Bryan Gomez

3.1 :

#7. Given $X(t) = C_1 \cdot \cos(wt) + C_2 \sin(wt)$ is the general solution of $x'' + w^2 x = 0$ on $(-\infty, \infty)$. The solution satisfies $X(0) = X_0$, $X'(0) = X_1$ is given by $X(t) = X_0 \cos(wt) + \frac{X_1}{w} \sin(wt)$

$$X'(t) = -wC_1 \cdot \sin(wt) + wC_2 \cdot \cos(wt)$$

$$X(0) = C_1 \cdot \cos(w \cdot 0) + C_2 \cdot \sin(w \cdot 0)$$
$$C_1 = X_0$$

$$X'(0) = -wC_1 \cdot \sin(w \cdot 0) + wC_2 \cdot \cos(w \cdot 0)$$
$$wC_2 = X_1, \quad C_2 = \frac{X_1}{w}$$

$$\therefore X = X_0 \cdot \cos(wt) + \frac{X_1}{w} \cdot \sin(wt)$$

#33. Verify 2-parameter family is a general solution of non-homogeneous DE on given interval

$$y'' - 4y' + 4y = 2e^{2x} + 4x - 12$$
$$y = C_1 e^{2x} + C_2 x e^{2x} + \underbrace{x^2 e^{2x} + x - 2}_{} \quad (-\infty, \infty)$$

y_c: $y'' - 4y' + 4y = 0$

$m^2 - 4m + 4 = 0$

$(m-2)(m-2)$, $m = 2$ twice

$y = C_1 e^{2x} + C_2 x e^{2x}$

$y_1 = e^{2x} \qquad y_2 = xe^{2x}$

Verify: $W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & (2x+1)e^{2x} \end{vmatrix} = (2x+1)e^{4x} - 2xe^{4x} = (1 + 2x - 2x)e^{4x}$
$= e^{4x} \neq 0$ on $(-\infty, \infty)$

y_p: let $y_p = x^2 e^{2x} + x - 2$

$y_p' = 2(x^2 + x)e^{2x} + 1$

$y_p'' = 2(2x^2 + 4x + 1)e^{2x}$

Continued ↓

$y_p'' - 4y_p' + 4y_p = [2(2x^2+4x+1)e^{2x}] - [4(2(2x)e^{2x}+1)]$
$+ 4(x^2e^{2x} + x - 2)$

$= (4x^2+8x+2)e^{2x} - [(8x^2+8x)e^{2x}$
$= (4x^2+8x+2)e^{2x} - (8x^2+8x)e^{2x}$
$(4x^2 - 8x^2 + 4x^2 + 8x - 8x + 2)e^{2x}$
$= 2e^{2x} + 4x - 12$

$\therefore y = c_1e^{2x} + c_2xe^{2x} + x^2e^{2x} + x - 2$

3.4:
#25: $y^{(4)} + 2y'' + y = (x-1)^2$ Solve by undetermine co

y_c: $y^{(4)} + 2y'' + y = 0$, $m^4 + 2m^2 + 1 = 0$
$m^4 + m^2 + m^2 + 1 = 0$, $m^2(m^2+1) + (m^2+1) = 0$
$(m^2+1)(m^2+1) = 0$
$m^2 + 1 = 0$ or $m^2 + 1 = 0$
$m = \pm i$ twice
$y_c = (c_1 + c_2)\cos(x) + (c_3 + c_4x)\sin x$

for y_p: $y_p = Ax^2 + Bx + C$
$y_p' = 2Ax + B$, $y_p'' = 2A$, $y_p''' = 0$, $y_p^{(4)} = 0$

Sub: $y_p^{(4)} + 2y_p'' + y_p = 0 + 4A + Ax^2 + Bx + C = x^2 - 2x + 1$
$A = 1$ $B = -2$
$4A + C = 1$, $C = 1 - 4A = 1 - 4 = -3$

$y(x) = y_c + y_p = c_1 \cdot \cos(x) + c_2 \cdot x \cdot \cos(x) + c_3 \cdot \sin(x) + c_4 \cdot x \cdot \sin(x)$
$+ x^2 - 2x - 3$

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3.4:

#33. Solve the IVP $\frac{d^2x}{dt^2} + w^2x = F_0 \sin$

X_c: $x'' + w^2x = 0$, $r^2 + w^2 = 0$

$(m + iw)(m - iw) = 0$

$m = \pm iw$

$X_c = C_1 \cdot \cos(wt) + C_2 \cdot \sin(wt)$

X_p: guess #1 : $X_p = AT \cdot \cos(wT) + BT \sin(wT)$

$X_p' = -AwT \cdot \sin(wT) + A\cos(wT) + BwT \cdot \cos(wT) + B\sin(wT)$

$X_p'' = -Aw(wT \cdot \cos(wT) + \sin(wT)) - Aw \cdot \sin(wt)$

$+ Bw(-wT \cdot \sin(wT) + \cos(wT)) + Bw \cdot \cos(wt)$

$X_p'' = -Aw^2T \cdot \cos(wT) - 2Aw\sin(wT) - Bw^2T \cdot \sin(wT) +$

$2Bw \cdot \cos(wT) + w^2(AT \cdot \cos(wt) + BT \cdot \sin(wt))$

$X_p'' + w^2X_p = -2Aw \cdot \sin(wT) + 2Bw \cdot \cos(wT) = F_0 \sin(wt)$

$-2Aw = F_0$, $A = -\frac{F_0}{2w}$, $2B = 0$, $B = 0$

$X_p(t) = -\frac{F_0}{2w} \cdot T \cdot \cos(wT)$

$X = C_1 \cdot \cos(wT) + C_2 \cdot \sin(wT) - \frac{F_0}{2w} \cdot T \cdot \cos(wT)$

for $X(0) = 0$, $X'(0) = 0$; $C_1 = 0$

$X'(t) = -wC_1 \cdot \sin(wt) + wC_2 \cdot \cos(wt) - \frac{F_0}{2w}(-wT \cdot \sin(wT) + \cos(wT))$

$X'(0) = wC_2 - \frac{F_0}{2w} = 0$, $C_2 = \frac{F_0}{2w^2}$

$X = \frac{F_0}{2w^2} \cdot \sin(wT) - \frac{F_0}{2w} \cdot T \cdot \cos(wT)$

#37. $y'' + y = x^2 + 1$, $y(0) = 5$, $y(1) = 0$

y_c : $y'' + y = 0$, $m^2 + 1 = 0$,

$y_c = C_1 \cdot \cos(x) + C_2 \cdot \sin(x)$

y_p : guess #1 : $y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$, $y_p'' = 2A$

Sub: $y_p'' + y_p = 2A + Ax^2 + Bx + C = Ax^2 + Bx + 2A + C$

$Ax^2 + Bx + 2A + C = x^2 + 1$

$A = 1$, $B = 0$, $C = -1$

$y_p = x^2 - 1$

$y = y_c + y_p = C_1 \cdot \cos(x) + C_2 \cdot \sin(x) + x^2 - 1$

for $y(0) = 5$: $5 = C_1 - 1$, $C_1 = 6$

for $y(1) = 0$: $0 = C_1 \cdot \cos(1) + C_2 \cdot \sin(1)$

$C_1 \cdot \cos(1) = -C_2 \cdot \sin(1)$

$C_2 = -6 \cdot \cot(1)$

$y = 6 \cdot \cos(x) - 6 \cdot \cot(1) \cdot \sin(x) + x^2 - 1$

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3.6:

#7. Solve the DE $x^2y'' - 3xy' - 2y = 0$

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$x^2y'' - 3xy' - 2y = x^2m(m-1)x^{m-2} - 3x \cdot m \cdot x^{m-1} - 2x^m = 0$

$x^2y'' - 3xy' - 2y = m(m-1)x^m - 3mx^m - 2x^m = 0$

$[m(m-1) - 3m - 2]x^m = 0$

$(m^2 - m - 3m - 2)x^m = 0$, $m^2 - 4m - 2 = 0$

$m = \frac{4 \pm \sqrt{16 - 4(1)(-2)}}{2} = \frac{4 \pm \sqrt{24}}{2}$, $m = 2 \pm \sqrt{6}$

$y = C_1 \cdot x^{2+\sqrt{6}} + C_2 \cdot x^{2-\sqrt{6}}$

#9.) $25x^2y'' + 25xy' + y = 0$

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$25x^2y'' + 25xy' + y = 25x^2m(m-1)x^{m-2} + 25xmx^{m-1} + x^m = 0$

$25x^2y'' + 25xy' + y = 25m(m-1)x^m + 25mx^m + x^m = 0$

$[25m(m-1) + 25m + 1]x^m = 0$

$(25m^2 - 25m + 25m + 1)x^m = 0$

$25m^2 + 1 = 0$, $m = \pm \frac{i}{5}$

$y = C_1 \cdot \cos(\frac{1}{5} \cdot \ln(x)) + C_2 \cdot \sin(\frac{1}{5} \cdot \ln(x))$

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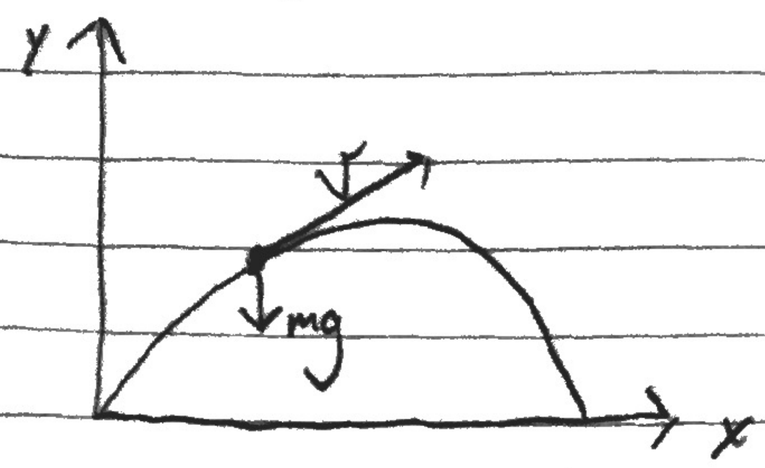
3.12:

#23. Determine a DE that that describes it's motion



$F_x = 0$, $m\vec{a}_x = 0$

$m \cdot \frac{d^2x}{dt^2} = 0$, $\frac{d^2x}{dt^2} = 0$

$\frac{dx}{dt} = C_1 \cdot T$

$x = C_1 T + C_2$

$m \cdot \frac{d^2y}{dt^2} = -mg$, $my'' = -mg$

$\frac{d^2y}{dt^2} = -g$, $\frac{dy}{dt} = -gT + C_3$

$y = -\frac{1}{2} g T^2 + C_3 \cdot T + C_4$