ENG272 FORMULAS FOR FINAL EXAM

ODE IN STANDARD FORM

$$\frac{dy}{dx} + P(x)y = f(x)$$

EXACT EQUATIONS

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

WRONSKIAN

$$W(f_1, \ldots, f_n) = \begin{vmatrix} f_1 & f_2 & \cdots & f_n \\ f_1' & f_2' & \cdots & f_n' \\ \vdots & \vdots & & \vdots \\ f_1^{(n-1)} & f_2^{(n-1)} & \cdots & f_n^{(n-1)} \end{vmatrix}$$

REDUCTION OF ORDER

$$y'' + P(x)y' + Q(x)y = 0$$

(Know how to get to the step below!!)

$$y_2 = y_1(x)\int \frac{e^{-\int P(x)dx}}{y_1^2(x)}dx$$

VARIATION OF PARAMETERS

(Know how to use these formulas!!)

$$u_1' = \frac{W_1}{W} \qquad u_2' = \frac{W_2}{W}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} \qquad W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

$$y_p = u_1 y_1 + u_2 y_2$$

UNDETERMINED COEFFICIENTS

TABLE 3.4.1 Trial Particular Solutions

$g(x)$	Form of y_p
1. 1 (any constant)	A
2. $5x+7$	$Ax+B$
3. $3x^2-2$	Ax^2+Bx+C
4. x^3-x+1	Ax^3+Bx^2+Cx+E
5. $\sin 4x$	$A\cos 4x+B\sin 4x$
6. $\cos 4x$	$A\cos 4x+B\sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x-2)e^{5x}$	$(Ax+B)e^{5x}$
9. x^2e^{5x}	$(Ax^2+Bx+C)e^{5x}$
10. $e^{3x}\sin 4x$	$Ae^{3x}\cos 4x+Be^{3x}\sin 4x$
11. $5x^2\sin 4x$	$(Ax^2+Bx+C)\cos 4x+(Ex^2+Fx+G)\sin 4x$
12. $xe^{3x}\cos 4x$	$(Ax+B)e^{3x}\cos 4x+(Cx+E)e^{3x}\sin 4x$

CRAMER'S RULE

$$\mathbf{A}_k=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1k-1} & b_1 & a_{1k+1} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2k-1} & b_2 & a_{2k+1} & \cdots & a_{2n} \\ \vdots & & & & \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nk-1} & b_n & a_{nk+1} & \cdots & a_{nn} \end{pmatrix}$$

$$x_1=\frac{\det \mathbf{A}_1}{\det \mathbf{A}},\quad x_2=\frac{\det \mathbf{A}_2}{\det \mathbf{A}},\quad \cdots\quad x_n=\frac{\det \mathbf{A}_n}{\det \mathbf{A}}$$

GRAM-SCHMIDT ORTHOGONALIZATION FOR ORTHONORMAL BASES

$$u_1=\frac{v_1}{\|v_1\|},\quad \boldsymbol{w_2=v_2-(v_2\bullet u_1)\,u_1},\quad u_2=\frac{w_2}{\|w_2\|}$$

CAYLEY-HAMILTON THEORM FOR 2 X 2 MATRICES

$$\boldsymbol{\lambda}^{\mathbf{m}}=\mathbf{C_0}+\mathrm{C}_1\boldsymbol{\lambda}$$
$$\mathbf{A}^{\mathbf{m}}=\mathbf{C_0 I}+\mathrm{C}_1\mathbf{A}$$

CONIC SECTIONS

$$ax^2+2bxy+cy^2=k$$

$$\mathbf{x}^T A\mathbf{x}=[x\quad y]\begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=k$$