8), the matrix $\mathbf{C}$ is singular. Therefore, the row and column of the reference bus are removed from the matrix $\mathbf{C}$ to obtain the invertible matrix $\mathbf{C}'$:

$$\begin{bmatrix} \boldsymbol{\theta}' \\ \mathbf{V} \end{bmatrix} = -\mathbf{C}'^{-1} \begin{bmatrix} \mathbf{P}' \\ \mathbf{Q} \end{bmatrix} \tag{9}$$

where the $\boldsymbol{\theta}'$ and $\mathbf{P}'$ denote $\boldsymbol{\theta}$ and $\mathbf{P}$ with reference bus removed respectively; $\mathbf{C}'^{-1}$ represents the inverse matrix of $\mathbf{C}'$.

For convenience, the row and the column of the reference bus of the matrix $-\mathbf{C}'^{-1}$ are filled with zeros to obtain a new matrix $\mathbf{X}$, then:

$$\begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{V} \end{bmatrix} = \mathbf{X} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \tag{10}$$

where the size of $\mathbf{X}$ is $2N \times 2N$.

Thus, the voltage at bus i is:

$$V_i = \sum_{k=1}^{N} X_{N+i,k} P_k + \sum_{k=1}^{N} X_{N+i,N+k} Q_k \tag{11}$$

where the X_{ij} denotes the elements of $\mathbf{X}$ in the *i*th row and the *j*th column.

Thus, $V_i - V_j$ can be calculated by:

$$V_i - V_j = \sum_{k=1}^{N}\left(X_{N+i,k} - X_{N+j,k}\right) P_k + \sum_{k=1}^{N}\left(X_{N+i,N+k} - X_{N+j,N+k}\right) Q_k \tag{12}$$

Similarly:

$$\theta_i - \theta_j = \sum_{k=1}^{N}\left(X_{i,k} - X_{j,k}\right) P_k + \sum_{k=1}^{N}\left(X_{i,N+k} - X_{j,N+k}\right) Q_k \tag{13}$$

Substituting Eqs. (12) – (13) into the Eq. (3), then:

$$\begin{aligned} P_{ij} &= \sum_{k=1}^{N}\Big(g_{ij}\left(X_{N+i,k} - X_{N+j,k}\right) - b_{ij}\left(X_{i,k} - X_{j,k}\right)\Big) P_k + \\ &\sum_{k=1}^{N}\Big(g_{ij}\left(X_{N+i,N+k} - X_{N+j,N+k}\right) - b_{ij}\left(X_{i,N+k} - X_{j,N+k}\right)\Big) Q_k \end{aligned} \tag{14}$$

$$Q_{ij}=\sum_{k=1}^{N}\Big(-g_{ij}\left(X_{i,k}-X_{j,k}\right)-b_{ij}\left(X_{N+i,k}-X_{N+j,k}\right)\Big)P_k+$$
$$\sum_{k=1}^{N}\Big(-g_{ij}\left(X_{i,N+k}-X_{j,N+k}\right)-b_{ij}\left(X_{N+i,N+k}-X_{N+j,N+k}\right)\Big)Q_k \tag{15}$$

Thus, the GSDF from bus k to line m are:

$$GSF_{m-k}^{P-P}=g_{ij}\left(X_{N+i,k}-X_{N+j,k}\right)-b_{ij}\left(X_{i,k}-X_{j,k}\right) \tag{16}$$

$$GSF_{m-k}^{P-Q}=g_{ij}\left(X_{N+i,N+k}-X_{N+j,N+k}\right)-b_{ij}\left(X_{i,N+k}-X_{j,N+k}\right) \tag{17}$$

$$GSF_{m-k}^{Q-P}=-g_{ij}\left(X_{i,k}-X_{j,k}\right)-b_{ij}\left(X_{N+i,k}-X_{N+j,k}\right) \tag{18}$$

$$GSF_{m-k}^{Q-Q}=-g_{ij}\left(X_{i,N+k}-X_{j,N+k}\right)-b_{ij}\left(X_{N+i,N+k}-X_{N+j,N+k}\right) \tag{19}$$

where GSF_{m-k}^{P-P} denotes the GSDF from the active power generation at bus k to the active power flow across line m; GSF_{m-k}^{P-Q} denotes the GSDF from the reactive power generation at bus k to the active power flow across line m; GSF_{m-k}^{Q-P} denotes the GSDF from the active power generation at bus k to the reactive power flow across line m; GSF_{m-k}^{Q-Q} denotes the GSDF from the reactive power generation at bus k to the reactive power flow across line m.

D.LMP Model and LMP Decompositon

According to the above derivations, the LMP model can be obtained as shown in Eq. (24). In Eq. (24), the λ_P, λ_Q, $u_k^{\min}$, $u_k^{\max}$, $v_i^{\min}$, $v_i^{\max}$ denote the Lagrange multiplier of the corresponding constraint respectively; f_P, f_Q, $h_k^{P\min}$, $h_k^{P\max}$, $h_i^{\min}$, $h_i^{\max}$ represent the shorthand for the left hand expression of corresponding constraint respectively; $\triangleq$ denotes definition; $P_k^{\lim}$ denotes the active power limits of line k; $V_i^{\min}$, $V_i^{\max}$ denote the minimum and maximum voltage at bus i respectively; $P_i^{G\min}$, $P_i^{G\max}$, $Q_i^{G\min}$, $Q_i^{G\min}$ represent the active and reactive power generation limits at bus i respectively; c_i, d_i represent the cost coefficients of generator at bus i. It should be noted that we assume that every bus has a generator in Eq. (24) for convenience.

The objective contains the generation cost of active power and reactive power. The generation cost function of reactive power is quadratic, due to both the generation and absorption of reactive power produce cost. However, it is can be linearized by the piecewise linear method. The constraints include active power balance constraints, reactive power balance constraints, active line flow constraints, voltage constraints, active power generation constraints, and reactive power generation constraints. As the reactive power flow of the line is relatively small in the real power system, the active power is close to the apparent power, so we only constrain the active power flow of the line. Therefore, the above model can be transformed into a linear optimization model.

After obtaining the optimal solution of generation dispatch, the LMP can be calculated with the Lagrangian function [1, 8]. The function can be written as:

$$\psi=\left(\sum_{i=1}^{N}c_iP_i^G+d_i(Q_i^G)^2\right)-\lambda_Pf_P-\lambda_Qf_Q-\sum_{k=1}^{M}\mu_k^{\min}h_k^{P\min}-$$
$$\sum_{k=1}^{M}\mu_k^{\max}h_k^{P\max}-\sum_{i=1}^{N}v_i^{\min}h_i^{V\min}-\sum_{i=1}^{N}v_i^{\max}h_i^{V\max} \tag{20}$$

According to the KKT conditions, the ψ in Lagrangian function represents the total cost of power generation at the optimal solution point and indicates how the constraints such as bus load and line flow limit affect the total cost. Considering the fact that the ALMP is defined as the minimum marginal cost of supplying the next increment of active load demand [11], the ALMP at bus B can be calculated as the partial derivative of Lagrangian function with respect to the load at bus B:

$$\begin{aligned}ALMP_B=\frac{\partial\psi}{\partial P_B^D}&=\left(-\lambda_P\frac{\partial f_P}{\partial P_B^D}\right)+\left(-\sum_{k=1}^{M}\mu_k^{\min}\frac{\partial h_k^{P\min}}{\partial P_B^D}-\sum_{k=1}^{M}\mu_k^{\max}\frac{\partial h_k^{P\max}}{\partial P_B^D}\right)\\&+\left(-\sum_{i=1}^{N}v_i^{\min}\frac{\partial h_i^{V\min}}{\partial P_B^D}-\sum_{i=1}^{N}v_i^{\max}\frac{\partial h_i^{V\max}}{\partial P_B^D}\right)\\&=\lambda_P+\left(\sum_{k=1}^{M}\mu_k^{\min}GSF_{k-B}^{P-P}+\sum_{k=1}^{M}\mu_k^{\max}GSF_{k-B}^{P-P}\right)\\&+\left(\sum_{i=1}^{N}v_i^{\min}X_{N+i,B}+\sum_{i=1}^{N}v_i^{\max}X_{N+i,B}\right)\end{aligned} \tag{21}$$

where the first term is the energy cost component $ALMP^{\mathrm{E}}$; the second term is the congestion cost component $ALMP^{\mathrm{C}}$; the third term is the voltage constraint cost component $ALMP^{V}$.

The RLMP is defined as the minimum marginal cost of supplying the next increment of reactive load demand. Thus, the RLMP at bus B can be calculated by:

$$\begin{aligned}QLMP_B=\frac{\partial\psi}{\partial Q_B^D}&=\lambda_Q+\left(\sum_{k=1}^{M}\mu_k^{\min}GSF_{k-B}^{P-Q}+\sum_{k=1}^{M}\mu_k^{\max}GSF_{k-B}^{P-Q}\right)\\&+\left(\sum_{i=1}^{N}v_i^{\min}X_{N+i,N+B}+\sum_{i=1}^{N}v_i^{\max}X_{N+i,N+B}\right)\end{aligned} \tag{22}$$

where the first term is the energy cost component $RLMP^{Q}$; the second term is the congestion cost component $RLMP^{\mathrm{C}}$; the third term is the voltage constraint cost component $RLMP^{V}$. Due to reactive power has a significant impact on the voltage, when voltage constraint is active, the voltage constraint component is the dominant component of RLMP. It should be noted that the proposed ALMP and QMLP does not contain the loss component. However, the impact of network loss can be considered in this model by using the Fictitious Nodal Demand method similar to the approach proposed in [1].

III. CASE STUDY

In this Section, the proposed linear LMP model is validated by using "case30Q" on MATPOWER [12]. The results are compared with LMP derived from DCOPF and ACOPF integrated into MATPOWER 5.1 [13].

The evaluation indices are average error of ALMP (AEA) and average error of RLMP (AER), which are defined as:

$$AEA=\frac{\sum_{i=1}^{N}\left|\left(ALMP_i-ALMP_i^{AC}\right)/ALMP_i^{AC}\right|}{N}$$
$$AER=\frac{\sum_{i=1}^{N}\left|\left(RLMP_i-RLMP_i^{AC}\right)/RLMP_i^{AC}\right|}{N} \tag{23}$$

where the $ALMP_i^{AC}$, $RLMP_i^{AC}$ denote the ALMP and RLMP derived from ACOPF respectively.

$$
\begin{aligned}
&Min\ \sum_{i=1}^{N} c_i P_i^G + d_i (Q_i^G)^2 \\
&s.t.\quad f_p \triangleq \sum_{i=1}^{N} P_i^G - \sum_{i=1}^{N} P_i^D = 0 \qquad \lambda_P \\
&\qquad f_Q \triangleq \sum_{i=1}^{N} Q_i^G - \sum_{i=1}^{N} Q_i^D = -\sum_{j=1}^{N} b_{jj} \qquad \lambda_Q \\
&\qquad h_k^{P\min} \triangleq \sum_{i=1}^{N} GSF_{k-i}^{P-P}\left(P_i^G - P_i^D\right) + \sum_{i=1}^{N} GSF_{k-i}^{P-Q}\left(Q_i^G - Q_i^D\right) \ge -P_k^{\lim}, \text{ for } k = 1, 2 \cdots M \qquad u_k^{\min} \\
&\qquad h_k^{P\max} \triangleq \sum_{i=1}^{N} GSF_{k-i}^{P-P}\left(P_i^G - P_i^D\right) + \sum_{i=1}^{N} GSF_{k-i}^{P-Q}\left(Q_i^G - Q_i^D\right) \le P_k^{\lim}, \text{ for } k = 1, 2 \cdots M \qquad u_k^{\max} \qquad (24) \\
&\qquad h_i^{V\min} \triangleq \sum_{k=1}^{N} X_{N+i,k}\left(P_k^G - P_k^D\right) + \sum_{k=1}^{N} X_{N+i,N+k}\left(Q_k^G - Q_k^D\right) \ge V_i^{\min}, \text{ for } i = 1, 2 \cdots N \qquad v_i^{\min} \\
&\qquad h_i^{V\max} \triangleq \sum_{k=1}^{N} X_{N+i,k}\left(P_k^G - P_k^D\right) + \sum_{k=1}^{N} X_{N+i,N+k}\left(Q_k^G - Q_k^D\right) \le V_i^{\max}, \text{ for } i = 1, 2 \cdots N \qquad v_i^{\max} \\
&\qquad P_i^{G\min} \le P_i^G \le P_i^{G\max}, \text{ for } i = 1, 2 \cdots N \\
&\qquad Q_i^{G\min} \le Q_i^G \le Q_i^{G\max}, \text{ for } i = 1, 2 \cdots N
\end{aligned}
$$

To emphasize the influence of voltage and reactive power, three scenarios with different voltage constraints are analyzed:

1) loose voltage constraint: $V_{\min} = 0.9, V_{\max} = 1.1$;

2) normal voltage constraint: $V_{\min} = 0.95, V_{\max} = 1.05$;

3) tight voltage constraint: $V_{\min} = 0.98, V_{\max} = 1.02$.

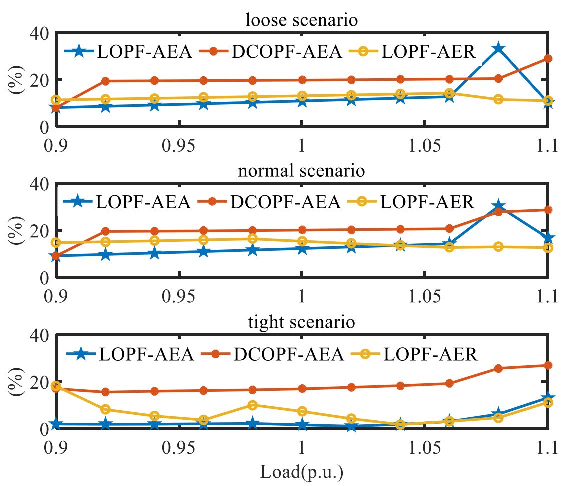

Figure 1 AEA and AER in three voltage scenarios

Fig.1 shows how the AEA and AER of the LOPF model, and AEA of DCOPF model change against the different load level with the three voltage constraint scenarios. The LOPF model outperforms the DCOPF on ALMP in all three scenarios especially in the tight voltage constraint scenario, except when the load level is 1.08 p.u. The average AEA of DCOPF is about 20% in the loose and the normal voltage constraint scenario, and is over 35% in the tight scenario. While the average AEA of the LOPF model is about 10% in the loose and normal scenario, and even lower in the tight scenarios. It implies the LOPF model is more accurate and more robust than DCOPF model on calculating the ALMP. As for RLMP, the AER of the LOPF model is about 10% lower than the AEA of DCOPF in the loose scenario and 5% lower in normal scenario while over 20% lower in the tight scenario. The difference between AEA of DCOPF and AER of LOPF increases as the active and reactive load grow. This indicates that the accuracy of RLMP derived from the proposed LOPF model is at least comparable with ALMP from DCOPF, which means the joint model is applicable in real electricity market like DCOPF.

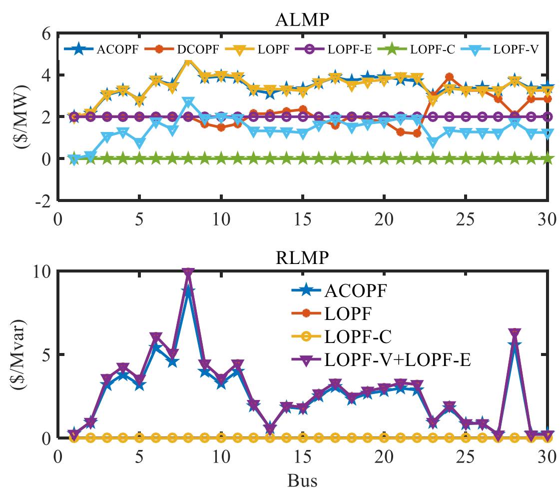

Figure 2 LMP and components of LMP

Note： the LOPF-E, LOPF-C, LOPF-V denotes the energy component, congestion component, voltage constraint component derived from LOPF respectively

Fig. 2 shows the ALMP with its components and RLMP with its components derived from the LOPF model, the ALMP derived from DCOPF, and the ALMP and RLMP derived from ACOPF in the tight scenario with load level 1 p.u. As shown, the joint model has two advantages compared with DCOPF. First, LMP derived from the LOPF model is nearly same as the LMP derived from ACOPF, while the LMP from the DCOPF shows great error. This is because the ALMP from DCOPF is unable to consider the influence of voltage and reactive power. For example, the ALMP in this scenario has remarkable voltage constraint components, nevertheless, the DCOPF gives the totally same LMP results for all different voltage constraint scenarios. Second, the LMP components from the LOPF model can be utilized as a tool for reactive compensation and congestion management. For example, the voltage component of RLMP at bus 8 is highest because the voltage at this point reaches the lower boundary, while the voltage component of RLMP at bus 13 is close to zero because the voltage at this bus reaches the upper boundary as shown in Fig. 3. Thus, the reactive power should be compensated at bus 8 while at bus 13 it can be used freely. Both the congestion components of ALMP and RLMP at all buses are nearly zeros because none of the lines is congested.

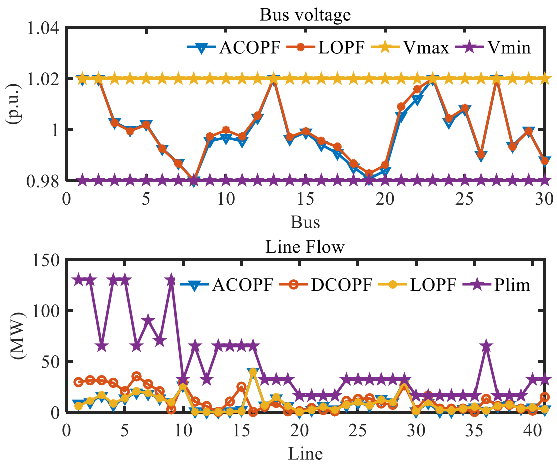

Figure 3 Bus voltage and line Flow

Note: the Vmax and Vmin denote the maximum and mininmum limit of bus voltage respectively; the Plim denotes the maximum line flow limit.

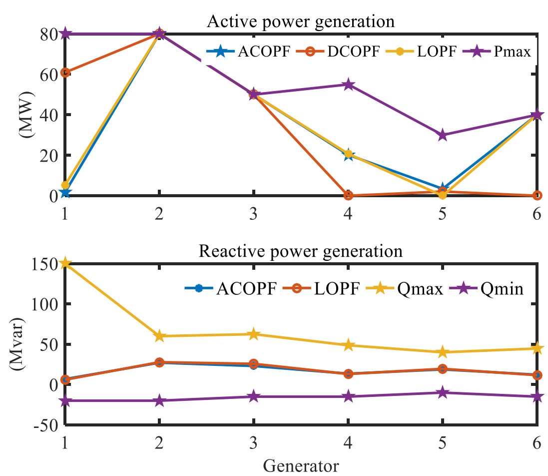

Figure 4 Active and reacitve power generation

Note: the Pmax denotes the maximum active power generation limit of generator; Qmin and Qmax denote the minimum and maximum reactive power generation limit of generator.

The Fig. 4 furtherly shows the active power generation and reactive power generation of the all six generators. The active power and reactive power from the LOPF model are almost same as those from ACOPF. However, the active power from DCOPF is 60 MW more at the first generator, 20MW less at the fourth generator, and 40 MW less at the sixth generator. This means totally different marginal generator for marginal load demand, and further results in distinct marginal price. This also explains why the proposed LOPF model outperforms DCOPF on ALMP, and why AER from the proposed LOPF model is acceptable.

It should be noted that the simulation platform is a PC with an Intel i7 CPU@2.60 GHz and 10 GB of RAM. With solver CPLEX 12.4, the CPU time of proposed LOPF is 0.05s while the CPU time of the ACOPF on MATPOWER 5.1 is 0.75s when solving the "case30Q".

IV. Conclusion

This paper proposes a new linear optimal power flow model for ALMP and RLMP with consideration of voltage. An augmented Generation Shift Distribution Factors is derived using the linearized power flow model considering bus voltage and reactive power. Furtherly, the linear optimal power flow model is constructed using GSDF, and the energy component, congestion component, and voltage component of ALMP and RLMP are derived. The case study on MATPOWER "case30Q" demonstrates that the joint model is more robust and accurate than DCOPF on ALMP and the error of RLMP is acceptable compared with DCOPF and ACOPF. The congestion component and voltage component are illustrated to be an effective tool for congestion management and reactive compensation. With limited space, the loss component of LMP is not described. Further analysis will focus on the loss component.

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