Assignment 5

Archit Ganvir (CS1BTECH11005)

June 5, 2022

Abstract

This document gives the solution for Assignment 5 (Papoulis ch.8 Problem 8.7).

Question

(Problem 8.7) Q.) (Estimation-prediction) The time to failure of electric bulbs of brand A is a normal random variable with $\sigma = 10$ hours and unknown mean. We have used 20 such bulbs and have observed that the average $\overline{x}$ of their time to failure is 80 hours. We buy a new bulb of the same brand and wish to predict with 95% confidence that its time to failure will be in the interval $80 \pm c$. Find c.

Solution

Solution : Let the random variable be x.
In this problem, we are given the average $\overline{x}$ of n samples of an N(η, σ) R.V. x and we wish to predict the value of x in a future trial with a confidence coefficient of γ, where
$\overline{x} = 80$,
$n = 20$,
$\sigma = 10$,
$\gamma = 0.95$
If η was known, then we would have an ordinary prediction problem. Hence, we must first estimate η.

To estimate η, we form a R.V. $w = x - \overline{x}$.
This R.V. has $N(0, \sigma_w)$, where

$$\sigma_w^2 = \sigma_{x-\overline{x}}^2 \tag{1}$$
$$= \sigma_x^2 + \sigma_{\overline{x}}^2 \tag{2}$$
$$= \sigma^2 + \frac{\sigma^2}{n} \tag{3}$$
$$= 100 + \frac{100}{20} \tag{4}$$
$$= 100 + 5 \tag{5}$$
$$= 105 \tag{6}$$
$$\Rightarrow \sigma_w = \sqrt{105} \tag{7}$$

We know that

$$c = z_{0.975}\sigma_w \tag{8}$$
$$= 2\sqrt{105} \tag{9}$$

We also know that

$$P(|w| < c) = \gamma \tag{10}$$

$$P(\overline{x} - c < x < \overline{x} + c) = 0.95 \tag{11}$$

Therefore, we get the value of c as $2\sqrt{105} \approx 20.494$.
The code in

Assignment5/codes/prob.py

verifies the solution.