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AME I Hw set #3

1. $Y_1(x) = 1$ is a solution to $x^2y'' + xy' = 0$, find another linearly independent solution using integrating factor method:

$\frac{1}{x^2}[x^2y'' + xy' = 0] \Rightarrow y'' + \frac{y'}{x} = 0$

$Y_2(x) = Y_1(x) \cdot \int \frac{e^{-\int P(x)dx}}{(Y_1(x))^2} = 1 \cdot \int \frac{e^{-\int \frac{1}{x}dx}}{(1)^2} = \int e^{-\ln(x)+c} = \int \frac{C_1}{x}dx$

$Y_2(x) = C_1 \cdot \ln(x) + C_2$

2. $Y_1(x) = e^{3x}$ is a solution to $Y'' - 6Y + 9Y = 0$

a.) use reduction of order to find a 2nd linearly independant solution $Y_2(x)$, use substitution $Y_2(x) = U(x) \cdot Y_1(x)$

b.) verify the solutions are linearly independant.

a.) $Y_2(x) = u \cdot e^{3x}$, $Y_2(x)' = u' \cdot e^{3x} + 3ue^{3x}$

$Y_2(x)'' = u''e^{3x} + 3u'e^{3x} + 9ue^{3x} + 3u'e^{3x} = u''e^{3x} + 6u'e^{3x} + 9ue^{3x}$

Substitute: $u''e^{3x} + 6u'e^{3x} + 9ue^{3x} - 6(u'e^{3x} + 3ue^{3x}) + 9(ue^{3x}) = 0$

$u''e^{3x} + 6u'e^{3x} + 9ue^{3x} - 6u'e^{3x} - 18ue^{3x} + 9ue^{3x} = 0$

$u'' \cdot e^{3x} = 0$, $u'' = 0$

let $w = u'$, $\therefore w' = u''$, $w' \cdot e^{3x} = 0$, $w' = 0 = \frac{dw}{dx}$

$dw = 0\,dx$, $\int dw = \int 0\,dx$, $w = c = \frac{du}{dx}$, $du = c\,dx$

$\int du = \int c\,dx$, $u = cx$

$Y_2(x) = u \cdot e^{3x} = cxe^{3x}$, $\boxed{Y_2(x) = cxe^{3x}}$

b.) $W = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & 3xe^{3x} + e^{3x} \end{vmatrix}$

$e^{3x}(3xe^{3x} + e^{3x}) - 3e^{3x}(xe^{3x})$

$= 3xe^{6x} + e^{6x} - 3xe^{6x}$

$e^{6x} \neq 0$

3. Use method of roots to find the general solution of $y''-y'-12y=0$

$a=1$, $b=-1$, $c=-12$, $b^2=1 > 4ac=-48$

$\therefore am^2+bm+c=0$

$m^2-m-12=0$, $(m+3)(m-4)=0$

$m=-3,4$, $y_2(x)=c_1e^{-3x}+c_2e^{4x}$

4. Use the method of roots to solve IVP $y''-2y'+5y=0$

$y(0)=-5$, $y'(0)=1$ $\qquad a=1$, $b=-2$, $c=5$

$b^2=4 < 20=4ac$ $\therefore$ $y=e^{\alpha x}(c_1\cdot\cos(\beta x)+c_2\cdot\sin(\beta x))$

$m_{1,2}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, $m_1=\dfrac{2+\sqrt{4-4(5)}}{2}=1+2i$ $\quad \alpha=1$, $\beta=2$

$m_2=\dfrac{2-\sqrt{4-4(5)}}{2}=\overset{\alpha}{1}-\overset{\beta}{2}i$

apply initial condition: $y(0)=-5$

$y=e^0(c_1\cdot\cos(0)+c_2\cdot\sin(0))=-5$, $c_1=-5$

take first derivative of solution:

$y'=e^x((-5)\cos(2x)+c_2\cdot\sin(2x))+e^x(10\sin(2x)+2c_2\cdot\cos(2x))$

apply $y'(0)=1$

$e^0(-5\cdot 1+0)+e^0(10\cdot\sin(0)+2c_2\cdot\cos(0))=1$

$-5+2c_2=1$

$2c_2=6$, $c_2=3$

final solution: $y=e^x(-5\cos(2x)+3\sin(2x))$