Final Semester Examination *

Signals and Systems (ELC 321-1)
Department of Electrical and Computer Engineering
The College of New Jersey.

Last Name: **First Name:**

Instructions:

1. This is a closed-book examination
2. Attempt all questions. Total score obtainable is 100%

Problem 1 (25 Marks)**.** *The simulation diagram of Fig.1 describes an echo generating system with input $x[n]$ and output $y[n]$. Each successive echo is represented by a delayed and scaled version of the output, which is fed back to the input. The coefficients a and b are attenuation factors while D denotes a unit delay operator.*

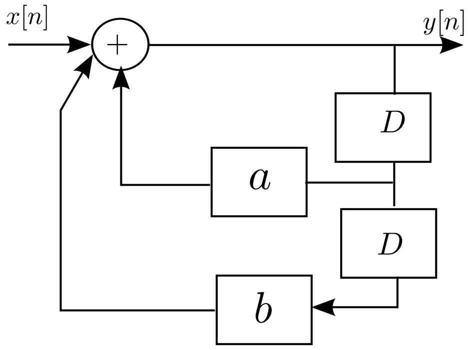

Figure 1: Sequence for Problem 1

a) *Write the difference equation describing the system.*

b) *Determine the transfer function $H(z)$ of the echo system.*

c) *Suppose that $a = 0.5$ and $b = 0.25$, determine the system stability and causality.*

d) *Suppose that we would like to recover the original signal $x[n]$ from the output $y[n]$ by using a system with transfer function $W(z)$(with input $y[n]$ and output $x[n]$), determine $W(z)$.*

□

*Dr. Ambrose A. Adegbege

Solution 1 (25 Marks).

Problem 2 (25 Marks)**.** *Consider the series RL circuit of Fig.2. The input is the applied voltage $v(t)$ and the output is the current $i(t)$ flowing through the inductor.*

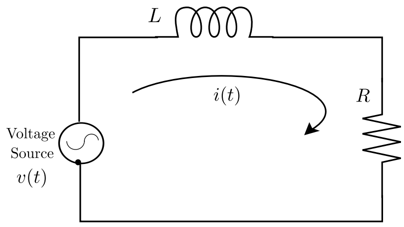

Figure 2: Sequence for Problem 1

a) *Derive the differential equation describing the system.*

b) *Draw a simulation diagram for the system.*

c) *Suppose that $L = 1H$ and $R = 2\Omega$, determine the system transfer function $H(s)$.*

d) *Show that for a unit step input ($v(t) = \mu(t)$),the expression for the current flowing through the circuit is given by;*

$$i(t) = \frac{1}{2}\left(1 - e^{-2t}\right), \ t > 0. \tag{1}$$

$\square$

Solution 2 (25 Marks).

Problem 3 (30 Marks)**.** *Figure 3 shows a half-wave rectifer circuit with sinusoidal signal input $V_S(t) = Asin(\omega t)$ as shown in Fig 4. The voltage measured across the load resistor R_L is shown in Fig 5 assuming ideal diode behavior.*

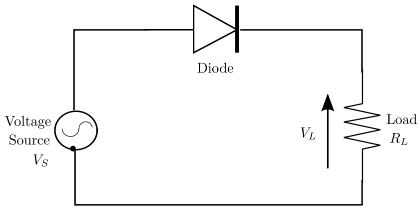

Figure 3: Half-wave Rectifier Circuit

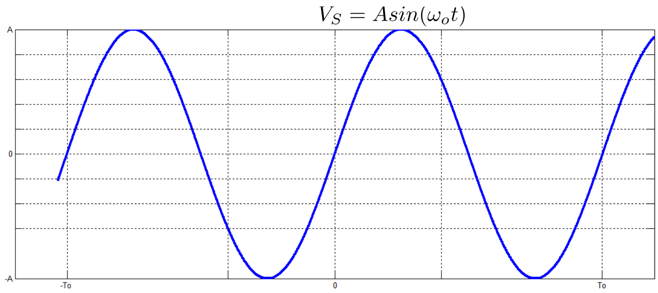

Figure 4: Sinusoidal Input

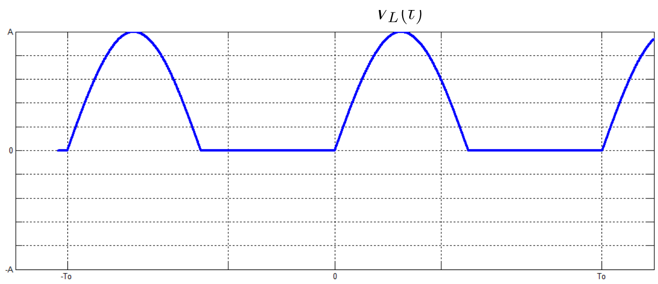

Figure 5: Half-wave rectified Signal

a) *Express the half-wave rectified signal of Fig 5 as an exponential Fourier series.*

b) *Determine the average value of the half-wave rectified signal.*

c) *The half-wave rectified signal of Fig 5 can be expressed as a product of a rectangular pulse train and the sinusoidal input of Fig. 5. Sketch the rectangular pulse train that can be used for this purpose and write a mathematical function describing the pulse train.*

□

Solution 3 (25 Marks).

Problem 4 (25 Marks)**.** *An infinite impulse response filter is decribed by the following difference equation:*

$$y[n] = (1 - \beta)y[n - 1] + \beta x[n] \tag{2}$$

where $x[n]$ and $y[n]$ represent the input and output sequences respectively. The coefficient β is an attenuation factor.

a) Determine if the system is a <u>causal</u> <u>linear</u> <u>time-invariant</u>.

b) Determine the impulse response $h[n]$ for the filter.

c) Suppose that $\beta = 0.4$, determine the system response to a unit step input i.e $x[n] = \mu[u]$ assuming zero initial condition, i.e. $y[0] = 0$.

d) Find the Discrete-time Fourier transform $H(\Omega)$ of $h[n]$

□

Solution 4 (25 Marks)**.**

1 Reference

The Fourier series of a continuous-time signal $x(t)$ is defined as:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t} \tag{3}$$

$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt \tag{4}$$

The continuous-time Fourier transform (inverse Fourier transform)of $x(t)$ is defined as

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt \tag{5}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega. \tag{6}$$

The magnitude and phase spectra of $X(\omega)$ are given by $|X(\omega)|$ and $\angle X(\omega)$ respectively.
The Laplace transform (two-sided or bilateral) of signal $x(t)$ is defined as

$$X(s) = \int_{t=-\infty}^{\infty} x(t)e^{-st}dt \tag{7}$$

For example $\mathcal{L}[\mu(t)] = \frac{1}{s}$ and $\mathcal{L}[e^{-at}\mu(t)] = \frac{1}{s+a}$
Given a discrete-time signal $x[n]$, its discrete-time Fourier transform (DTFT) (inverse DTFT)

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \tag{8}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}d\Omega \tag{9}$$

$X(\Omega)$ is said to be periodic with respect Ω if $X(\Omega + kT) = X(\Omega)$ where T is the period and k is any integer.
The z-transform (two-sided or bilateral) of signal $x[n]$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{10}$$

For example $\mathcal{Z}(\mu[n]) = \frac{z}{z-1}$, $\mathcal{Z}(a^n\mu[n]) = \frac{z}{z+a}$ and $\mathcal{Z}(x[n-1]) = z^{-1}X(z)$.
Given that an LTI system has an impulse response $h[n]$, the output response of the system $y[n]$ for an input $x[n]$ is given by

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{k=\infty} h[n-k]x[k] = \sum_{k=-\infty}^{k=\infty} h[k]x[n-k] \tag{11}$$

The system function is given by

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=-\infty}^{k=\infty} h[n]z^{-n} \tag{12}$$