ITP20002-02 Discrete Mathematics, Fall 2017

Midterm Exam

Name and Student Number: _____

- Total **10 pages** including this cover page and 3 blank pages for notes (pp.8—10).
- You have **120 minutes** to complete **8 problems** (100 points).
- Write answers only on given boxes. Write them clearly. No point for illegible answers.
- Write all answers in English. Korean is allowed only for commenting your English expressions.
- Remind the following quoted from Handong CSEE Standard and put your signature below as a mean to show your agreement to keep the standard in taking this exam.

> Examination
>
> 1. Examination is an educational act necessary for evaluation of the students' achievement and for encouraging the students to absorb the material in the process of preparation.
> 2. Student should do their best to prepare for exams in order to improve her/his own knowledge and skill, and should fully engage in the test during examination hour.
> 3. Accessing or providing unauthorized information, including other students' answer sheets, is regarded as cheating. The use of electronic devices, including cell phones and computers, without permission is strictly prohibited.
> 4. Entering or leaving the classroom during the examination before the finish time without permission is regarded as cheating.

I agree to uphold Handong Honor Code and Handong CSEE Standard in taking this exam.

Signature: _____

1. Define the satisfiability and the validity of a propositional formula (10 points)

2. Give a result of converting the following propositional formula into a DIMACS representation (8 points)

$$(\neg p \vee \neg q) \rightarrow (r \leftrightarrow \neg q)$$

3. Show that the power set of a countably infinite set is uncountable (16 points).

4. Give a tautology of the following six rules of inference (18 points)

Modus ponens	
Modus tollens	
Resolution	
Simplification	
Addition	
Disjunctive syllogism	

5. Explain what is a theorem and also explain what is a proof (9 points)

6. Use mathematical induction to show that $\neg(\boldsymbol{p_1} \vee \boldsymbol{p_2} \vee \ldots \vee \boldsymbol{p_n})$ is equivalent to $\neg\boldsymbol{p_1} \wedge \neg\boldsymbol{p_2} \wedge \ldots \wedge \neg\boldsymbol{p_n}$ (14 points).

7. Give two versions of proofs – one as a proof by contradiction, and the other as a proof by contraposition – that show that a positive integer n is odd if n^3+5 is odd (10 points)

(a) a proof by contradiction

(b) a proof by contraposition

8. Give an answer to each of the following questions (15 points)

(a) For two finite sets A and B of the same universe U, list the followings in order of increasing size:

$$|A|, \quad |A \cup B|, \quad |A \cap B|, \quad |\emptyset|, \quad |U|$$

(b) For two finite sets A and B, list the followings in order of increasing size:

$$|A - B|, \quad |A| + |B|, \quad |A \cup B|, \quad |\emptyset|, \quad |A \oplus B|$$

(c) Let f be a function from $\{1, 2, 3, 4\}$ to $\{a, b, c, d\}$, and let g be a function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$.
Suppose that $f(1) = d, f(2) = c, f(3) = a, f(4) = b,$ and $g(a) = 2, g(b) = 1, g(c) = 3, g(d) = 2.$
Find the inverse of f and also the inverse of g.

(d) Specify the condition that a relation $R \subset A \times B$ represents a function from a finite set A to a finite set B.

For your note

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