Name: Solutions

MAT 128
Quiz 8

1. Find the radius of convergence and interval of convergence of the series:

$\sum_{n=1}^{\infty} \frac{5^n x^n}{n^3}$ $\quad \lim_{n\to\infty} \left| \frac{5^{n+1} x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{5^n x^n} \right| = \lim_{n\to\infty} \left| 5x \cdot \left(\frac{n}{n+1}\right)^3 \right| = |5x| < 1$

$\Rightarrow |x| < \frac{1}{5}$

If $x = \frac{1}{5}$ $\sum \frac{1}{n^3}$ conv.

If $x = -\frac{1}{5}$ $\sum \frac{(-1)^n}{n^3}$ conv $\Rightarrow$ $R = \frac{1}{5}$ $\left[-\frac{1}{5}, \frac{1}{5}\right]$

2. Find the radius of convergence and interval of convergence of the series

$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2+1}$ $\quad \lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{n^2+2n+2} \cdot \frac{n^2+1}{(x-3)^n} \right| = \lim_{n\to\infty} |x-3| \left| \frac{n^2+1}{n^2+2n+2} \right| = |x-3| < 1$

$\Rightarrow -1 < x-3 < 1 \Rightarrow 2 < x < 4$

If $x = 4$ $\sum \frac{1}{n^2+1} < \sum \frac{1}{n^2}$ conv.

If $x = 2$ $\sum \frac{(-1)^n}{n^2+1}$ conv alt series test

$R = 1$ $[2, 4]$

3. Find a power series representation of the function and the interval of convergence

$f(x) = \frac{x}{9+x^2} = \frac{x}{9} \cdot \frac{1}{1-\left(-\left(\frac{x}{3}\right)^2\right)} = \frac{x}{9} \cdot \sum (-1)^n \frac{x^{2n}}{3^{2n}} = \sum (-1)^n \frac{x^{2n+1}}{3^{2n+2}}$

$\left|\frac{x}{3}\right| < 1$ $\quad |x| < 3$, $-3 < x < 3$

4. Find a power series representation of the function and the radius of convergence

$f(x) = \frac{x}{(1+4x)^2}$

$\frac{1}{1+4x} = \sum_{n=0} (-4x)^n \Rightarrow \frac{-4}{(1+4x)^2} = \sum_{n=1} n(-4x)^{n-1}$

$\Rightarrow \frac{x}{(1+4x)^2} = \frac{-x}{4} \sum_{n=1} n(-4x)^{n-1} = -\sum_{n=1} (-1)^{n-1} 4^{n-2} n x^n$

$= \sum_{n=1} (-1)^n 4^{n-2} n x^n$ $\quad |-4x| < 1 \Rightarrow |x| < \frac{1}{4} = R$