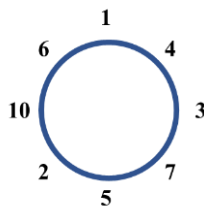


ITP20002-01 Discrete Mathematics, 2018 Fall

## Midterm Exam: Problems

Write an answer to each problem on the corresponding box in your answer sheets.

1. The order of nested quantifiers may or may not change the meaning of a statement.  
Give two examples of statements with nested quantifiers: one where the order of nested quantifiers changes the meaning of the statement, and another one where the order of nested quantifiers does not change the meaning (14 points)
2. Show that  $(B - A) \cup (C - A) = (B \cup C) - A$  (18 points)
3. Let  $A$  be a set of ten integers 1, 2, 3, 4, ..., 10. We are picking eight numbers from  $A$  without duplication, and then arranging the eight numbers around a circle, for example, as the figure below:



- Prove that in every such arrangement, there are three consecutively-located numbers whose sum is greater than 13 (e.g.,  $\langle 10, 6, 1 \rangle$  in the figure) (18 points)
4. Let  $P$  be the set of all finite sequences of computer instructions, each of which represents a program that receives an input from the set of all inputs  $I$ . Let  $T$  be a predicate such that, for  $p \in P$  and  $i \in I$ ,  $T(p, i)$  holds if and only if  $p$  terminates with a given input  $i$  in a finite number of steps.  
  
Express the Halting Problem Theorem as a predicate formula using  $P$ ,  $I$  and  $T$ .  
In addition, state all theorems, assumptions, and definitions that should be additionally given for the current setting to make your answer precise (16 points)
  5. Show that  $f_1(x)f_2(x)$  is  $O(g_1(x)g_2(x))$  if  $f_1(x)$  is  $O(g_1(x))$  and  $f_2(x)$  is  $O(g_2(x))$  (14 points)
  6. Prove or disprove that the set of all functions mapping  $\mathbb{N}$  to  $\mathbb{N}$  is countable (20 points)
  7. (Bonus) Name three persons whose biographies are presented in Chapters 1 to 3 of the textbook (up to 3 points in extra)