Assignment 4

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Abstract

This document gives the solution for Assignment 4 (Papoulis ch.2 Example 2-9).

LaTeX

Question

(Example 2-9) Q.) Two players A and B draw balls one at a time alternately from a box containing m white balls and n black balls. Suppose the player who picks the first white ball wins the game. What is the probability that the player who starts the game will win?

Solution

Solution : We obtain the following distribution of balls in the box :-

Number of White Balls	m
Number of Black Balls	n
Total Number of Balls	m + n

Table:

Suppose A starts the game. The game can be won by A if he extracts a white ball at the start or if A and B draw a black ball each and then A draws a white one, or if A and B extract two black balls each and then A draws a white one and so on. Let

$X_k = \{$A and B alternately draw k black balls each and then A draws a white ball$\}$

where, k = 0, 1, 2, ...

Here, the X_ks represent mutually exclusive events and moreover the event

$$\{\text{A wins}\} = X_0 \cup X_1 \cup X_2 \cup \dots \tag{1}$$

Hence

$$P_A \triangleq P(\text{A wins}) = P(X_0 \cup X_1 \cup X_2 \cup \dots) \tag{2}$$

The axiom of infinite additivity states that if the events A_1, A_2, ... are mutually exclusive, then

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots \tag{3}$$

From eq.s (2), (3), we get

$$P_A = P(X_0) + P(X_1) + P(X_2) + \ldots \tag{4}$$

Now,

$$P(X_0) = \frac{m}{m+n} \tag{5}$$

$$P(X_1) = \frac{n}{n+m} . \frac{n-1}{m+n-1} . \frac{m}{m+n-2} \tag{6}$$

$$= \frac{n(n-1)m}{(m+n)(m+n-1)(m+n-2)} \tag{7}$$

and

$$P(X_2) = \frac{n(n-1)(n-2)(n-3)m}{(m+n)(m+n-1)(m+n-2)(m+n-3)} \tag{8}$$

So,

$$P_A = \frac{m}{m+n}\left(1 + \frac{n(n-1)}{(m+n-1)(m+n-2)} + ...\right) \tag{9}$$

The code in

Assignment4/codes/prob.py

verifies the solution.