ITP20002-02 Discrete Mathematics, Fall 2017

Final Exam

Name and Student Number: ____________________________

- Total **10 pages** including this cover page and 4 blank pages for notes (pp.7—10).
- You have **120 minutes (strict)** to complete **8 problems** (total 100 points).
- Write answers only on given boxes. Write them clearly as no point is given to illegible answers.
- Write all answers in English. Korean is allowed only for commenting your English expressions.
- Remind the following quoted from Handong CSEE Standard and put your signature below as a mean to show your agreement to keep the standard in taking this exam.

Examination

1. Examination is an educational act necessary for evaluation of the students' achievement and for encouraging the students to absorb the material in the process of preparation.
2. Student should do their best to prepare for exams in order to improve her/his own knowledge and skill, and should fully engage in the test during examination hour.
3. Accessing or providing unauthorized information, including other students' answer sheets, is regarded as cheating. The use of electronic devices, including cell phones and computers, without permission is strictly prohibited.
4. Entering or leaving the classroom during the examination before the finish time without permission is regarded as cheating.

I agree to uphold Handong Honor Code and Handong CSEE Standard in taking this exam.

Signature: ____________________________________

1. Suppose that a relation $R \subseteq A \times A$ is an equivalent relation. Prove the following two statements on R (15 points).

(a) for any $a \in A$ and $b \in A$, $(a, b) \in R \rightarrow [a] = [b]$

(b) for any $a \in A$ and $b \in A$, $[a] \cap [b] \neq \emptyset \rightarrow (a, b) \in R$

2. Give answers to the following questions (12 points)

(a) State what is the Halting problem.

(b) State what is the Halting problem theorem.

3. Prove the following identity using a combinatorial proof (16 points).

$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{r-k} \binom{n}{k}$ for non-negative integers m, n, and r where $r \le m$ and $r \le n$

4. Suppose that $f(x), g(x),$ and $h(x)$ are functions from real numbers to real numbers.
Show that $f(x)$ is of $O(h(x))$ if $f(x)$ is of $O(g(x))$ and $g(x)$ is of $O\big(h(x)\big)$. (14 points)

5. Give a recursive definition of a function f that returns the reversal of a given string (7 points)

6. A binary tree T is a full binary tree iff T meets one of the following two conditions:

- Case 1. T has only one vertex which is the root and leaf at the same time.
- Case 2. There are two disjoints full binary trees T_1 and T_2 such that T has a root node r together with two edges connecting r to the roots of T_1 and T_2 as left and right subtrees, respectively.

Use structural induction to prove that, for a full-binary tree T, $n(T) \geq 2h(T) + 1$ where $n(T)$ is the number of nodes in T and $h(T)$ is the height of T (i.e., the maximum number of edges from the root to a leaf). (18 points)

7. Suppose that there is a poset (S, R) where R is a partial order relation on S. Give an answer to each of the following questions. (12 points)

(a) Specify mathematically the condition that poset (S, R) is a lattice.

(b) Give a recursive definition of a function that receives a lattice (S, R) and returns the greatest element (the maximal).

8. How many solutions are there to the equation $|x_1| + |x_2| + |x_3| = 12$ for three integers $x_1, x_2,$ and x_3? (6 points)

For your note

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