Richard Nakka's *Experimental Rocketry* Web Site

Technical Notepad #4 -- Convective Heat Transfer Coefficient Calculation

The *convective heat transfer coefficient, h*, relates the amount of heat transferred between a moving bulk fluid (liquid or gas) and a bounding surface. The moving fluid itself offers no resistance to heat transmission, rather, a thin film of fluid adjacent to the wall offers resistance. Therefore, the convective coefficient is also referred to as the film coefficient.

The important physical properties of the fluid which affect the convection coefficient are thermal conductivity, viscosity, density (proportional to pressure), and specific heat of the fluid. Other parameters that affect the coefficient are the fluid velocity, geometry of the bounding surface, and pressure (to which fluid density is directly proportional).

In order to estimate this coefficient for conditions within a rocket motor, the casing may be considered to be a smooth tube with conditions of turbulent internal flow, with the following expression being applicable:

$$h = 0.023\ k/Di\ Re^{0.8}\ Pr^{0.33}$$

where *Re* and *Pr* represent the *Reynolds number* and *Prandtl number* of the fluid properties, given by

$$Re = \frac{v\ D_i\ \rho}{\mu} \qquad Pr = \frac{C_p\ \mu}{k}$$

This equation is valid for

2300 < Re < 106 and 0.6 < Pr < 500

In these two equations, *v*, *Di*, *Cp*, μ, ρ and *k* are the fluid velocity, tube diameter, fluid heat capacity, fluid dynamic viscosity, fluid density and thermal conductivity of the fluid, respectively. These fluid properties are all a function of temperature, but only density varies with pressure.

Physically, the Reynolds number may be considered to be a ratio of fluid *inertia forces* to *viscous forces*. The Prandtl number is the ratio of *momentum diffusivity* to *thermal diffusivity*, where diffusivity is the ability to facilitate heat transfer.

An alternative equation, more readily applied to rocket motor analysis, is

$$h = C\ \frac{C_p\ G^{0.8}}{D_i^{\ 0.2}} \left[1 + \left(\frac{D_i}{L}\right)^{0.7}\right]$$

(Ref. Mark's Hdbk. for Mechanical Engineers, 8th ed.)

where C is a coefficient, with a value dependant upon whether English units or S.I. units are used in the equation
C = 0.024 (English units shown)
C = 3.075 (S.I. units shown), and where

C_p =	Specific heat of combustion mixture	BTU/lb-R	J/g-K (or kJ/kg-K)
$G = w/S$ =	Mass velocity	lb/hr-ft^2	kg/sec-m^2
$w = w_p/\ t_b$	Mass flow rate though chamber, avg.	lb/sec.	kg/sec.
w_p =	Propellant weight	lb	kg.
t_b =	Burn time	sec.	sec.
S =	Chamber cross-section area	ft^2	m^2
D_i =	Chamber inside diameter	in.	metre
L =	Chamber length	in.	metre

The units of *h* are BTU/hr-ft^2-F or W/m^2-K.

<u>Example:</u>
Estimate the convection coefficient for a rocket motor powered by 1.50 kg. of 65/35 KN-Sorbitol propellant and a burn time of

1.1 seconds. Casing chamber dimensions are are 65 mm diameter by 400 mm length. The propellant grain is free-standing (unrestricted burning).

From Technical Notepad # 3, Cp' = 69.41 J/mol-K at the combustion temperature of 1600 K. From the same reference, the molecular weight of the combustion mixture is given as MW = 39.86 g/mol. Therefore

Cp = 69.41/39.86 = 1.74 J/g-K.

The average mass flow rate is w = 1.50/ 1.1 = 1.36 kg/sec., and the chamber cross-section area is S = π / 4 $(65/1000)^2$ = 0.0033 m^2 , giving the mass velocity

G = 1.36/ 0.0033 = 410 kg/sec-m^2

This gives a convection coefficient of

$$h = 3.075\ \frac{1.74\,(410)^{0.8}}{(0.065)^{0.2}}\left[1+\left(\frac{0.065}{0.4}\right)^{0.7}\right]$$ = **1457 W/m^2-K.**

- This represents the average convection coefficient nearest the nozzle entrance, where the flow velocity is greatest.
- At the forward (bulkhead) end of the motor, the value would be much less, as the flow velocity is (ideally) zero.
- The example of a free-standing grain was chosen, as this would be the case most closely representing the condition of "turbulent fluid flow through a tube".

Last updated January 8, 2023

Originally posted February 26, 1999
