Assignment 2

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Contents

***Abstract*—This document gives the solution for Assignment 2 (ICSE Class 12 Maths 2019 Q.17(a)).**

1 ICSE Class 12 Maths 2019

1.1. (Q.17(a)) Q.) Find the equation of the plane passing through the intersection of the planes $2x+2y-3z-7=0$ and $2x+5y+3z-9=0$ such that the intercepts made by the resulting plane on the x-axis and the z-axis are equal.

Solution: The given planes are :-

$$\begin{pmatrix} 2 & 2 & -3 \end{pmatrix} \mathbf{x} = 7 \tag{1.1.1}$$

$$\begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \mathbf{x} = 9 \tag{1.1.2}$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1.1.3}$$

The equation of a plane passing through the intersection of planes given by $\mathbf{n_1}^\top \mathbf{x} = c_1$ and $\mathbf{n_2}^\top \mathbf{x} = c_2$ respectively is given by :-

$$(\mathbf{n_1} + \lambda \mathbf{n_2})^\top \mathbf{x} = c_1 + \lambda c_2 \tag{1.1.4}$$

which can also be stated as :-

$$\frac{1}{c_1 + \lambda c_2} (\mathbf{n_1} + \lambda \mathbf{n_2})^\top \mathbf{x} = 1 \tag{1.1.5}$$

The input parameters are :-

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \tag{1.1.6}$$

$$\mathbf{n_2} = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} \tag{1.1.7}$$

$$c_1 = 7 \tag{1.1.8}$$

$$c_2 = 9 \tag{1.1.9}$$

From eq.s (1.1.6), (1.1.7), we get :-

$$\mathbf{n} = \mathbf{n_1} + \lambda \mathbf{n_2} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} \tag{1.1.10}$$

$$= \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 2\lambda \\ 5\lambda \\ 3\lambda \end{pmatrix} \tag{1.1.11}$$

$$= \begin{pmatrix} 2+2\lambda \\ 2+5\lambda \\ -3+3\lambda \end{pmatrix} \tag{1.1.12}$$

$$\Rightarrow \mathbf{n}^\top = (\mathbf{n_1} + \lambda \mathbf{n_2})^\top = \begin{pmatrix} 2+2\lambda & 2+5\lambda & -3+3\lambda \end{pmatrix} \tag{1.1.13}$$

From eq.s (1.1.8), (1.1.9), we get :-

$$c = c_1 + \lambda c_2 = 7 + 9\lambda \tag{1.1.14}$$

From eq.s (1.1.13), (1.1.14), we get :-

$$\frac{\mathbf{n}^\top}{c} = \frac{(\mathbf{n_1} + \lambda \mathbf{n_2})^\top}{c_1 + \lambda c_2} = \frac{\begin{pmatrix} 2+2\lambda & 2+5\lambda & -3+3\lambda \end{pmatrix}}{7+9\lambda} \tag{1.1.15}$$

$$= \begin{pmatrix} \frac{2+2\lambda}{7+9\lambda} & \frac{2+5\lambda}{7+9\lambda} & \frac{-3+3\lambda}{7+9\lambda} \end{pmatrix} \tag{1.1.16}$$

From eq.s (1.1.5) (1.1.16), we get the equation of the required plane as :-

$$\begin{pmatrix} \frac{2+2\lambda}{7+9\lambda} & \frac{2+5\lambda}{7+9\lambda} & \frac{-3+3\lambda}{7+9\lambda} \end{pmatrix} \mathbf{x} = 1 \tag{1.1.17}$$

Let the basis vectors (direction vectors of the coordinate axes) be given by :-

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{1.1.18}$$

The x, y and z-intercepts of the plane whose equation is $\mathbf{n_0}^\top \mathbf{x} = 1$ are given by :-

$$a_0 = \mathbf{n_0}^\top \mathbf{e_1}, b_0 = \mathbf{n_0}^\top \mathbf{e_2}, c_0 = \mathbf{n_0}^\top \mathbf{e_3} \tag{1.1.19}$$

Hence, using eq. (1.1.17), we get the x-intercept of the required plane as :-

$$a = \begin{pmatrix} \frac{2+2\lambda}{7+9\lambda} & \frac{2+5\lambda}{7+9\lambda} & \frac{-3+3\lambda}{7+9\lambda} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{1.1.20}$$
$$= \frac{2+2\lambda}{7+9\lambda} \tag{1.1.21}$$

Similarly, the z-intercept of the required plane is given by :-

$$c = \begin{pmatrix} \frac{2+2\lambda}{7+9\lambda} & \frac{2+5\lambda}{7+9\lambda} & \frac{-3+3\lambda}{7+9\lambda} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{1.1.22}$$
$$= \frac{-3+3\lambda}{7+9\lambda} \tag{1.1.23}$$

Since the required plane makes equal intercepts on the x-axis and the z-axis, using eq.s (1.1.21), (1.1.23), we get :-

$$a = c \tag{1.1.24}$$
$$\Rightarrow \frac{2+2\lambda}{7+9\lambda} = \frac{-3+3\lambda}{7+9\lambda} \tag{1.1.25}$$
$$\Rightarrow 2+2\lambda = 3\lambda - 3 \tag{1.1.26}$$
$$\Rightarrow \lambda = 5 \tag{1.1.27}$$

Substituting the value of λ from eq. (1.1.27) in eq. (1.1.17), we get the equation of the required plane as :-

$$\begin{pmatrix} \frac{2+2\times 5}{7+9\times 5} & \frac{2+5\times 5}{7+9\times 5} & \frac{-3+3\times 5}{7+9\times 5} \end{pmatrix} \mathbf{x} = 1 \tag{1.1.28}$$
$$\equiv \begin{pmatrix} \frac{2+10}{7+45} & \frac{2+25}{7+45} & \frac{-3+15}{7+45} \end{pmatrix} \mathbf{x} = 1 \tag{1.1.29}$$
$$\equiv \begin{pmatrix} \frac{12}{52} & \frac{27}{52} & \frac{12}{52} \end{pmatrix} \mathbf{x} = 1 \tag{1.1.30}$$
$$\equiv \frac{1}{52} \begin{pmatrix} 12 & 27 & 12 \end{pmatrix} \mathbf{x} = 1 \tag{1.1.31}$$

which can also be stated as :-

$$\begin{pmatrix} 12 & 27 & 12 \end{pmatrix} \mathbf{x} = 52 \tag{1.1.32}$$

Therefore, the equation of the required plane is $12x + 27y + 12z - 52 = 0$.

The code in

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Assignment2/codes/plane.py
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verifies the solution.