Assignment 2

Archit Ganvir (CS21BTECH11005)*

CONTENTS

***Abstract*—This document gives the solution for Assignment 2 (ICSE Class 12 Maths 2019 Q.17(a)).**

1 ICSE Class 12 Maths 2019

1.1. (Q.17(a)) Q.) Find the equation of the plane passing through the intersection of the planes $2x+2y-3z-7=0$ and $2x+5y+3z-9=0$ such that the intercepts made by the resulting plane on the x-axis and the z-axis are equal.

Solution: The given planes are :-

$$\begin{pmatrix} 2 & 2 & -3 \end{pmatrix} \mathbf{x} = 7 \tag{1.1.1}$$
$$\begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \mathbf{x} = 9 \tag{1.1.2}$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1.1.3}$$

The equation of a plane passing through the intersection of planes given by $\mathbf{n_1}^\top \mathbf{x} = c_1$ and $\mathbf{n_2}^\top \mathbf{x} = c_2$ respectively is given by :-

$$(\mathbf{n_1} + \lambda \mathbf{n_2})^\top \mathbf{x} = c_1 + \lambda c_2 \tag{1.1.4}$$

If the plane $\mathbf{n}^\top \mathbf{x} = c$ makes equal intercepts on the x-axis and the z-axis, then

$$\mathbf{n}^\top \mathbf{e_1} = \mathbf{n}^\top \mathbf{e_3} \tag{1.1.5}$$

where, $\mathbf{e_1}$, $\mathbf{e_2}$, $\mathbf{e_3}$ are the basis vectors (direction vectors of the coordinate axes) :-

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{1.1.6}$$

From eq.s (1.1.4), (1.1.5), we get the condition for the required plane to make equal intercepts on the x-axis and z-axis as :-

$$(\mathbf{n_1} + \lambda \mathbf{n_2})^\top \mathbf{e_1} = (\mathbf{n_1} + \lambda \mathbf{n_2})^\top \mathbf{e_3} \tag{1.1.7}$$
$$\Rightarrow \mathbf{n_1}^\top \mathbf{e_1} + (\lambda \mathbf{n_2})^\top \mathbf{e_1} = \mathbf{n_1}^\top \mathbf{e_3} + (\lambda \mathbf{n_2})^\top \mathbf{e_3} \tag{1.1.8}$$
$$\Rightarrow \mathbf{n_1}^\top \mathbf{e_1} + \lambda \mathbf{n_2}^\top \mathbf{e_1} = \mathbf{n_1}^\top \mathbf{e_3} + \lambda \mathbf{n_2}^\top \mathbf{e_3} \tag{1.1.9}$$

Rearranging, we get :-

$$\lambda = \frac{\mathbf{n_1}^\top \mathbf{e_3} - \mathbf{n_1}^\top \mathbf{e_1}}{\mathbf{n_2}^\top \mathbf{e_1} - \mathbf{n_2}^\top \mathbf{e_3}} \tag{1.1.10}$$
$$= \frac{\begin{pmatrix} 2 & 2 & -3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 & 2 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}} \tag{1.1.11}$$
$$= \frac{-3-2}{2-3} \tag{1.1.12}$$
$$= \frac{-5}{-1} \tag{1.1.13}$$
$$= 5 \tag{1.1.14}$$

The various parameters considered in this solution are :-

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \tag{1.1.15}$$
$$\mathbf{n_2} = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} \tag{1.1.16}$$
$$c_1 = 7 \tag{1.1.17}$$
$$c_2 = 9 \tag{1.1.18}$$
$$\lambda = 5 \tag{1.1.19}$$

Substituting the value of λ in eq. (1.1.4), we get the equation of the required plane as :-

$$(\mathbf{n_1} + \mathbf{5n_2})^\top \mathbf{x} = c_1 + 5c_2 \tag{1.1.20}$$

Substituting the values of $\mathbf{n_1}$, $\mathbf{n_2}$, c_1 and c_2, we get the equation of the required plane as :-

$$\left(\begin{pmatrix}2\\2\\-3\end{pmatrix}+5\begin{pmatrix}2\\5\\3\end{pmatrix}\right)^\top \mathbf{x} = 7+5\times 9 \tag{1.1.21}$$

$$\Rightarrow \left(\begin{pmatrix}2\\2\\-3\end{pmatrix}+\begin{pmatrix}5\times 2\\5\times 5\\5\times 3\end{pmatrix}\right)^\top \mathbf{x} = 7+45 \tag{1.1.22}$$

$$\Rightarrow \left(\begin{pmatrix}2\\2\\-3\end{pmatrix}+\begin{pmatrix}10\\25\\15\end{pmatrix}\right)^\top \mathbf{x} = 52 \tag{1.1.23}$$

$$\Rightarrow \begin{pmatrix}2+10\\2+25\\-3+15\end{pmatrix}^\top \mathbf{x} = 52 \tag{1.1.24}$$

$$\Rightarrow \begin{pmatrix}12\\27\\12\end{pmatrix}^\top \mathbf{x} = 52 \tag{1.1.25}$$

$$\Rightarrow \begin{pmatrix}12 & 27 & 12\end{pmatrix} \mathbf{x} = 52 \tag{1.1.26}$$

$$\tag{1.1.27}$$

Therefore, the equation of the required plane is $12x+27y+12z-52=0$.

The code in

```
Assignment2/codes/plane.py
```

verifies the solution.