Univerza v Ljubljani

ViCOS visual cognitive systems lab

Advanced CV methods
Linearization – Homework

Laboratorij za Umetne Vizualne Spoznavne Sisteme,
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Homework:

1. Take derivative of the following function:

   $f(x) = (x + x^2)^3 - x$

2. Linearize the following function at x_0: $f(x) = x^2 + 2x^3$

   - Write the equation of a line tangent at $x_0 = 3$:

     First derive the form: $f(x_0 + \delta) =?$
     Then write the linear approximation of $f(x)$ at x_0, i.e., $f(x)|_{x_0} \approx ?$

   - In Matlab or Python (or your favorite math program) plot the original function $f(x)$ and its linear approximation (line) for values $x = [-5{:}\,5]$.

Homework:

3. Linearize the following function $f\left(x_1(\boldsymbol{p}), x_2(\boldsymbol{p})\right)$ for small changes in parameters $\boldsymbol{p}$, i.e., $\boldsymbol{p_0} + \boldsymbol{\delta}$, using the Jacobian:

$$f\left(x_1(\boldsymbol{p}), x_2(\boldsymbol{p})\right) = 4x_1^2(\boldsymbol{p}) + x_2^4(\boldsymbol{p})$$

with parameter vector defined as:

$$\boldsymbol{p} = [p_1, p_2]^T$$

and with parameters of $f(x_1, x_2)$ defined as:

$$x_1(\boldsymbol{p}) = p_1\,, x_2(\boldsymbol{p}) = p_1 + 3p_2$$

Homework:

4. Assume a vector $x \in \mathbb{R}^d$ and a symmetric positive definite matrix $A \in \mathbb{R}^{d \times d}$.
   1. Take the function $f(x) = x^T A x \cdot 3$
   2. And compute its gradient $\nabla f(x) = \frac{\partial f}{\partial x} = ?$

Tips:

1. Note that $f(x)$ is a scalar so it is *equal to its determinant* $f(x) = |f(x)|$
2. Use the matrix Cookbook, equation (53) and simplify
3. This is basic matrix algebra!