EE 302 - Controls

Tut - 1

1 a) $2e^{-2t}\sin(2t)1(t)$

$$\mathcal{L}(2e^{-2t}\sin(2t)1(t)) = 2 \cdot \frac{2}{(s+2)^2+2^2}$$

$$= \frac{4}{s^2+4s+8}$$

b) $t^2 e^{-2t} 1(t)$

$$\mathcal{L}(t^2 e^{-2t} 1(t)) = \frac{2!}{(s+2)^{2+1}}$$

$$= \frac{4}{(s+2)^3}$$

d) $e^{2t}u(t) + e^{-3t}1(-t)$

$$\mathcal{L}(e^{2t}u(t) + e^{-3t}1(-t))$$

$$= \int_{-\infty}^{\infty} e^{-st}(e^{2t}u(t) + e^{-3t}1(-t))\,dt$$

$$= \int_{0}^{\infty} e^{-(s-2)t}\,dt + \int_{-\infty}^{0} e^{-(s+3)t}\,dt$$

for $s > 2$ $\qquad s+3 < 0 \quad s < -3$

$$\frac{1}{s-2} + \frac{-1}{s+3}$$

Function's Laplace transform does not exists.

f $\left[e^{-4t}\sin 3t + 2e^{4t}\sin 3t\right] 1(t)$

$\mathcal{L}(e^{-4t}\sin 3t + 2e^{4t}\sin 3t)$

$$= \frac{3}{(s+4)^2+3^2} + \frac{6}{(s-4)^2+3^2}$$

$$= 3\left\{\frac{(s-4)^2+3^2+(s+4)^2+3^2}{\{(s+4)^2+3^2\}\{(s-4)^2+3^2\}}\right\}$$

$$= 3\left\{\frac{2s^2+50}{\{s^2+8s+25\}\{s^2-8s+25\}}\right\}$$

$$= 3\,\frac{2s^2+50}{(s^2+25)^2-(8s)^2}$$

$$= \frac{3\{2s^2+50\}}{\{s^4+625-14s\}}$$

g $t^2 e^t \sin(5t)\, 1(t)$

$\mathcal{L}(t^2 e^t \sin(5t))$

$e^t \to$ causes $s \to s-1$

$t^2 \to$ causes $\frac{d^2}{ds^2}F(s-1)$

$\therefore \mathcal{L}(t^2 e^t \sin(5t))$

$$= \frac{d^2}{ds^2}\left\{\frac{5}{(s-1)^2+25}\right\}$$

$$= \frac{d^2}{ds^2}\left\{\frac{5(s-1)2(-1)}{((s-1)^2+25)^2}\right\}$$

$$= \frac{d}{ds}\left\{ \frac{-10(\cancel{s-1})}{((s-1)^2+25)^2} + \frac{(-2)(-10)\cdot 2(s-1)^2}{((s-1)^2+25)^3} \right\}$$

$$= \frac{40(s-1)^2}{((s-1)^2+25)^3} - \frac{10}{((s-1)^2+25)^2}$$

$$= \frac{40(s-1) - 10\{s^2-2s+26\}}{(s^2-2s+26)^3}$$

$$= \frac{30(s-1)^2 \cancel{+20s} - 250}{(s^2-2s+26)^3}$$

5 ⓐ $\dfrac{2s+1}{s^4+8s^3+16s^2+s} = F(s)$

$$F(s) = \frac{2s+1}{(s^3+8s^2+16s+1)\,s}$$

$s=0$ is a pole
$s^3+8s^2+16s+1$ has no positive pole (root)

$\therefore$ F(s) is a ~~stable and~~ converging.
since all ~~roots a~~
poles are in the negative.

Using Final Value Thm:-

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) = 0$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} \frac{\cancel{s}\,(2s+1)}{\cancel{s}\,(s^3+8s^2+16s+1)}$$

$$= 1$$

$$\lim_{t\to\infty} f'(t) = \lim_{s\to 0} s^2(sF(s) - F(0^-))$$

$$= \lim_{s\to 0} s\,(sF(s) - 0)$$

$$\lim_{s\to 0} \frac{s\,(2s+1)}{(s^3+8s^2+16s+1)}$$

$$= 0$$

b) $F(s) = \dfrac{2}{s(s^2 - s - 2)}$

$= \dfrac{2}{s(s-2)(s+1)}$

Assuming $\dfrac{A}{s} + \dfrac{B}{s-2} + \dfrac{C}{s+1}$

$A(s-2)(s+1) + Bs(s+1) + C(s(s-2)) = 2$

s^2 : $A + B + C = 0$

s : $-A + B - 2C = 0$

$B = A + 2C$

1 : $-2A = 2$

$A = -1$

$-1 + -1 + 2C + C = 0$

$3C = 2$ $\quad C = \dfrac{2}{3}$

$B = +\dfrac{1}{3}$

$F(s) = \dfrac{-1}{s} + \dfrac{1}{3(s-2)} + \dfrac{2}{3(s+1)}$

$f(t) = \mathcal{L}(F(s))$

$= 1(t)\left(-1 + \frac{1}{3}e^{2t} + \frac{2}{3}e^{-t}\right)$

$f(\infty) = \infty \qquad f'(\infty) = \infty$

c) $F(s) = \frac{5(s+2)}{s(s^2+4)}$

$$= \frac{5}{s^2+4} + \frac{10s}{s^2(s^2+4)}$$

$$= \frac{5}{s^2+4} + \frac{10s}{4}\left(\frac{1}{s^2} - \frac{1}{s^2+4}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{5}{s^2+4} + \frac{5}{2s} - \frac{5s}{2(s^2+4)}\right)$$

$$= \left(\frac{5\sin 2t}{2} + \frac{5}{2} - \frac{5}{2}\cos 2t\right) 1(t)$$

Oscillating $\Rightarrow$ does not converge.

d) $F(s) = \frac{4s^2+3s}{s^2+as-4}$

$s^2+as-4 \rightarrow \frac{-a \pm \sqrt{a^2+16}}{2}$

↳ roots are +ve and −ve

Since the pole is +ve there exsists proper roots.

$$= \frac{4s^2+3s}{s^2+as+\frac{a^2}{4} - \left(4+\frac{a^2}{4}\right)}$$

$$= \frac{4(s^2+as-4) - 4as + 16 + 3s}{\left(s^2+as+\frac{a^2}{4}\right) - \left(4+\frac{a^2}{4}\right)}$$

$$= 4 + \frac{s(3-4a)+16}{\left(s^2+as+\frac{a^2}{4}\right) - \left(4+\frac{a^2}{4}\right)}$$

Page No.:
Date:

$$= 4 + \frac{\left(s+\frac{a}{2}\right)(3-4a) + 16 - \frac{a}{2}(3-4a)}{\left(s+\frac{a}{2}\right)^2 - \left(4+\frac{a^2}{4}\right)}$$

$$= 4 + \frac{\left(s+\frac{a}{2}\right)(3-4a)}{\left(s+\frac{a}{2}\right)^2 - \left(4+\frac{a^2}{4}\right)} + \frac{16 - \frac{3a}{2} + 2a^2}{\left(s+\frac{a}{2}\right)^2 - \left(4+\frac{a^2}{4}\right)}$$

$= \cancel{4 +}$

$$f(t) = \mathcal{L}^{-1}\left\{ 4 + \frac{\left(s+\frac{a}{2}\right)(3-4a)}{\left(s+\frac{a}{2}\right)^2 - \left(4+\frac{a^2}{4}\right)} + \frac{16 - \frac{3a}{2} + 2a^2}{\left(s+\frac{a}{2}\right)^2 - \left(4+\frac{a^2}{4}\right)} \right\}$$

$$= 4\delta(t) + \frac{e^{-\frac{at}{2}}(3-4a)}{\cancel{\sqrt{4+\frac{a^2}{4}}}} \cosh\left(t\sqrt{4+\frac{a^2}{4}}\right)$$

$$+ \frac{\left(16 - \frac{3a}{2} + 2a^2\right)}{\sqrt{4+\frac{a^2}{4}}} e^{-\frac{at}{2}} \sinh\left(t\sqrt{4+\frac{a^2}{4}}\right)$$

~~sin~~

$t \to \infty$

$f(\infty) = 0 + \infty = \infty$

$\cosh x$ is $\frac{e^x + e^{-x}}{2}$ $\qquad \sqrt{\frac{4+a^2}{4}} > \frac{a}{2}$

$\sinh x$ is $\frac{e^x - e^{-x}}{2}$

Both tend to ∞

$f''(\infty)$ is also ∞

Page No.:
Date:

f) $\cancel{\dfrac{6s-7}{s(s^2+\varepsilon s+4)}}$

e) $\dfrac{2s-a}{s(s^2+\varepsilon s+4)}$

$$s^2+\varepsilon s+4 \Rightarrow -\varepsilon \pm \sqrt{\varepsilon^2-16}$$

both roots are neg.

Can use final value thm

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} s\frac{(2s-a)}{s(s^2+\varepsilon s+4)}$$

$$= -\frac{a}{4}$$

$$\lim_{t\to 0^+} f(0^+) = \lim_{s\to\infty} sF(s)$$

$$= \lim_{s\to\infty} \frac{2s^2-as}{s^2+\varepsilon s+4}$$

$$= \cancel{\lim_{s\to\infty}} = 0$$

$$\lim_{t\to\infty} f'(t) = \lim_{s\to 0} s(sF(s)-f(0^+))$$

$$= \lim_{s\to 0} s\left(\frac{2s-a}{s^2+\varepsilon s+4} - 0\right)$$

$$= 0$$

Page No.:
Date:
YOUVA

f $F(s) = \frac{6s-31}{s^2(4s+3)}$

all neg roots

$= 4\left\{\frac{6}{4s(4s+3)}\right\} - \frac{31}{s^2(4s+3)}$

$= 8\left\{\frac{1}{4s} - \frac{1}{4s+3}\right\} - \frac{31}{s^2(4s+3)}$

$= \frac{2}{s} - \frac{8}{4s+3} - \frac{31}{s^2(4s+3)}$

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s F(s)$

$= \lim_{s\to 0} \frac{6s-31}{s(4s+3)}$

$= \lim_{s\to 0} \frac{6}{8s+3}$

$4\left\{\frac{6}{4s(4s+3)}\right\} - \frac{31}{s^2(4s+3)}$

$8\left\{\frac{1}{4s} - \frac{1}{4s+3}\right\} - \frac{31\times 4}{3s \; 3}\left\{\frac{1}{4s} - \frac{1}{4s+3}\right\}$

$= \frac{2}{s} - \frac{8}{4s+3} - \frac{31}{3s^2} + \frac{31\times4\times4}{3\times3}\left\{\frac{1}{4s} - \frac{1}{4s+3}\right\}$

$= \frac{2}{s} - \frac{8}{4s+3} - \frac{31}{3s^2} + \frac{31\times4}{9s} - \frac{16\times31}{9\{4s+3\}}$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{2}{s} - \frac{8}{4s+3} - \frac{31}{3s^2} + \frac{31\times 4}{9s} - \frac{16\times 31}{9(4s+3)}\right\}$$

$$= \left\{2 - 2e^{-\frac{3t}{4}} - \frac{31}{3}t + \frac{31\times 4}{9} - \frac{31\times 4}{9} e^{-\frac{3t}{4}}\right\} \mathbb{1}(t)$$

$f(\infty) = \infty$ as proportional of t

$$f(0^+) = 2 - 2 + \frac{31\times 4}{9} - \frac{31\times 4}{9} = 0$$

$$f'(t) = \mathbb{1}(t)\left\{+\frac{3}{2}e^{-3/4t} - \frac{31}{3} + \frac{31\times 4}{3}\cdot\frac{3}{4}e^{-3/4t}\right\}$$

$$= \mathbb{1}(t)\left\{\frac{3}{2}e^{-3/4t} - \frac{31}{3} + \frac{31}{3}e^{-3/4t}\right\}$$

$$f'(\infty) = -\frac{31}{3}$$

Page No.:
Date:
YOUVA

g) $$\frac{s^2+4s+7}{s^2+s+5} = 1 + \frac{3s+2}{s^2+s+5}$$

$$= 1 + \frac{3s+2}{\left(s+\frac{1}{2}\right)^2 + \frac{99}{4}}$$

$$= 1 + \frac{3s+\frac{3}{2}+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{99}{4}}$$

$$= 1 + 3\left\{\frac{s+1/2}{(s+1/2)^2+99/4}\right\}$$

$$+ \frac{1}{2\sqrt{\frac{99}{4}}} \frac{\sqrt{99/4}}{\left(s+\frac{1}{2}\right)^2+\frac{99}{4}}$$

$$f(t) = \mathcal{L}^{-1}\left(1 + \frac{3\{s+1/2\}}{\{s+1/2\}^2+99/4} + \frac{1}{\sqrt{\frac{99}{4}}}\frac{\sqrt{99/4}}{\left(s+\frac{1}{2}\right)^2+\frac{99}{4}}\right)$$

$$= \delta(t) + 3e^{-1/2t}\cos\frac{3\sqrt{11}t}{2} + \frac{1}{3\sqrt{11}}e^{-1/2t}\sin\frac{3\sqrt{11}t}{2}$$

$t \to \infty \quad f(\infty) = 0$

as $e^{-1/2t}$ is decreasing while cos and sin are only oscillating

h) $$f'(t) = \delta\delta(t) + -\frac{1}{2}\left\{3e^{-1/2t}\cos\frac{3\sqrt{11}t}{2} + e^{-1/2t}\sin\frac{3\sqrt{11}t}{2}\right\}$$

$$+ \frac{3\sqrt{11}}{2}\left\{-3e^{-1/2t}\sin\frac{3\sqrt{11}}{2}t + \frac{e^{-1/2t}}{3\sqrt{11}}\cos\frac{3\sqrt{11}t}{2}\right\}$$

$f'(\infty) \to 0$ as again the oscillating function dies

M T W T F S S
Page No.:
Date:
YOUVA

h) $F(s) = \frac{2}{s^2+4s+7}$

$\frac{2}{(s+2)^2+3}$

$F(s) = \frac{2}{\sqrt{3}} \frac{\sqrt{3}}{(s+2)^2+3}$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$

$f(t) = \frac{2}{\sqrt{3}} \sin\sqrt{3}t \quad e^{-2t}$

$f(\infty) = 0 \qquad f'(\infty) = 0$

as the oscillating is suppressed by exponential decay.

i) $F(s) = \left(\frac{s^2+3s-2}{a}\right)\left\{\frac{1}{s} - \frac{1}{s+a}\right\}$

$= \frac{1}{a}\left(\frac{s}{a} + \frac{3}{a} - \frac{2}{s} - \left\{\frac{s^2+as+3s-as-2}{s+a}\right\}\right)$

$= \frac{1}{a}\left(\frac{s}{a} + 3 - \frac{2}{s} - s + \frac{(a-3)s+2}{s+a}\right\}$

$= \frac{1}{a}\left\{3 - \frac{2}{s} + \right.$

i) $\frac{s^2+3s-2}{s(s+a)}$

The pole is neg.

~~FVT~~

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} \frac{s^2+3s-2}{s+a}$$

$$= \frac{-2}{a}$$

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$$

$$= \frac{s^2+3s-2}{s+a}$$

$$= \infty$$

$$f(t) = \mathcal{L}^{-1} \frac{s^2+as+(3-a)s-2}{s(s+a)}$$

$$= \mathcal{L}^{-1}\ 1 + \frac{\cancel{s}(3-a)}{s+a} - \frac{2}{s^2+as+\frac{a^2}{4}-\frac{a^2}{4}}$$

$$= \mathcal{L}^{-1}\ 1 + \frac{3-a}{s+a} - \frac{2}{a/2}\frac{\frac{a}{2}}{\left(s+\frac{a}{2}\right)^2-\frac{a^2}{4}}$$

$$= \mathcal{L}^{-1}\left\{1 + \frac{3-a}{s+a} - \frac{4}{a}\frac{a/2}{\left(s+\frac{a}{2}\right)^2-a^2/4}\right\}$$

$$= \delta(t) + (3-a)e^{-at} - \frac{4}{a}e^{-a/2\,t}\sinh\frac{a}{2}t$$

M T W T F S S
Page No.: YOUVA
Date:

~~i)~~ $\cancel{\frac{as+5}{s^2+3s-2}}$

$$f(t) = \left\{ \delta(t) + (3-a)e^{-at} + e^{-a/2\,t}\left(-\frac{4}{a}\right)\sinh\left(\frac{a}{2}t\right) \right\}$$

$$f(\infty) = \left\{ 0 + 0 + \left(\frac{-4}{a}\right) \frac{1-e^{-at}}{2} \right\}$$

$$= \frac{-2}{a}$$

$$f'(t) = \delta\delta(t) + -a(3-a)e^{-at}$$
$$- \frac{a}{2} e^{-a/2\,t} \left(\frac{-4}{a}\right)\sinh\left(\frac{a}{2}t\right)$$
$$+ \frac{a}{2} e^{-a/2\,t} \left(\frac{-4}{a}\right)\cosh\left(\frac{a}{2}t\right) \Big\}$$

$$f'(\infty) = 0 + 0$$
$$+ 2 \times \frac{1}{2} - 2 \times \frac{1}{2} = 0$$

j) $F(s) =$

M T W T F S S	
Page No.:	YOUVA
Date:	

j) $F(s) = \frac{as+5}{s^2+3s-2}$

$= \frac{as+5}{s^2+3s+\frac{9}{4}-\frac{17}{4}}$

$= \frac{as}{\left(s+\frac{3}{2}\right)^2-\frac{17}{4}} + \frac{5}{s^2+3s+\frac{9}{4}-\frac{17}{4}}$

$= \frac{a\left(s+3/2\right)}{\left(s+\frac{3}{2}\right)^2-\frac{17}{4}} + \frac{5-3a/2}{\left(s+\frac{3}{2}\right)^2-\frac{17}{4}}$

$\Rightarrow f(t) = \mathcal{L}^{-1}(F(s))$

$= a \cosh \sqrt{\frac{17}{4}}\, t \; e^{-3/2 t}$

$+ 2\left(5-\frac{3a}{2}\right)\frac{e^{-3/2 t}}{\sqrt{17}} \sinh \sqrt{\frac{17}{4}}\, t$

$\sqrt{\frac{17}{4}} > \frac{3}{2}$

$\therefore f(\infty)$ diverges

so does $f'(\infty)$

10 a) $G(s) = \frac{k(s-z)}{s-p}$

$= \frac{k(s-p+p-z)}{s-p}$

$= k + \frac{k(p-z)}{s-p}$

$= \cancel{ks}$

$\mathcal{L}^{-1}\left(k + \frac{k(p-z)}{s-p}\right)$

$g(t) = k\delta(t) + k(p-z)e^{+pt}1(t)$

b) for 0^+ $\delta(t) = 0$

$\cancel{y(x) = 0 \quad y(0^+) = k(p-z)}$

For step response

$Y(s) = \frac{1}{s}G(s)$

$= \cancel{\frac{1}{s}\left\{k\delta(t) + k(p-z)e^{+pt}1(t)\right\}}$

$= \frac{1}{s}\left(k + \frac{k(p-z)}{s-p}\right)$

$= \frac{k}{s} + \frac{k(p-z)}{-p}\left\{\frac{1}{s} - \frac{1}{s-p}\right\}$

$= \frac{k}{s} + \frac{k(z-p)}{p}\left\{\frac{1}{s} - \frac{1}{s-p}\right\}$

$y(t) = k + \left(\frac{k(z-p)}{p}\right) - e^{pt}\left(\frac{k(z-p)}{p}\right)$

M T W T F S S
Page No.:
Date:
YOUVA

$$y(t) = \frac{kt}{p} - e^{pt}\left(\frac{k(t-p)}{p}\right)$$

$$= \frac{kt}{p} - e^{pt}\left(\frac{kt}{p} - kt\right)$$

c) ~~$y(\infty) = \frac{kt}{p} \to neg$~~

$$y(t) = k\left(\frac{t}{p} - e^{pt}\left(\frac{t}{p} - t\right)\right)$$

$y(\infty) = \frac{t}{p}$ is neg

$y(0^+) = t$ is pos

~~Th~~ By IVP

11 a) $Y_1(s) = \frac{16}{(s+4)^2}$

$\mathcal{L}^{-1}(Y_1(s))$

$= 16\, t e^{-4t}$

b) $= \frac{256}{(s^2+4s+16)^2}$

$= \frac{256}{(s^2+4s+4+12)^2}$

$= \frac{256}{((s+2)^2+12)^2}$

$$\frac{\cancel{256}}{\cancel{144}} \qquad \frac{2a^3}{(s^2+a^2)^2}$$

$$= \frac{256}{2(12)^{3/2}} \quad \frac{12^{3/2}}{((s+2)^2+(\sqrt{12})^2)^2}$$

$$L^{-1}\left\{ \frac{256}{2(12)^{3/2}} \quad \frac{12^{3/2}}{((s+2)^2+(\sqrt{12})^2)^2} \right\}$$

$$= \frac{256\, e^{-2t}}{2(12)\sqrt{12}} \left\{ \sin(t\sqrt{12}) - \sqrt{12}\, t\cos(t\sqrt{12}) \right.$$

M T W T F S S
Page No.:
Date:
YOUVA

1B (a) $y(t) = 2t\, r(t) + 3r(t-3)$

Linear, time-varying, causal, dynamic

$y_1(t) = 2t\, r_1(t) + 3r_1(t-3)$

time-varying → presence of t term

$r(t)$ can be added or multiplied

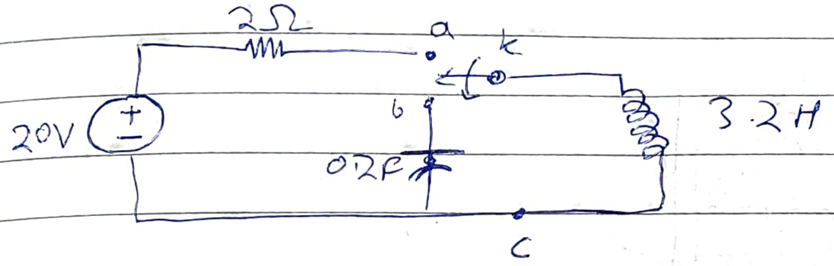
depends on past & present → dynamic and causal

b) $\dot{x} + x(t) = r(t)$ $\quad y(t) = 7\dot{x}(t) + 2x(t)$

$r(t)$ ⇒ linear, time-independent, dynamic causal ~~and~~ depends on defn of differentia

$y(t)$ = linear, time-independent, dynamic causal depends on defn of diff.

16



$I(0^-) = 10 \text{ Amp}$ $\qquad Q(0^-) = 0\,C$

Equivalent in Laplace

$$i(t) = C \frac{d\,V_{cb}}{dt}$$

$$I(s) = C\left(s V_{cb}(s) - \cancel{V_{cb}(0^-)}^{\,0}\right)$$

$$V_{cb}(s) = \frac{I(s)}{Cs}$$

$$V_{bc}(t) = L \frac{dI(t)}{dt}$$

$$\therefore V_{bc}(s) = L\left(s\,I(s) - I(0^-)\right)$$

$$= Ls\,I(s) - 10L$$

KVL: $V_{bc}(s) + V_{cb}(s) = 0$

$$-10(3.2) + 3.2s\,I(s) + \frac{I(s)}{0.2s} = 0$$

$$I(s) = \frac{\cancel{10\times}\ \cancel{32}\ 6.4s}{0.64s^2 + 1}$$

$$= \frac{10s}{s^2 + \cancel{\frac{100}{64}}\frac{25}{16}}$$

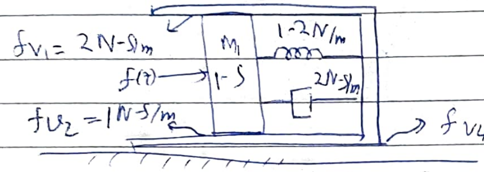
$$I(t) = 10\cos\left(\frac{5}{4}t\right) = 10\cos(1.25t)\,A$$

M T W T F S S

Page No.: | YOUVA

Date:

19



Initially at rest

a) For body 2

$$M_2 \frac{d^2 x_2(t)}{dt^2} + f_{v_4} \frac{dx_2(t)}{dt}$$

$$= k(x_1(t) - x_2(t))$$

$$+ (f_{v_1} + f_{v_2} + f_{v_3})\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right)$$

For body 1

$$M_1 \frac{d^2 x_1(t)}{dt} + k(x_1(t) - x_2(t))$$

$$+ (f_{v_1} + f_{v_2} + f_{v_3})\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right)$$

$$= f(t)$$

b) $$M_2 (s^2 X_2(s)) + f_{v_4}\, s X_2(s)$$

$$= k(X_1(s) - X_2(s))$$

$$+ (f_{v_1} + f_{v_2} + f_{v_3})(s(X_1(s) - X_2(s)))$$

$M_1 s^2 X_1(s)$

$+ (f_{v_1} + f_{v_2} +$

$\frac{3}{2} s^2 X_1(s)$

$+ 5$

$X_1(s) \{$

$3\cdot 1 s^2 X_2(s)$

$+$

$X_2(s) \{ 3\cdot 1 s^2$

$X_2 \{ 3\cdot 1 s^2 + 6$

$\frac{X_2(s)}{F(s)} = \{($

$=$

c) ~~Roots = 0~~ Po

$$M_1 s^2 X_1(s) + k(X_1(s) - X_2(s))$$
$$+ (f_{v_1} + f_{v_2} + f_{v_3})(s X_1(s) - s X_2(s))$$
$$= F(s)$$

$$\frac{3}{2} s^2 X_1(s) + 1.2(X_1(s) - X_2(s))$$
$$+ 5 s X_1(s) - 5s X_2(s) = F(s)$$

$$X_1(s) \left\{ \frac{3}{2} s^2 + 1.2 + 5s \right) = F(s) + 5s X_2(s) + 1.2 X_2(s)$$

$$3.1 s^2 X_2(s) + 1.1 s X_2(s) + 1.2 X_2(s)$$
$$+ 5s X_2(s) = X_1(s) \{ 5s + 1.2 \}$$

$$X_2(s) \{ 3.1 s^2 + 6.1 s + 1.2 \} = \frac{\{5s + 1.2\}}{(1.5s^2 + 1.2 + 5s)} \times (F(s) + 5s X_2(s) + 1.2 X_2(s))$$

$$X_2 \left\{ 3.1 s^2 + 6.1 s + 1.2 - \frac{(5s + 1.2)^2}{1.5s^2 + 5s + 1.2} \right\} = \frac{F(s)\{5s + 1.2\}}{1.5s^2 + 1.2 + 5s}$$

$$\frac{X_2(s)}{F(s)} = \frac{5s + 1.2}{\{(3.1s^2 + 6.1s + 1.2)(1.5s^2 + 5s + 1.2) - (5s + 1.2)^2\}}$$

$$= \frac{5s + 1.2}{4.65 s^4 + 24.65 s^3 + 11.02 s^2 + 1.32 s}$$

c) ~~Roots = 0~~ Poles = 0, −4.82 Zeroes = −0.24

21 $\cancel{J_1 \frac{d^2\theta_1}{dt^2} + D_1}$

$$T(t) = J_1 \frac{d^2\theta_1}{dt^2} + D_1 \frac{d\theta_1}{dt} + k(\theta_1(t) - \theta_2(t))$$

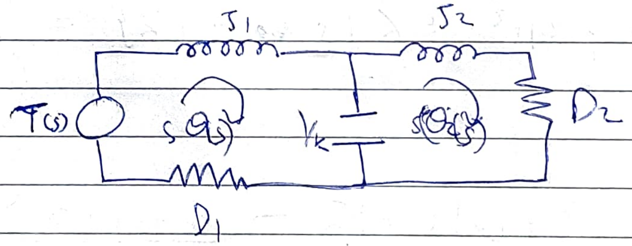
~~kcl~~

$$J_2 \frac{d^2\theta_2}{dt^2} + k(\theta_2 - \theta_1(t)) + D_2 \frac{d\theta_2}{dt} = 0$$

$$T(s) = J_1 s^2 \theta_1(s) + D_1 s \theta_1 + k\theta_1(s) - k\theta_2(s)$$

$$J_2 s^2 \theta_2 + D_2 s \theta_2 + k\theta_2 - k\theta_1(s) = 0$$

Series $V \to T(s)$



KVL in loop 1

$$T(s) - sJ_1(s\theta_1(s)) - \frac{(s\theta_1(s) - s\theta_2(s))}{\frac{s}{k}}$$

$$- s\theta_1(s) D_1 = 0$$

$$T(s) = J_1 s^2 \theta_1(s) + k(\theta_1(s) - \theta_2(s)) + D_1 s \theta_1(s)$$

θ_1

θ_2

$- \theta$

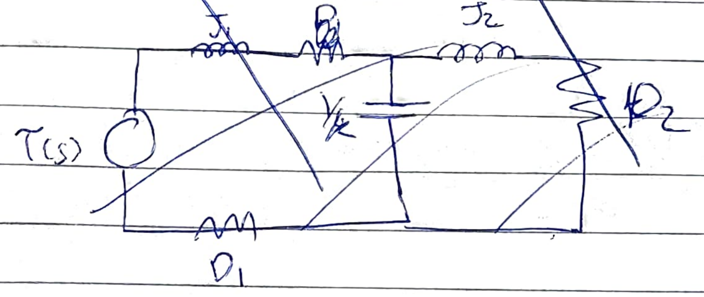
Parallel

$T(s)$

$T(s)$

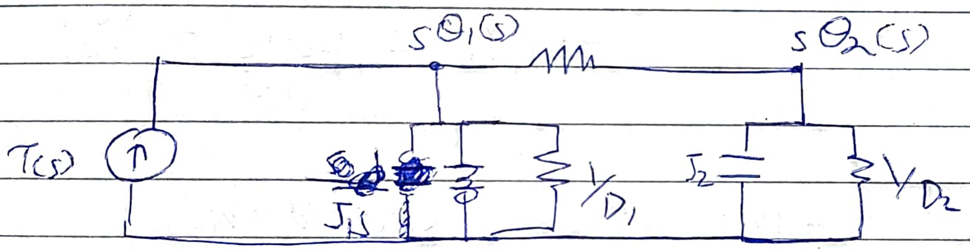
$$\theta_1 \left(J_1 s^2 + D_1 s + k\right) - k\theta_2 = T(s)$$

$$-\theta_1 (k) + \theta_2 \left(J_2 s^2 + D_2 s + k\right) = 0$$



Rough $\left(\frac{1}{J_1 s +}\right)$

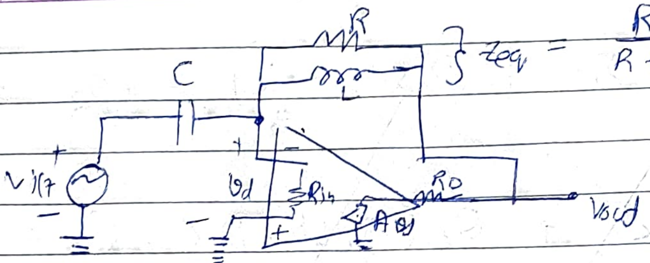
Parallel ~~Vs~~ $T(s) =$ I source



$$T(s) = s\,\theta_1(s) \Big\{ \frac{1}{\cancel{J_1}} \quad \frac{1}{\frac{1}{J_1 s}}$$

Page No.: Date: YOUVA

Q24



$$Z_{eq} = \frac{RLs}{R+Ls}$$

$$\cancel{C\frac{d(V_d - V_{in}(t))}{dt}} +$$

KVL

$$+Av_d - i_1 R_o - i_2 R_o$$
$$- i_1 R - L\frac{di_2}{dt} = v_d$$

$$i_1 + i_2 = \frac{v_d}{R_{in}} + \frac{C\,d(V_d - V_{in})}{dt}$$

$$i_1 R = L\frac{di_2}{dt}$$

Applying ~~k~~ Laplace

$$A V_d(s) - I_1(s) R_o - I_2(s) R_o$$
$$- I_1(s) R - Ls\, I_2 = V_d(s)$$

$$I_1(s) + I_2(s) = \frac{V_d(s)}{R_{in}} + Cs\,V_d(s) - Cs\,V_{in}(s)$$

$$I_1(s) R = Ls\, I_2(s)$$

$$\frac{Ls I_2(s)}{R}$$

$$AV_d(s) -$$

$$(A-1)V_d$$

$$\frac{Ls}{R}(A-1)V$$
$$Cs\left\{\frac{R_o Ls}{R}\right.$$

$$V_d(s)$$

$$V_{in} =$$

$$A V_d$$

M	T	W	T	F	S	S

Page No.:
Date:
YOUVA

$$\frac{Ls I_2(s)}{R} + I_2(s) = \frac{V_d(s)}{R_{in}} + Cs V_d(s) - Cs V_{in}(s)$$

$$A V_d(s) - I_2(s)\left\{\frac{R_o Ls}{R} + R_o + \frac{R Ls}{R} - Ls\right\}$$

$$= V_d(s)$$

$$(A-1) V_d(s) = I_2(s)\left\{\frac{R_o Ls}{R} + R_o\right\}$$

$$\frac{\frac{Ls}{R}(A-1)V_d(s) + (A-1)V_d(s)}{Cs\left\{\frac{R_o Ls}{R} + R_o\right\}} = Cs V_d - Cs V_{in}$$

$$V_d(s)\left\{\frac{\frac{Ls}{R}(A-1)\left(\frac{Ls}{R}+1\right)}{Cs R_o} - 1\right\} = -V_{in}$$

$$V_{in} = V_{ds}\left\{1 - \frac{(A-1)}{Cs R_o}\right\}$$

$$A V_d(s) - V_{out}(s) = R_o\left\{\frac{Ls}{R} + 1\right\}\frac{(A-1)V_{ins}}{R_o\left(1+\frac{Ls}{R}\right)}$$

$$V_d(s) = V_{out}(s)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 - \frac{(A-1)}{Cs R_o}}$$