Assignment 9

Archit Ganvir (CS21BTECH11005)

June 20, 2022

Abstract

This document gives the solution for Assignment 9 (Papoulis ch.15 Example 15-9).

(Example 15-9) Here the two end-boundary states e_0 and e_{N-1} loop together to form a circle so that e_{N-1} has neighbors e_0 and e_{N-1} (Fig. 15-3). The random walk continues on this circular boundary by passing from one state either to the right or left neighbor and this corresponds to the following $N \times N$ transition matrix

$$P = \begin{pmatrix} 0 & p & 0 & 0 & . & . & . & 0 & q \\ q & 0 & p & 0 & . & . & . & . & 0 \\ 0 & q & 0 & p & 0 & . & . & . & 0 \\ . & & & & & & & & . \\ . & & & & & & & & . \\ . & & & & & & & & . \\ 0 & 0 & . & . & . & . & q & 0 & p \\ p & 0 & . & . & . & . & 0 & q & 0 \end{pmatrix} \tag{1}$$

More generally, if we permit transition between any two states $e_0, e_1, ..., e_{N-1}$, then since moving k steps to the right on a circle is the same as moving $N-k$ to the left (Fig 15-3). we obtain the following circulant transition matrix

$$P = \begin{pmatrix} q_0 & q_1 & q_2 & . & . & . & q_{N-1} \\ q_{N-1} & q_0 & q_1 & . & . & . & q_{N-2} \\ q_{N-2} & q_{N-1} & q_0 & . & . & . & q_{N-3} \\ . & & & & & & . \\ . & & & & & & . \\ . & & & & & & . \\ q_1 & q_2 & . & . & . & q_{N-1} & q_0 \end{pmatrix} \tag{2}$$

Here

$$q_k = P\{X_{n+1} = e_{i+k} | X_n = e_i\} = P\{X_{n+1} = e_{i-(N-k)} | X_n = e_i\} \qquad (3)$$

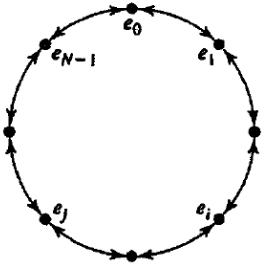

FIGURE 15-3
Cyclic Random Walk in (15-23).