Minimal distance between parametric curves and surfaces

Given two parametric curves in space $\mathbb{R}^3$, find two points one on each curve, such that the distance between them is minimal. If the curves are given by

$$\begin{aligned} \mathbf{r}_1 &= F(t); \quad t \in \mathbb{R} \\ \mathbf{r}_2 &= G(s); \quad s \in \mathbb{R}, \end{aligned}$$

we search for parameters

$$(t, s) = \text{argmin}_{t,s} \|F(t) - G(s)\|$$

namely the values (t, s) for which the distance

$$\|\mathbf{r}_1 - \mathbf{r}_2\| = \|F(t) - G(s)\|$$

is minimal.

Assignement

1. (3 points) Find the equations for parameters t, s at the points where the minimal distance is attained. Use the fact that the segment between points is perpendicular to the both tangents. If possible, write the equations in vector form without the use of the coordinates.

2. (7 points) Find the solution to the previous equations with Newton's method. In `octave` write a function `[d, t, s] = razdalja(F, G, t0, s0)`, where F and G are parametric descriptions of the curves, `t0` and `s0` are initial estimates for the Newton method. Function `razdalja` should return the distance `d` and values of the parameters `t` and `s`.

3. (3 points) Generalize the problem for parametric surfaces. Derive the equations and update the function `razdalja` so that will work for surfaces as well.

4. (1 points) Generalize the problem to arbitrary dimensions, so that the functions can be $F : \mathbb{R}^m \to \mathbb{R}^k$ and $G : \mathbb{R}^n \to \mathbb{R}^k$, for arbitrary natural numbers m, n and k.

Submission

Use the online classroom to submit the following:

1. a file **razdalja.m** which sould be well-commented and contain at least one test,

2. a report file **solution.pdf** which contains the necessary derivations and answers to questions.

While you can discuss solutions of the problems with your colleagues, the programs and report must be your own creation. You can use all `octave` functions from problem sessions.

Detailed specification

Arguments `F` and `G` of the function `[d, t, s] = razdalja(F, G, t0, s0)` that describe the curves should be function handles for a function of the form `[r, dr, ddr] = F(t)`. The function `F` should take a parameter `t` and should return 3 values:

1. a vector (collumn) `r` containing the coordinates of a point on the curve,
2. a vector (collumn) `dF` containing the first derivatives with respect to the parameter and
3. a vector (collumn) `ddF` containing the second derivatives with respect to the parameter

Example: For the circle $x = \cos(t), y = \sin(t)$ *the function would be*

```
function [r, dr, ddr] = circle(t)
  r = [sin(t); cos(t)];
  dr = [cos(t); -sin(t)];
  ddr = -r; % [-sin(t); -cos(t)]
endfunction
```