Assignment 8

Archit Ganvir (CS21BTECH11005)

June 17, 2022

Abstract

This document gives the solution for Assignment 8 (Papoulis ch.12 Problem 12-21).

Question

(Problem 12-21) Q.) If the reflection coefficients satisfy $s_k = p^k \cdot k = 1 \to \infty$ where $|p| < 1$, then show that all zeros of the Levinson polynomial $P_n(z)$, $n = 1 \to \infty$ lie on the circle of radius $1/p$.

Solution

Solution: We know the recursion :-

$$\sqrt{1-s_n^2}P_n(z) = P_{n-1}(z) - zs_n\tilde{P}_{n-1}(z) \tag{1}$$

where, $n = 1 \to \infty$
Substituting $s_n = p^n$, $|p| < 1$ in eq.(1), we get :-

$$\sqrt{1-p^{2n}}P_n(z) = P_{n-1}(z) - (zp)^n P_{n-1}^*(z^*) \tag{2}$$

Let $x = zp$ and

$$P_n(z) = P_n(x/p) \triangleq A_n(x) \tag{3}$$

so that the above iteration reduces to :-

$$\sqrt{1-p^{2n}}A_n(x) = A_{n-1}(x) - x^n A^*_{n-1}(1/x^*) \tag{4}$$
$$= A_{n-1}(x) - x\tilde{A}_{n-1}(x) \tag{5}$$

We know that the polynomial $A_n(x)$ has all its zeros on the unit circle (since $s_n = 1$), i.e.

$$x_k = e^{j\theta_k} = z_k p \tag{6}$$

Hence, the zeros $z_k = (1/p)e^{j\theta_k}$ or $|z_k| = 1/p$. (The zeroes of $P_n(z)$ lie on a circle of radius $1/p$).