1)a) M - Sensor in Maine

T - Daily high temperature of 80° or more

$P(M) = 0.05$ $\quad$ $P(\neg M) = 0.95$

$P(T/M) = 0.2$ $\quad$ $P(T/\neg M) = 0.9$

To find

$P(M/\neg T)$

$\langle P(M/\neg T) \quad P(\neg M/\neg T) \rangle$

$= \alpha \langle P(\neg T/M) P(M) \quad P(\neg T/\neg M) P(\neg M) \rangle$

$= \alpha \langle 0.8 \times 0.05 \quad 0.1 \times 0.95 \rangle$

$= \alpha \langle 0.04 \quad 0.095 \rangle$

$= \langle 0.2963 \quad 0.7037 \rangle$ $\quad$ $\alpha = \frac{1}{0.135}$

$P(M/\neg T) = 0.2963$

29.63% probability of sensor in Maine

b) $P(\neg T_2/\neg T_1) = \dfrac{P(\neg T_2 \wedge \neg T_1)}{P(\neg T_1)}$

$P(\neg T_1) = P(\neg T_1/M) P(M) + P(\neg T_2/\neg M) P(\neg M)$

$P(\neg T_2 \wedge \neg T_1) = P(\neg T_2 \wedge \neg T_1/M) P(M) + P(\neg T_2 \wedge \neg T_1/\neg M) P(\neg M)$

$= P(\neg T_2/M) P(\neg T_1/M) P(M) + P(\neg T_2/\neg M) P(\neg T_1/\neg M) P(\neg M)$

$= 0.8 \times 0.8 \times 0.05 + 0.1 \times 0.1 \times 0.095$

$= 0.0415$

$$P(\neg T_2/\neg T_1) = \frac{0.0415}{0.135} = 0.3074$$

c) $P(\neg T_3 \wedge \neg T_2 \wedge \neg T_1)$

$= P(\neg T_3 \wedge \neg T_2 \wedge \neg T_1/M)\, P(M) + P(\neg T_3 \wedge \neg T_2 \wedge \neg T_1/\neg M)\, P(\neg M)$

$= P(\neg T_3/M)\, P(\neg T_2/M)\, P(\neg T_1/M)\, P(M) +$
$\quad P(\neg T_3/\neg M)\, P(\neg T_2/\neg M)\, P(\neg T_1/\neg M)\, P(\neg M)$

$= (0.8 \times 0.8 \times 0.8 \times 0.005) + (0.1 \times 0.1 \times 0.1 \times 0.95)$

$= 0.0256 + 0.00095$

$= 0.02655$

2)

a) $P(A, B_1, B_2, \ldots, B_{10})$

A can have 5 values each B can have 7values
Joint probability needs $5^1 \times 7^{10}$ numbers or $5^1 \times 7^{10} - 1$ number

b) using conditional probability:

$P(A, B_1, B_2, \ldots, B_{10}) = P(B_1/A)\, P(B_2/A) \ldots P(B_{10}/A)\; P(A)$

Each $P(B_i/A)$ needs $5 \times (7-1) = 5 \times 6 = 30$ values
$P(A)$ needs $5 - 1 = 4$ values
So we needs $\cancel{30\times(6+4) = 30\times}$
$(30 \times 10) + 4 = 300 + 4 = 304$ values

5) a) G, P, Q, K, M

b) $P(A,F) = P(F,A) = P(F/A)\,P(A)$
$= 0.8 \times 0.8$
$= 0.64.$

c) $P(M, \neg C/H) = P(M, \neg C, H) / P(H)$

$P(M, \neg C, H) = P(M, H, \neg C)$
$= P(M/H)\, P(H/\neg C)\, P(\neg C)$
$= 0.1 \times 0.1 \times 0.4$
$= 0.004$

$P(H) = P(H/C)\,P(C) + P(H/\neg C)\,P(\neg C)$
$= (0.6 \times 0.6) + (0.1 \times 0.4)$
$= 0.36 + 0.04$
$= 0.40$

$P(M, \neg C/H) = \dfrac{0.1 \times 0.1 \times 0.4}{0.4}$

$= 0.01.$