

Midterm Exam

Write an answer to each problem on the corresponding box in your answer sheets.

1. Compare the cardinalities of a range of real numbers $[0, 1)$ and $\mathcal{P}(\mathbb{N})$ (18 points)
2. Use set-builder notation to define the set of all enumerations of a set X without redundancy (20 points).
3. Answer each of the following questions (12 points)
 - a. Is it possible to write a program A that receives a program p as input, and returns True if there exists an input that makes p do not halt?
 - b. Is it possible to write a program B that receives a program p as input, and returns True if p always halt for every input?
 - c. Assume that we have A and B . Can we give a solution of the Halting Problem by using A and B ?
4. Prove that x is irrational if x^3 is irrational (16 points)
5. Show that $f(x)$ is $\Theta(h(x))$ if $f(x)$ is $\Theta(g(x))$ and $g(x)$ is $\Theta(h(x))$ (16 points)
6. Prove that $\forall x(P(x) \rightarrow Q(x)), \forall x(\neg Q(x) \vee S(x)), \forall x(R(x) \rightarrow \neg S(x))$ and $\exists x R(x)$ imply that $\exists x \neg P(x)$, while declaring the corresponding rule of inferences at each step. (18 points)