Univerza *v Ljubljani*

Advanced CV methods
Optical flow – Homework

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Task 1

- Show that, in the derivation of the Lucas Kanade optical flow constraint equation, the Jacobian is indeed an identity matrix.

$$I(X+\delta) \doteq I(X) + \nabla I^T \underline{\underline{J}}\, \delta$$

The Jacobian

- Tips:
  - Note that $I(\boldsymbol{X}) = I(x, y, t)$
  - And the *parameters* over which the derivatives need to be taken are $[x, y, t]$ as well.

Task 2

- The similar motion assumption in Lucas Kanade leads to the following system of equations:

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_9) & I_y(\mathbf{x}_9) \end{bmatrix}_{9\times2} \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}_{2\times1} = - \begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_9) \end{bmatrix}_{9\times1}$$

- Show that we can rewrite this in the following form:

$$\begin{bmatrix} \sum_{i=1:9} I_x(\mathrm{x}_i)^2 & \sum_{i=1:9} I_x(\mathrm{x}_i) I_y(\mathrm{x}_i) \\ \sum_{i=1:9} I_x(\mathrm{x}_i) I_y(\mathrm{x}_i) & \sum_{i=1:9} I_y(\mathrm{x}_i)^2 \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} = - \begin{bmatrix} \sum_{i=1:9} I_x(\mathrm{x}_i) I_t(\mathrm{x}_i) \\ \sum_{i=1:9} I_y(\mathrm{x}_i) I_t(\mathrm{x}_i) \end{bmatrix}$$

- See next slide for the tips!

Tips for task 2

- A straight forward approach is to show that multiplication with

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_9) & I_y(\mathbf{x}_9) \end{bmatrix}$$

  from the left on both sides results in the 2x2 and 2x1 matrices, respectively, with summations.
- A different, but mathematically more interesting (offers nice a practice) approach is to minimize the cost function directly. See tips on the next slide.

Tips for task 2

- First note that

$$\tilde{\mathbf{d}} = \arg\min_{\mathbf{d}} \|\mathbf{A}\mathbf{d} - \mathbf{b}\|^2$$

- Is really

$$\tilde{\mathbf{d}} = \arg\min_{\mathbf{d}} \epsilon(\mathbf{d})$$

$$\epsilon(\mathbf{d}) = \sum_{i=1:9} (I_x(\mathbf{x}_i)\delta_x + I_y(\mathbf{x}_i)\delta_y + I_t(\mathbf{x}_i))^2$$

- Proceed by:
  - Get two equations from $\frac{\partial \varepsilon(d)}{\partial \delta_x} = \emptyset, \ \frac{\partial \varepsilon(d)}{\partial \delta_y} = \emptyset$
  - Then manipulate terms to get the matrix form.