Math 127 Exam 2 (70 points)

Name: ______________________

(1) (10 points) For each of the following statements, decide whether the statement is true (T) or false (F). Circle either T or F.

(a) $\dfrac{d}{dx}(\arctan x) = \dfrac{1}{x^2+1}$ T F

(b) $\dfrac{d}{dx}e^{2-9x^2} = e^{2-9x^2}$ T F

(c) $\dfrac{d}{dx}\ln(7) = \dfrac{1}{7}$ T F

(d) $\dfrac{d}{dx}\ln(2-9x^2) = \dfrac{1}{2-9x^2}$ T F

(e) If $f(x)$ is a differentiable function whose first derivative is continuous, decreasing, and negative for all real numbers x, then the graph of $y = f(x)$ in the xy-plane is concave up. T F

(f) The absolute maximum of $f(x) = x^2$ on every closed interval is at one of the endpoints of the interval. T F

(g) If you zoom in with your calculator on the graph of $y = f(x)$ in a small interval around $x = 10$ and see a straight line, then the slope of that line equals the derivative $f'(10)$. T F

(h) If f is a differentiable function and $f'(p) = 0$, then $f(p)$ is either a local maximum or a local minimum. T F

(i) If $f'(x) \leq 0$ for all x, then $f(a) \leq f(b)$ whenever $a \leq b$. T F

(j) If f' is increasing, then f is increasing. T F

(2) (12 points)

(a) On the grid below sketch a well-labeled graph of a function $y = f(x)$ that satisfies all of the following conditions. Assume that the function is defined and continuous for all real numbers x.

x	$(-\infty, 3)$	3	$(3, +\infty)$
$f'(x)$	2	2	$+$
$f''(x)$	0	0	$+$

TABLE 1. Sign table of $f'(x)$ and $f''(x)$

(b) On the grid below sketch a well-labeled graph of a function $y = f(x)$ that satisfies all of the following conditions. Assume that the function is defined and continuous for all real numbers x.

x	$(-\infty,-3)$	-3	$(-3,3)$	3	$(3,+\infty)$
$f'(x)$	$+$	0	$+$	$+$	$+$
$f''(x)$	$-$	0	$+$	0	$-$

TABLE 2. Sign table of $f'(x)$ and $f''(x)$

(c) On the grid below sketch a well-labeled graph of a function $y = f(x)$ that satisfies all of the following conditions. Assume that the function is defined and continuous for all real numbers $x \neq -3$.

x	$(-\infty, -3)$	-3	$(-3, 3)$	3	$(3, +\infty)$
$f'(x)$	$+$	undefined	$-$	0	$+$
$f''(x)$	$+$	undefined	$+$	$+$	$+$

TABLE 3. Sign table of $f'(x)$ and $f''(x)$

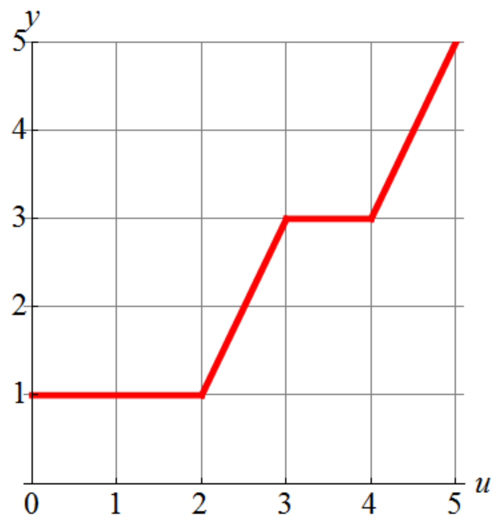
(3) (4 points) Suppose that the population of trout in a certain area of Delaware River is approximated by $f(t) = 20e^{0.1t}$, where t is measured in months since the trout first arrive in the area. (Note that e is approximately 2.71, that $e^{0.1}$ is approximately 1.105, and that e^{10} is approximately 22,026.31.)

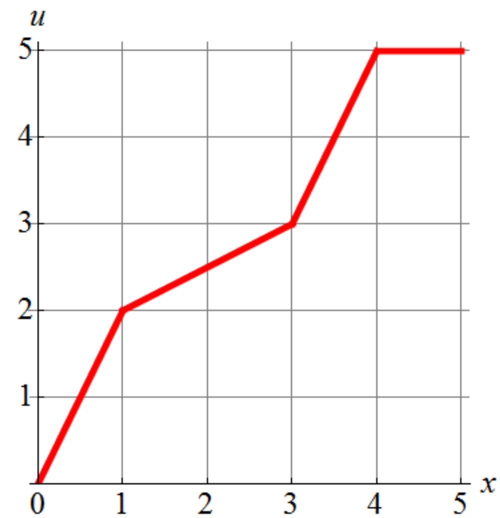
(a) Calculate $f(10)$. Interpret the result in practical terms, giving units.

(b) Calculate $f'(10)$. Interpret the result in practical terms, giving units.

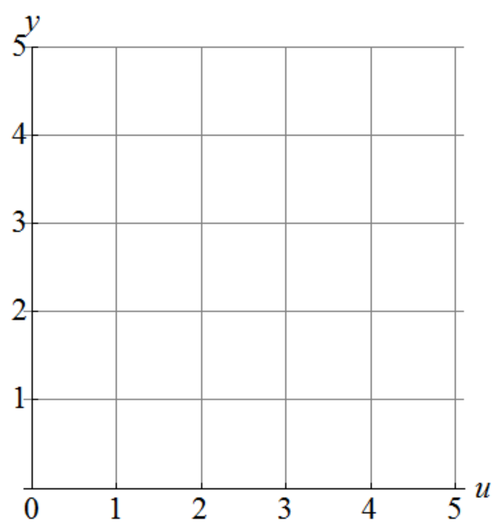
(4) (4 points) Let $y = u^4$ and $u = \dfrac{x+1}{x-1}$. Using the chain rule, compute $\dfrac{dy}{dx}$ when $x = 0$.

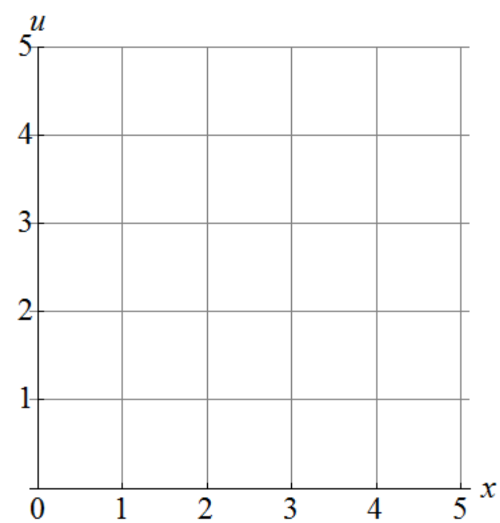
(5) (4 points) Consider functions $y = f(u)$ and $u = g(x)$, where the graph $y = f(u)$ is shown in the left image below and the graph of $u = g(x)$ is shown in the right image below.





(a) Graph $f'(u)$ in the left gridline below and graph $g'(x)$ in the right gridline below.

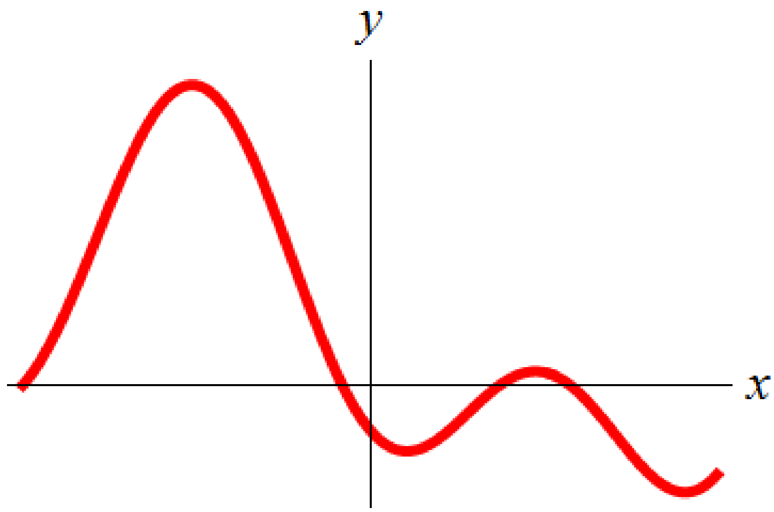




(b) Find u when $x = 2$.

(c) Using the chain rule, compute dy/dx when $x = 2$. (Make sure you evalute dy/du at the correct value of u.)

(6) (4 points) The graph of the function $f(x)$ is shown in the figure below.



(a) Indicate all the critical points on the graph. Determine which critical points correspond to relative minima, relative maxima, absolute minima, absolute maxima, or none of these.

(b) Indicate all the inflection points on the graph.

(7) (15 points)

(a) Compute the derivatives of the following six functions.

$$y = \arcsin x$$

$$y = \tan(8 \sin x)$$

$$y = (x^3 + e^{2x})^4 - 1$$

$$y = \frac{1}{2} \ln(x^2 + 1)$$

$$y = \sqrt{2x - x^2}$$

$$y = (\ln(x + 1))^2$$

(b) In the xy-plane the graphs of all six functions above pass through the origin. Imagine you are zooming in on these six graphs near the origin. Which of them look the same? Group together those functions which become indistinguishable, and give the equation of the line they look like.

(8) (5 points) Find the absolute maximum and the global minimum of the function $f(x) = x^3 - 12x^2 - 60x + 50$ on the following intervals.

(a) $-1 \leq x \leq 1$

(b) $0 \leq x \leq 20$

(9) (4 points) The equation $x^3 + y^3 - xy^2 = 5$ defines a curve in the xy-plane.

(a) Verify that the point $(x, y) = (1, 2)$ is on the curve.

(b) Calculate dy/dx at $(x, y) = (1, 2)$.

(c) Find the equation of the tangent line to the curve at the point $(x, y) = (1, 2)$.

(10) (4 points) Calculate dy/dx using logarithmic differentiation. (No credit will be given if another method is used.)

$$y = \sqrt{\frac{(x-2)^3 x^5 e^x}{x^2+5}}$$

(11) (4 points) Let $f(x) = \dfrac{1}{\sqrt{1+x}}$ be a function defined on the interval $(-1, 1)$.

(a) Find the linear approximation of $f(x)$ near $x = 0$. In other words, find the equation of the tangent line to the graph of f at $x = 0$.

(b) (Extra Credit) What is the relative position of the tangent line at $x = 0$ relative to the graph of f? Is the tangent line above or below the graph of f to the left of $x = 0$? Is the tangent line above or below the graph of f to the right of $x = 0$? (Hint: Compute and interpret $f''(0)$.)