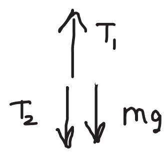
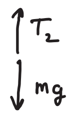
SOLUTIONS TO REVIEW PROBLEMS 2 FOR MIDTERM 1

① Top mass:



$$T_1 - T_2 - mg = ma$$

Bottom Mass:



$$T_2 - mg = ma$$

a) Adding the two equations, we get:

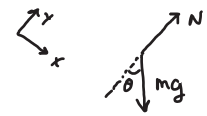
$$T_1 - 2mg = 2ma \Rightarrow T_1 = 2m(g+a)$$
$$= 2 \times 3.50\,kg\,(9.8\,m/s^2 + 1.60\,m/s^2)$$
$$= \boxed{79.8N}$$

$$T_2 = m(g+a) = \boxed{39.9\,N}$$

b) Breaks when $T_1 = 85.6N = 2m(g+a)$

$$\Rightarrow \quad a = \frac{T_1}{2m} - g = \boxed{2.34\,m/s^2}$$

②



$$\Sigma F_x = ma_x$$

$$mg\sin\theta = ma$$

$$\Rightarrow \quad (a) \quad a = g\sin\theta = \boxed{4.9\,m/s^2}$$

b) $v^2 = v_0^2 + 2a(s - s_0)$

$$v^2 = 0 + 2a\frac{h}{\sin\theta} \Rightarrow v = \sqrt{\frac{2ah}{\sin\theta}} = \boxed{3.13\,m/s}$$

c) $R = v_{0x}t = v_0\cos\theta\, t$ where v_0 here is the final velocity from b) and t is the time in the air.

We can get t from looking at the y-motion:

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$-H = 0 - v_0\sin\theta\, t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad \frac{1}{2}gt^2 + v_0\sin\theta\, t - H = 0$$

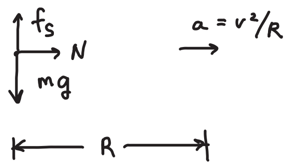
$$\Rightarrow \quad 4.9t^2 + 1.57t - 2 = 0$$

$$t = \frac{-1.57 \pm \sqrt{(1.57)^2 - 4(4.9)(-2)}}{2(4.9)} = 0.50\,s, -0.82\,s$$

$$R = v_0\cos\theta\, t = (3.13)(\cos 30°)(0.50) = \boxed{1.35\,m}$$

d) $t_{TOT} = t_{ramp} + t_{flight} = v/a + 0.5s$

$$= 3.13\,m/s / 4.9\,m/s^2 + 0.5\,s = \boxed{1.14\,s}$$

③ 

$$\Sigma F_x = ma_x \implies N = \frac{mv^2}{R}$$

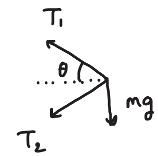
$$\Sigma F_y = ma_y \implies f_s - mg = 0 \implies f_s = mg$$

Max. period occurs when $f_s = f_{s,max}$

$$\implies f_s = mg = \mu_s N = \mu_s \frac{mv^2}{R}$$

$$\implies v^2 = gR/\mu_s \implies v = \sqrt{gR/\mu_s}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{gR/\mu_s}} = \boxed{2\pi\sqrt{\mu_s R/g}}$$

④ 

$\theta = 30°$ (b/c of equilateral triangle)

Choosing 

$$\Sigma F_x = ma_x$$

$$-T_1\cos\theta - T_2\cos\theta = m\left(\frac{-v^2}{r}\right) \quad \text{where } r = L\cos\theta$$

$$\implies T_1\cos\theta + T_2\cos\theta = \frac{mv^2}{L\cos\theta} \implies T_1 + T_2 = \frac{mv^2}{L\cos^2\theta} = \frac{mv^2}{L\left(\sqrt{3}/2\right)^2}$$

$$\Sigma F_y = ma_y \qquad = \frac{4}{3}\frac{mv^2}{L}$$

$$T_1\sin\theta - T_2\sin\theta - mg = 0$$

$$\implies T_1 - T_2 = \frac{mg}{\sin\theta} = \frac{mg}{1/2} = 2mg$$

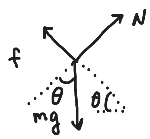
Adding the above equations:

$$2T_1 = \frac{4}{3}\frac{mv^2}{L} + 2mg \implies \boxed{T_1 = m\left(\frac{2}{3}\frac{v^2}{L} + g\right)}$$

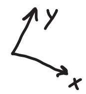
Subtracting:

$$2T_2 = \frac{4}{3}\frac{mv^2}{L} - 2mg \implies \boxed{T_2 = m\left(\frac{2}{3}\frac{v^2}{L} - g\right)}$$

(5)



a) When $\theta = 30°$; $f = f_{s,max} = \mu_s N$

Letting 

$$\Sigma F_x = ma_x$$

$$-f_{s,max} + mg\sin\theta = 0$$

$$\Rightarrow \quad f_{s,max} = mg\sin\theta$$

$$\Rightarrow \quad \mu_s N = mg\sin\theta \Rightarrow \mu_s = \frac{mg\sin\theta}{N}$$

$$\Sigma F_y = ma_y$$

$$N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$$

Thus putting in for N: $\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \boxed{0.58}$

b) $L = 2.5\,m$, $t = 4.0\,s \Rightarrow L = \frac{1}{2}at^2 \Rightarrow a = 0.31\,m/s^2$

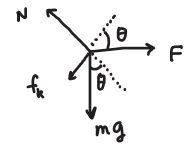
Since its moving, the friction is now kinetic friction:

$$\Sigma F_x = ma_x \Rightarrow -f_k + mg\sin\theta = ma$$

$$\Rightarrow \mu_k mg\cos\theta - mg\sin\theta = -ma$$

$$\Rightarrow \quad \mu_k = \frac{g\sin\theta - a}{g\cos\theta} = \tan\theta - \frac{a}{g\cos\theta} = \boxed{0.54}$$

(6)



$$\Sigma F_x = ma_x$$

$$-mg\sin\theta + F\cos\theta - f_k = ma \quad \& \quad f_k = \mu_k N$$

$$\Sigma F_y = ma_y$$

$$N - F\sin\theta - mg\cos\theta = 0 \Rightarrow N = F\sin\theta + mg\cos\theta$$

$$= 69\ N$$

$$\Rightarrow \quad f_k = \mu_k N = (0.30)(69\ N) = 21\ N$$

From above, $a = \frac{F\cos\theta - f_k}{m} - g\sin\theta$

$$= 3.8\ m/s^2 - 5.9\ m/s^2 = \boxed{-2.1\ m/s^2}$$

b) $$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$0 = (4m/s)^2 + 2(-2.1\,m/s^2)(x - 0)$$

$$\Rightarrow \quad \boxed{x = 3.8\,m}$$

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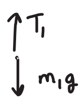
Looking carefully at what's happening, we see that the distance string 1 moves equals twice the distance string 2 moves. This must be true of the accelerations as well:

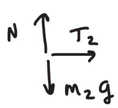
$\Rightarrow$ a) $\boxed{a_2 = 2a_1}$

m_1:



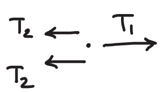
$$T_1 - m_1 g = -m_1 a_1$$

m_2:



$$T_2 = m_2 a_2$$

P2:



$$-T_2 - T_2 + T_1 = \underset{\substack{\shortparallel \\ 0}}{m_p} a_1$$

b/c considered "light"

$$\Rightarrow \quad 2T_2 = T_1$$

Now we can solve for T_1:

$$T_1 = m_1 g - m_1 a_1 = m_1 g - m_1 \frac{a_2}{2} = m_1 g - \frac{m_1}{2}\left(T_2/m_2\right)$$

$$= m_1 g - \frac{m_1}{2m_2}\frac{T_1}{2} \Rightarrow \boxed{T_1 = \frac{4m_1 m_2}{m_1 + 4m_2} g} \quad \text{b)}$$

b) $\boxed{T_2 = \frac{T_1}{2} = \frac{2m_1 m_2 g}{m_1 + 4m_2}}$

c) $\boxed{a_2 = \frac{T_2}{m_2} = \frac{2m_1 g}{m_1 + 4m_2}}$

$\boxed{a_1 = \frac{a_2}{2} = \frac{m_1 g}{m_1 + 4m_2}}$