Control Systems (EE302)
Tutorial - 1

Note:
If a particular method has been specified in a question, you will only get credit if you use that method.

Ques num. 1: Find the Laplace transform of the following functions:

(a) $\mathbf{2e^{-2t}\sin(2t)1(t)}$

(b) $\mathbf{t^2e^{-2t}1(t)}$

(c) $\mathbf{(1-e^{-10t})1(t)}$

(d) $\mathbf{e^{2t}u(t)+e^{-3t}1(-t)}$

(e) $\mathbf{(t\sin(2t)+e^{-2t})1(t)}$

(f) $\mathbf{(e^{4t}\sin(3t)+2e^{4t}\sin(3t))1(t)}$

(g) $\mathbf{t^2e^{t}\sin(5t)1(t)}$

(h) $\mathbf{t^3 1(t)}$

Ques num. 2: Derive the Laplace transform of the following (from definition):

(a) $\mathbf{t^2 1(t)}$

(b) $\mathbf{\sin(\omega)1(t)}$

(c) $\mathbf{e^{-at}\cos(\omega t)1(t)}$

Verify each case using MATLAB.

Ques num. 3: Use initial value theorem to get $\boldsymbol{f(0^+)}$ and $\boldsymbol{f'(0^+)}$

for $\boldsymbol{F(s)=\frac{s^2}{s^2-4s+9}}$.

Ques num. 4: Use final value theorem for obtaining $\boldsymbol{f(\infty)}$ for $\boldsymbol{F(s)}$ given as:

(a) $\dfrac{\mathbf{2s-7}}{\mathbf{s(s^2+0.1s+4)}}$ (b) $\dfrac{\mathbf{2s+7}}{\mathbf{s(s^2+4)}}$

Ques num. 5: Find the final values of $\boldsymbol{x(t), \dot{x}(t)}$ as $\boldsymbol{t\to\infty}$ for the following Laplace transforms, $\boldsymbol{X(s)}$, given by the following.

(a) $\frac{2s+1}{s^4+8s^3+16s^2+s}$

(b) $\frac{2}{s(s^2-s-2)}$

(c) $\frac{5(s+2)}{s(s^2+4)}$

(d) $\frac{(4s^2+3s)}{(s^2+as-4)}, a \in \mathbf{R}$

(e) $\frac{(2s-a)}{s(s^2+\epsilon s+4)}, a \in \mathbf{R}$ and $\epsilon > 0$

(f) $\frac{6s-31}{s^2(4s+3)}$

(g) $\frac{s^2+4s+7}{s^2+s+5}$

(h) $\frac{2}{s^2+4s+7}$

(i) $\frac{s^2+3s-2}{s(s+a)}, a \in \mathbf{R}$

(j) $\frac{as+5}{s^2+3s-2}, a \in \mathbf{R}$

Ques num. 6: Find the inverse Laplace transforms of the following transfer functions:

(a). $G(s) = \frac{5(s+2)}{s(s^2+6s+9)}$

(b). $G(s) = \frac{s}{(s^2+1)^2}$

(c). $G(s) = \frac{s^2+s+2}{s+1}$

Show the steps required for hand-computation and also write the code required (in the language of your choice) to verify your computation on a computer.

Ques num. 7: Find the impulse response of the following differential equation:

$$\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 2y(t) = 2\frac{d}{dt}r(t) + r(t)$$

(Assume zero initial conditions)

Ques num. 8: : Solve following initial value problem using Laplace transform:

$\ddot{y} - \dot{y} - 6y = 2; y(0) = 1, \dot{y}(0) = 0$

Ques num. 9: An input $x(t) = e^{-2t}u(t) + \delta(t-6)$ is applied to an LTI system with impulse response $h(t) = u(t)$. Find the output $y(t)$ using Laplace transform. Verify your answer by computing $y(t)$ in time domain.

Ques num. 10: Consider first order transfer function $G(s) = \frac{k(s-z)}{s-p}$

with $p < 0$ and $k \neq 0$.

(a) Find step response in terms of k, z, p.

(b) Show that the step response of $G(s)$ can be written as

$y(t) = y(\infty) - (y(\infty) - y(0^+))e^{pt}$.

(c) Show that when $z > 0$, there exists some $t_1 \in (0, \infty)$ such that $y(t_1) = 0$.

Ques num. 11: Find inverse Laplace transform to get $y_1(t)$ and $y_2(t)$ for:

(a) $Y_1(s) = \dfrac{16}{(s+4)^2}$ (b) $Y_2(s) = \dfrac{256}{(s^2+4s+16)^2}$

Ques num. 12: A system with input $u(t)$ and output $y(t)$ is governed by the relation:

$$\frac{d}{dt}u + u^2 = \frac{d}{dt}y - \frac{d^2}{dt^2}y.$$

(a) Is the system linear? Give a brief reason.

(b) Is the system time-invariant? Give a brief reason.

(c) Is the system causal? Give a brief reason.

Ques num. 13: (4 marks) Determine if the following two systems are linear/nonlinear, time invariant/time-varying, static/dynamic, causal/non-causal. Provide brief justifications for your answer in each case.

(a) $y(t) = 2tr(t) + 3r(t-3)$

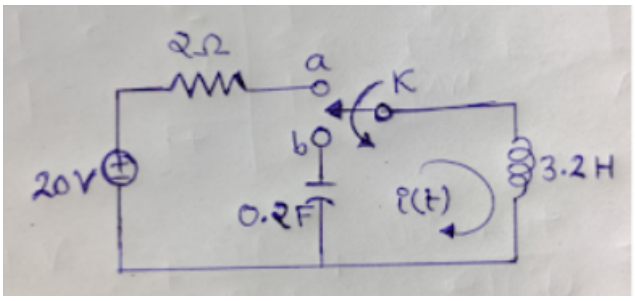
(b) $\ddot{x}(t) + x(t) = r(t)$ and $y(t) = 7x(t) + 2x(t), t \geq 0$

Ques num. 14: For the system in qn. 1(b) above, assume zero initial conditions. If possible, design bounded inputs (i.e., $\exists M \in \mathbf{R}$ s.t. $|r(t)| < M \forall t$), such that
(a) $y(t) \to \infty$ as $t \to \infty$
(b) $y(t) \to 0$ as $t \to \infty$
(c) $\lim_{t\to 0} y(t)$ does not exist

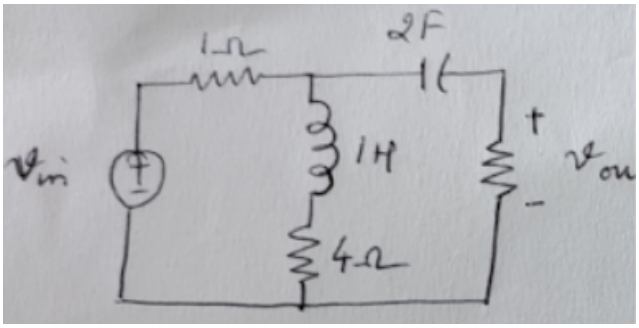
Ques num. 15: Design an electrical circuit (with any components of your choice) with the transfer function from input voltage $V_i(s)$ to output voltage $V_o(s)$ being

$$\frac{V_o(s)}{V_i(s)} = \frac{5(s+1)}{s+2}$$

Ques num. 16: In the circuit shown in the figure, switch **K** is moved from position 'a' to position 'b' at $\boldsymbol{t = 0}$ (a steady state existed at position 'a' prior to $\boldsymbol{t = 0}$). Solve for current $\boldsymbol{i(t)}$, using the Laplace transform.



Ques num. 17: (a) Find $\boldsymbol{G(s) = \frac{V_{on}(s)}{V_{in}(s)}}$ for the given RLC network (second figure below).



(b) Use series-parallel-based simplification rules for RLC.

(c) Write KCL/KVL in Laplace transformed variables (instead of time-domain variables)

(d) Find $\boldsymbol{\frac{I(s)}{V_{in}(s)}}$ and $\boldsymbol{\frac{V_{in}(s)}{I(s)}}$, $\boldsymbol{I}$ is the current through the resistor where $\boldsymbol{V_{on}}$ is measured across.

Ques num. 18:

(a) The switch **K** in fig(a) is initially closed at 'a' till steady state is attained. Then at t=0, it is thrown to position 'b'. Obtain $\boldsymbol{v_{out}}$(t) for $\boldsymbol{t > 0}$ using the Laplace transform suitably.

(b) For the circuit in fig(b), obtain the transfer function $\boldsymbol{\frac{V_{out}(s)}{V_{in}(s)}}$.

(c) For the active network in fig(c), write down suitable equations in transformed variables, using KCL at nodes 1, 2, 3 and 4. Hence, obtain $\boldsymbol{\frac{V_{out}(s)}{V_{in}(s)}}$.

Ques num. 19: For the given translational mechanical system in the figure (on next page),

(a) Use a free body diagram to obtain the differential equations in terms of $\boldsymbol{x_1(t), x_2(t)}$ and their derivatives.

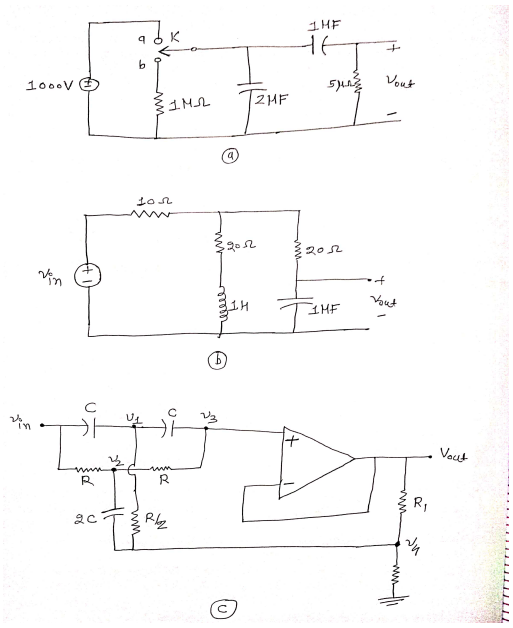
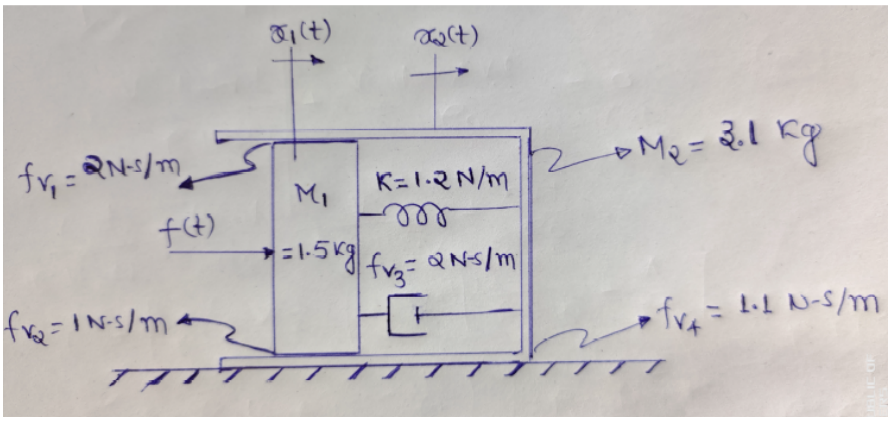


Figure 1: Figure for Q18



(b) Hence, obtain the transfer function $\boldsymbol{G(s) = \frac{X_2(s)}{F(s)}}$

(c) What are the poles and zeros of $\boldsymbol{G(s)}$?

Ques num. 20:

Consider the relevant figure for this question.

(a) Find transfer function from $\boldsymbol{f_1}$ to $\boldsymbol{x_2}$.

(b) Find transfer function from $\boldsymbol{f_2}$ to $\boldsymbol{x_1}$.

(c) Write units for each of $\boldsymbol{k_i}$,$\boldsymbol{f_i}$,$\boldsymbol{x_i}$,$\boldsymbol{d_i}$ explicitly.

Ques num. 21: For the rotational mechanical system given in the figure, draw both "series" and "parallel" analog circuits and explain using dynamical equations for the system.

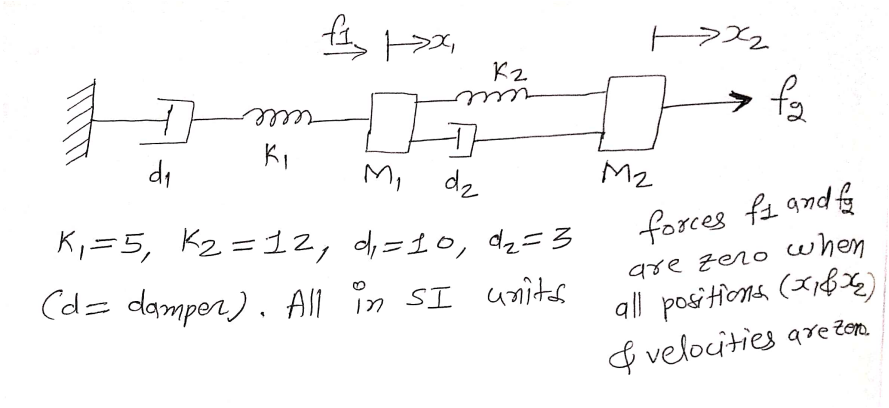
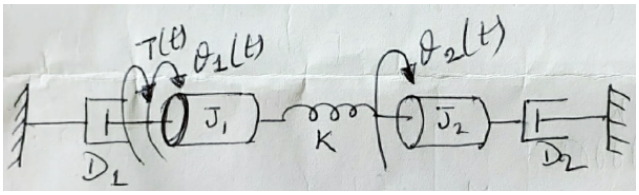


Figure 2: Figure for Q20



Ques num. 22: Rotational spring/mass/(moment of)inertia/torque.

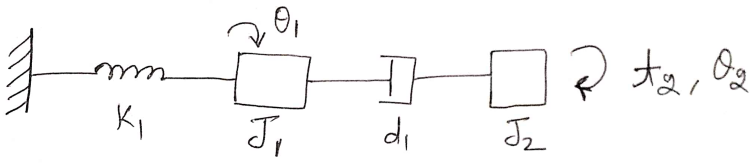


Figure 3: Figure for Q22

(a) List units of $\boldsymbol{k_1}$, $\boldsymbol{d_1}$, $\boldsymbol{\theta_2}$, $\boldsymbol{J_2}$, $\boldsymbol{J_1}$ (SI units).

(b) Find transfer function from $\boldsymbol{T_2}$ to $\frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{\theta_2}$(rate of change of $\boldsymbol{\theta_2}$).

Ques num. 23: Motor/rotational motion example:

Find the transfer function from armature voltage $\boldsymbol{e_a}$(t) to $\boldsymbol{\theta_L}$(t)(load angle).

Ques num. 24: For the Op-amp circuit shown in figure below, find transfer function $\frac{\boldsymbol{V_{out}(s)}}{\boldsymbol{V_{in}(s)}}$

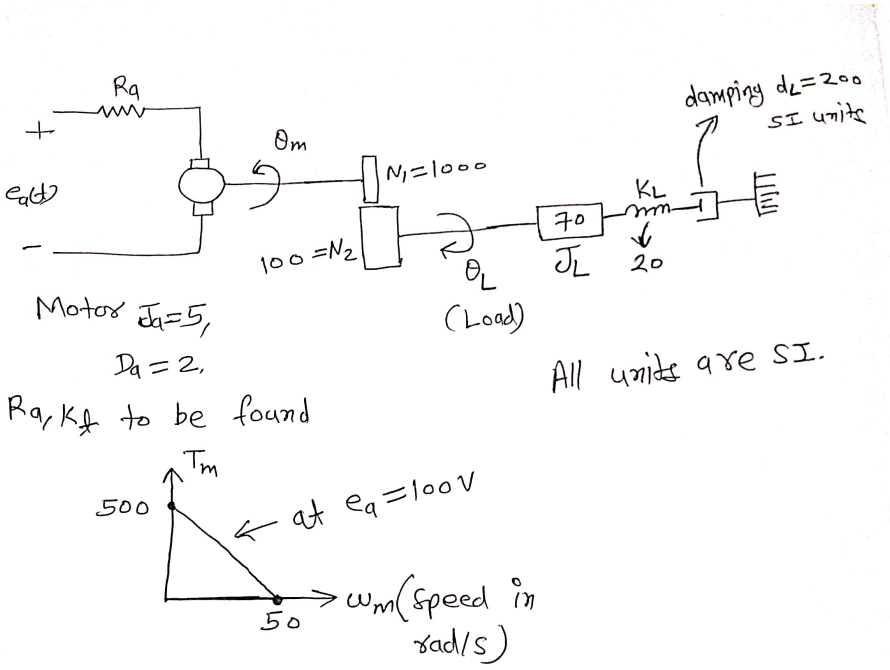


Figure 4: Figure for Q23

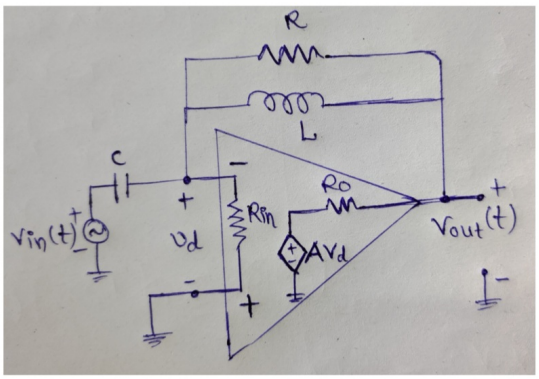


Figure 5: Figure for Q24