

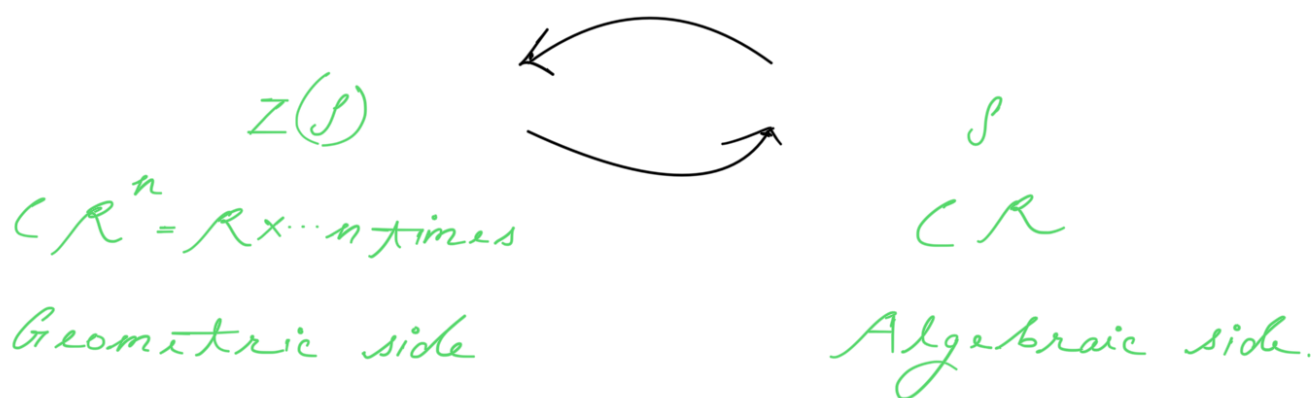
Algebraic Geometry is the study of the set of common zeroes of a set of polynomials.

$$S = \{ f_\lambda(x_1, \dots, x_n) \in R[x_1, \dots, x_n] \mid \lambda \in \Lambda \}$$

$R$  is a commutative polynomial ring.

The set of all common zeroes of  $S$  is denoted by

$$Z(S) = \{ (r_1, \dots, r_n) \in R^n \mid f_\lambda(r_1, \dots, r_n) = 0 \forall \lambda \}$$



We are only interested in scenarios when  $Z(S) \neq \emptyset$ . Variety theory is the stream of Algebraic Geometry where  $R$  is algebraically closed field so

$$Z(S) \neq \emptyset.$$

If polynomials on field  $F$ , have their zeroes always in  $F$ , then  $F$  is called an algebraically closed field. An example is  $\mathbb{C}$ .  $Z(S) \neq \emptyset$  then

If  $S$  is such that  $\nexists$  a finite subset

$\{f_{\alpha_1}, \dots, f_{\alpha_m}\} \subset S$  and polynomials

$g_{\alpha_1}, \dots, g_{\alpha_m} \in \mathbb{R}[x_1, \dots, x_n]$  such that

$$\sum_{i=1}^m f_{\alpha_i} g_{\alpha_i} = 1.$$

Since  $Z(S) \neq \emptyset$ ,  $\exists (r_1, \dots, r_n) \in \mathbb{R}^n$  for which

$$\forall i \quad f_{\alpha_i} = 0 \Rightarrow \sum_{i=1}^m f_{\alpha_i} g_{\alpha_i} = 0 \neq 1.$$

The converse is true and is a form of the Hilbert's Nullstellensatz (weak version).

From now on, we will represent an

algebraically closed field  $K$ .

arbitrarily chosen from  $K$ .

$K^n$  is a vector space but we will not consider the vector space structure of  $K^n$ . We will consider all points in  $K^n$  to be alike.

$K^n$  along with the Zariski topology is called an Affine  $n$ -space and represented by  $A_K^n$ .