Assignment 6

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June 13, 2022

Abstract

This document gives the solution for Assignment 6 (Papoulis ch.9 Problem 9.10).

Question

(Problem 9.10) Q.) The process x(t) is normal WSS and $E\{x(t)\} = 0$. Show that if $z(t) = x^2(t)$, then $C_{zz}(\tau) = 2C_{xx}^2(\tau)$.

Solution

Solution : We shall show that if x is a normal process with zero mean and z(t) = x^2(t), then $C_{zz}(\tau) = 2C_{xx}^2(\tau)$.
We know that if the random variables x_i are jointly normal with zero mean, and E$\{x_i x_j\}$ = C_{ij}, then

$$E\{x_1x_2x_3x_4\} = C_{12}C_{34} + C_{13}C_{24} + C_{14}C_{23} \tag{1}$$

Hence, if the R.V.s x_k are normal and $\mathrm{E}\{x_k\} = 0$, then

$$E\{x_1x_2x_3x_4\} = E\{x_1x_2\}E\{x_3x_4\} + E\{x_1x_3\}E\{x_2x_4\} + E\{x_1x_4\}E\{x_2x_3\} \tag{2}$$

With $x_1 = x_2 = x(t+\tau)$ and $x_3 = x_4 = x(t)$, we conclude that the autocorrelation of z(t) equals

$$E\{x^2(t+\tau)x^2(t)\} = E^2\{x^2(t+\tau)\} + 2E^2\{x(t+\tau)x(t)\} \tag{3}$$

$$= R_{xx}^2(0) + 2R_{xx}^2(\tau) \tag{4}$$

And since $R_{xx}(\tau) = C_{xx}(\tau)$, and $E\{z(t)\} = R_{xx}(0)$, the above yields

$$C_{zz}(\tau) = R_{zz}(\tau) - E^2\{z(t)\} = 2C_{xx}^2(\tau) \tag{5}$$