EE302

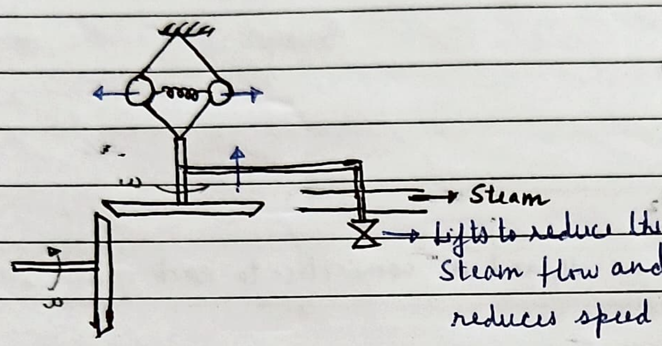
Lecture - 1

Analysis | Synthesis

STEPS

1. Model the system
2. Set performance Metrics
3. Check if the System meets the Desired performance criteria/metric
4. Suppose it doesn't; Decide on the type of Controller you want.
5. Synthesize / Design the Chosen controller
6. ☆☆ Carry out extensive simulation of the designed controller and if unsatisfactory go to step 5
7. Place the controller in actual system & carry out experiment

Ex:



Lecture - 2

Feedback

→ Open Loop (Absence of Feedback) Vs Closed Loop (Presence of Feedback)

→ Linear System Vs Non Linear System

If $u_1 \rightarrow \boxed{G} \rightarrow G(u_1) = y_1$

$u_2 \rightarrow \boxed{G} \rightarrow G(u_2) = y_2$

, then $\alpha_1 u_1 + \alpha_2 u_2 \rightarrow \boxed{G} \rightarrow G(\alpha_1 u_1 + \alpha_2 u_2)$

$\Downarrow$

$\alpha_1 y_1 + \alpha_2 y_2$

$\Downarrow$

"Linear"

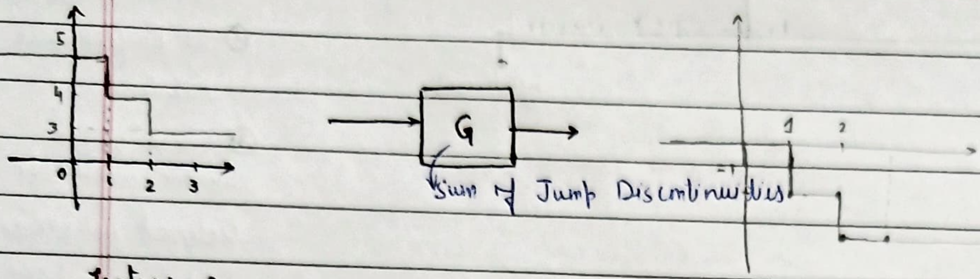
Ex: ① $y = 3u + 5$ (Non Linear)

② $y = 3u^2 + u$ (Non Linear)

③ $u = \begin{bmatrix} a \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ b \end{bmatrix}$ ⇒ Non Linear → Domain is punctured

$y = 5u$

④ $y = 7u$ $\quad u = \{0, 2, 4, 6 \dots\}$ ⇒ Non Linear → Domain is punctured



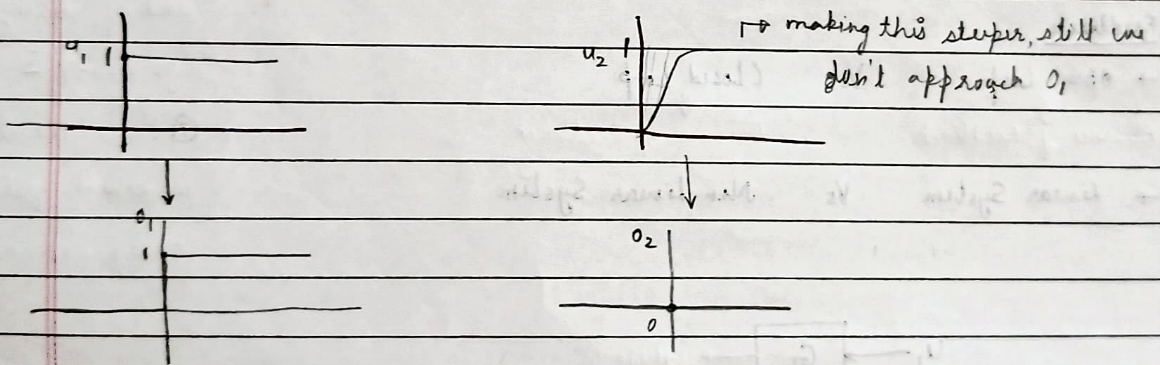
Lecture - 2

\#

$u_1 \rightarrow \boxed{G} \rightarrow y_1$

$u_2 \rightarrow \boxed{G} \rightarrow y_2$

- A useful property is that as u_1 and u_2 come close to each other, y_1 and y_2 also approach each other
- Mostly linear function has this property. but the above example violates this



$u_1 \rightarrow y_1$

$u_2 \rightarrow y_2$

⋮

$u_k \rightarrow y_k$

⋮

$$\hat{u} = \sum_{i=1}^{\infty} \alpha_i u_i \longrightarrow G(\hat{u}) = \sum_{i=1}^{\infty} \alpha_i y_i$$

and G is smooth

$\boxed{\text{Only when } \hat{u} = \sum_{i=1}^{\infty} \alpha_i u_i \text{ converges}}$

- Smoothness is important; mostly taken for granted.

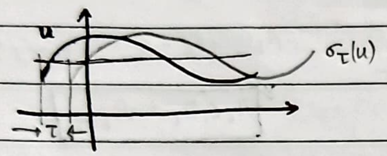
Static Vs Dynamic $\longrightarrow$ • Example: $y(t) = u(\cos(t))$

- ~~Memoryless System and Causal~~
- Output depends on input at that time

Example: $V(t) = R\,I(t)$

Time Invariant Vs Time Variant

Shift Operator $= \sigma_\tau$



$u \rightarrow \boxed{\sigma_\tau} \xrightarrow{u(t-\tau)} \boxed{G} \rightarrow G(u(t-\tau))$

$u \rightarrow \boxed{G} \rightarrow \boxed{\sigma_\tau} \rightarrow *$

} If these are equal then G is time Invariant

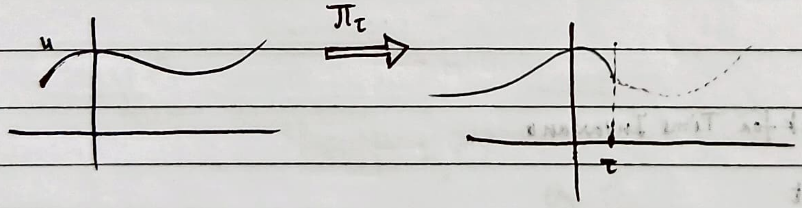
$\Downarrow$

$\boxed{\text{If } G\sigma_\tau = \sigma_\tau G \ \forall \tau \text{ \& for all inputs, then G is time variant}}$

$\pi_t G \pi_t = \pi_t G$

Causal Vs Non-Causal

π_τ = Truncation operator



Suppose

$\boxed{\pi_\tau G = \pi_\tau G \ \forall \tau \text{ and for all inputs then it is called causal}}$ P1

$u \rightarrow \boxed{G} \rightarrow \boxed{\pi_\tau} \rightarrow *$

$u \rightarrow \boxed{\pi_\tau} \rightarrow \boxed{G} \rightarrow \boxed{\pi_\tau} \rightarrow *$

$\Rightarrow$ Whether you truncate the input or not, it doesn't matter

- Check for left Invertibility and Right Invertible $\Rightarrow \sigma_{-\tau}$ is right / left inverse of σ_τ
- ROC and Causality.

  $\Downarrow$

  If ROC has ∞ then causal

- Suppose; the system is time-invariant: We can ~~change~~ check for finite τ, and infer about Causality.

$\Downarrow$

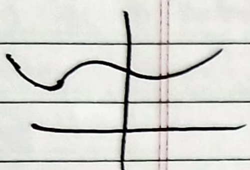
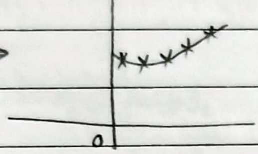
Let it be true for $\underbrace{\tau = 0}_{(\text{w.l.g})}$ i.e. $\Pi_0 G \Pi_0 = \Pi_0 G$

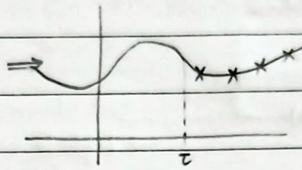
$$\sigma_\tau \Pi_0 G \Pi_0 = \sigma_\tau \Pi_0 G = \Pi_\tau \sigma_\tau G \Pi_0 = \Pi_\tau \sigma_\tau \Pi_0 G \quad \forall \tau$$

Claim: $\sigma_\tau \Pi_0 = \Pi_\tau \sigma_\tau$

$$= \Pi_\tau G \sigma_\tau \Pi_0 = \Pi_\tau G \sigma_\tau = \Pi_\tau G \Pi_\tau \sigma_\tau = \Pi_\tau G \sigma_\tau \quad \forall \tau$$

since full line sig... σ_τ is invertible

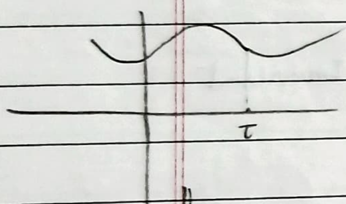

$\Rightarrow$


$\Rightarrow$


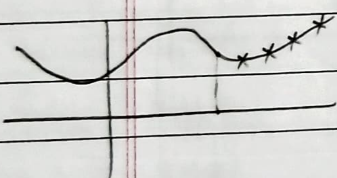
$$\Pi_\tau G \Pi_\tau \sigma_\tau \sigma_\tau^{-1} = \Pi_\tau G \sigma_\tau \sigma_\tau^{-1}$$

$$\boxed{\Pi_\tau G \Pi_\tau = \Pi_\tau G}$$

$\Downarrow$



$\Downarrow$



Exercise: - $= u'(t)$

1) $y(t) = u(3t)$ Check for Time Invariance
2) $y(t) = \int_0^t \sqrt{u(t)}\, dt$
3) $y(t) = \int_{t-3}^{t+3} u^2(t)\, dt$

① $u(t-\tau)$

$y_1(t) = u(3t - 3\tau)$

$y_2(t) = u(3t - \tau)$

} Not Time Invariant

② $u(t-\tau)$

$\downarrow$

$$y_1(t) = \int_0^t \sqrt{u(\alpha - \tau)} \cdot d\alpha = \int_{-\tau}^{t-\tau} \sqrt{u(\alpha')} \cdot d\alpha'$$

$t - \tau = \alpha'$

$$y_2(t) = y(t-\tau) = \int_0^{t-\tau} \sqrt{u(\alpha)}\, d\alpha$$

} ~~Not time invar~~

$-50 + 10 + 45$

Lecture - 3

Discrete time Vs Continous time

Deterministic Vs Stochastic → Some Randomness is there

⇓

- No uncertainity/ Randomness in output
- Everytime we give same inputs we get the same output

$\dddot{y} + a_1\ddot{y} + a_2\dot{y} + a_3 y = b_0\dddot{u} + b_1\ddot{u} + b_2\dot{u} + b_3 u$ → Order of derivative of $(u \leq y)$

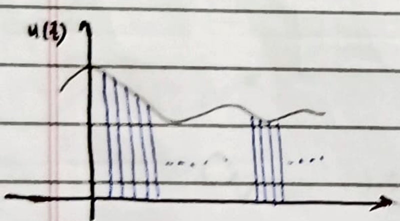
↓

Linear (Excercise proof)

$\dot{u} = \lim_{\delta \to 0} \frac{u(t) - u(t-\delta)}{\delta}$

$\ddot{u} = \lim_{\delta \to 0} \frac{\frac{u(t) - u(t-\delta)}{\delta} - \frac{u(t-\delta) + u(t-2\delta)}{\delta}}{\delta} = \frac{u(t) - 2u(t-\delta) + u(t-2\delta)}{\delta^2}$

$\dddot{u} = \lim_{\delta \to 0} \frac{u(t) - 3u(t-\delta) + 3u(t-\delta) - u(t-3\delta)}{\delta^2}$



Define: $\bar{u}(t) = \sum_{k=0}^{\infty} u(t_k) \cdot \delta_\Delta(t_k) \cdot \Delta$

$\bar{u}(t) = \sum_{k=0}^{\infty} u(t_k) \cdot \underline{(1(t - t_k) - 1(t - t_k - \Delta))}$

If: $y(t) = G(u(t))$

$\bar{y}(t) = G\left(\sum_{k=0}^{\infty} u(t_k)\, \delta_\Delta(t - t_k)\Delta\right)$

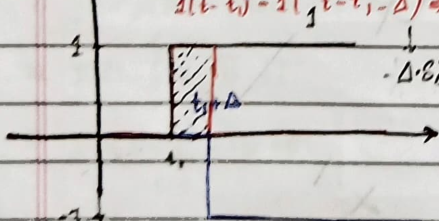
$= \sum_{k=0}^{\infty} G(\delta_\Delta(t - t_k)) \cdot u(t_k) \cdot \Delta$

$\boxed{\text{as } \Delta \to 0}$: $= \boxed{\int_0^{t} G(\delta(t-\tau))\, u(\tau)\, d\tau = y(t)}$

$1(t-t_1) - 1(t-t_1-\Delta) \Rightarrow \mathcal{L}(1(t-t_1) - 1(t-t_1-\Delta)) = \frac{1}{s}\left(e^{-st_1} - e^{-(t_1+\Delta)s}\right)$

↓

$\cdot\,\Delta \cdot \delta_\Delta(t - t_1) =$



$g(t-\tau) : g(t) =$ Impulse response

- $y(t) = \int_0^{\infty} \underbrace{G(\delta(t-\tau))}_{g(t-\tau)} \cdot u(\tau)\, d\tau$

↓

- o/p of the system to a impulse applied at τ
- Time Invariance is implicit

⇓

$y(t) = \int_0^{t} g(t-\tau) \cdot u(\tau)\, d\tau$

$= \int_0^{t} u(t-\tau)\, g(\tau)\, d\tau$

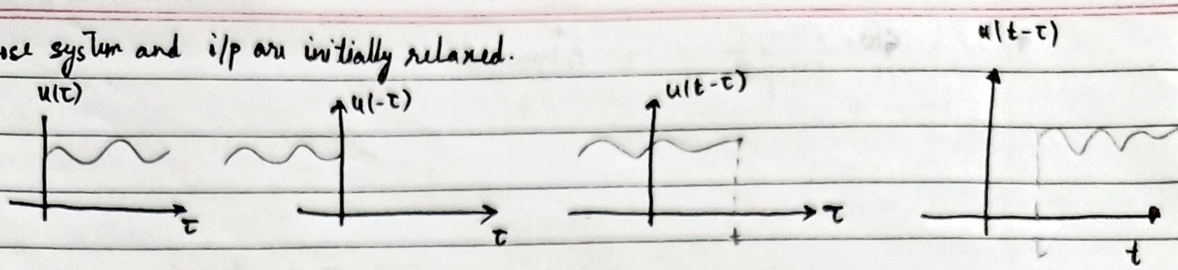
Dirac Delta

- Not a function, but a generalized Distribution
- $\int_{-\infty}^{\infty} f(t) \cdot \delta(t - t_1)\, dt = f(t_1)$: Shifting Property
- $\delta_\Delta(t - t_1) \xrightarrow{\Delta \to 0} \delta(t - t_1)$

$\mathcal{L}(g(t)) = G(s) = \int_{0^-}^{\infty} g(t) \cdot e^{-st}\, dt$

"Transfer Func^{n}"

- Suppose system and i/p are initially relaxed.



$$Y(s) = \mathcal{L}\left[\int_{0^-}^{\infty} g(\tau).u(t-\tau)d\tau\right] = \int_{0^-}^{\infty}\int_{0^-}^{\infty} g(\tau).u(t-\tau).e^{-st}d\tau.dt$$

$$= \int_{0^-}^{\infty} g(\tau).\left[\int_{0^-}^{\infty} u(t-\tau).e^{-st}dt\right]d\tau = \int_{0^-}^{\infty} g(\tau).e^{-s\tau}\left[\int_{\tau}^{\infty} u(t-\tau)e^{-s(t-\tau)}dt\right]$$

$$= \int_{0^-}^{\infty} g(\tau).U(s).e^{-s\tau}d\tau = G(s).U(s) = \underbrace{\mathcal{L}(g(t))}_{\text{Transfer Function}}.\mathcal{L}(u(t))$$

$\int_{0^-}^{\infty} \frac{d}{dt}(f(t))e^{-st}dt = sF(s) - f(0^-)$

$$\Downarrow$$

$$\int_{-0}^{0^+} \frac{d}{dt}(f(t))e^{-st}dt + \int_{0^+}^{\infty} \frac{d}{dt}(f(t))e^{-st}dt = f(0^+) - \cancel{f(0^-)} + \int_{0^+}^{\infty} \frac{d}{dt}(f(t))e^{-st}dt = sF(s) - \cancel{f(0^-)}$$

$$\boxed{\lim_{s\to\infty} sF(s) = f(0^+)}$$

$\dot{x}(t) + x(t) = \delta(t)$ $\quad x(0^-) = 1$, Obtain $x(t)$ for $t \geq 0$

① Let us consider, $x(0^+)$

$\dot{x}(t) + x(t) = 0 \Rightarrow x(t) = e^{-t}x(0^+) = 2e^{-t}$

$\dot{x}(t)$ should produce a δ, hence it has a unit jump discontinuity

$$\Downarrow$$

$$x(0^+) - x(0^-) = 1$$

$$x(0^+) = 2$$

② $\int_{0^+}^{\infty} f(t)e^{-st}dt = F_+(s)$: unilateral ~~Fourier~~ Laplace Transform

$\mathcal{L}(\delta(t)) = 1$

$\mathcal{L}_+(\delta(t)) = 0$

Taking the unilateral Laplace Transform

$sX(s) - \underbrace{x(0^+)}_{\text{needs to be infered from the eq in time domain; suffers the same pitfall as in classical method.}} + X(s) = 0$

③ Taking LT;

$sX(s) - x(0^-) + X(s) = 1$

$X(s) = \frac{2}{s+1}$

$\mathcal{L}^{-1}(X(s)) = \mathcal{L}^{-1}\left(\frac{2}{s+1}\right) = 2e^{-t}.1(t)$

• Note: Soluⁿ of a Linear ODE

⇓

Z. State Soluⁿ + Z. Input Soluⁿ — This is different to Forced + Zero

→ Rational Functions: $\frac{N(s)}{D(s)}$ (N(s): Polynomial, D(s): Polynomial), deg(N) = m, deg(D) = n
- n ≥ m = proper
- m = n = biproper
- n > m = strictly proper

$G(s) = \frac{\mathcal{L}(\text{Output})}{\mathcal{L}(\text{Input})}$

$\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y = b_1\dot{u} + b_0 u$, $\ddot{y}(0^-) = \gamma$, $\dot{y}(0^-) = \beta$, $y(0^-) = \alpha$

~~① $G(s) = \frac{L(O)}{L(I)}$ with all IV set to 0~~ } Zero State Soluⁿ ⇓ Both natural and forced modes included.

$G(s) = \frac{b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$

② Obtain U(s)

③ $\mathcal{L}^{-1}(G(s) \cdot U(s)) = y_{\text{Zero.State}}(t)$

④ Set u(t) = 0

Find the Zero Input Soluⁿ

$\boxed{\text{⑤ Soluⁿ = Z. Input Soluⁿ + Z. State Soluⁿ}}$

Lecture 4

Transfer Functions

⇓

Soluⁿ divided as Z State Soluⁿ + Z. Input Soluⁿ

$G(s) = \frac{N(s)}{D(s)}$; Zeros = $\{z \in \mathbb{C} : N(z) = 0\}$

Poles = $\{p \in \mathbb{C} : D(p) = 0\}$

$\ddot{y} + 3\dot{y} + 2y = \dot{u}$; $y(0^-) = 0$

$\dot{y}(0^-) = -5$

$u(t) = 10 \cdot e^{-3t} \mathbb{1}(t)$

$s^2 Y(s) - s\cancel{y(0)} - \dot{y}(0) + 3(sY(s) - \cancel{y(0)}) + 2Y(s) = \frac{10s}{s+3}$ initial

$Y(s)(s^2 + 3s + 2) + 5 = \frac{10s}{s+3}$ → From initial State

$Y(s)(s^2 + 3s + 2) = \frac{10s}{s+3} - 5$ ← From Input

$= \frac{10s}{(s+3)(s+2)(s+1)} - \frac{5}{(s+2)(s+1)}$

$\frac{C}{s+1} + \frac{D}{s+2} + \frac{E}{s+3} - \left(\frac{A}{s+2} + \frac{B}{s+1}\right) = \frac{-5}{s+1} + \frac{20}{s+2} - \frac{15}{s+3} + \frac{5}{s+2} - \frac{5}{s+1}$

$= \frac{25}{s+2} - \frac{10}{s+1} - \frac{15}{s+3}$

Z. State Soluⁿ: $\underbrace{(20e^{-2t} - 5e^{-t} - 15e^{-3t})}$ + Z. Input Soluⁿ: $\underbrace{(5e^{-2t} - 5e^{-t})}$

$E = \frac{10 \times -3}{-1 \times -2} = -15$

$C = \frac{10 \times -1}{2 \times 1} = -5$

$D = \frac{10 \times -2}{1 \times -1} = 20$

A = 5

B = −5

$y(t) = (25e^{-2t} - 10e^{-t} - 15e^{-3t}) \mathbb{1}(t)$

Natural Response — forced

Exercise

$$\frac{ps+q}{(s+a)^2+b^2} = \frac{ps+q}{(s+a+jb)(s+a-jb)} = \frac{k_1+jk_2}{(s+a+jb)} + \frac{k_3+k_4 j}{(s+a-jb)}$$

$-a+jb$

$-a-jb$

$$ps+q = (k_1+jk_2)(s+a-jb) + (k_3+jk_4)(s+a+jb)$$

$k_2+k_4=0$

$k_2 = -k_4$, $p = k_1+k_3$

$q =$

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* $y(t) = \int_0^t g(\tau)u(t-\tau)d\tau = \int_0^\infty g(t-\tau)u(\tau)d\tau$ (what if $u(\tau) = e^{\alpha\tau}$)

$$= \int_{0^-}^\infty g(t-\tau)\cdot e^{\alpha\tau}d\tau \text{ or } \int_0^\infty g(\tau)\cdot e^{\alpha(t-\tau)}d\tau = e^{\alpha t}\cdot\int_0^\infty g(\tau)\cdot e^{-\alpha\tau}d\tau$$

$= y(t) = G(\alpha)\cdot e^{\alpha t}$

* $\dddot{y} + a_2\ddot{y} + a_1\dot{y} + a_0 y = \dot{u} + \beta u$, $u = e^{\alpha t}$

$(\alpha^3 + a_2\alpha^2 + a_1\alpha^2 + a_0)G(\alpha) = (\alpha+\beta)$

$$\boxed{G(\alpha) = \frac{\alpha+\beta}{\alpha^3 + a_2\alpha^2 + a_1\alpha^2 + a_0}}$$

* $$\frac{2se^{-5s} + 3se^{-2s} + s}{(s+1)(s+2)(s+3)(s+4)} = G(s) = e^{-5s}\left[\frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} + \frac{D}{s+4}\right] + e^{-2s}\left[\quad\right] + 1\cdot\left[\quad\right]$$

$\cancel{e^{-5s}\left(\frac{A}{(s+1)(s+2)(s+3)}\right)}$

Q $\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t)$, $y(0^-) = 0$ $\dot{y}(0^-) = 0$

$Y(s).s^2 + 12Y(s).s + 32Y(s) = \frac{32}{s}$

$Y(s) = \frac{32}{s(s^2+12s+32)} = \frac{32}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8}$

$= \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$

A = 1

B = -2

C = 1

$y(t) = u(t) - 2e^{-4t}u(t) + e^{-8t}u(t)$

$= u(t)(1 - 2e^{-4t} + e^{-8t})$

Q $F(s) = \frac{10}{s(s+2)(s+3)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+3)} = \frac{5}{9s} - \frac{5}{s+2} + \frac{40}{9(s+3)} + \frac{10}{3(s+3)^2}$

$\frac{10}{s^2+2s} = C + D(s+3) \Rightarrow C = \frac{10}{9-6} = \frac{10}{3}$

$= \left(\frac{5}{9} - 5e^{-2t} + \frac{40}{9}e^{-3t} + \frac{10t}{3}e^{-3t}\right)u(t)$

$\frac{-10}{(s^2+2s)^2} \times (2s+2) = D = \frac{-10 \times -4}{9} = \frac{40}{9}$

$\frac{10}{2\times 9} = A = \frac{5}{9}$

$B = \frac{10}{-2\times 1} = -5$

Lecture 5

$Y(s) = G(s).U(s)$

$y(t) = \mathcal{L}^{-1}[G(s).U(s)]$

- If we assume U(s) to be proper ⇒ It doesnot have impulse; bounded

⇓

① Different roots $= e^{p_i t}$

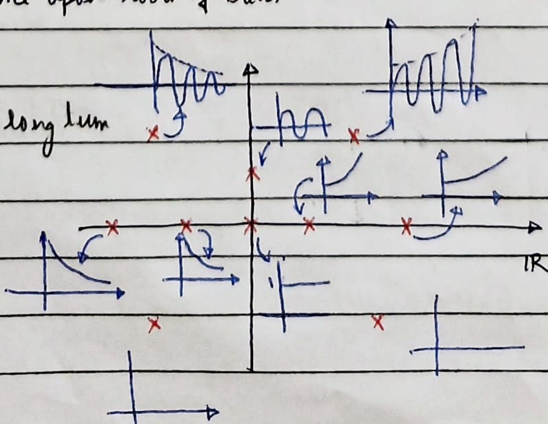
② Repeat roots $= \frac{t^{k-1}}{k!} e^{p_i t}$

Dependence of response upon roots of Du(s)

- $G(s).U(s) = \frac{N_u(s)}{D_u(s)}$
- Due to repeated roots, long term response is unaffected

⇓

~~except when it lies~~ on Im-axis. Then it can blow up.



Analysis of Laplace Transforms of Electrical circuits

- **Resistance**

$$R \quad = V = RI \Rightarrow V(s) = I(s)\cdot R$$

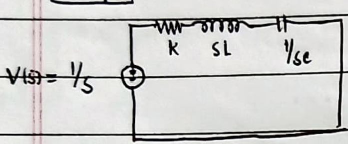
- **Inductor**

$$= V = L\frac{di}{dt} \Rightarrow V(S) = L\cdot s\, I(s)$$

- **Capacitors**

$$= i = C\frac{dv}{dt} \Rightarrow I(s) = C\, s\, V(s)$$

Example:



$$\left(R + SL + \frac{1}{SC}\right) I(s) = V(s) = \frac{1}{S}$$

$$I(s) = \frac{1}{RS + S^2 L + \frac{1}{c}} = \frac{1/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{1/L}{\left(s^2 + \frac{R}{L}s + \frac{R^2}{4L^2}\right) + \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)} \cdot \frac{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$$

$$I(s) = \frac{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}{\left[\left(s^2 + \frac{R}{L}s + \frac{R^2}{4L^2}\right) + \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)\right]\cdot\left(\sqrt{\frac{L}{c} - \frac{R^2}{4}}\right)}$$

Assume; $\frac{1}{LC} - \frac{R^2}{4L^2} > 0$

$$I(t) = \frac{e^{-\frac{R}{2L}t}}{\left(\frac{L}{c} - \frac{R^2}{4}\right)^{1/2}} \cdot \sin\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\; t\right)$$

\# $\quad \Leftrightarrow \quad R$ (a to b)

Time Domain $\qquad$ S-Domain

\# $\quad (a \text{ to } b) \Leftrightarrow \quad I(s),\ Li(0^-)$

$$V(s) = L\left(L\frac{di}{dt}\right) \to L\left(s\, I(s) - I(0^-)\right)$$

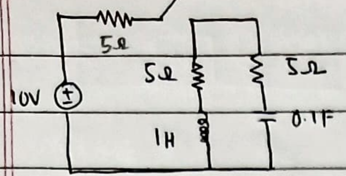
$$V_{ab} = L\frac{di}{dt} \qquad V_{ab}(s) = sLI(s) - Li(0^-)$$

\# $\quad i,\ c \Leftrightarrow \quad +\ -\quad V(0^-)/s$

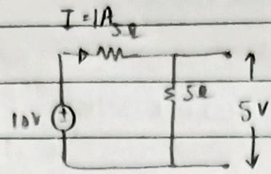
$$C\frac{dv}{dt} = i \qquad CsV(s) - CV(0^-) = I(s)$$

$$V(s) = \frac{I(s)}{Cs} + \frac{1}{s}V(0^-)$$

#

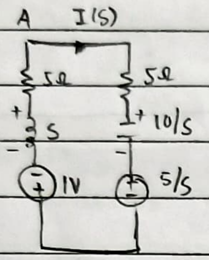


Before the current being thrown off,



$V_c(0) = 5V$

$I_L(0) = 1A$

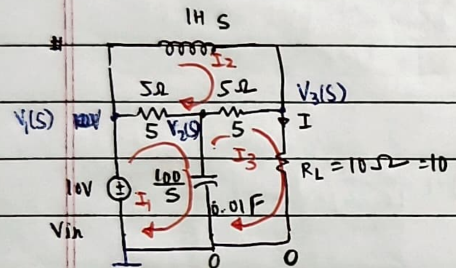


$V_A - I(S)\times 5 - I(S)\times\frac{10}{S} - \frac{5}{S} - 1 - S\,I(S) - 5\times I(S) = V_A$

$I(S)\left[10 + \frac{10}{S} + S\right] = 1 + \frac{5}{S}$

$I(S) = \dfrac{1+\frac{5}{S}}{10+\frac{10}{S}+S} = \dfrac{S+5}{S^2+10S+10}$

#



T.fn between $V_{in}(S)$ & $I_{RL}(S)$ ✓

① By KVL

$$\begin{array}{ccccc} B & & A & & X \\ \begin{bmatrix} V(S) \\ 0 \\ 0 \end{bmatrix} & = & \begin{bmatrix} \frac{100}{S}+5 & -5 & -100 \\ -5 & 10+S & -S \\ -100 & -5 & 15+\frac{100}{S} \end{bmatrix} & & \begin{bmatrix} I_1(S) \\ I_2(S) \\ I_3(S) \end{bmatrix} \end{array}$$

$I_3 = \det \begin{bmatrix} \frac{100}{S}+5 \\ \\ \end{bmatrix}$

• Diagonal elements are sum of all impedances
• Other element have −(common impedances)

⇓

~~Transfer function is not improper~~

~~We~~

② By KCV

$V_1(S) = V_{in}(S) = 10$

$I_{RL} = V_3(S)/R_L$

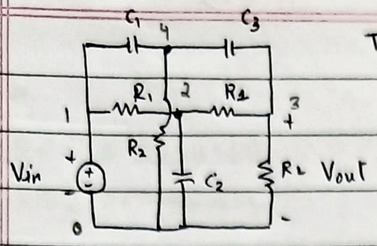
$\dfrac{V_2(S)-0}{\frac{100}{S}} + \dfrac{V_2-V_1}{5} + \dfrac{V_2(S)-V_3(S)}{S} = 0$

$\dfrac{V_3(S)-V_1(S)}{5} + \dfrac{V_3(S)-V_2(S)}{S} = I_{R_L}$

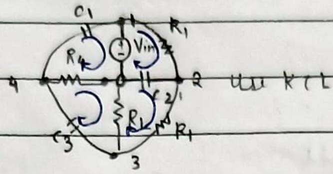
$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & & \\ & -\frac{1}{5} & \\ & & -\frac{1}{10} \end{bmatrix} \begin{bmatrix} 10 \\ V_2 \\ V_3 \end{bmatrix}$

$\frac{S}{100} + \frac{1}{5\times 10}$

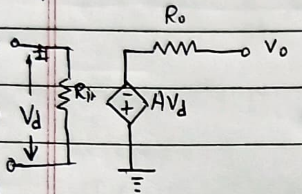
Lecture-6

* 

Twin T



use KCL

\# 

$A \to \infty$

$R_0 \to 0$

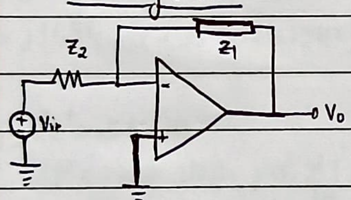
$R_{in} = \infty$

$V_0 = AV_d = A(V_+ - V_-)$

$V_+ - V_- = \frac{V_0}{A} = 0$ (as $A \to \infty$)

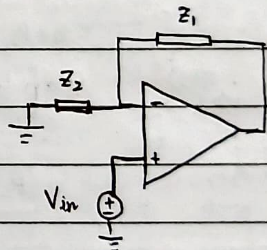
$V_+ = V_-$ [Virtual Short]

Inverting Mode



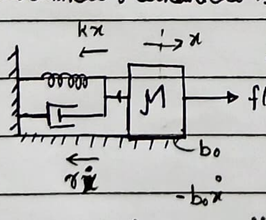
$$\boxed{\frac{V_0(S)}{V_{in}(S)} = -\frac{Z_1(S)}{Z_2(S)}}$$

Non Inverting Mode



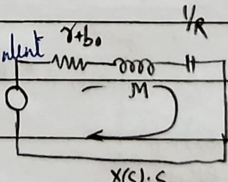
$$\boxed{\frac{V_0(S)}{V_{in}(S)} = 1 + \frac{Z_1(S)}{Z_2(S)}}$$

\# Translational Mechanical System:-



$\rightarrow F(S)$

Series Equivalent



$f(t) - \gamma\dot{x} - kx = m\ddot{x}$

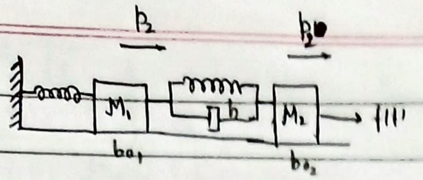
$f(t) = m\ddot{x} + kx + \gamma\dot{x} + b_0\dot{x} = m\ddot{x} + (b_0+\gamma)\dot{x} + kx$

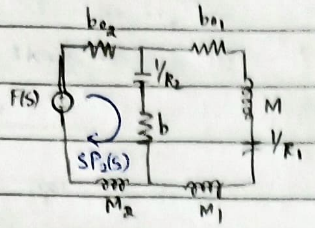
$F(S) = (MS^2 + (b_0+\gamma)S + k)X(S)$

$F(S) = SX(S)\left[MS + (b_0+\gamma) + \frac{k}{S}\right]$ $\quad V(S) \leftrightarrow F(S)$

$V(S) = I(S)\cdot\left[SL + R + \frac{1}{CS}\right] \Rightarrow$

Note: For a mech system we can always get a elec system, Conversely it not true
↓
when Inductor is shared

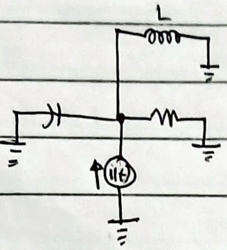




Parallel Analogy: Protects the Physical Insight

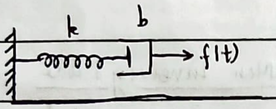
$I(S)=V(S)\left[\frac{1}{R}+\frac{1}{SL}+SC\right]$

$F(S)=SP(S)\left(b+MS+\frac{k}{s}\right)$



$I(S)=\left(\frac{1}{R}+CS+\frac{1}{SL}\right)V(S)$

$F(S)=\left(b+MS+\frac{k}{s}\right)SP(S)$

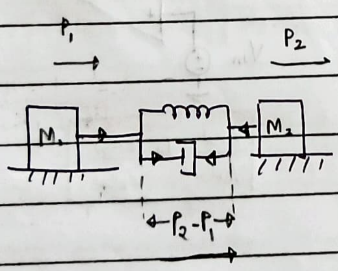
#


$F(S)=bs\,P_2(S)$

$F(S)=k\,P_1(S)$

$P_1+P_2=P(S)$

$F(S)\left[\frac{1}{k}+\frac{1}{bs}\right]$

#


$\cancel{M_2\ddot{p}_2=-k(p_2-p_1)-b(\dot{p}_2-\dot{p}_1)}$

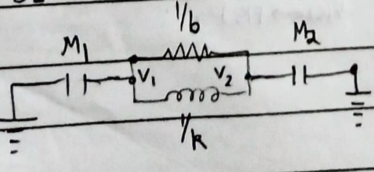
$\cancel{s^2M_2P_2(S)+k(P_2(S)-P_1(S))+bs(P_2(S)-P_1(S))=0}$

$s^2M_1P_1(S)-k(P_2(S)-P_1(S))-bs(P_2(S)-P_1(S))=0$

$M_1\ddot{p}_1=k(p_2-p_1)+b\cdot$

$(S\,P_2(S))SM_2+\frac{k}{s}\cdot S(P_2(S)-P_1(S))+b\,(S)(P_2(S)-P_1(S))$

$V_2(S)SC_2+\frac{1}{SL}(V_2-V_1)+\frac{1}{R}(V_2-V_1)$



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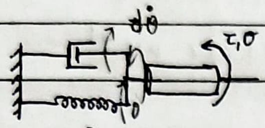
Lecture-7

Rotational Mechanical System

- Inertia — J $\boxed{J\ddot{\theta} = \tau}$, eqv. to mass
- d $\boxed{d\dot{\theta} = \tau}$
- $\boxed{K\theta = \tau}$



$$\tau_1 - d_1\dot{\theta}_1 - k\theta_1 = J\ddot{\theta}_1$$

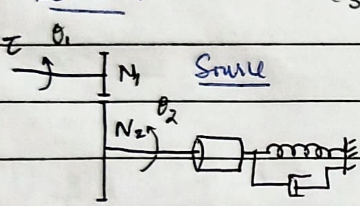
$$\tau(s) = (Js^2 + ds + k)\,\theta(s)$$

$$= \left(sJ + d + \frac{k}{s}\right) s\,\theta(s) = \left(sJ + d + \frac{k}{s}\right)\Omega(s) = \tau(s)$$

$$\left(sL + R + \frac{1}{Cs}\right) I(s) = V(s) \text{ (Series Equivalent)}$$

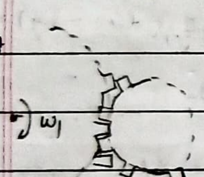
$$\left(\frac{1}{sL} + \frac{1}{R} + Cs\right) V(s) = I(s) \text{ (Parallel Equivalent)}$$

Destination



Source

→ This is Analogous to a transform

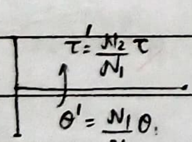


$$N_1 = \frac{2\pi r_1}{2c}$$

$$N_2 = \frac{2\pi r_2}{2c}$$

$$\boxed{\frac{N_1}{N_2} = \frac{r_1}{r_2} = \frac{\omega_2}{\omega_1} = \frac{\tau_1}{\tau_2}}$$

1. Tooth width $= c$
2. $\omega_1 r_1 = \omega_2 r_2$
3. $\tau_1\omega_1 = \tau_2\omega_2$



$$\tau' = \frac{N_2}{N_1}\tau$$

$$\theta' = \frac{N_1}{N_2}\theta_1$$

$$\frac{N_2}{N_1}\tau_1 - k\frac{N_1}{N_2}\theta_1 - d\dot{\theta}_1\frac{N_1}{N_2} = J_L\ddot{\theta}_1\frac{N_1}{N_2}$$

$$\left(\frac{N_2}{N_1}\right)^2 \tau(s) = \left(\frac{k}{s} + d + sJ_L\right)\Omega(s)$$

$$\tau(s) = \left[\underbrace{J_L\left(\frac{N_1}{N_2}\right)^2}_{J_L^{eff}} s^2 + \underbrace{d\left(\frac{N_1}{N_2}\right)^2}_{d^{eff}} s + \underbrace{k\left(\frac{N_1}{N_2}\right)^2}_{k^{eff}}\right]\theta(s)$$

$$\left(\frac{\text{Dest}^{n}}{\text{Source}}\right)^2$$

$\frac{N_L}{N_m} = \frac{T_L}{T_M}$

classmate

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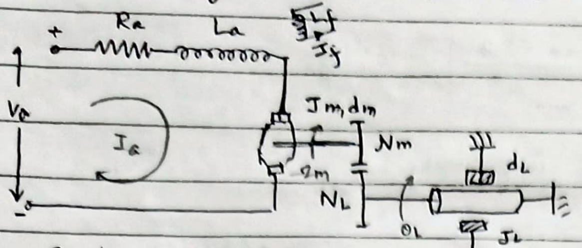
Electromechanical System (Armature Control of DC Motors)



$\frac{\theta_m}{\theta_L} = \frac{N_L}{N_m}$

- Transfer function : $V_a(S) \rightarrow \theta_L(S)$

$V_a(S) = (R_a + SL_a) I_a(S) + \boxed{V_b(S)}$ → Back EMF : Electrical Part

$V_a(S) = (R_a + SL_a) \frac{T_m(S)}{K_a} + K_b \cdot S\, \theta_L(S) \frac{N_L}{N_m}$

$V_b(S) = K_b\, \Omega_m(S)$

$J_L + J_m \times \left(\frac{N_L}{N_m}\right)^2 = J_{eqv}$

$d_L + d_m \left(\frac{N_L}{N_m}\right)^2 = d_{eqv}$

$\left[\left(J_L + J_m\left(\frac{N_L}{N_m}\right)^2\right)s^2 + \left(d_L + d_m\left(\frac{N_L}{N_m}\right)^2\right)s\right] \theta_L(S) = T_L(S)$: Mech^n Part

$T_M(S) = K_a\, I_a(S)$ Electromagnetic Part

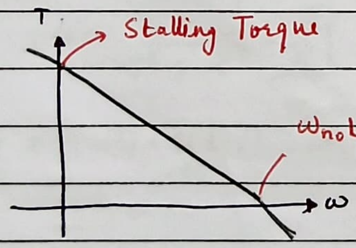
$V_a(S) = \frac{N_m}{N_L} \frac{(R_a + SL_a)}{K_a} (J_{eq}s^2 + d_{eq}s)\,\theta_L(S) + K_b S \cdot \theta_L(S) \frac{N_L}{N_m}$

$\frac{\theta_L(S)}{V_a(S)} = \frac{1}{\left(\frac{N_m}{N_L}\left(\frac{R_a + SL_a}{K_a}\right)(J_{eq}s^{\cancel{2}} + d_{eq}\cancel{s})\,\theta + K_b \frac{N_L}{N_m}\cancel{s}\right)s} = \frac{1}{\left(\frac{R_a N_m}{K_a N_L}\left(1 + \frac{S L_a}{R_a}\right)(J_{eq}s + d_{eq}) + K_b \frac{N_L}{N_m}\right)s}$

Generally
$R_a >> L_a S$

$= \frac{\frac{N_L}{N_m} \cdot \frac{K_a}{J_{eq} R_a}}{s\left(s + \frac{d_{eq}}{J_{eq}} + \frac{K_b K_a}{J_{eq} R}\right)}$

$v_a(t) = K_b\, \omega(t) + R_a \frac{T_m(t)}{K_a}$



From these two we get $\frac{K_a}{R_A}$

(Ramp Response
Parab' Response)

Lecture-8

* **Step Response**
$\Downarrow$
Transient Response
$\Downarrow$
① 1st Order System
② 2nd Order System

1st Order System

- $G(S) = \frac{a}{S+a}$ $\boxed{a>0}$

Step Resp^n: $Y(S) = \frac{G(S)}{S} = \frac{a}{(S+a)S} \Rightarrow y(t) = 1(t)[1 - e^{-at}]$

$\lim_{t\to\infty} y(t) = \lim_{S\to 0} S \cdot \frac{1}{S} \times \frac{a}{S+a} = 1$ G(S) has dc gain equal to unity

- **Time Constant** (t_c)
$t_c = \frac{1}{a}$

- ~~Settling~~ Rise Time (t_r) : 10% to 90% of the steady state value

$$\left.\begin{array}{l}(1 - e^{-at_1}) = 0.1 \\ 1 - e^{-at_2} = 0.9\end{array}\right\} \begin{array}{l} t_1 = -\frac{1}{a}\ln(0.9) \\ t_2 = -\frac{1}{a}\ln(0.1)\end{array} = t_2 - t_1 = \frac{\ln 9}{a} = \frac{2.2}{a} = t_r$$

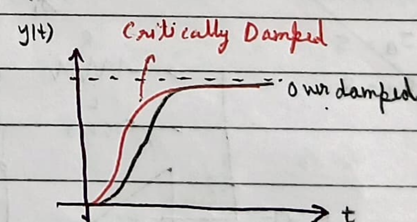
- Settling Time (t_s) : when the system reaches 2% of the Steady-State value.

$$1 - e^{-at_s} = 0.98$$
$$0.02 = e^{-at_s}$$
$$\boxed{t_s = \frac{\ln 50}{a}} \approx \frac{4}{a}$$

2nd Order System

- $G(S) = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$

ξ → Damping Coeff.
ω_n → Natural Frequency



① $\xi > 1$ (Overdamped)

$p_1, p_2 = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} = -(\xi \pm \sqrt{\xi^2 - 1})\omega_n$
$\Downarrow$
LHP

$G(S) = \frac{p_1 p_2}{(S-p_1)(S-p_2)}$

root closer to origin are dominant
$\Downarrow$
Closer the pole to the origin slower the response

$$\mathcal{L}^{-1}\left[\frac{1}{S}G(S)\right] = \mathcal{L}^{-1}\left(\frac{p_1 p_2}{(S-p_1)(S-p_2)S}\right) = \mathcal{L}^{-1}\left[\frac{\frac{p_2}{p_1 - p_2}}{S-p_1} + \frac{\frac{p_1}{p_2 - p_1}}{S-p_2} + \frac{1}{S}\right]$$

$$y(t) = 1(t)\left[1 + \frac{1}{p_1 - p_2}\left[p_2 e^{p_1 t} - p_1 e^{p_2 t}\right]\right]$$

② $\zeta = 1$ (Critically Damped)

$$\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{1}{s} + \frac{-1}{s-p_1} + \frac{p}{(s-p_1)^2}\right] = 1(t)\left[1 - e^{p_1 t} + pte^{p_1 t}\right]$$

$$\frac{1}{s}\frac{p^2}{(s-p)^2} = \frac{1}{s} + \frac{B}{s-p} + \frac{C}{(s-p)^2}$$

$$p^2 = (s-p)^2 + B(s-p)s + C$$

$$\downarrow$$

$$0 = 1 + B$$

$$B = -1$$

③ $\zeta < 1$ (Underdamped)

$$p_1, p_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$|p_1| = |p_2| = \omega_n$$

$$\frac{1}{s}G(s) = \frac{1}{s} \times \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\zeta\omega_n + \omega_n^2} = \frac{1}{s} - \frac{(s + 2\zeta\omega_n)}{(s + \zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

$$= \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s+\zeta\omega_n)^2 + (\omega_n\sqrt{1-\zeta^2})^2}$$

$$= \frac{1}{s} - \frac{(s + \zeta\omega_n)}{(s+\zeta\omega_n)^2 + (\omega_n(\sqrt{1-\zeta^2})^2} - \frac{\zeta\omega_n}{(\quad)^2 + (\quad)^2}$$

$$1(t)\left(1 - e^{-\zeta\omega_n}\cos(\omega_n\sqrt{1-\zeta^2}t) - \frac{e^{-\zeta\omega_n s}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}t)\right)$$

$$\boxed{A = 1}$$

$$\omega_n^2 = A(s^2 + 2\zeta\omega_n s + \omega_n^2) + (Bs + C)s$$

$$0 = A + B \Rightarrow B = -1$$

$$0 = 2\zeta\omega_n A + C \Rightarrow -2\zeta\omega_n = C$$

Let $\frac{\zeta}{\sqrt{1-\zeta^2}} = \tan\phi$

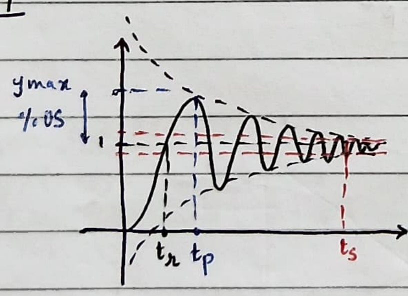
$$y(t) = 1(t)\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \phi)\right]$$

$$\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}} \qquad \boxed{\phi = \sin^{-1}(\zeta)}$$

$$\Downarrow$$

$$\phi > 0$$

Lecture 9



- Time Constant makes no sense here.
- Settling Time: Time after which the curve remains in the 2% error band (how soon the curve settles down)
- Rise Time: 0 to 100% for the first time

⇓

how soon does the system react after application of input

- $\%OS(M_p) = \frac{y_{max} - y_{ss}}{y_{ss}} \times 100$
- Peak time: Time when $y = y_{max}$

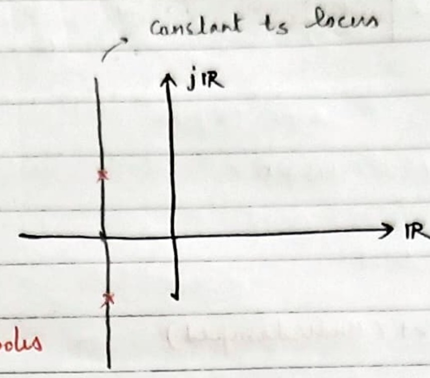
[Consider only the envelope only as a approximation

- For Settling time: is a function of real part of the poles

$$\frac{1-e^{-\zeta\omega_n t_s}}{\sqrt{1-\zeta^2}} = 0.98$$

$$\frac{e^{-\zeta\omega_n t_s}}{\sqrt{1-\zeta^2}} = 0.02$$

$$\boxed{t_s = \frac{1}{\zeta\omega_n}\underbrace{\ln\left(\frac{50}{\sqrt{1-\zeta^2}}\right)}_{x}} \approx \frac{4}{\zeta\omega_n}$$



For $0.1<\zeta<0.85$: $3.9<x<4.7$, so $x\approx 4$

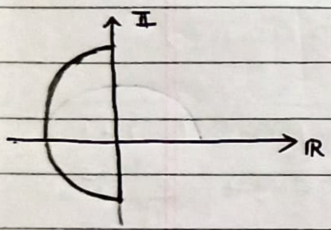
- For Rising time: related to the modulus of the poles

$$\cos(\omega_d t_r - \phi) = \cos(\pi/2)$$

$$\omega_d t_r - \phi = \pi/2$$

$$\omega_n\sqrt{1-\zeta^2}\, t_r = \frac{\pi}{2} + \phi$$

$$t_r = \frac{\frac{\pi}{2} + \sin^{-1}(\zeta)}{\omega_n\sqrt{1-\zeta^2}} \approx \frac{\frac{\pi}{2}+\zeta}{\omega_n\sqrt{1-\zeta^2}} \approx \frac{\pi}{2\omega_n} \approx \frac{1.6}{\omega_n}$$



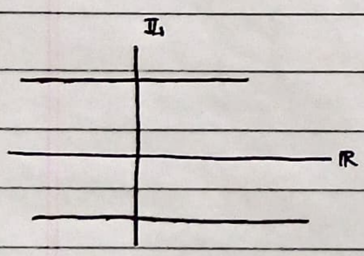
- For %OS (M_p):

$\dot{y}(t) = 0$

$\mathcal{L}^{-1}(G(s)) = 0$

$$\mathcal{L}^{-1}\left(\frac{\omega_n}{\sqrt{1-\zeta^2}} \times \frac{\omega_n\sqrt{1-\zeta^2}}{(s+\zeta\omega_n)^2 + (\omega_n\sqrt{1-\zeta^2})^2}\right) = \frac{\omega_n}{\sqrt{1-\zeta^2}} \times e^{-\zeta\omega_n} \times \sin(\omega_n\sqrt{1-\zeta^2}\, t) = 0$$

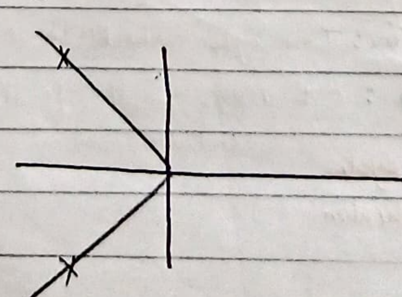
$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$



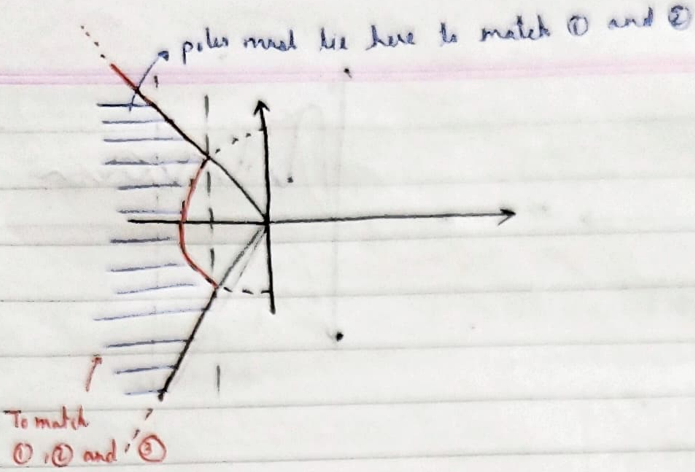
$$y_{max} = \left[1 - e^{-\zeta\omega_n \times \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}} \times \cos\left(\sqrt{1-\zeta^2}\,\omega_n \times \frac{\pi}{\sqrt{1-\zeta^2}\,\omega_n} - \phi\right)\right]$$

$$y_{max} = \left(1 + e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}\cos\phi\right)$$

$$\%OS = 100\, e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$



We have : ① t_r given
② % OS
③ t_s



Padé Approximation

$$P(m,n) = \frac{a_0 s^m + a_1 s^{m-1} \cdots a_m}{s^n + b_1 s^{n-1} \cdots b_n}, \quad n \geq m$$

$\approx e^{-sT}$

$$P(1,1) = \frac{a_0 s + a_1}{s + b_1} \approx e^{-sT}$$

$$\frac{(a_0 s + a_1)}{b_1}\left(1 + \frac{s}{b_1}\right)^{-1} = 1 - sT + \frac{s^2 T^2}{2!}$$

⇓

$$P(1,1) = \frac{1 - sT/2}{1 + sT/2}$$

$$P(1,0) = \frac{1}{1 + sT}$$

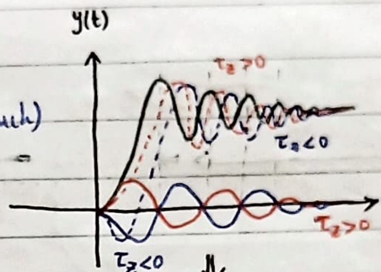
effect of a RHP zero has a delay effect

---

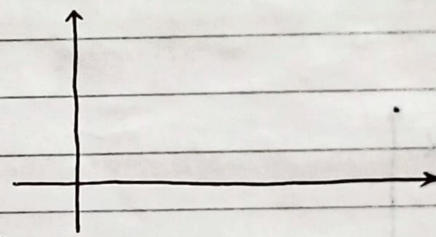
* **Effect of Additional poles and Zeros**

① $G_1(s) = \dfrac{\omega_n^2 (1 + \tau_z s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ (Doesn't change t_s much) but really effect t_r

$y_1(t) = y(t) + \tau_z \dot{y}(t)$



⇓



Case I $\tau_z > 0$

More the τ_z, closer the poles and more the contribution

Case II $\tau_z < 0$ We see a undershoot

② $G_2(s) = \dfrac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(1 + \tau_p s)}$

$$\frac{1}{s} G_2(s) = \underbrace{\frac{A}{s} + \frac{Bs + D}{s^2 + 2\zeta\omega_n s + \omega_n^2}}_{\text{Qualitatively known}} + \frac{D}{s + \frac{1}{\tau_p}} \xrightarrow{\mathcal{L}^{-1}} D \cdot e^{-t/\tau_p} \, 1(t)$$

↓

can be ignored if

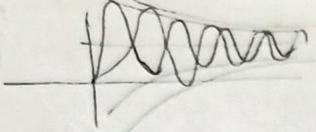
① D is small ($D << \min(A, B, C)$)

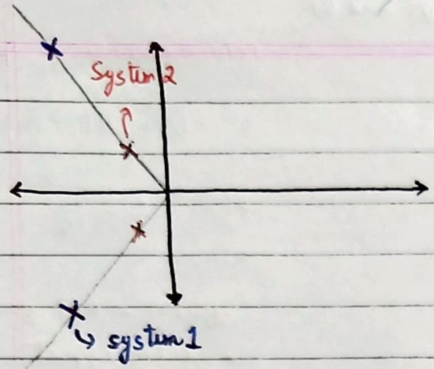
② $\tau_p << \frac{1}{\omega_n \zeta}$ (Difference of 5σ)

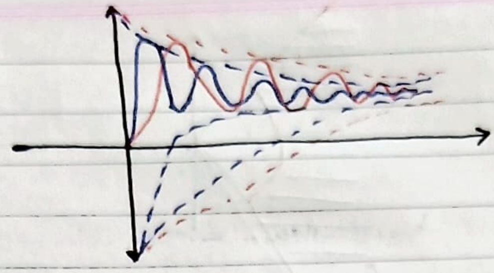
③ $G_3(s) = \dfrac{\omega_n^2 (1 + \tau_z s)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(1 + \tau_p s)}$

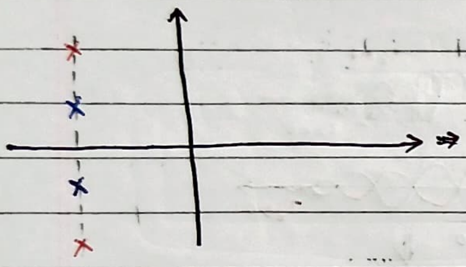
⇓

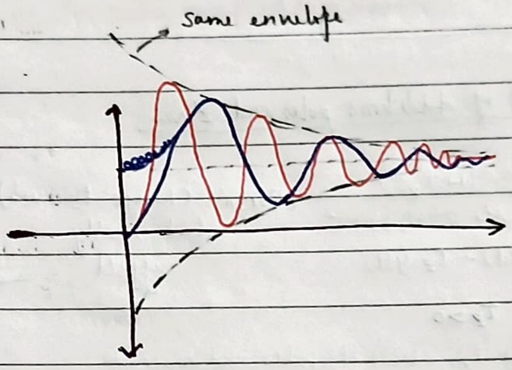
If τ_z and τ_p are close, we get a very good approximate 2nd Order system

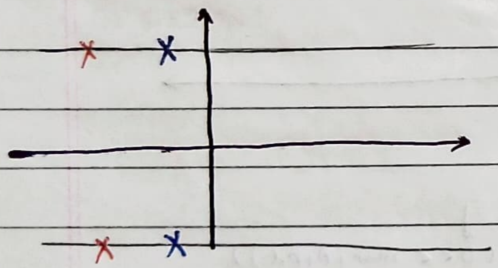


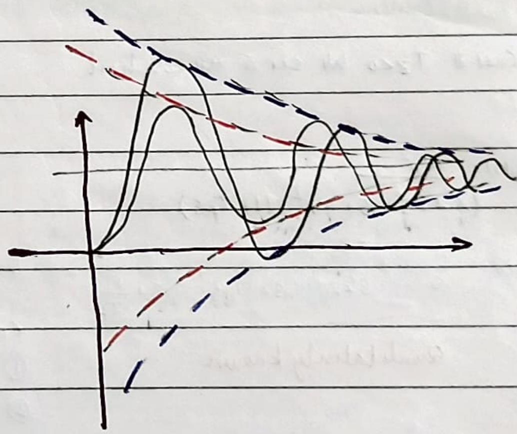

System 2
system 1



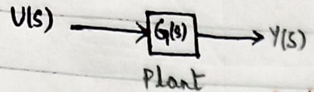



same envelope

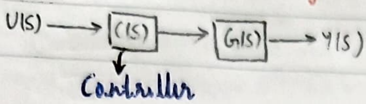




Close loop Control



⇓ For a badly designed Plant, we need a controller



→ This is called Open Loop Control.

Let $G(s) = \frac{(s+2)}{(s+1)(s+3)(s+4)}$ ↳ do not want this pole/zero

↓

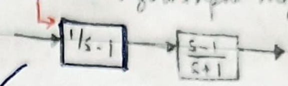
we can define $C(s)$ as $\frac{s+4}{s+2}$ to remove this.

Unobservable

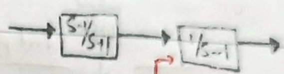
This is a bad idea

⇓

Ex: x_0 ⇒ This zero input might "burn" our system



or

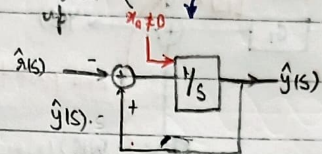


x_0 = uncontrollable

⇓

A zero input will blow this up.

Zero initial input can flow up



$$\frac{\hat{r}(s) + \hat{y}(s)}{s} = \hat{y}(s)$$

$$\frac{\hat{y}(s)}{\hat{r}(s)} = \frac{1}{s-1}$$

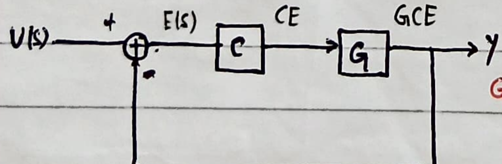
- **Sensitivity:**

x ⇒ Indep. Variable

y ⇒ Dependent Variable

$$\text{Sensitivity} = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} \approx \left(\frac{\partial y}{\partial x}\cdot\frac{x}{y}\right)$$

$$S_{\%OS/\zeta} = \frac{\partial(\%OS)}{\partial \zeta}\cdot\frac{\zeta}{\%OS} = -100\cdot e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times \pi \times \partial\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) \times \frac{\zeta}{100\times e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$



GC is the open loop TF.

$$S_{G_{cl}/G_{ol}} = \frac{\partial}{\partial GC}\left(\frac{GC}{1+GS}\right) \times \frac{GC}{GC}(1+GC)$$

$$= \frac{1}{(1+GC)^{\cancel{2}}} \times \cancel{(1+GC)} = \frac{1}{1+GC} = S$$

$$= \frac{1}{1+G_{OL}}$$

$E = U - GCE$

$E = U - GC(U-Y)$

$\cancel{U} + Y = \cancel{U} + GC(U-Y)$

$(1+GC)Y = GCU$

$\frac{Y}{U} = \frac{GC}{1+GC}$ => $\boxed{\text{Closed Loop Transfer Function}}$

Is $\frac{G_{OL}}{1+G_{OL}}$ Stable?

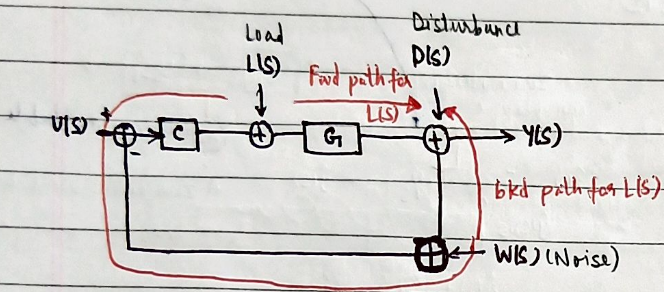
Ex: Take G_{OL} as $k\frac{S-2}{(S+1)(S+2)}$

Poles $\Rightarrow$ $1+\frac{k(S-2)}{(S+1)(S+2)}=0 \Rightarrow (S+2)(S+1)+\underset{\downarrow}{k}(S-2)=0$

can cannot be arbitrarily large

$C(S)=\frac{S+a}{S+b} \Rightarrow |C(j\omega)| = \left|\frac{j\omega+a}{j\omega+b}\right|$

We choose a band of frequencies, and then chose a, b for robustness



$Y(S) = \frac{U(S)\cdot CG}{1+GC} + \frac{G}{1+GC}L(S) + \frac{1}{1+GC}D(S) - \frac{GC}{1+GC}W(S)$

we want this to be unity

① at lower freq

we donot wish this to be unity

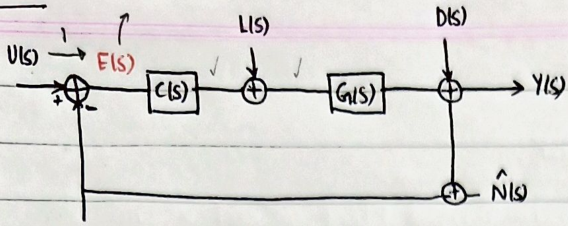
At Higher frequency

$Y(S) = \frac{G_{fw}}{1+G_{fw}\cdot G_{bk}}$

C(S) is such that it is High at lower frequencies and low at Higher frequencies

- Order: Degree of Denominator of TF
- Type of a TF: # of Poles of TF at the origin

Lecture 11 $\quad \mathcal{L}^{-1}(E(S)) = e(t)$



$$G(s) = \frac{50}{(s+1)(s+2)\underbrace{(s+25)}_{}} = \frac{50^{2}}{25\times2(1+s)(1+\frac{s}{2})(1+\frac{s}{25})} \approx \frac{2}{(1+s)(2+s)}$$ → Donot do this if G(s) is in a closed loop form

Very Fast Mode; can be removed — freq will be much less than 25 and can be ignored

$s = \sigma + j\omega$ → freq^n

Neper frequency

$$Y(s) = \underbrace{\frac{GC}{1+GC}}_{} U(s) + \frac{L(s)\cdot G}{1+GC} + \frac{D(s)}{1+GC} - \frac{N(s)GC}{1+GC}$$

DC gain < 1 ; tends to 1

Tracking problem

\# How to ensure : $\lim_{t\to\infty} u(t) - y(t) \to 0$

- If we apply a DC signal, then if we plant a integrator just in front of E(s) to get zero e(t).

- $E(s) = U(s)\cdot\frac{1}{1+GC}$ (Assuming Stable) Suppose $G(0)\,C(0)$ is finite → Type-0

$\Downarrow$

G and C donot have a pole at origin

3 Input:

① $U(s) = 1/s$, $u(t) = 1(t)$

② $U(s) = 1/s^2$, $u(t) = t\,1(t)$

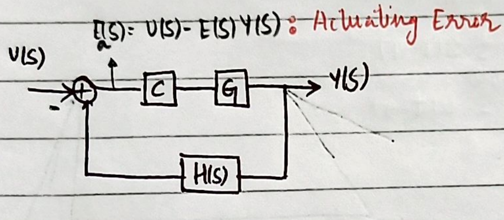
③ $U(s) = 1/s^3$, $u(t) = \frac{t^2}{2} 1(t)$

FVT ① $\lim_{s\to0} sE(s) = \frac{s\times 1/s}{1+GC} = e(\infty)$	FVT ② $\lim_{s\to0} sE(s) = \frac{\cancel{s}\times\frac{1}{s^{\cancel{2}}}}{1+G(s)C(s)} = e(\infty)$	FVT ③ $\lim_{s\to0} sE(s) = \frac{s\times\frac{1}{s^3}}{1+G(s)C(s)} = e(\infty)$
$= \frac{1}{1+G(0)C(0)}$ = constant; $G(0)C(0)$ → K_p = Position Const	$= \lim_{s\to0}\frac{1}{s(1+GC)} \to \boxed{\infty}$	→ $\boxed{\infty}$
	$\lim_{s\to0} sG(s)C(s) = K_v$ = Velocity Const	$\lim_{s\to0} s^2G(s)C(s) = K_a$ = accel^r Const
Now Consider G(s) C(s) to be Type 1 :— $C(s)G(s) = \frac{1}{s}\bar{C}(s)\bar{G}(s)$ ⇒		$\bar{G}(0)\bar{C}(0)$ = finite
$\frac{1}{1+\frac{1}{s}\bar{G}\cdot\bar{C}} = \boxed{0}$ ✓	$= \lim_{s\to0}\frac{1}{s+\bar{C}(s)\bar{G}(s)} = \boxed{\frac{1}{K_v}}$	$= \lim_{s\to0} sE(s) = \boxed{\infty}$
Now Consider Type 2		
$\boxed{\text{Step} = 0}$	$\boxed{\text{Ramp} = 0}$	$\boxed{\text{Parabola} = 1/K_a}$

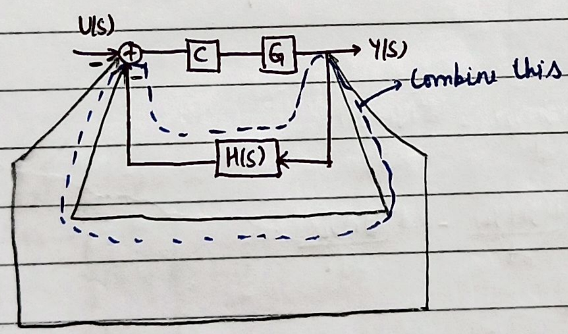
Summary:

i/p \ Type	Step	Ramp	Para
1	$1/1+K_p$	∞ $K_v=0$	∞ $K_a=0$
2	$0, K_p\to\infty$	$1/K_v$	∞ $K_a=0$
3	0 $K_p\to\infty$	0 $K_v\to\infty$	$1/K_a$

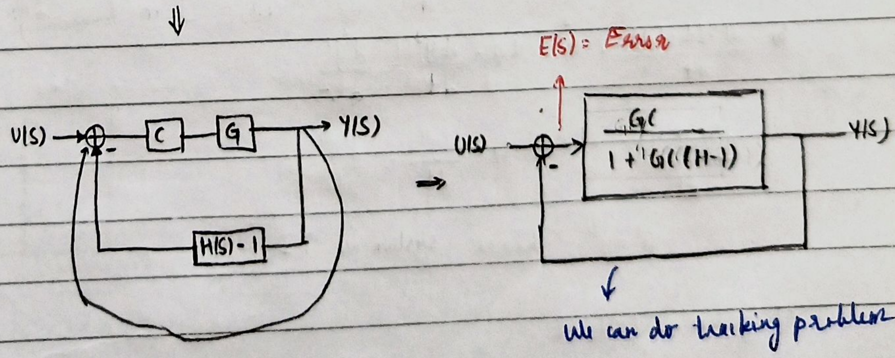
- Suppose we are told the finite value (finite, non zero) of one of the error const, what all info can be infered?
  - Closed loop System is stable
  - The kind of Input that I am interested in
  - The Type of Loop TF.
  - The value of steady State Error



Tracking Problem: Care about error not actuating Error



⇓

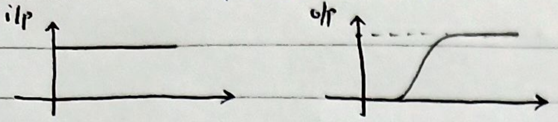


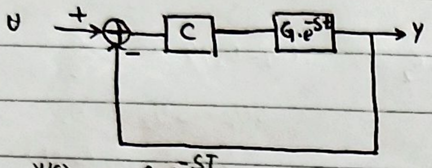
We can do tracking problem

Lecture 12

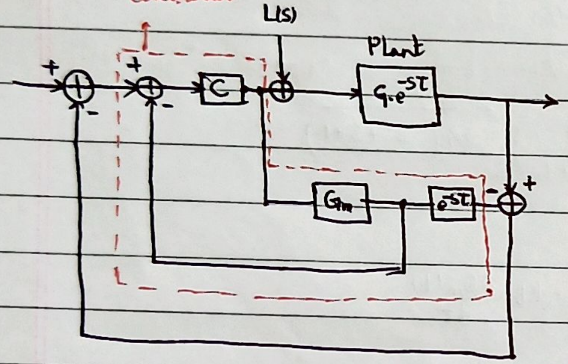
SMITH PREDICTOR

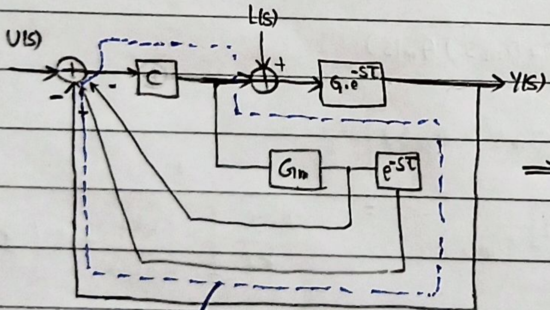
(O.J Smith (1950s))



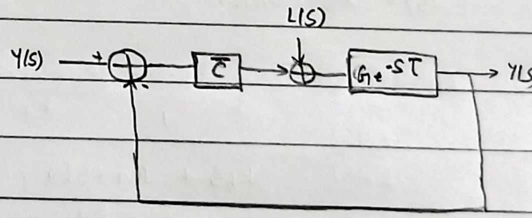


$$\frac{Y(s)}{U(s)} = \frac{G.e^{-sT}C}{1+Ge^{-sT}C}$$ → Trancedental: ∞ root, hence ∞ poles

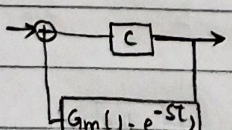




⟹



$$Y(s) = U(s).\left[\frac{Ge^{-sT}\bar{C}}{1+Ge^{-sT}\bar{C}}\right] + L\left[\frac{Ge^{-sT}}{1+e^{-sT}\bar{C}}\right]$$



⟹

Let $C(s) = K_p + \frac{K_I}{s}$ (PI controller)

$$Y(s) = \frac{\dfrac{G.e^{-sT}.c}{1+CG(1-e^{-sT})}}{1+\dfrac{Ge^{-sT}c}{1+CG(1-e^{-sT})}}U(s) + \frac{Ge^{-sT}}{1+\dfrac{e^{-sT}c}{1+CG(1-e^{-sT})}}L(s)$$

Assumpt

$G_m = G$

$\bar{T} = T$

$$\bar{C} = \frac{c}{1+CG_m(1-e^{-sT})}$$

$$Y(S) = \frac{GC \cdot e^{-ST}}{1+CG} U(S) + \frac{G e^{-ST}(1+CG(1-e^{-ST}))}{1+CG} L(S)$$

$\left.\begin{array}{c}\text{Assume } G(S) \text{ is type } 0 \\ \hline GC \text{ is type } 1\end{array}\right\}$ Then we have tracking property; can also reject load disturbance

• Note: If $G(S)$ is Type 1: Cannot reject Load Disturbance

$$E(S) = Y(S) - U(S)$$

$$= \frac{GC(e^{-ST}-1)-1}{1+GC} U(S) + \frac{G \cdot e^{-ST}(1+CG(1-e^{-ST}))}{1+CG} L(S)$$

$$\lim_{S \to 0} \left[\frac{G \cdot (K_P + \frac{K_I}{S})(e^{-ST}-1)-1}{1+(K_P+\frac{K_I}{S})G} \right] + \lim_{S \to 0} \frac{G e^{-ST}(1+G(1-e^{-ST}) \cdot (K_P + \frac{K_I}{S}))}{1+G(K_P+\frac{K_I}{S})}$$

$$= \lim_{S \to 0} \frac{G \cdot (K_P S + K_I)(e^{-ST}-1)-1}{S+(K_P S + K_I)G} + \cancelto{0}{\lim_{S \to 0} \frac{G e^{-ST}\left(1+\frac{G(1-e^{-ST})(K_P S + K_I)}{S}\right) \cdot S}{S + G(S K_P + K_I)}}$$

$$= \lim_{S \to 0} \frac{-1}{G(0) K_I}$$

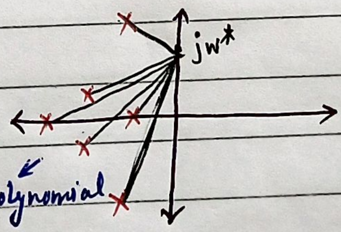
ROUTH- HORWITZ CRITERION

• If a Polynomial has all its roots in open LHP, then it is said to be Horwitz

• $P(S) = a_0 + a_1 S + a_2 S^2 - - - a_n S^n$

$$= a_n \prod_{i=1}^{n} (S - \lambda_i), \ \lambda_i \in \mathbb{R}$$

$$P(jw^*) = a_n \prod_{i=1}^{n} (jw^* - \lambda_i)$$



For a Horwitz Polynomial

$$\sum_{i=1}^{n} \Delta\theta \Big|_{\omega=0}^{\omega=\infty} = \frac{n\pi}{2}$$

$\Downarrow$

Montonically Increasing