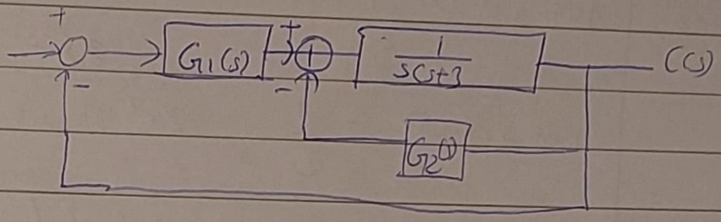
EE302 - Controls Assignement -2

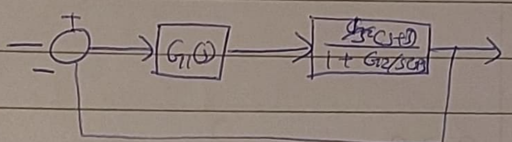
Question 2

Part 4



~~$C(s) = \frac{1}{s(s+3)} G_1(s)$~~

$\Downarrow$



$$C(s) = R(s)\left\{ \frac{G_1(s)\left\{\dfrac{\frac{1}{s(s+3)}}{1+\frac{G_2(s)}{s(s+3)}}\right\}}{1+G_1(s)\left\{\dfrac{\frac{1}{s(s+3)}}{1+\frac{G_2(s)}{s(s+3)}}\right\}} \right\}$$

$$= R(s)\left\{ \frac{\dfrac{G_1(s)}{s(s+3)+G_2(s)}}{\dfrac{s(s+3)+G_2(s)+G_1(s)}{s(s+3)+G_2(s)}} \right\}$$

$$= \frac{R(s) \cdot G_1(s)}{G_1(s) + s(s+3) + G_2(s)}$$

Part 1:

$$C(s) = \frac{R(s)\, G_1(s)}{G_1(s) + G_2(s) + s(s+3)}$$

$\lim_{s\to 0} sC(s) = 1$ for $R(s) = 1$

$$\lim_{s\to 0} s\left(\frac{1}{s}\right)\frac{G_1(s)}{G_1(s)+G_2(s)+s(s+3)}$$

$$\Rightarrow \lim_{s\to 0} \frac{G_1(s)}{G_1(s)+G_2(s)+s(s+3)} = 1$$

$G_1(s) = as + 1$

$$\lim s\left(\frac{1}{s^2} - \frac{1}{s^2}\frac{G_1(s)}{G_1(s)+G_2(s)+s(s+3)}\right) = 1$$

$$\lim \frac{1}{s} \cdot \frac{G_2(s)+s(s+3)}{G_1(s)+G_2(s)+s(s+3)} = 1$$

$G_2(s) = s(s-2)$

Assume.

~~Part 2:-~~

~~for $R(s) = \frac{1}{s}$~~

$\lim s\,C(s) = \frac{1}{2}$

b) $\frac{as+1}{as+1 + s(s-2) + s(s+3)}$ $a = 1$

$\frac{as+1}{2s^2+2s+1} = \frac{s+1}{2s^2+2s+1}$

$\%OS = 4.33\%$

$T_s = 8s$ $T_r = 2.22$

$T_p = 6.28s$

Part 2

The unit ramp and unit step response is exactly $\frac{1}{2}$

$\lim_{s\to 0} s \frac{1}{s} \frac{G_1(s)}{G_1(s) + G_2(s) + s(s+3)} = \frac{1}{2}$

$G_1(s) = \cancel{s(s+3)}$ $s+5$

$G_2(s) = \cancel{s(2s+2)}$ $2s+5$

$C(s) = R(s) \frac{s+5}{s+5+2s+5+s(s+3)}$

$= R(s) \frac{s+5}{s^2+6s+10}$

b)

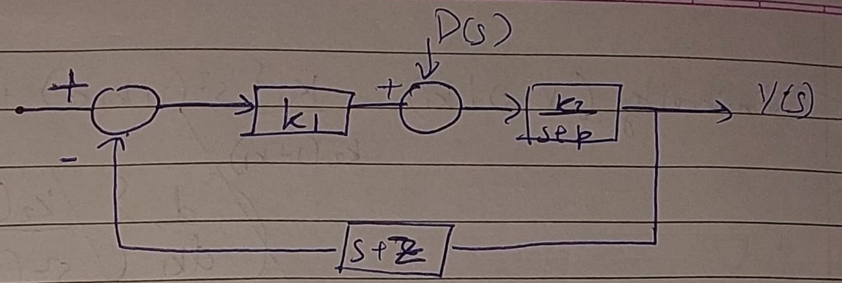
$\%OS = 0.94\%$

Settling time $= 1.33$

peak $= 3.141$

Rise time $= 0.496s$

3



$$Y(s) = \frac{k_2}{s+p}\left\{ D(s) + k_1\left\{ R(s) \mp Y(s)(s+z) \right\} \right\}$$

$$= \frac{k_2 D(s)}{s+p} + \frac{k_1 k_2}{s+p}\left\{ R(s) - Y(s)(s+z) \right\}$$

$$Y(s)\left\{ 1+(s+z)\frac{k_1 k_2}{s+p} \right\} = \frac{k_2}{s+p}\left\{ \frac{1}{s} + \frac{k_1}{s} \right\}$$

$$Y(s) = \frac{k_2(1+k_1)}{\cancel{(s+p)}\, s \left\{ \frac{(s+p)+k_1k_2(s+z)}{\cancel{(s+p)}} \right\}}$$

$$Y(s) = \frac{k_2(1+k_1)}{s\left\{ s(1+k_1k_2) + p + k_1k_2 z \right\}}$$

Steady state error = $\lim_{s\to 0} s\left\{ \frac{1}{s} - \frac{k_2(1+k_1)}{s\{s(1+k_1k_2)+p+k_1k_2 z\}} \right\}$

$$\lim_{s\to 0} 1 - \frac{k_2(1+k_1)}{p+k_1k_2 z}$$

$$1 - \frac{0.1\times 101}{2+10}$$

$$1 - \frac{10.1}{12} = \frac{1.9}{12} = 0.1583.$$

~~a) Sensitivity (Y(s) | K1) =~~ $\cancel{\left\{ \frac{k_2}{s(s(1+k_1k_2)+p+k_1k_2 z} \right.}$

d) ~~Sensitivity~~ $(Y(s) | k_1) = \frac{k_1 \{ s^2(1+k_1k_2) + s(p+k_1k_2 z)\}}{k_2(1+k_1)} \left\{ \frac{d}{dk_1} \left\{ \frac{k_2(1+k_1)}{s^2(1+k_1k_2)+ps+sk_1k_2z} \right\} \right\}$

$$= \frac{k_1 \{ s^2(1+k_1k_2) + s(p+k_1k_2 z)\}}{k_2(1+k_1)} \left\{ \frac{k_2 - \{s^2 k_2}{\{ s^2(1+k_1k_2) + ps + sk_1k_2z\}^2} \right\}$$

a) Sensitivity $(E_{rror} | k_1) = \frac{d}{dk_1}\left(1 - \frac{k_2(1+k_1)}{p+k_1k_2 z}\right) \times \frac{k_1}{err}$

$$= \frac{k_1}{err} \left\{ -\frac{k_2}{p+k_1k_2z} + \frac{k_2(1+k_1)k_2 z}{(p+k_1k_2z)^2} \right\}$$

$$= \frac{k_1}{err} \left\{ \frac{(1+k_1)k_2^2 z - k_2(p+k_1k_2z)}{(p+k_1k_2z)^2} \right\}$$

$$= \frac{100 \times 12}{1.9} \left\{ \frac{1.01 \times 1 - 0.1\{2+10)}{12 \times 12} \right\}$$

$$= \frac{100}{1.9} \left\{ \frac{1.01 - 1.2}{12} \right\}$$

$$= -\frac{100}{1.9} \left\{ \frac{0.19}{12} \right\} = -\frac{10}{12} = -\frac{5}{6} = -0.83$$

b) Sensitivity $(E_{rror} | k_2) = \frac{d}{dk_2}\left(1 - \frac{k_2(1+k_1)}{p+k_1k_2z}\right) \frac{k_2}{err}$

$$= \left\{ -\frac{(1+k_1)}{p+k_1k_2z} + \frac{k_2(1+k_1)k_1 z}{(p+k_1k_2z)^2} \right\} \frac{k_2}{err}$$

$$= \left\{ \frac{-101 \times 12 + 0.1 \times 101 \times 100 \times 1}{12 \times 12} \right\} \quad \frac{0.1}{1.9} \times 12$$

$$= \left\{ \frac{-101 \times \cancel{2}}{\cancel{12}6} \right\} \frac{1}{19}$$

$$= -\frac{101}{1\cancel{0}4} = -0.886$$

Q $P(s) = s^6 - s^4 - 7s^2 + 7s - 6$

Reverse co-eff

s^6	-6	-7	-1	1
s^5	$\cancel{8}1$	0	0	0
s^4	-7	-1	1	
s^3	-1	1	0	
s^2	$-\cancel{8}$	$+1$		
s	$\frac{7}{8}$			
1	-1			

3 R.H.P
3 L.H.P

Epsilon method

s^6	1	-1	-7	-6
s^5	ε	0	7	0
s^4	-1	$\frac{-7\varepsilon-7}{\varepsilon}$	-6	
s^3	$-7\varepsilon-7$	$\frac{6\varepsilon-7}{-1}$		
s^2	$\frac{(7\varepsilon+7)^2-(6\varepsilon-7)}{-(7\varepsilon+7)}$	-6		

$$s \quad \frac{(7-6\varepsilon)\left\{\frac{(7\varepsilon+7)^2-(6\varepsilon-7)}{-(7\varepsilon+7)}\right\} - 6(7\varepsilon+7)}{\frac{(7\varepsilon+7)^2-(6\varepsilon-7)}{-(7\varepsilon+7)}}$$

$s^0 \quad -6$

11(a) Routh table

a) $s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$

s^5	1	3	5
s^4	2	6	3
s^3	ε	$\frac{7}{2}$	0
s^2	$\frac{6\varepsilon - 14}{\varepsilon}$	3	
s	$\frac{7\left(6 - \frac{14}{\varepsilon}\right) - 3\varepsilon}{\left(6\varepsilon - \frac{14}{\varepsilon}\right)}$		
s^0	3		

Reversal

s^5	3	6	2
s^4	5	3	1
s^3	$\cancel{\frac{21}{5}}$ 3	$\cancel{\frac{7}{5}}$ 1	0
s^2	$4/3$	3	
s	$-5/4$		
s^0	3		

2 RHP - 3 LHP

b) $3s^7 + 9s^6 + 6s^5 + 4s^4 + 7s^3 + 8s^2 + 2s + 6$

s^7	3	6	7	2
s^6	9	4	8	6
s^5	$\frac{14}{\cancel{3}}$ 1	$\frac{13}{\cancel{3}}$ 1	0	0
s^4	$\frac{-61}{\cancel{14}}$ 1	14×8	6×14	
s^3	$\frac{2361}{\cancel{61}}$ 1	$\frac{6}{\cancel{61}}$ 1		
s^2	$\frac{112056}{\cancel{787}}$	84×787		
s^1	-216.88			
s^0	84×787			

4 RHP
3 LHP

c) $s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$

s^5	1	6	8
s^4	$\cancel{7} \to 1$	$\cancel{42}$ 6	$\cancel{56}$ 8
s^3	$\cancel{4} \to 1$	$\cancel{12} \to 3$	
s^2	3	8	
s	$\frac{1}{3}$		
s^0	8		

$(s+7)(s^4 + 6s^2 + 8)$

~~stable.~~ $\cancel{(s^2+3)} \to 1$

0 RHP 5 LHP

14

$$\frac{1}{(s+2)(s+4)}$$

$$\frac{s+8}{(s+7)(s+6)(s+2)}$$

15

a) $\frac{1}{(s+2)(s+4)}$

b) $\frac{s-2}{s+2}$

c) $\frac{s-3}{(s-2)(s+3)}$

d) $\frac{(s+2)}{(s+3)(s+7)(s+8)}$

e) $\frac{s+8}{(s+7)(s+2)(s+3)}$

f) $\frac{(s+8)(s+7)}{(s+2)(s+3)}$

g) $\frac{1}{(s+2)(s+3)(s+5)}$

h) $\frac{s^2+4s+5}{(s+2)(s+4)}$

i) $\frac{s^2+4s+5}{s^2+6s+10}$

j)

18

$$y = \frac{CG}{1+CG} = \frac{k}{(s+a)(s+1)+k} = \frac{k}{s^2+(a+1)s+a+k}$$

$\omega_n^2 = a+k$ $\qquad$ $t_s = \dfrac{4}{\zeta\omega} = 5$

$2\zeta\omega_n = a+1$ $\qquad$ $\zeta\omega = \dfrac{4}{5}$

$\dfrac{8}{5} = a+1$ $\qquad$ $\%OS = 10\%$

$a = \dfrac{3}{5}$ $\qquad$ $10 = 100\, e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

$\dfrac{\zeta\pi}{\sqrt{1-\zeta^2}} = \ln 10$

$\omega = \dfrac{4}{\zeta \times 0.591}$ $\qquad$ $\dfrac{\zeta^2\pi^2}{(\ln 10)^2} = 1-\zeta^2$

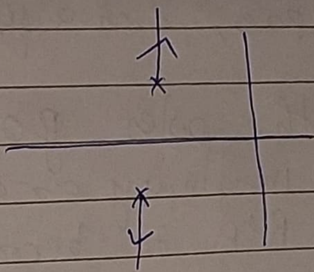
$\omega_n = 1.35$

$\zeta = \dfrac{1}{\sqrt{1+\dfrac{\pi^2}{(\ln 10)^2}}}$

$k = 1.2225$

$\zeta = 0.591$

19 $G(s) = \dfrac{\omega_n^2}{s^2 + 2\delta_n \omega_n s + \omega_n^2}$

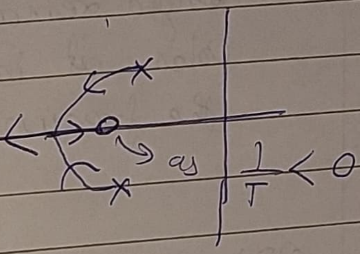


$\omega_n = 2 \quad \delta_n = 0.5$

$p_1 = -1 \pm j\sqrt{0.75}$

$p_2 = -1 - j\sqrt{0.75}$

a) A break in pt becomes.



Poles go towards $-\infty$ till damping increases.

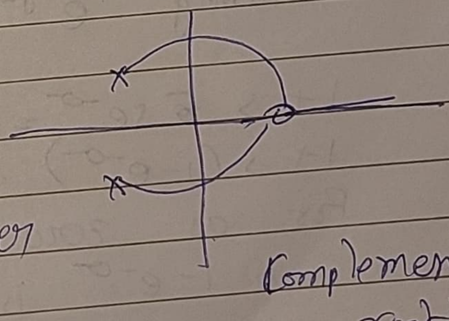
as $\frac{1}{T} < 0$

Then it reaches critical damping.
Then over damped. One pole moves towards zero.
It gets nullified by the zero there by changing the system to 1st order

Settling time is ~~proportional to~~ inversely proportional to the pole nearer to the imaginary axis

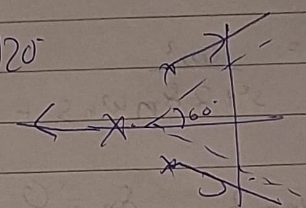
k inc $\rightarrow$ t_s dec till critical time, then increases, untill pole is cancelled by zero,

b) On increasing k, the settling time inc till the poles reach imaginary axis, and then it oscillates ifinitely, after that it is unstable



Complementary root locus

c) 3 poles, each asymptote at 120°



the poles go ~~the~~ to imaginary and then to RHP to become unstable ~~if p~~

d if p_3 moves further left, the median inc, angle remaining same the intersection of poles at imaginary axis increases increasing the frequency of oscillations

Q2)

a) $1 + s + se^{-s} = 0$

assume s to be $\sigma + j\omega$

$1 + s + s e^{-\sigma} e^{-j\omega} = 0$

$|e^{-j\omega}| = 1$ and $e^{-j\omega}$ can be at min -1

assuming $e^{-j\omega}$ to be -1

$1 + s - se^{-\sigma} = 0$ for real part,

$1 + s(1 - e^{-\sigma}) = 0 \Rightarrow 1 + \sigma(1 - e^{-\sigma})$

For as zero in RHP $\sigma > 0$

so $1 - e^{-\sigma}$ is always +ve

so $\sigma(1 - e^{-\sigma})$ is always +ve

$\therefore$ No zero in RHP

b) $p(s) =$

s^3

s^2

s

s^0

For

$2q_1 q_2 + 3$

$(q_1 + 3)$

b) $p(s) = (2.04 + 6q_1 + 6q_2 + 2q_1q_2) + (2+q_1+q_2)s$
$+ (2+q_1+q_2)s^2 + s^3$

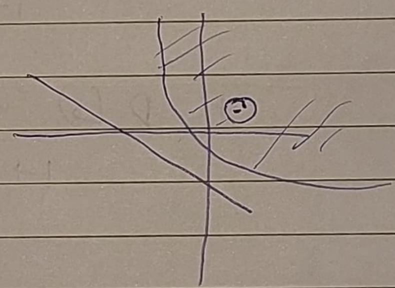
s^3 1 $2+q_1+q_2$
s^2 $2+q_1+q_2$ $2.04+6q_1+6q_2+2q_1q_2$
s $4+q_1^2+q_2^2+2q_1q_2+4q_1+4q_2-2.04-6q_1-6q_2-2q_1q_2$
$\Rightarrow 1.96 + q_1^2 + q_2^2 - 2q_1 - 2q_2$
$(q_1-1)^2 + (q_2-1)^2 - 0.04$
s^0 $2q_1q_2 + 6q_1 + 6q_2 + 2.04$

For statibity

$2+q_1+q_2 > 0$

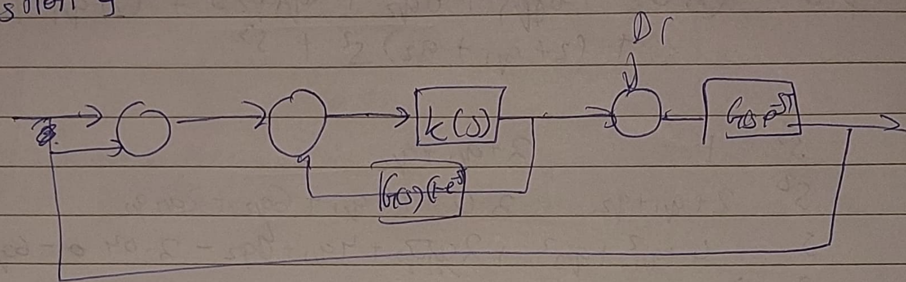
$2(q_1q_2 + 3q_1 + 3q_2 + 9) > 15.96$

$(q_1+3)(q_2+3) > 7.98$



Question 4

a)



$$Y(s) = R(s)\,\frac{\dfrac{k\,\cancel{G(1-e^{-sT})}\,G(s)e^{-sT}}{1+kG(1-e^{-sT})}}{1+\dfrac{k\,\cancel{G(1-e^{-sT})}\,G(s)e^{-sT}}{1+kG(1-e^{-sT})}}$$

$$+\; D(s)\,\frac{G(s)e^{-sT}}{1+\dfrac{k(s)\,G(s)\,e^{-sT}}{1+kG(1-e^{-sT})}}$$

$$Y(s) = R(s)\,\frac{kGe^{-sT}}{1+kG} + D(s)\,\frac{Ge^{-sT}(1+kG(1-e^{-sT}))}{1+kG}$$

$$\lim s\left\{R(s) - R(s)\frac{kGe^{-sT}}{1+kG}\right\} = 0$$

$$\lim \; 1 - \frac{kGe^{-sT}}{1+kG}$$

$$\lim_{s\to 0} \; 1 - \frac{(sk_p+k_i)Ge^{-sT}}{s+(sk_p+k_i)G} = 1-1=0$$

$\lim_{s\to 0}$ G

$\lim_{s\to 0}$ G

$\lim_{s\to 0}$ s

It will n

b)

R(s)

R(s)

Y(s)=

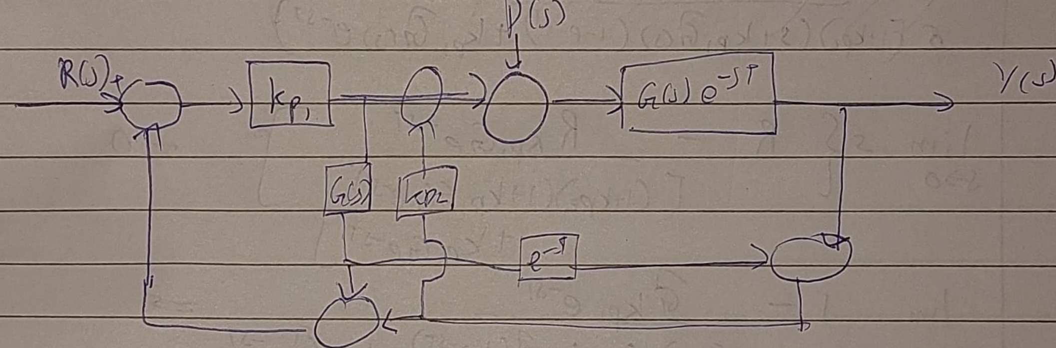
$$\lim_{s\to 0} \frac{G e^{-sT}\left(1+kG(1-e^{-sT})\right)}{1+kG}$$

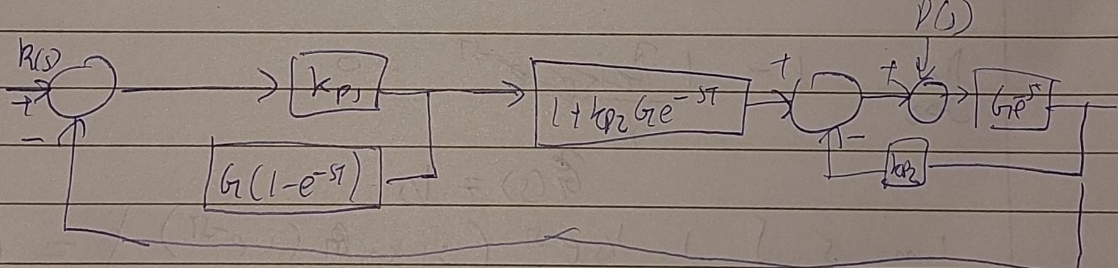
$$\lim_{s\to 0} \frac{G e^{-sT}\left(s+(sk_p+k_i)G\left(sT-(sT)^2 \ldots\right)\right)}{s+(sk_p+k_i)G}$$

$$\lim_{s\to 0} \frac{s\, G e^{-sT}\left(1+(sk_p+k_i)G\left(T-sT^2 \ldots\right)\right)}{s+(sk_p+k_i)G} = 0$$

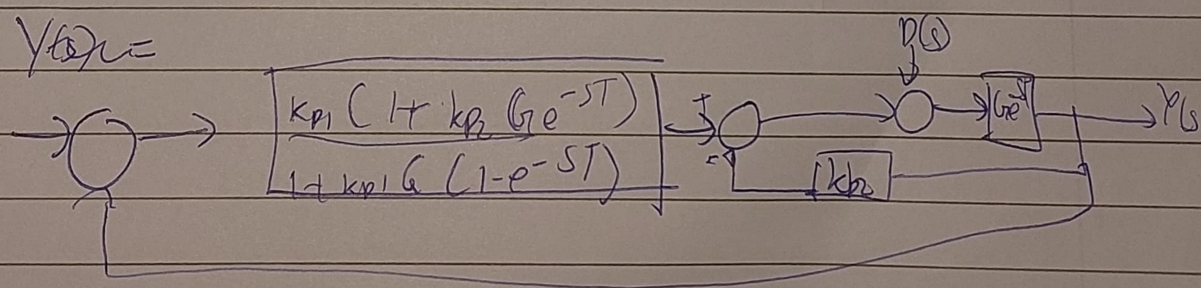
It will not work for G of type 1

b)





Y(s) =



M T W T F S S
Page No.:
Date:
YOUVA

$$Y = \frac{(R-Y)\,k_{p_1} G e^{-sT}}{(1+k_{p_2})\left(1+k_{p_1} G (1-e^{-sT})\right)} + \frac{D G e^{-sT}}{1+k_{p_2}}$$

$$Y(s)\left\{1 + \frac{k_{p_1} G e^{-sT}}{(1+k_{p_2})(1+k_{p_1} G(1-e^{-sT}))}\right\} = \frac{R\,k_{p_1} G e^{-sT}}{(1+k_{p_2})(1+k_{p_1} G(1-e^{-sT}))}$$

$$+\frac{D G e^{-sT}}{1+k_{p_2} G e^{-sT}}$$

$$Y(s) = \frac{R\,k_{p_1} G e^{-sT}}{\left[(1+k_{p_2})(1+k_{p_1} G(1-e^{-sT})) + k_{p_1} G e^{-sT}\right)} + \frac{D G e^{-sT}\,(1+k_{p_2})(1+k_{p_1} G(1-e^{-sT}))}{(1+k_{p_2} G e^{-sT})\left\{(1+k_{p_2})(1+k_{p_1} G(1-e^{-sT})) + k_{p_1} G e^{-sT}\right\}}$$

$$Y(s) = \frac{R(s)\,k_{p_1}\,\tilde{G}(s)\,e^{-sT}}{s\left\{(1+k_{p_2})(s+k_{p_1}\tilde{G}(s))(1-e^{-sT}) + k_{p_1}\tilde{G}(s)\,e^{-sT}\right\}}$$

$$\lim_{s\to 0} s\left\{R - \frac{R\,k_{p_1} G e^{-sT}}{\left[(1+k_{p_2})(1+k_{p_1} G(1-e^{-sT})) + k_{p_1} G e^{-sT}\right]}\right\} = 0$$

$$\lim_{s\to 0} 1 - \frac{\tilde{G}\,k_{p_1} e^{-sT}}{(1+k_{p_2})(s+k_{p_1}\tilde{G}(1-e^{-sT})) + k_{p_1}\tilde{G}\,e^{-sT}} = 0$$

$$\lim_{s\to 0} 1 - \frac{1\,\cancel{\tilde{G}k_{p_1}}\,e^{-sT}}{\cancel{\;}} = 0$$

$$\tilde{G}(s) = 1/s$$

$$\lim s\left\{\frac{1}{s}\;\frac{1/s\; e^{-sT}}{(s+k_{p_1}\tilde{G}(1-e^{-sT})) \ldots}\right\} = 0$$