Assignment 8

Task 1

Let M – Season in Maine

T – Daily High $\geq$ 80 degrees.

$P(M) = 0.05$ $\qquad$ $P(\neg M) = 0.95$

$P(T/M) = 0.2$ $\qquad$ $P(T/\neg M) = 0.9$

$P(\neg T/M) = 0.8$ $\qquad$ $P(\neg T/\neg M) = 0.1$

a) $$P(M/T) = \frac{P(M \cap T)}{P(T)}$$

$$= \frac{P(T/M)P(M)}{P(T/M)P(M) + P(T/\neg M)P(\neg M)}$$

$$= \frac{0.2 \times 0.05}{0.2 \times 0.05 + 0.9 \times 0.95} = 0.011561$$

1.15% Chance.

b/ Let

T_1 be first email, T_2 be second email.

$$P\left(T_2 / T_1\right) = \frac{P\left(T_2 \wedge T_1\right)}{P(T_1)}$$

$$P(T_2 \wedge T_1) = P\left(T_2 \wedge T_1 / M\right) P(M) + P\left(T_2 \wedge T_1 / \neg M\right) P(\neg M)$$

Given M, T_1 & T_2 are conditionally independent.

So

$$P(T_2 \wedge T_1) = P\left(T_2 / M\right) P\left(T_1 / M\right) P(M) + P\left(T_2 / \neg M\right) P\left(T_1 / \neg M\right) P(\neg M)$$

$$= 0.2 \times 0.2 \times 0.05 + 0.9 \times 09 \times 0.95$$

$$= 0.7715$$

$$P(T_1) = P(T_1 \cap T_2 \cap M) + P(T_1 \cap T_2 \cap \neg M)$$
$$+ P(T_1 \cap \neg T_2 \cap M)$$
$$+ P(T_1 \cap \neg T_2 \cap \neg M)$$

$$= P(T_1/M)\, P(T_2/M)\, P(M) + P(T_1/\neg M)\, P(T_2/\neg M)\, P(\neg M)$$
$$+ P(T_1/M)\, P(\neg T_2/M)\, P(M) + P(T_1/\neg M)\, P(\neg T_2/\neg M)\, P(\neg M)$$

$$= P(T_1/M)\, P(M) \left[P(T_2/M) + P(\neg T_2/M) \right]$$
$$+ P(T_1/\neg M)\, P(\neg M) \left[P(T_2/M) + P(\neg T_2/M) \right]$$

$$= P(T_1/M)\, P(M) + P(T_1/\neg M)\, P(\neg M)$$

$$= 0.2 \times 0.05 + 0.9 \times 0.95$$
$$= 0.865$$

So $$P(T_2/T_1) = \frac{0.7715}{0.865} = 0.8919$$

a/

$$P(T_3 \cap T_2 \cap T_1)$$

$$= P\left(T_3 \wedge T_2 \wedge T_1 / M\right) P(M)$$

$$P\left(T_3 \wedge T_2 \wedge T_1 / \neg M\right) P(\neg M)$$

$$= P\left(T_3/M\right) P\left(T_2/M\right) P\left(T_1/M\right) P(M)$$

$$+ P\left(T_3/\neg M\right) P\left(T_2/\neg M\right) P\left(T_1/\neg M\right) P(\neg M)$$

$$= 0.2 \times 0.2 \times 0.2 \times 0.05 + 0.9 \times 0.9 \times 0.9 \times 0.95$$

$$= 0.69295$$

Task 2

A can have 6 values

Each B can have 5 values.

(a)

Joint probability needs

$6^1 \times 5^{10} = 6 \times 5^{10}$ numbers in theory

$(6 \times 5^{10}) - 1$ numbers in practice.

(b) The JPD,

$P(A, B_1, B_2, \dots B_{10}) = P(B_1/A)\, P(B_2/A) \cdot$

$\dots P(B_{10}/A)\, P(A)$

Each $P(B_i/A)$ needs $6 \times (5-1) = 24$ values

$P(A)$ needs $6-1 = 5$ values

in total $24 \times 10 + 5 = 240 + 5 = 245$

TASK 5

(a) For N:

Parents : I

Children : R, S.

Other Parents : M, O

These form the Markov blanket.

(b)

$$P(I, D) = P(I/D)\,P(D)$$

$$= 0.5 \times 0.5$$

$$= 0.25$$

(c)
$$P(M, \neg C / H) = \frac{P(M, \neg C, H)}{P(H)}$$

$$= \frac{P(M/H)\,P(H/\neg C)\,P(\neg C)}{P(H/C)\,P(C) + P(H/\neg C)\,P(\neg C)}$$

$$= \frac{0.1 \times 0.1 \times 0.4}{0.6 \times 0.6 + 0.1 \times 0.4}$$

$$= 0.01$$