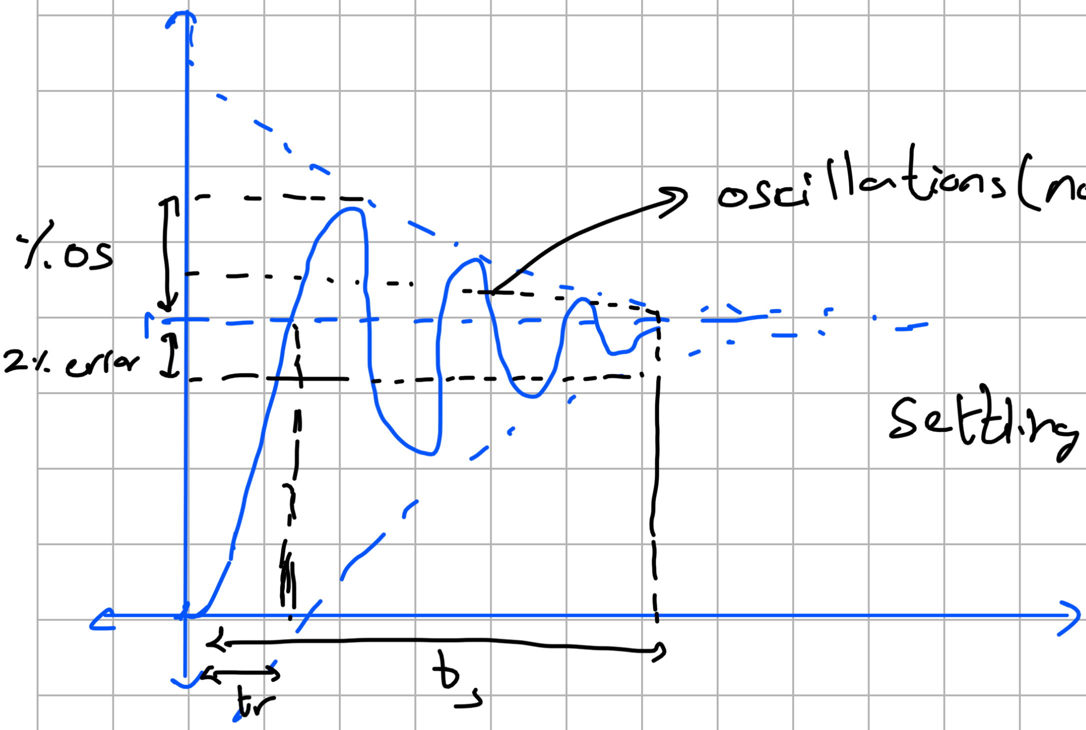
Lecture-10

$$y(t) = \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t - \phi)\right] 1.(t)$$

$$\phi = \sin^{-1}(\xi) \; ; \; \omega_d = \omega_n \sqrt{1-\xi^2}$$



Settling time, rise time?

What about rising time?
0 → 100% value.

$$\%\text{Overshoot} = \frac{y_{max} - y_{steady\ state}}{y_{steady\ state}}$$

$$t_p = (y = y_{max})$$

Derivations:

Settling time (t_s): $\quad 1 - \dfrac{e^{-\xi\omega_n t_s}}{\sqrt{1-\xi^2}} = 0.98$

$$\boxed{-\frac{1}{\xi\omega_n}\ln\left(\frac{\sqrt{1-\xi^2}}{50}\right) = t_s} \longleftarrow \text{Envelope setting}$$

Approximation: for $0.1 < \xi < 0.85$

$3.9 < k < 4.7$

$$\Rightarrow \boxed{t_s \approx \frac{k}{\xi\omega_n}} \qquad t_s = f(\mathrm{Re}(p))$$

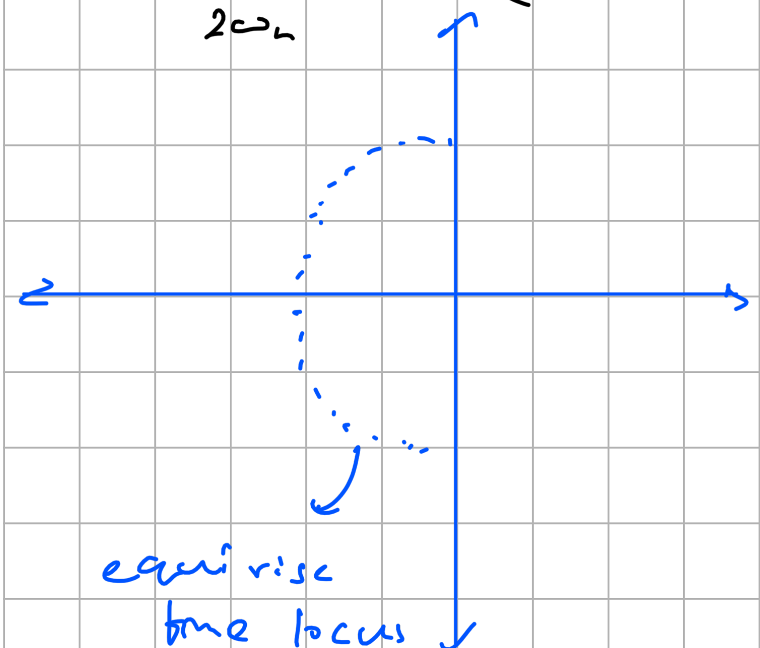
Rise time (t_r):

when $\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \underbrace{\cos(\omega_d t - \phi)}_{0} = 0$

i.e. $\omega_d t_r - \phi = \frac{\pi}{2} \Rightarrow \boxed{t_r = \frac{\sin^{-1}\xi + \pi/2}{\omega_n\sqrt{1-\xi^2}}}$

$\approx \frac{\xi + \pi/2}{\omega_n\sqrt{1-\xi^2}} \longleftarrow \sin^{-1}x \approx x$ approximation

$\approx \frac{\pi}{2\omega_n}$ (This has to be check via computation)



Peak time: $\dot{y}(t) = 0 \Rightarrow L^{-1}[G(s)] = 0$

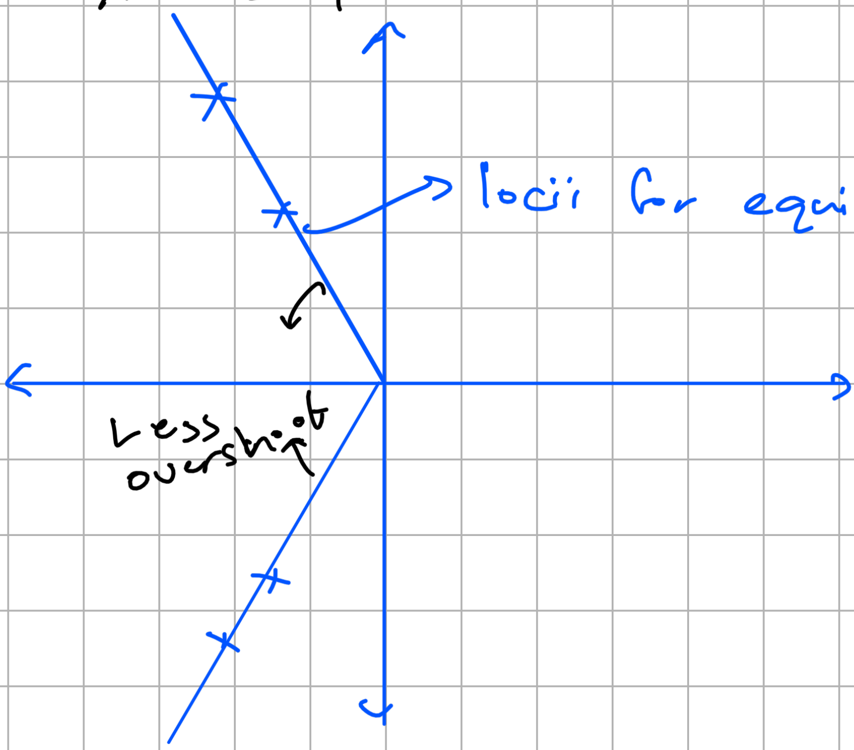
$$0 = \frac{\omega_n^2}{\sqrt{1-\xi^2}} \cdot L^{-1}\left[\frac{\omega_n\sqrt{1-\xi^2}}{(s+\xi\omega_n)(s+\xi\omega_n)}\right]$$

$$\hookrightarrow e^{-\xi\omega_n t_p} \cdot \sin\omega_d t_p = 0$$

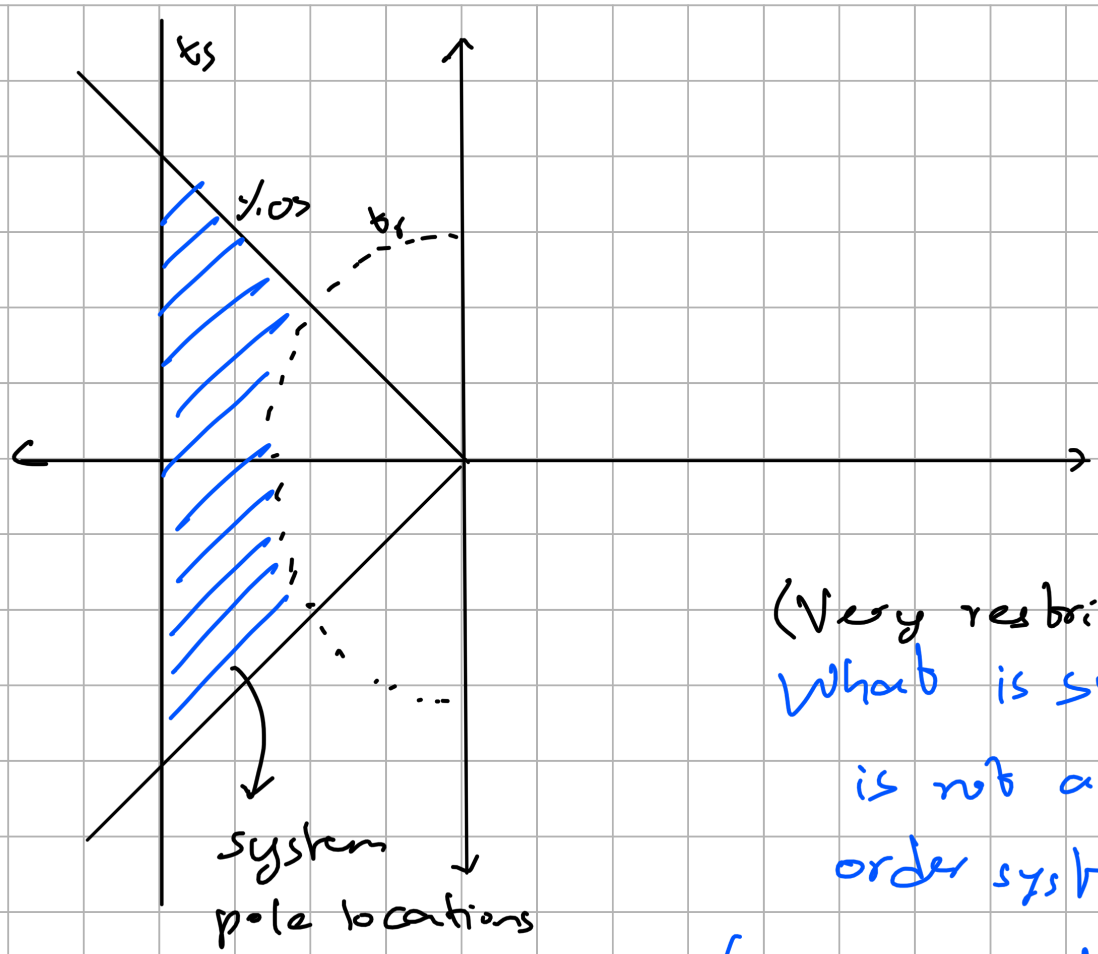
$$\therefore \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\therefore \quad \%OS = 100 \cdot \left[1 - \frac{e^{-\xi\omega_n\pi/\omega_n\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \cdot \cos\left(\pi - \cos^{-1}\sqrt{1-\xi^2}\right)\right]$$

$$\%OS = 100\, e^{-\xi\pi/\sqrt{1-\xi^2}}$$



User: Design Details $\longrightarrow$ You: o o what to modify?



(Very restricted class)

What is system is not a 2nd order system?

(Is our analysis is useless?)

Effects of additional poles and/or zeroes:

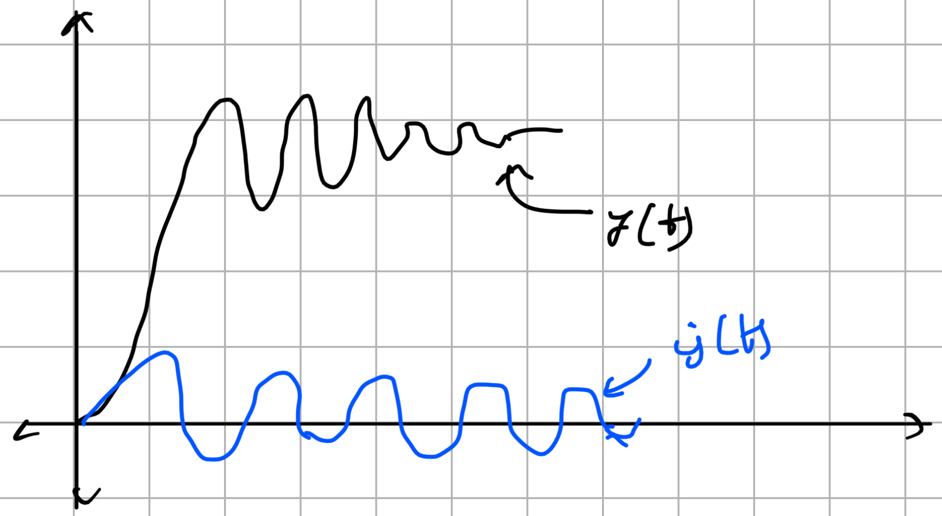
$$G_1(s) = \frac{\omega_n^2 (1 + T_2 s)}{s^2 + 2\zeta\omega_n + \omega_n^2} \longrightarrow y_1(t)$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \longrightarrow y(t)$$

Now $y_1(t) = y(t) + \underbrace{T_2 \dot{y}(t)}$

what does this look like?

$T_2 > 0$



→ Addition of zero doesn't affect the settling time much.

→ If $T_2 \ll 0$ then there is not much effect on the rise time.

→ RHP zero will cause an undershoot at the start.

Effect of a pole:

$$G_1(s) = \frac{\omega_n^2}{(s^2+2\xi\omega_n s+\omega_n^2)(1+T_p s)}$$

$$\left[\frac{1}{s}G_1(s)\right] = \underbrace{\frac{A}{s} + \frac{Bs+C}{s^2+2\xi\omega_n s+\omega_n^2}}_{\text{known qualitatively}} + \frac{D}{s+1/T_p}$$

$D \cdot e^{-t/T_p} \cdot 1(t)$

can be ignored

→ if $T_p \ll \dfrac{1}{\xi\omega_n}$

→ D is small.

(Tough to find)

$$G_1(s) = \frac{\omega_n^2 (1 + sT_z)}{(s^2 + 2\xi\omega_n s + \omega_n^2)(1 + sT_p)}$$

These are good for approximating.

The Padé approximation

$$P(m,n) = \frac{a_0 s^m + a_1 s^{m-1} + \ldots + a_m}{s^n + b_1 s^{n-1} + \ldots + a_n} \approx e^{-sT}$$

$n \geq m$

$$P(1,1) = \frac{a_0 s + a_1}{s + b_1} \approx e^{-sT} \quad \left(\frac{1 - sT/2}{1 + sT/2}\right)$$

$$= \left(\frac{a_0 s + a_1}{b_1}\right)\left(1 + \frac{s}{b_1}\right)^{-1} = 1 - sT + \frac{s^2 T^2}{2} - \frac{s^3 T^3}{6} + \ldots$$

equate coefs.