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• For a 1 Ω resistive load, design a periodic waveform w average power of 1 watt, that minimizes Power in the 5th harmonic with the restriction that the average value is Zero.
→ Compare your result to the 5th harmonic power of a square wave with $0 = V_{avg}$.

$\rightarrow$ Let $V(t) = A \cdot \cos(wt)$ $\qquad w = \frac{2\pi}{T}$

$\therefore I(t) = \frac{A}{r = 1\Omega} \cdot \cos(wt) = A\cos(wt)$

let $T = 2\pi$, $w = 1$

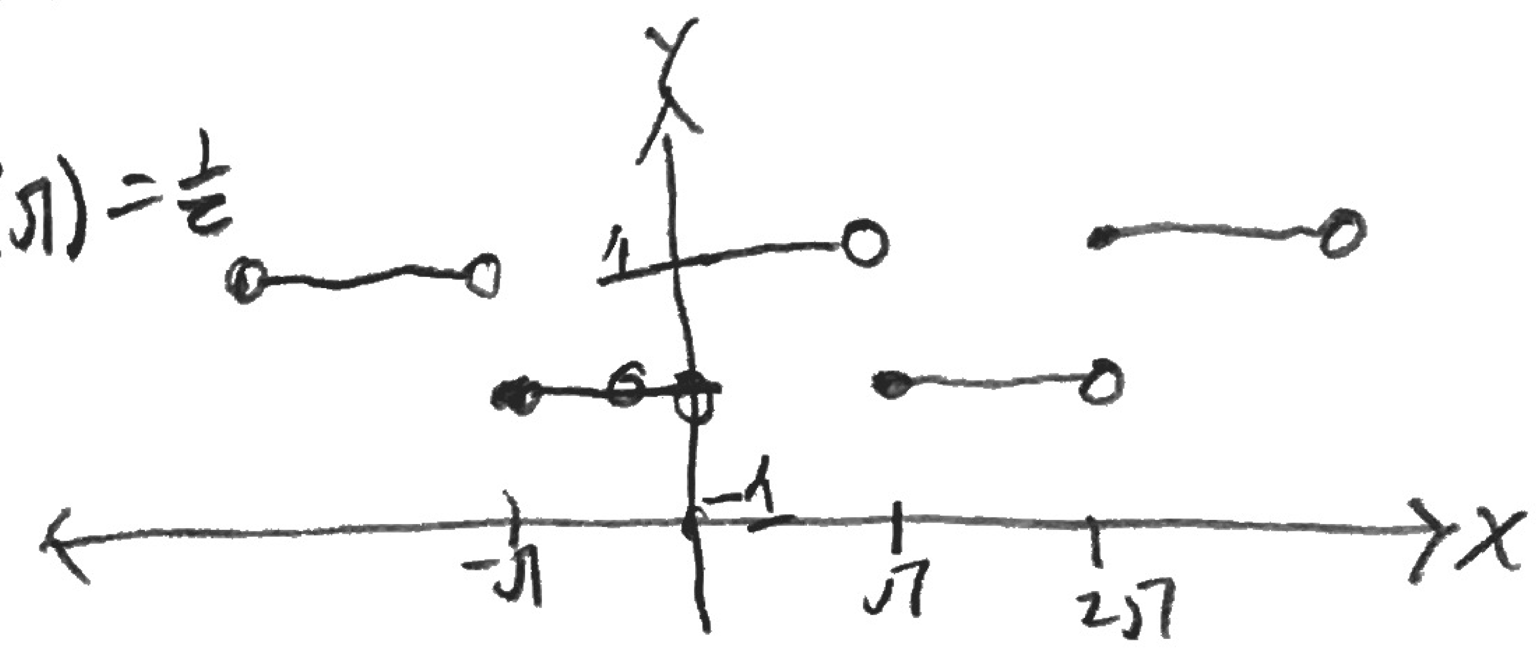
$$1 = P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} A^2 \cdot \cos^2(t)\, dt = \frac{A^2}{2}$$

$$A^2 = 2, \quad A = \sqrt{2} \approx 1.4142$$

$$V(t) = \sqrt{2} \cdot \cos(t)$$ $\leftarrow$ Cos(t) is even function $\therefore$ Odd harmonics including 5th have value of 0

for a Square wave: $f(x) = \begin{cases} 0 & \text{if } -\pi \le x < 0 \\ 1 & \text{if } 0 \le x < \pi \end{cases}$ & $f(x + 2\pi) = f(x)$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{2\pi}\int_{-\pi}^{0} 0\, dx + \frac{1}{2\pi}\int_0^{\pi} 1\, dx = 0 + \frac{1}{2\pi}(\pi) = \frac{1}{2}$$



for $n \ge 1$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cdot \cos(nx)\, dx = \frac{1}{\pi}\int_{-\pi}^{0} 0\, dx + \frac{1}{\pi}\int_0^{\pi} \cos(nx)\, dx$$

$$= 0 + \frac{1}{\pi} \cdot \frac{\sin(nx)}{n}\Big]_0^{\pi} = \frac{1}{n\pi}(\sin(n\pi) - \sin(0)) = 0$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} \sin(nx)\, dx = -\frac{1}{\pi} \cdot \frac{\cos(nx)}{n}\Big]_0^{\pi} = -\frac{1}{n\pi}(\cos(n\pi) - \cos(0))$$

$$= \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{2}{n\pi} & \text{if } n \text{ is odd} \end{cases}, \qquad f(x) = \frac{1}{2} + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2}{n\pi} \cdot \sin(nx)$$

5th harmonic: $\frac{2}{5\pi} \cdot \sin(5x)$ $\rightarrow$ to compare to above waveform let $x = t$

$\frac{2}{5\pi} \cdot \sin(5t)$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} \frac{4}{25\pi^2} \cdot \sin^2(5t)\, dt$$

$$= \frac{1}{2\pi} \cdot \frac{4}{25\pi} = \frac{4}{50\pi^2} > 0$$