Assignment 2

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***Abstract*—This document gives the solution for Assignment 2 (ICSE Class 12 Maths 2019 Q.17(a)).**

1 ICSE Class 12 Maths 2019

1.1. (Q.17(a)) Q.) Find the equation of the plane passing through the intersection of the planes $2x+2y-3z-7=0$ and $2x+5y+3z-9=0$ such that the intercepts made by the resulting plane on the x-axis and the z-axis are equal.

Solution: The given planes are :-

$$\begin{pmatrix}2 & 2 & -3\end{pmatrix}\mathbf{x}=7 \tag{1.1.1}$$

$$\begin{pmatrix}2 & 5 & 3\end{pmatrix}\mathbf{x}=9 \tag{1.1.2}$$

where

$$\mathbf{x}=\begin{pmatrix}x\\y\\z\end{pmatrix} \tag{1.1.3}$$

The equation of a plane passing through the intersection of planes given by $\mathbf{n_1}^\top\mathbf{x}=c_1$ and $\mathbf{n_2}^\top\mathbf{x}=c_2$ respectively is given by :-

$$(\mathbf{n_1}+\lambda\mathbf{n_2})^\top\mathbf{x}=c_1+\lambda c_2 \tag{1.1.4}$$

which can also be stated as :-

$$\frac{1}{c_1+\lambda c_2}(\mathbf{n_1}+\lambda\mathbf{n_2})^\top\mathbf{x}=1 \tag{1.1.5}$$

The input parameters are :-

$$\mathbf{n_1}=\begin{pmatrix}2\\2\\-3\end{pmatrix} \tag{1.1.6}$$

$$\mathbf{n_2}=\begin{pmatrix}2\\5\\3\end{pmatrix} \tag{1.1.7}$$

$$c_1=7 \tag{1.1.8}$$

$$c_2=9 \tag{1.1.9}$$

From eq.s (1.1.6), (1.1.7), we get :-

$$\mathbf{n}=\mathbf{n_1}+\lambda\mathbf{n_2}=\begin{pmatrix}2\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\5\\3\end{pmatrix} \tag{1.1.10}$$

$$=\begin{pmatrix}2\\2\\-3\end{pmatrix}+\begin{pmatrix}2\lambda\\5\lambda\\3\lambda\end{pmatrix} \tag{1.1.11}$$

$$=\begin{pmatrix}2+2\lambda\\2+5\lambda\\-3+3\lambda\end{pmatrix} \tag{1.1.12}$$

$$\Rightarrow \mathbf{n}^\top=(\mathbf{n_1}+\lambda\mathbf{n_2})^\top=\begin{pmatrix}2+2\lambda & 2+5\lambda & -3+3\lambda\end{pmatrix} \tag{1.1.13}$$

From eq.s (1.1.8), (1.1.9), we get :-

$$c=c_1+\lambda c_2=7+9\lambda \tag{1.1.14}$$

From eq.s (1.1.13), (1.1.14), we get :-

$$\frac{\mathbf{n}^\top}{c}=\frac{(\mathbf{n_1}+\lambda\mathbf{n_2})^\top}{c_1+\lambda c_2}=\frac{\begin{pmatrix}2+2\lambda & 2+5\lambda & -3+3\lambda\end{pmatrix}}{7+9\lambda} \tag{1.1.15}$$

$$=\begin{pmatrix}\frac{2+2\lambda}{7+9\lambda} & \frac{2+5\lambda}{7+9\lambda} & \frac{-3+3\lambda}{7+9\lambda}\end{pmatrix} \tag{1.1.16}$$

From eq.s (1.1.5) (1.1.16), we get the equation of the required plane as :-

$$\begin{pmatrix}\frac{2+2\lambda}{7+9\lambda} & \frac{2+5\lambda}{7+9\lambda} & \frac{-3+3\lambda}{7+9\lambda}\end{pmatrix}\mathbf{x}=1 \tag{1.1.17}$$

Since the required plane makes equal intercepts on the x-axis and the z-axis, using eq. (1.1.17), we get :-

$$\frac{2+2\lambda}{7+9\lambda}=\frac{-3+3\lambda}{7+9\lambda} \tag{1.1.18}$$

$$\Rightarrow 2+2\lambda=3\lambda-3 \tag{1.1.19}$$

$$\Rightarrow \lambda=5 \tag{1.1.20}$$

Substituting the value of λ from eq. (1.1.20) in eq. (1.1.17), we get the equation of the required plane as :-

$$\begin{aligned}
\begin{pmatrix} \frac{2+2\times5}{7+9\times5} & \frac{2+5\times5}{7+9\times5} & \frac{-3+3\times5}{7+9\times5} \end{pmatrix} \mathbf{x} &= 1 && (1.1.21)\\
\equiv \begin{pmatrix} \frac{2+10}{7+45} & \frac{2+25}{7+45} & \frac{-3+15}{7+45} \end{pmatrix} \mathbf{x} &= 1 && (1.1.22)\\
\equiv \begin{pmatrix} \frac{12}{52} & \frac{27}{52} & \frac{12}{52} \end{pmatrix} \mathbf{x} &= 1 && (1.1.23)\\
\equiv \frac{1}{52} \begin{pmatrix} 12 & 27 & 12 \end{pmatrix} \mathbf{x} &= 1 && (1.1.24)
\end{aligned}$$

which can also be stated as :-

$$\begin{pmatrix} 12 & 27 & 12 \end{pmatrix} \mathbf{x} = 52 \qquad (1.1.25)$$

Therefore, the equation of the required plane is $12x + 27y + 12z - 52 = 0$.

The code in

Assignment2/codes/plane.py

verifies the solution.