2. Homework

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Question 1

Show that, in the derivation of the lucas-Kanade optical flow constraint equation, the Jacobian is indeed an identity.

The change in the image is approximated as:

$$I(X + \delta) \approx I(X) + \nabla I^T J \delta$$

Where X is a parameter vector $X = \begin{bmatrix} x & y & t \end{bmatrix}^t$. The Jacobian can now be simply written as:

$$J_X = \begin{bmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial y} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial y} & \frac{\partial y}{\partial t} \\ \frac{\partial t}{\partial x} & \frac{\partial t}{\partial y} & \frac{\partial t}{\partial t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2

Show that the system of equation obtained by the similar motion assumption in the Lukas-Kanade can be rewritten in an equivalent form (see questions sheet).

The solution to the original system of equations is a vector that minimizes the squared sum of errors:

$$\tilde{d} = \underset{d}{\operatorname{argmin}} ||Ad - b||^2.$$

This can be written equivalently as:

$$\tilde{d} = \underset{d}{\operatorname{argmin}} \ \epsilon(d)$$

The error function for the i-th term of the vector d can be written as:

$$\epsilon(d)^{(i)} = (I_x(x_i)\delta_x + I_y(x_i)\delta_y + I_t(x_i))^2$$

The vector satisfying the constraint can be obtained by minimizing the error function. This is achieved by differentiating the function with respect to x and y and setting the derivatives to 0. Let us compute the derivative of the error function with respect to x and obtain the first part of the desired system of linear equations.

$$\frac{\partial \epsilon(d)}{\partial x} = 2\sum_{i=1}^{9}(I_x(x_i)\delta_x + I_y(x_i)\delta_y + I_t(x_i))I_x = 0$$

$$2\sum_{i=1}^{9} I_x(x_i)^2\delta_{x_i} + 2\sum_{i=1}^{9} I_x(x_i)I_y(x_i)\delta_y + 2\sum_{i=1}^{9} I_x(x_i)I_t(x_i) = 0$$

Rearranging the equations and writing them in terms of matrix (vector) multiplication, we obtain:

$$2\sum_{i=1}^{9} I_x(x_i)^2\delta_{x_i} + 2\sum_{i=1}^{9} I_xI_y\delta_y = -2\sum_{i=1}^{9} I_x(x_i)I_t(x_i)$$

$$\begin{bmatrix}\sum_{i=1}^{9} I_x(x_i)^2 & \sum_{i=1}^{9} I_x(x_i)I_y(x_i)\end{bmatrix}\begin{bmatrix}\delta_x \\ \delta_y\end{bmatrix} = -\begin{bmatrix}\sum_{i=1}^{9} I_x(x_i)I_t(x_i)\end{bmatrix}$$

Similarly, let us compute the derivative of the error function with respect to y and obtain the second part of the desired system of linear equations.

$$\frac{\partial \epsilon(d)}{\partial y} = 2\sum_{i=1}^{9}(I_x(x_i)\delta_x + I_y(x_i)\delta_y + I_t(x_i))I_y = 0$$

$$2\sum_{i=1}^{9} I_x(x_i)I_y(x_i)\delta_{x_i} + 2\sum_{i=1}^{9} I_y(x_i)^2\delta_y + 2\sum_{i=1}^{9} I_y(x_i)I_t(x_i) = 0$$

Rearranging the equations and writing them in terms of matrix (vector) multiplication, we obtain:

$$2\sum_{i=1}^{9} I_x(x_i)I_y(x_i)\delta_x + 2\sum_{i=1}^{9} I_y(x_i)^2\delta_y = -2\sum_{i=1}^{9} I_y(x_i)I_t(x_i)$$

$$\begin{bmatrix}\sum_{i=1}^{9} I_x(x_i)I_y(x_i) & \sum_{i=1}^{9} I_y(x_i)^2\end{bmatrix}\begin{bmatrix}\delta_x \\ \delta_y\end{bmatrix} = -\begin{bmatrix}\sum_{i=1}^{9} I_y(x_i)I_t(x_i)\end{bmatrix}$$

Putting together the equations obtained from the partial derivatives, we obtain the desired system of equations:

$$\begin{bmatrix}\sum_{i=1}^{9} I_x(x_i)^2 & \sum_{i=1}^{9} I_x(x_i)I_y(x_i) \\ \sum_{i=1}^{9} I_x(x_i)I_y(x_i) & \sum_{i=1}^{9} I_y(x_i)^2\end{bmatrix}\begin{bmatrix}\delta_x \\ \delta_y\end{bmatrix} = -\begin{bmatrix}\sum_{i=1}^{9} I_x(x_i)I_t(x_i) \\ \sum_{i=1}^{9} I_y(x_i)I_t(x_i)\end{bmatrix}$$