ITP 20002-03 Discrete Mathematics, 2021 Fall

Test #4

December 17, 2021

You are given five problems with total 100 points to be solved in 75 minutes.

Write an answer of each of the following questions on the corresponding answer box in the answer sheet.

1. Let R be the relation on the set of ordered pairs of integers such that $((a, b), (c, d)) \in R$ iff $a \times d = b \times c$. Prove or disprove that R is equivalence relation (20 points)

2. Suppose that there is a partial order relation R_1 from set A to set B, and an equivalence relation R_2 from set set A to set B. Using the two relations, we are defining a new relation $R_3 \subseteq (A \times B) \times (A \times B)$ such that $((a_1, b_1), (a_2, b_2)) \in R_3$ if and only if $(a_1, a_2) \in R_1$ and $(b_1, b_2) \in R_2$.

   Prove or disprove that R_3 is a partial order relation (20 points)

3. Show that every nonempty finite subset of a lattice has a least upper bound and a greatest lower bound. (20 points)

4. Suppose that G_1 and H_1 are isomorphic, and G_2 and H_2 are isomorphic. Prove or disprove that $G_1 \cap G_2$ and $H_1 \cap H_2$ are isomorphic if both $H_1 \cap H_2$ and $G_1 \cap G_2$ are nonempty. (20 points)

5. Give an algorithm that constructs an Euler path in a directed graph (20 points).

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