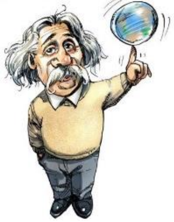
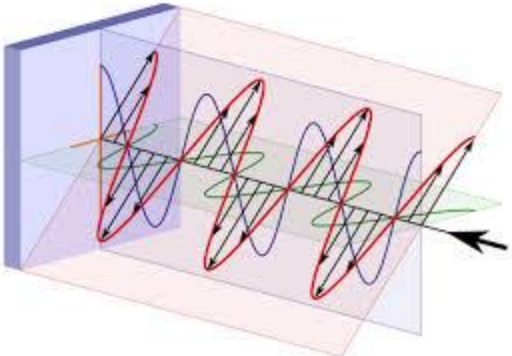
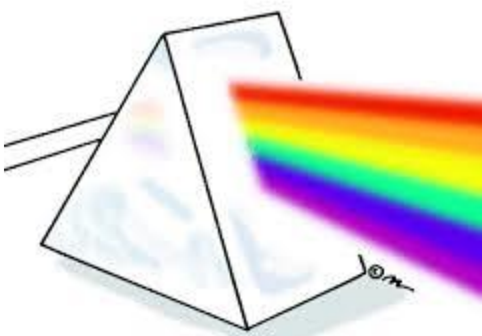
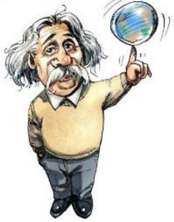


Thin Lenses and Optical Instruments

- Dr. Taryl L. Kirk







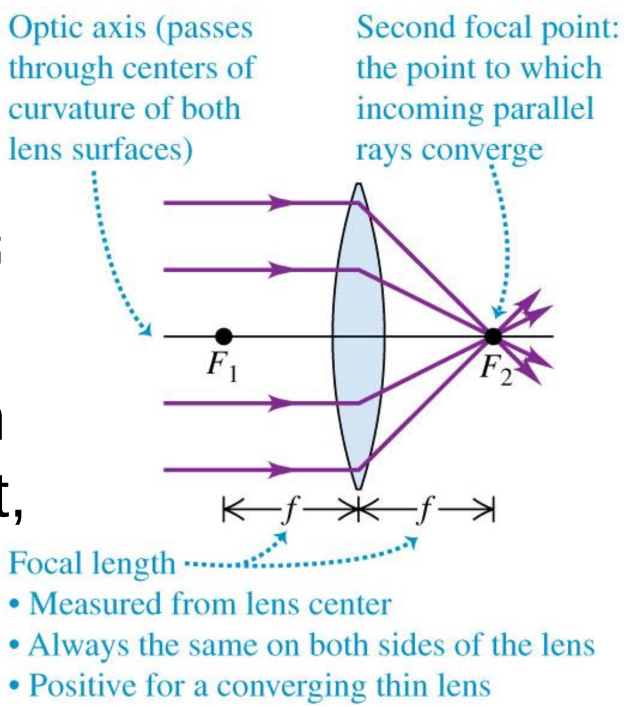
Learning Goals

- Thin Lenses
  - Properties of a Lens
  - Image of an Extended Object: Converging Lens
  - Diverging Lenses
  - The Lensmaker’s Equation
- Optical Instruments
  - The Magnifier
  - Microscopes and Telescopes

Thin Lenses

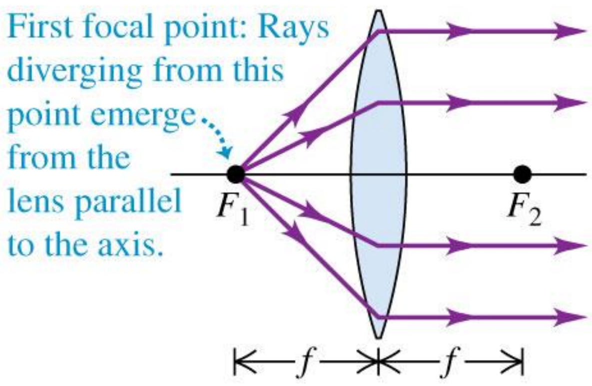
- Similar to the plane mirror, the lens is widely used and well-known.
- A lens in an optical system with two refracting surfaces.
- A **thin lens** is two refracting surfaces separated by a negligible distance.

- **Properties of a Lens**
  - The most important property of a lens is refraction.
  - If paraxial rays converge to a point on the opposite side of the incoming light, the lens is called a **converging lens**.

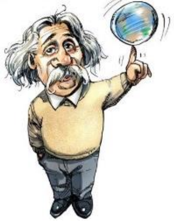


Thin Lenses

- This forms a real image at the second focal point.
- Alternatively, if an object is placed at the first focal point, the refracted light will exit the opposite side as paraxial rays.



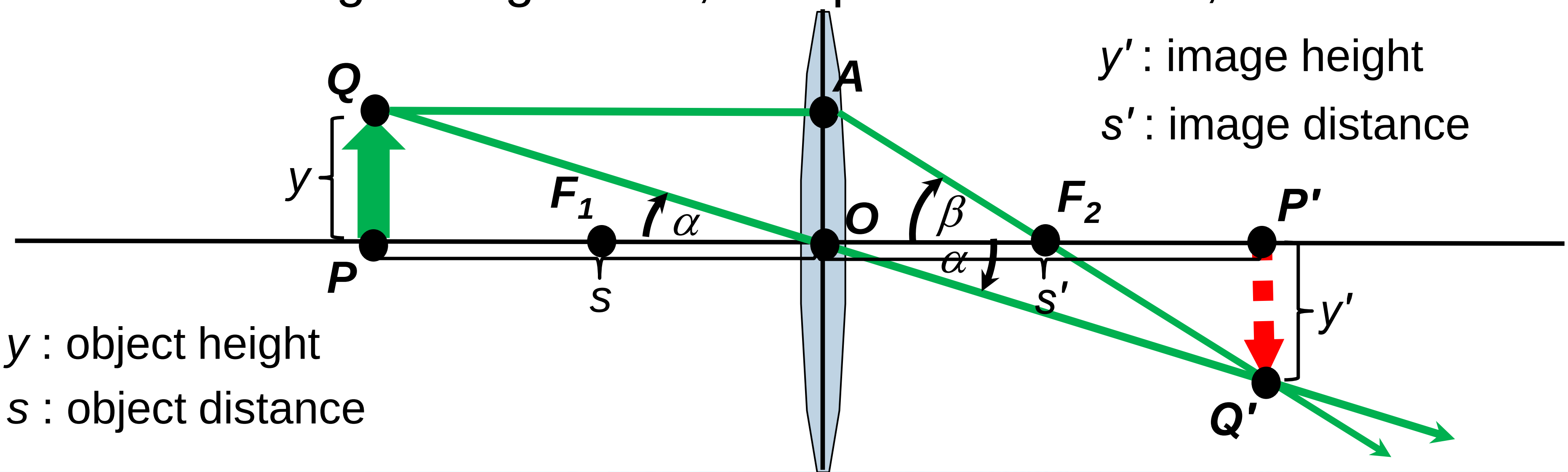
- The *focal lengths* are defined by the distances from the center of the lens to the *focal points*.
- The focal length of a converging lens is by definition positive, so it is called a positive lens.
- The centers of curvature of the two spherical surface define the line of the optic axis.
- The focal lengths are always equal for a thin lens.

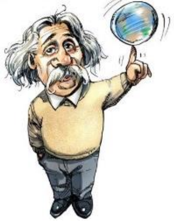


Thin Lenses

- This is still the case even if the two sides have different curvatures.

- **Image of an Extended Object: Converging Lens**
  - Similar to a concave mirror, a converging lens can form an image of an extended object.
  - The original sign rules, for spherical mirrors, are still valid.





Thin Lenses

- The two right triangles $\Delta\, PQO$ and $\Delta\, P'Q'O$ are similar, because the angles α are the same. It follows:

$$\frac{y}{s} = -\frac{y'}{s'} \qquad\qquad \frac{y'}{y} = -\frac{s'}{s}$$

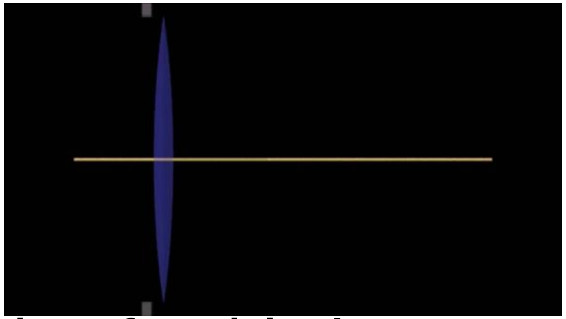
- Recall: the negative sign indicates that the image is below the optic axis.
- The angle β must be the same as the angle opposite to the image height, and the two associated right triangles are similar. Accordingly:

$$\frac{y}{f} = -\frac{y'}{s'-f} \qquad\qquad \frac{y'}{y} = -\frac{s'-f}{f}$$

Thin Lenses

- If the ratio of the image height to object height equations are set equal to each other, and the equations are normalized by the image distance.

$$\textit{object-image relationship} \qquad \frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$



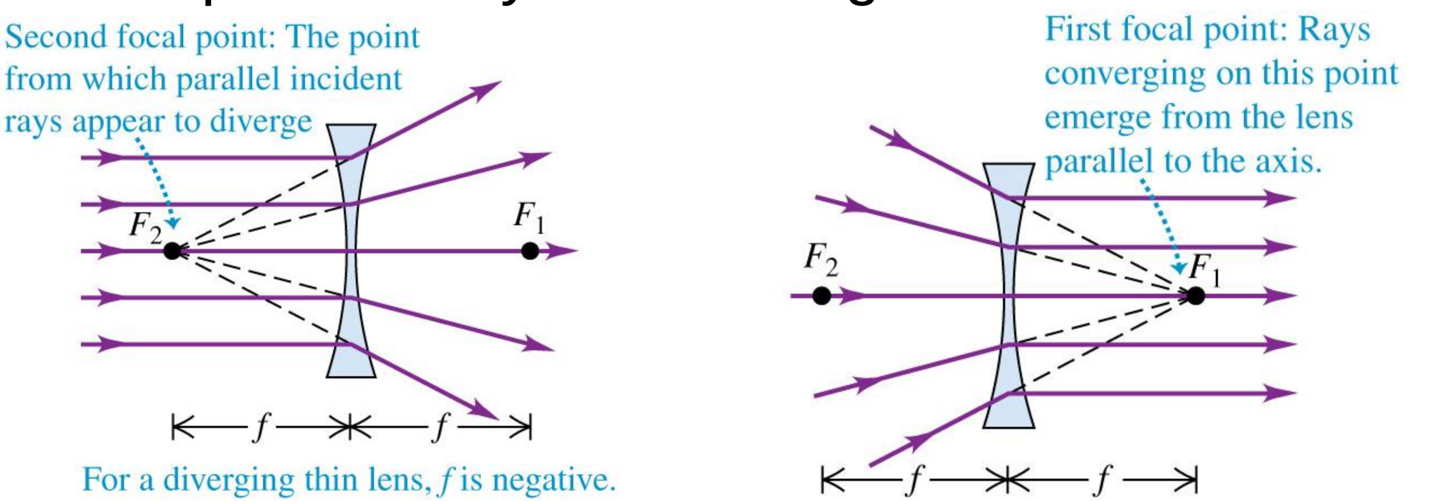
- This also yields the lateral magnification for thin lens:

$$m=-\frac{s'}{s}$$

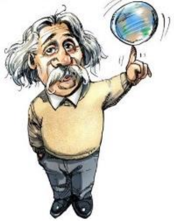
- If both *s* and *s′* are positive the image will be *inverted*.
- A three-dimensional image is not reversed.

Thin Lenses

- **Diverging Lenses**
  - Consider paraxial rays that diverge at a lens after refraction.

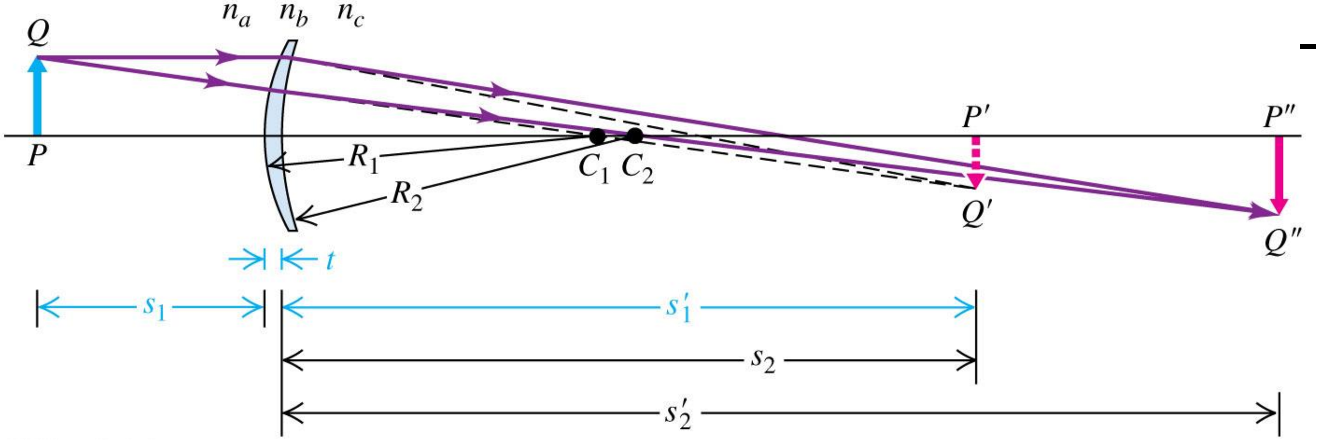


  - This is called a diverging (negative) lens, and the focal points are reversed *cf*. a positive lens.
  - Both the object-image relationship and the lateral magnification for thin lenses is applicable



Thin Lenses

- **The Lensmaker's Equation**
  - Examine the object-image relationship for thin lens.
  - Derive a relationship involving the focal length, *f*, the index of refraction, *n*, of the lens and the radii of curvature R_1 and R_2 of the two refracting surfaces.
  - Note: apply notion that the image of a reflected/refracted light rays can be object for another image

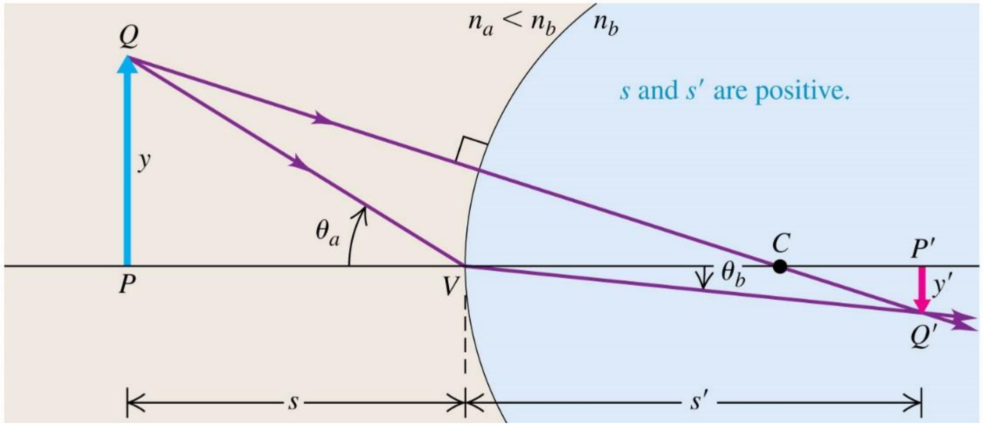


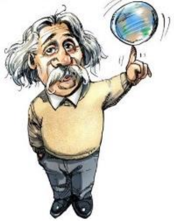
- Consider two spherical interfaces separating three materials

Thin Lenses

- The object and image distance for the first/second interface are s_1/s_2 and s_1'/s_2'.
- Assume that the thickness of the lens is very small, *cf*. s, s_1'.
- It follows that s_2 and s_1' have the same magnitude but opposite sign: $s_2 = - s_1'$.
- Consider the object-image relationship for spherical refracting surfaces.

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$





Thin Lenses

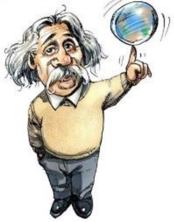
$$\frac{n_a}{s_1}+\frac{n_b}{s_1'}=\frac{n_b-n_a}{R_1} \qquad \frac{n_b}{s_2}+\frac{n_c}{s_2'}=\frac{n_c-n_b}{R_2}$$

- Typically, the first and third material are air (n ~ 1) and the lens has an index of refraction of n. $s_2 = - s_1'$

$$\frac{1}{s_1}+\frac{n}{s_1'}=\frac{n-1}{R_1} \qquad -\frac{n}{s_1'}+\frac{1}{s_2'}=\frac{1-n}{R_2}$$

- Next, add the two equations together:

$$\frac{1}{s_1}+\frac{1}{s_2'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$



Thin Lenses

- This can be generalized for any object distance, *s* and the final image.

$$\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

- Compare this equation to the object-image relationship for thin lenses, so the focal length can be incorporated:

lensmaker's equation $$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

- This equation can be generalized further for situations that the lens is not in air (n = unity), but another medium, *e.g.* water.

Question 1

A thin lens has focal length $f = -12$ cm.

If an object 9 cm tall is placed 24 cm from the lens, what is the height of the image?

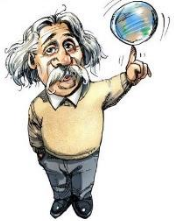
A. 3 cm tall.

B. 4.5 cm tall.

C. 9 cm tall.

D. 18 cm tall.

E. 27 cm tall.



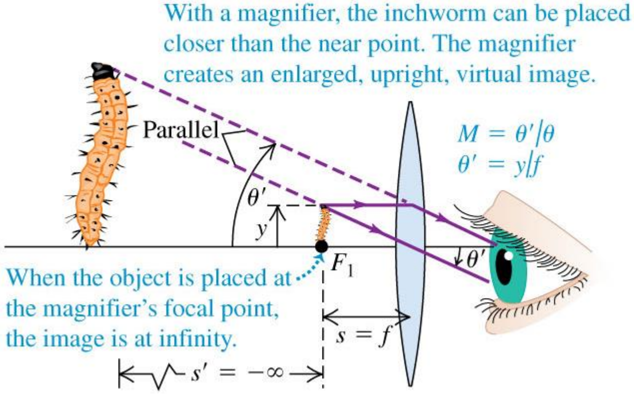
Optical Instruments

- The human eye is limited in its capabilities, so it is sometimes important to implement optical instruments.
- Optical instruments can be used to probe ranges beyond the visible light spectrum or enhance the resolution capabilities of the small or far objects, *e.g.* microscope.
- Typical microscopes do not have thin lenses, and they implement multiple (compound) lens configurations.
- I will consider various optical instruments and I will assume the thin lens formulas are valid.

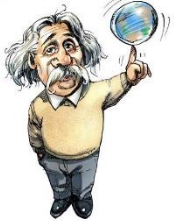
- **Magnifier**
  - A converging lens can be employed to form a virtual image that is larger and farther from the eye than the object.

Optical Instruments

- This converging lens acts as a **magnifier**.



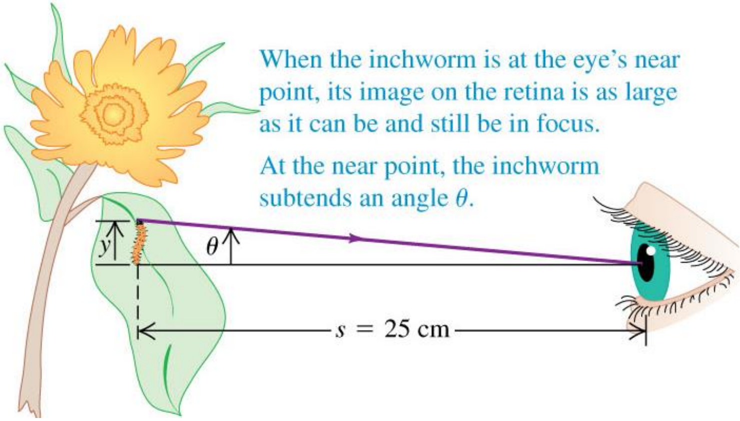
- A converging lens can be employed to form a virtual image that is larger and farther from the eye than the object.
- This image can be easily viewed at infinity, because the ciliary muscles of the eyes will be relaxed. The object must be placed at the magnifier's focal point, F_1, for this to occur.
- The image at infinity subtends an angle of θ' at the magnifier.



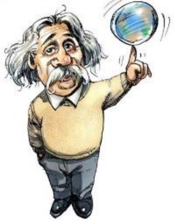
Optical Instruments

- The **angular magnification**, *M*, is given by the ratio of θ' to the angle θ, without the magnifier:

$$M = \frac{\theta'}{\theta}$$



- Do not confuse angular magnification with lateral magnification. The image distance is at infinity but $m \neq \infty/s$.
- You must assume that the angles are small enough to apply the small angle approximation: $\theta \sim \sin\theta$.



Optical Instruments

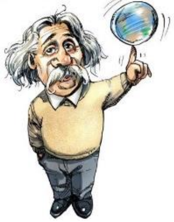
- Note that the rays are drawn undeviated through the lens.
- An expression of for the angles can easily be written:

$$\theta = \frac{y}{25\,cm} \qquad\qquad \theta' = \frac{y}{f}$$

- These expressions can be combined to find the angular magnification:

$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25\,cm} = \frac{25\,cm}{f}$$

- Magnifiers have maximum magnification of about 20x.

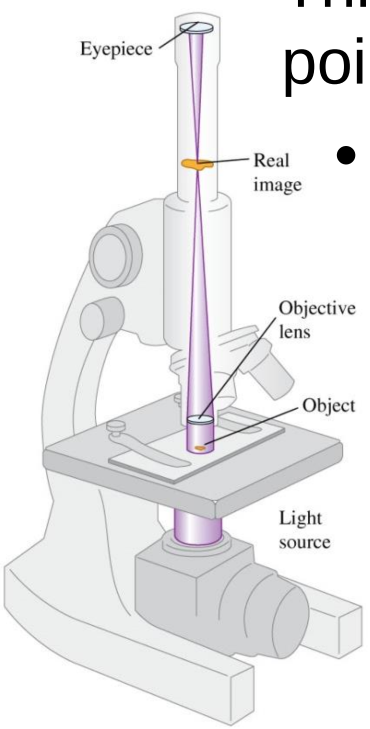


Optical Instruments

- **Microscopes and Telescopes**
  - Two of the most important scientific instruments uses two lenses to enhance spatial resolution capabilities.
  - Microscopes and telescopes both use a primary lens, or *objective*, that form a real image, and a secondary lens, *i.e*. the *eyepiece*, to make an enlarged virtual image.
  - **Microscopes**
    - This device is used to magnify objects greater than a magnifier, and it is sometimes called a *compound microscope*.
    - Here, the image formed by one optical element will be the object for the subsequent optical element.

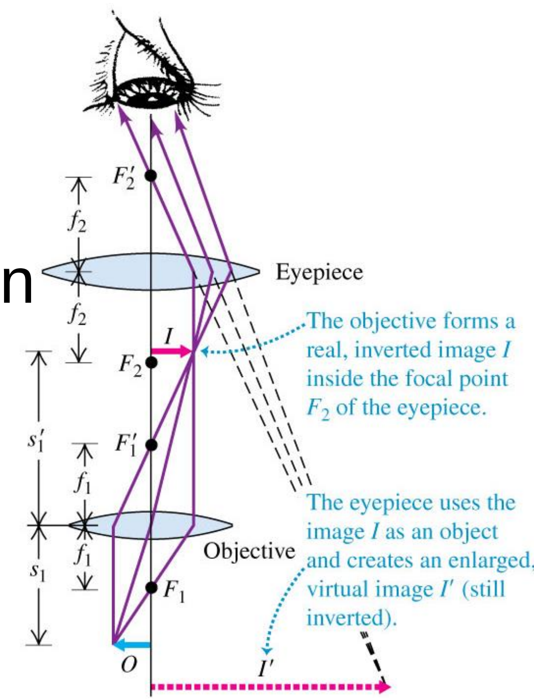
Optical Instruments

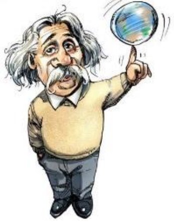
- Elements of a microscope.
- The object is placed just beyond the focal point, F_1, of the **objective** above it.
- This forms a real and enlarged image, I, within the focal point, F_2, of the eyepiece, or ocular.
  - The real image generated by the objective is then enlarged by the eyepiece forming an image between the near and far points of the eye.



m_1 : lateral magnification of object

M_2 : angular magnification of the eyepiece





Optical Instruments

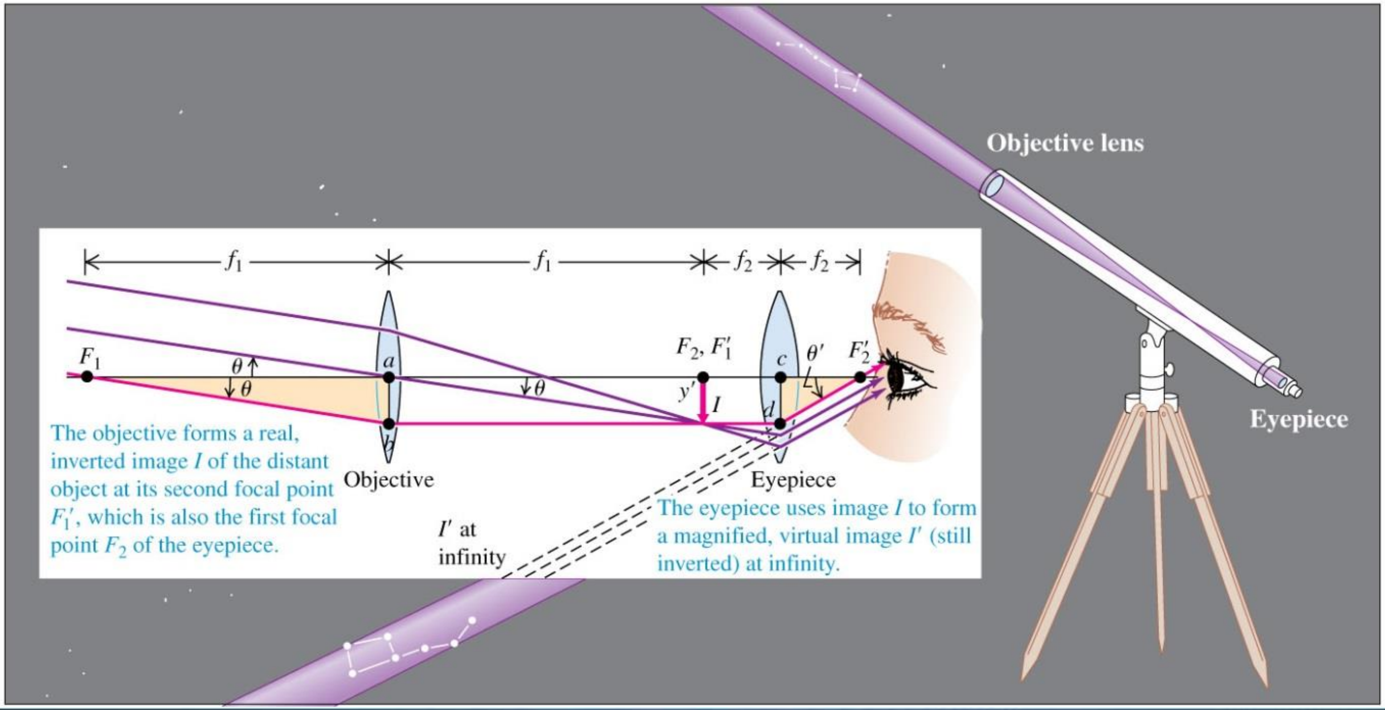
$$M_2 = \frac{25\,cm}{f_2} \qquad M = m_1 M_2 = \frac{(25\,cm)s_1'}{f_1 f_2}$$

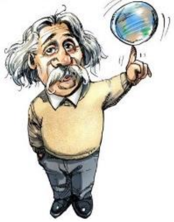
- The final image is inverted with respect to the object.

- **Telescopes**
  - The anatomy of a microscope is similar to telescopes.
  - Many telescopes implement curved mirrors, not a lens, as an objective.
  - ***Refracting telescopes*** **use lenses.**
  - The lens forms a real reduced image of the object.
  - This image is the object for the eyepiece lens, which forms an enlarged virtual image of *I*.

Optical Instruments

- Objects viewed by a telescope form an image *I* at the second focal point of the objective lens.
- The distance between the objective and the eyepiece lenses is the sum of the focal lengths, $f_1 + f_2$.





Optical Instruments

- The angular magnification, *M*, is defined similar to a microscope.
- The magnification of the telescope can be expressed in terms of the these focal lengths.
- If θ and θ' are small, they can be approximated by their tangents:

$$\theta = \frac{-y'}{f_1} \qquad \theta' = \frac{y'}{f_2}$$

- It follows that the angular magnification, *M* is:

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2}$$

Question 2

You are choosing lenses for a telescope that you will use to look at the Moon and planets. You should select

A. an objective lens with a short focal length and an eyepiece lens with an even shorter focal length.

B. an objective lens with a short focal length and an eyepiece lens with a longer focal length.

C. an objective lens with a long focal length and an eyepiece lens with a shorter focal length.

D. an objective lens with a long focal length and an eyepiece lens with an even longer focal length.

Question 3

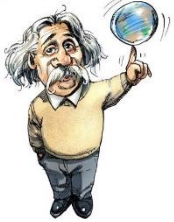
You are designing a telescope that will use a single lens. The purpose of the telescope is to take photographs of the Moon. The linear magnification m of the image will be in the range

A. $m < -1$.

B. $-1 < m < 0$.

C. $0 < m < 1$.

D. $m > 1$.



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