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MDL- Assignment 3 - Part I

POMDP

- Roll-Number used for calculations - 2019101053
- Last four digit of roll-number 1053

$$x = 1 - ((1053 \% 30 + 1)/100)$$

$$x = 1 - 4/100$$

$$x = 0.96$$

- Last two digit of roll-number 53

$$y = (53) \% 4 + 1$$

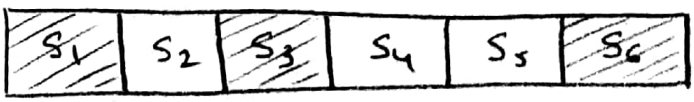
$$y = 1 + 1$$

$$y = 2$$

- Observation table used - Table 2

P(Observation = Red | State = Red)	0.90
P(obervation = Green | State = Green)	0.85

- Initial position S_1, S_3, S_6
- States



For simplicity lets generate 2 tables

Table - 1

State	Colour	Observed	
		R	G
S_1	Red	0.90	0.10
S_2	Green	0.15	0.85
S_3	Red	0.90	0.10
S_4	Green	0.15	0.85
S_5	Green	0.15	0.85
S_6	Red	0.90	0.10

Table 2 : Initial belief states

State	Probability
S_1	1/3
S_2	0
S_3	1/3
S_4	0
S_5	0
S_6	1/3

Action - Observation pairs (Given)

Equation Using the equation mentioned below

$$b'(s) = \alpha P(o|a,s)\left(\sum_{s' \in S} P(s|a,s')\, b(s')\right)$$

for finding $b(s_i)$

Pair 1 Agent took right and observed green

$\Rightarrow$ Finding raw probabilities

$$b'(s_1) = \alpha(0.10)\left(\frac{1}{3}(0.04) + 0\cdot(0.04)\right)$$

$$= \alpha(0.10)(0.04)(1/3)$$

$$= \alpha\left(\frac{4}{3000}\right)$$

$$b'(s_2) = \alpha(0.85)\left(\frac{1}{3}(0.96) + \frac{1}{3}(0.04)\right)$$

$$= \alpha\left(\frac{85}{300}\right)$$

$$b'(s_3) = \alpha(0.10)\left(0(0.96) + 0(0.04)\right)$$

$$= \alpha(0.10)\cdot(0)$$

$$= 0$$

$$b'(s_4) = \alpha(0.85)\left(\frac{1}{3}(0.96) + 0(0.04)\right)$$

$$= \alpha(0.85)(0.96)(1/3)$$

$$= \alpha\left(\frac{816}{3000}\right)$$

$$b'(S_s) = \alpha(0.85)\left(0(0.96) + \frac{1}{3}(0.04)\right)$$

$$= \alpha(0.85)(0.04)(1/3)$$

$$= \alpha\left(\frac{34}{3000}\right)$$

$$b'(S_c) = \alpha(0.10)\left(0(0.96) + \frac{1}{3}(0.96)\right)$$

$$= \alpha(0.10)(0.96)(1/3)$$

$$= \alpha\left(\frac{96}{3000}\right)$$

$$\sum_{i=1}^{6} b'(S_i) = 1 = \alpha\left(\frac{4}{3000}\right) + \alpha\left(\frac{850}{3000}\right) + 0 + \alpha\left(\frac{816}{3000}\right) + \alpha\left(\frac{34}{3000}\right) + \alpha\left(\frac{96}{3000}\right)$$

$$= \alpha\left(\frac{4 + 850 + 0 + 816 + 34 + 96}{3000}\right)$$

$$= \alpha\left(\frac{1800}{3000}\right) = \alpha\left(\frac{3}{5}\right)$$

$$\Rightarrow \boxed{\alpha = \frac{5}{3}}$$

⇒ After normalization. the belief is :

	Fraction	Decimal
$b(S_1)$	$\frac{5}{3}\left(\frac{4}{3000}\right)$	0.00222
$b(S_2)$	$\frac{5}{3}\left(\frac{85}{300}\right)$	0.47222
$b(S_3)$	$\frac{5}{3}(0)$	0
$b(S_4)$	$\frac{5}{3}\left(\frac{816}{3000}\right)$	0.45333
$b(S_5)$	$\frac{5}{3}\left(\frac{34}{3000}\right)$	0.01888
$b(S_6)$	$\frac{5}{3}\left(\frac{96}{3000}\right)$	0.05333

Pair 2 Agent took left and observed Red

⇒ Finding raw probability.

$$b'(S_1) = \alpha(0.90)\left(\frac{2}{900}(0.96) + \frac{425}{900}(0.96)\right)$$

$$= \alpha(0.90)\left(\frac{427}{900}\right)(0.96) = \alpha\left(\frac{40992}{100000}\right)$$

$$b'(S_2) = \alpha(0.15)\left(\frac{2}{900}(0.04) + 0.(0.96)\right)$$

$$= \alpha\left(\frac{4}{300000}\right)$$

$$b'(s_3) = \alpha(0.90)\left(\frac{425}{900}(0.04) + \frac{136}{300}(0.96)\right)$$

$$= \alpha(0.90)\left(\frac{40868}{90000}\right) = \alpha\left(\frac{40868}{100000}\right)$$

$$b'(s_4) = \alpha(0.15)\left(0\cdot(0.04) + \frac{17}{900}(0.96)\right)$$

$$= \alpha\left(\frac{272}{100000}\right)$$

$$b'(s_5) = \alpha(0.15)\left(\frac{408}{900}(0.04) + \frac{48}{900}(0.96)\right)$$

$$= \alpha\left(\frac{164}{10000}\right)$$

$$b'(s_6) = \alpha(0.90)\left(\frac{17}{900}(0.04) + \frac{48}{900}(0.04)\right)$$

$$= \alpha\left(\frac{26}{10000}\right)$$

$$\sum_{i=1}^{6} b'(s_i) = 1 = \frac{\alpha}{300000}\Big(3(40992) + 4 + 3(40868) + 3(272) + 30(164) + 30(26)\Big)$$

$$= \alpha\left(\frac{250360}{300000}\right)$$

$$\Rightarrow \alpha = \frac{3000}{2503}$$

⇒ After normalisation belief is:

	Fraction	Decimal
$b(s_1)$	$\frac{3000}{2503}\left(\frac{40992}{100000}\right)$	0.491314
$b(s_2)$	$\frac{3000}{2503}\left(\frac{4}{300000}\right)$	0.000015
$b(s_3)$	$\frac{3000}{2503}\left(\frac{40868}{100000}\right)$	0.489828
$b(s_4)$	$\frac{3000}{2503}\left(\frac{272}{100000}\right)$	0.003260
$b(s_5)$	$\frac{3000}{2503}\left(\frac{104}{10000}\right)$	0.012465
$b(s_6)$	$\frac{3000}{2503}\left(\frac{26}{10000}\right)$	0.003116

Pair 3 Agent took action left and observed green

$$b'(s_1) = \alpha(0.10)\left(\frac{3000}{2503}\left(\frac{40992}{100000}\right)(0.96) + \frac{3000}{2503}\left(\frac{4}{300000}\right)(0.96)\right)$$

$$= \alpha(0.10)\left(\frac{3000}{2503}\right)(0.96)\left(\frac{3(40992)+4}{300000}\right)$$

$$= \alpha\left(\frac{11806080}{250300000}\right)$$

$$b'(s_2) = \alpha(0.85)\left(\frac{3000}{2503}\left(\frac{40992}{100000}\right)(0.04) + \frac{3000}{2503}\left(\frac{40868}{100000}\right)(0.96)\right)$$

$$= \alpha\left(\frac{(34742016)_{13}}{250300000}\right) = \alpha\left(\frac{104226048}{250300000}\right)$$

$$b'(S_3) = \alpha(0.10)\left(\frac{3000}{2503}\left(\frac{4}{300000}(0.04) + \frac{272}{100000}(0.96)\right)\right)$$

$$= \alpha(0.10)\left(\frac{3000}{2503}\right)\left(\frac{16 + 26112}{300000000}\right)$$

$$= \alpha\left(\frac{78384}{250300000}\right)$$

$$b'(S_4) = \alpha(0.85)\left(\frac{3000}{2503}\left(\frac{40868}{100000}(0.04) + \frac{104}{10000}(0.96)\right)\right)$$

$$= \alpha(0.85)\left(\frac{3000}{2503}\right)\frac{(3)(263312)}{300000000}$$

$$= \alpha\left(\frac{6714456}{250300000}\right)$$

$$b'(S_5) = \alpha(0.85)\left(\frac{3000}{2503}\left(\frac{272}{100000}(0.04) + \frac{26}{10000}(0.96)\right)\right)$$

$$= \alpha(0.85)\left(\frac{3000}{2503}\right)\left(\frac{26048}{100000000}\right)$$

$$= \alpha\left(\frac{664224}{250300000}\right)$$

$$b'(S_6) = \alpha(0.10)\left(\frac{3000}{2503}(0.04)\left(\frac{104}{10000} + \frac{26}{10000}\right)\right)$$

$$= \alpha\left(\frac{15600}{250300000}\right)$$

$$\sum_{i=1}^{6} b'(S_i) = 1 = \alpha\left(\frac{123504792}{250300000}\right)$$

$$\Rightarrow \alpha = \frac{250300000}{123504792} = 2.02664$$

=> After normalization belief is

	Fraction	Decimal
$b(s_1)$	$\frac{11806080}{123504792}$	0.095592
$b(s_2)$	$\frac{104226048}{123504792}$	0.843902
$b(s_3)$	$\frac{78384}{123504792}$	0.000634
$b(s_4)$	$\frac{6714456}{123504792}$	0.054365
$b(s_5)$	$\frac{664224}{123504792}$	0.005378
$b(s_6)$	$\frac{15600}{123504792}$	0.000126

Final Answers upto 4 decimal not round off

Number	$b(s_1)$	$b(s_2)$	$b(s_3)$	$b(s_4)$	$b(s_5)$	$b(s_6)$
1	0.0022	0.4722	0	0.4533	0.0188	0.0533
2	0.4913	0.000015 *	0.4898	0.0032	0.0124	0.0031
3	0.0955	0.8439	0.0006	0.0543	0.0053	0.0001