Algebraic Geometry is the study of the set of common zeroes of a set of polynomials.

$$S = \{ f_\lambda (x_1, \dots, x_n) \in R[x_1, \dots, x_n] \mid \lambda \in \Lambda \}$$

R is a commutative polynomial ring. The set of all common zeroes of S is denoted by

$$Z(S) = \{ (r_1, \dots, r_n) \in R^n \mid f_\lambda (r_1, \dots, r_n) = 0 \ \forall \lambda \}$$

$$\begin{tikzcd} Z(S) \arrow[r, bend right] & S \arrow[l, bend right] \end{tikzcd}$$

$\subset R^n = R \times \cdots n$ times $\qquad\qquad \subset R$

Geometric side $\qquad\qquad$ Algebraic side.

We are only interested in scenarios when $Z(S) \neq \phi$. Variety theory is the stream of Algebraic Geometry where R is algebraically closed field so

$Z(S) \neq \phi$.

If polynomials on field F, have their zeroes always in F, then F is called an algebraically closed field. An example is $\mathbb{C}$.

If S is such that $Z(S)\neq\phi$ then $\nexists$ a finite subset

$\{f_{\lambda_1}, \dots, f_{\lambda_m}\} \subset S$ and polynomials

$g_{\lambda_1}, \dots, g_{\lambda_m} \in R[x_1, \dots, x_n]$ such that

$\sum_{i=1}^{m} f_{\lambda_i} g_{\lambda_i} = 1$.

Since $Z(S) \neq \phi$, $\exists (r_1, \dots, r_n) \in R^n$ for which

$\forall i \; f_{\lambda_i} = 0 \Rightarrow \sum_{i=1}^{m} f_{\lambda_i} g_{\lambda_i} = 0 \neq 1$.

The converse is true and is a form of the Hilbert's Nullstellensatz (weak version).

From now on, we will represent an

algebraically closed field by K.
K^n is a vector space but we will not consider the vector space structure of K^n. We will consider all points in K^n to be alike.

K^n along with the Zariski topology is called an Affine n-space and represented by $\mathbb{A}^n_K$.