Univerza v Ljubljani

Advanced CV methods
Optical flow 2– Homework

Matej Kristan

Laboratorij za Umetne Vizualne Spoznavne Sisteme,
Fakulteta za računalništvo in informatiko,
Univerza v Ljubljani

Task 1

- Using the Euler-Lagrange equations

$$\frac{\partial L}{\partial u} - \frac{d}{dx}\left(\frac{\partial L}{\partial u_x}\right) - \frac{d}{dy}\left(\frac{\partial L}{\partial u_y}\right) = 0 \text{ and } \frac{\partial L}{\partial v} - \frac{d}{dx}\left(\frac{\partial L}{\partial v_x}\right) - \frac{d}{dy}\left(\frac{\partial L}{\partial v_y}\right) = 0$$

- Show that the following functional

$$E_c = \iint_D \left(I_x u + I_y v + I_t\right)^2 + \alpha\left(u_x^2 + u_y^2 + v_x^2 + v_y^2\right) dx\, dy$$

- Is minimized at ***u*** and ***v*** that satisfy equations (1) and (2)

$$I_x(\mathrm{I}_x u + I_y v + I_t) - \alpha(\mathrm{u}_{xx} + \mathrm{u}_{yy}) = 0 \qquad (1)$$

$$I_y(\mathrm{I}_x u + I_y v + I_t) - \alpha(v_{xx} + v_{yy}) = 0 \qquad (2)$$

Task 1: TIPS

- When taking derivative $\left(\frac{\partial L}{\partial u_y}\right)$ treat u_y as being independent of u. For example, $\frac{\partial u}{\partial u_y} = 0$.

- But u_y does depend on y. For example, $\frac{\partial u_y}{\partial y} \neq 0$.
- The above extends to other variables

Task 2

- Using the definition $\Delta u = \bar{u} - u$, show that the Horn Schunck equations

$$I_x(\mathrm{I}_x u + I_y v + I_t) - \alpha \Delta u = 0$$
$$I_y(\mathrm{I}_x u + I_y v + I_t) - \alpha \Delta v = 0$$

can be rewritten into the following matrix form.

$$\begin{bmatrix} (I_x{}^2 + \alpha) & I_x I_y \\ I_x I_y & (I_y{}^2 + \alpha) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \alpha\bar{u} - I_x I_t \\ \alpha\bar{v} - I_y I_t \end{bmatrix}$$

- What is the difference to the Lucas Kanade form?