# Test #2

October 22, 2021

You are given six problems with total 100 points to be solved in 75 minutes.

Write an answer of each of the following questions on the corresponding answer box in the answer sheet.

1. Prove that $x^n - 1$ is divisible by $x - 1$ for $x > 1$ and $n \geq 1$ using mathematical induction (15 points).

2. For two functions $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$, state $f$ is $O(g)$ as a predicate statement (15 points).

3. Prove or disprove that $g: X \longrightarrow Y$ is injective if $f \cdot g$ is bijective for $f: Y \longrightarrow Z$ (15 points).

4. Give a recursive definition of the function that returns the set of all possible subsequences for a given string (20 points)

5. Prove or disprove that $|\mathcal{P}(\mathbb{N})| = |F|$ where $F$ is the set of all functions each of which maps $\mathcal{P}(\mathbb{N})$ to $\mathbb{N}$ (20 points).

6. Suppose that there are two algorithms $A$ and $B$ for the same kind of the problems. To achieve better time performance, John chose $A$ over $B$ to implement a part of this software product because he found that $A$ is $O(B)$ for worst-case time complexity. Discuss the limitations of the John's decision (15 points)

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