ELC251: Electronics I
The College of New Jersey
Department of Electrical and Computer Eng (ECE)
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Quiz and Exam Cheat Sheet

Electrical Basics

$$\mathbf{i}(t) = \frac{d\mathbf{q}(t)}{dt} \textit{ in Amps}$$

$$Ampere = Columb/Second$$

$$Volt = \frac{kg(m^2)}{As} = A\Omega = \frac{W}{A} = \frac{J}{C}$$

$$W = Ft$$

$$P = \frac{d\mathbf{w}(t)}{dt} = ma = I^2R = \frac{V^2}{R} \textit{ in Watts}$$

$$\underbrace{P = VI}_{\substack{\textit{remember passive sign convention,} \\ \textit{current flows from+to-}}}$$

$$\textit{Ohm's Law}: V = IR$$

Note: passive sign convention states that current flows through a passive element (like a resistor) from positive to negative terminal. This is required for proper implementation of both equations above.

$$\textit{Tellegen's Theorem}: \sum_{k=1}^{N} P_k = 0$$

$$\textit{Kirchoff's Current Law (KCL)}: \sum_{k=1}^{N} I_{km} = 0$$

$$\textit{Kirchoff's Voltage Law (KVL)}: \sum_{k=1}^{N} V_k = 0$$

$$\textit{Impedance}: \vec{Z} = R + jX = \frac{1}{\vec{Y}} = \frac{1}{G + jB}$$

$$V_{ab} = V_a - V_b$$

$$\textit{Volt Divider}: V_{Out} = V_{In}\left(\frac{R_2}{R_1 + R_2}\right) \textit{ when } R_L \gg R_2$$

$$\textit{Current Divider}: I_{Out} = I_{In}\left(\frac{R_1}{R_1 + R_2}\right)$$

$$\textit{Series R}: R_{eq} = \sum_{k=1}^{N} R_k$$

$$\textit{Series V}: V_{eq} = \sum_{k=1}^{N} V_k$$

$$\textit{Parallel R}: R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

$$\textit{Parallel G}: G_{eq} = \sum_{k=1}^{N} G_k$$

$$\textit{Parallel I}: I_{eq} = \sum_{k=1}^{N} I_k$$

Note: voltage sources add in series, cannot be combines in parallel. Current sources add in parallel, cannot be combined in series.

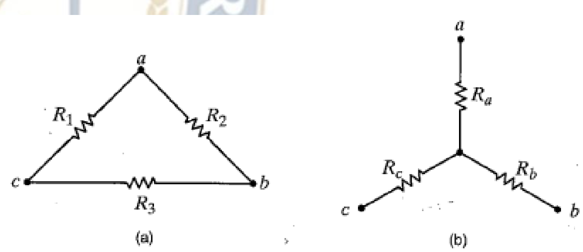


Figure 2.35
Delta and wye resistance networks.

$$R_a = \frac{R_1R_2}{R_1 + R_2 + R_3} \qquad R_1 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b}$$

$$R_b = \frac{R_2R_3}{R_1 + R_2 + R_3} \qquad R_2 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_c}$$

$$R_c = \frac{R_1R_3}{R_1 + R_2 + R_3} \qquad R_3 = \frac{R_aR_b + R_bR_c + R_aR_c}{R_a}$$

Nodal Analysis

$$\#\ variables/eqns = N - 1$$

Simple Nodal Analysis By Inspection: $\begin{pmatrix} I_{S1} \\ I_{S2} \\ I_{S3} \end{pmatrix} =$

$$= \underbrace{\begin{pmatrix} +all\ adj\ Ys & -Y_{12} & -Y_{13} \\ -Y_{12} & +all\ adj\ Ys & -Y_{23} \\ -Y_{13} & -Y_{23} & +all\ adj\ Ys \end{pmatrix}}_{admittance\ matrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$\text{to solve: } \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \left[\underline{Y}\right]^{-1} \begin{pmatrix} I_{S1} \\ I_{S2} \\ I_{S3} \end{pmatrix}$$

Note: that the basis of nodal analysis is KCL.

Note: that only circuits with independent current sources only can be solved via inspection-based nodal analysis. For others, KCL and KVL must be used.

Note: that symmetry of $\left[\underline{Y}\right]$ matrix is lost when circuit contains dependent sources.

Note: that two nodes separated by voltage source may be considered to be supernode.

Table 1: Mesh vs. Nodal Analysis

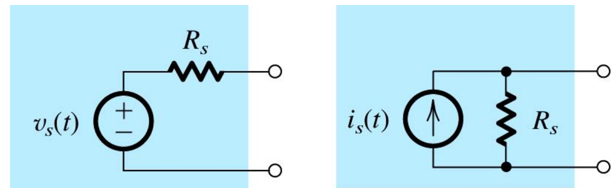
	nodal	loop/mesh
use?	KCL	KVL
solve for?	voltages	currents
easiest when?	ind current sources only	ind voltage sources only
unit?	nodes (N)	loops (L)
equation?	I=YV	V=ZI
# unknowns	N-1	B-N+1

Nodal Analysis

Rules for choosing loops:
- must have $B - N + 1$ branches.
- must include all nodes and branches.

Suggestions for choosing loops:
- any current sources should be isolated and adjacent to only one loop.
- any remaining loops should include no current sources.

Thevenin vs. Norton Equivalent Sources



(a) (b)

Figure 1: (Left) Thevenin Source and (Right) Norton Source

$$\text{equiv: } V_s = I_s R_s \text{ and } R_s = R_s$$

Note: that for superposition, independent sources are considered individually. This theorem can only be applied for linear circuits. Sources are removed as follows:
- voltage sources become "shorts"
- current sources become "open ckts"

$$\mathbf{f}(\alpha_1 x_1 + \alpha_2 x_2) = \alpha_1 \mathbf{f}(x_1) + \alpha_2 \mathbf{f}(x_2)$$

How can one find Thevenin Equivalent for a linear circuit? Use two of following four steps:
- Calculate open-ckt term voltage ($V_{OC} = V_{TH}$).
- Calculate short-ckt term current ($I_{SC} = I_{No}$).
- Calculate equivalent resistance by removing all independent sources ($R_{eq} = R_{Th}$).
- Apply external source and calculate output current.

Note: that you may need nodal / mesh analysis to solve Thevenin / Norton problems.

Note: that inductors act like resistors when it comes to series and parallel connections. For capacitors, this behavior is reversed.

Complex Numbers

$$(a + jb) + (c + jd) = (a + c) + j(b + d)$$
$$|A|\boldsymbol{exp}(j\theta) \times |B|\boldsymbol{exp}(j\theta_B) = |AB|\boldsymbol{exp}\big(j(\theta + \theta_B)\big)$$
$$|A|\boldsymbol{exp}(j\theta) = \underbrace{|A|\boldsymbol{cos}(\theta)}_{real} + j\,\underbrace{|A|\boldsymbol{sin}(\theta)}_{imag}$$

$$a + jb = \underbrace{\left|\sqrt{a^2 + b^2}\right|}_{magnitude} \boldsymbol{exp}\left(j\, \underbrace{\boldsymbol{tan}^{-1}\left(\frac{b}{a}\right)}_{angle\ in\ rads} \right)$$

Introduction to Electronics

Fourier Series Representation of $\mathbf{f}(x)$

$$\mathbf{f}(x) = \frac{a_0}{2} + \underbrace{\sum_{k=1}^{\infty}\left[a_k \cos(kx) + b_k \sin(kx)\right]}$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} \mathbf{f}(x)\cos(kx)dx,\ n \geq 0$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} \mathbf{f}(x)\sin(kx)dx,\ n \geq 1$$

	One Filter	The Other
Transfer Function $T(s)$	$\frac{K}{1+(s/\omega_0)}$	$\frac{Ks}{s+\omega_0}$
Magnitude Response $\lvert T(s)\rvert$	$\frac{\lvert K\rvert}{\sqrt{1+(\omega/\omega_0)^2}}$	$\frac{\lvert K\rvert}{\sqrt{1+(\omega_0/\omega)^2}}$
Phase Response $\lvert T(s)\rvert$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$	K	0
Transmission at $\omega = 0$	0	K

Introduction to Operational Amplifiers

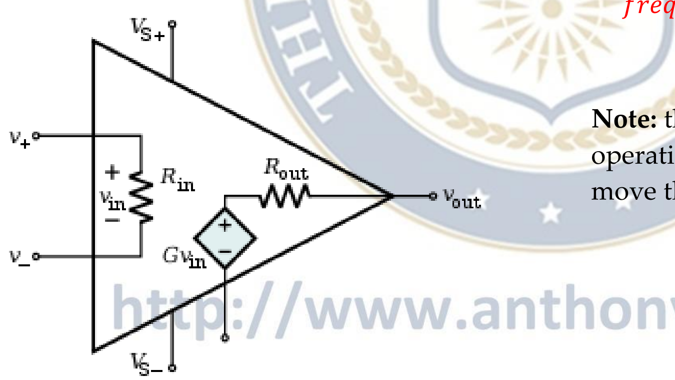


Figure 2: Operational Amplifier Where $G = A_0$ and

$$V_{CC+} = V_{S+}$$

$$\text{ideal: } v_{Out} = \underbrace{A_0(v_{In+} - v_{In-})}_{if\ R_{Out}=0,\ R_{In}=\infty}$$

$$\text{actual: } v_{Out} = A_0\underbrace{(v_{In+} - v_{In-})}_{v_{In}} - i_{Out}R_{Out}$$

Note: that output of amplifier cannot exceed supplied voltages $V_{CC+} = V_{S+}$ and $V_{CC-} = V_{S-}$.

Ideal Op-Amp Characteristics:

- $A_0 = \infty \rightarrow (v_{In+} - v_{In-}) = 0$
- $R_{In} = \infty \rightarrow i_{In} = 0$
- $R_{Out} = 0 \rightarrow v_{Out} = A_0 v_{In}$
- $V_{CC+} = \infty, V_{CC-} = -\infty$

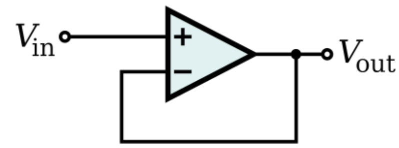


Figure 3: Voltage Follower / Buffer Configuration (Vout = Vin)

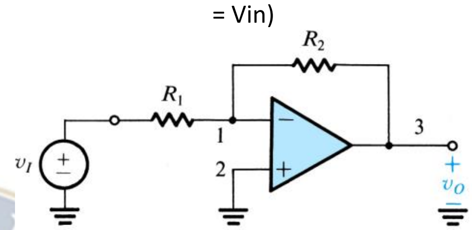


Figure 4: Inverting Op Amp

$$\text{ideal inverting: } v_O = \left(\frac{-R_2}{R_1}\right)v_I$$

$$\text{for } A \ll \infty\text{: } v_O = \left(\frac{-R_2/R_1}{1 + \left(\frac{1+R_2/R_1}{A}\right)}\right)v_I$$

$$\text{freq dependent: } v_O = \left(\frac{-R_2/R_1}{1 + s\left(\frac{1+R_2/R_1}{\omega_t}\right)}\right)v_I$$

Note: that, to calculate the gain of a closed-loop operational amplifier configuration, one may remove the amplifier from the circuit all together.

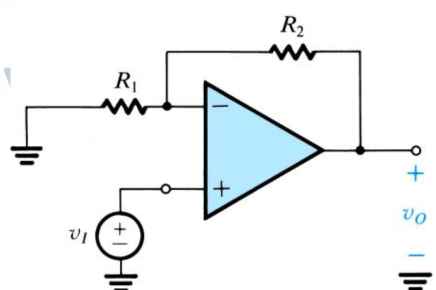


Figure 5: Non-Inverting Amplifier Configuration

$$\text{ideal non inverting: } v_O = \left(1 + \frac{R_2}{R_1}\right)v_I$$

$$\text{for } A \ll \infty\text{: } v_O = \left(\frac{1+R_2/R_1}{1 + \left(\frac{1+R_2/R_1}{A}\right)}\right)v_I$$

$$freq\ dependent: v_O = \left(\frac{1 + R_2/R_1}{1 + s\left(\frac{1 + R_2/R_1}{\omega_t}\right)} \right) v_I$$

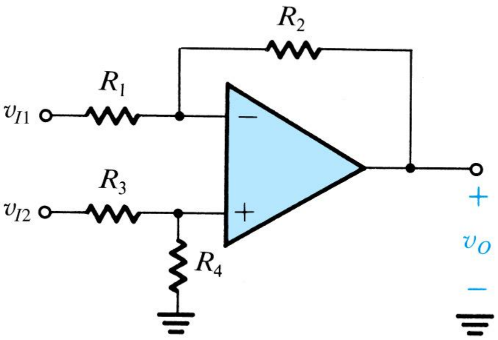


Figure 6: DIfference Amplifier Configuration

$$ideal\ difference: v_O = \frac{(R_1 + R_2)R_4}{(R_3 + R_4)R_1} v_{I2} - \frac{R_2}{R_1} v_{I1}$$
$$= \frac{R_2}{R_1}(v_{I2} - v_{I1})\ if \begin{pmatrix} R_1 = R_3 \\ R_2 = R_4 \end{pmatrix}$$

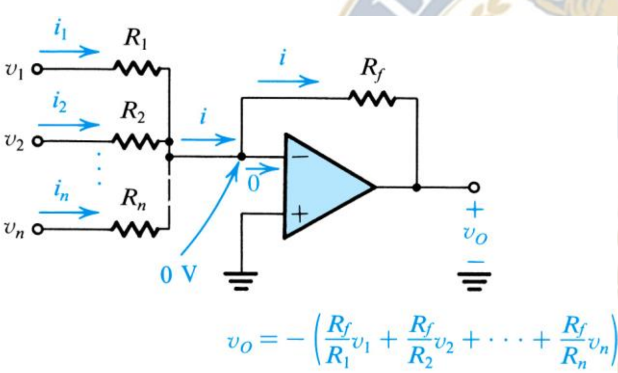


Figure 7: Summing Amplifier Configuration

Differential vs. Common-Mode

$$v_O = A_D v_{DIn} + A_{CM} v_{CMIn}$$
$$v_{CMIn} = \frac{1}{2}(v_{In1} + v_{In2})\ and\ v_{In1,2} = v_{CMIn} \pm \frac{v_{DIn}}{2}$$
$$CMRR = 20\boldsymbol{log}_{10}\left(\frac{A_D}{A_{CM}}\right)$$

Integrators and Differentiating Op-Amps

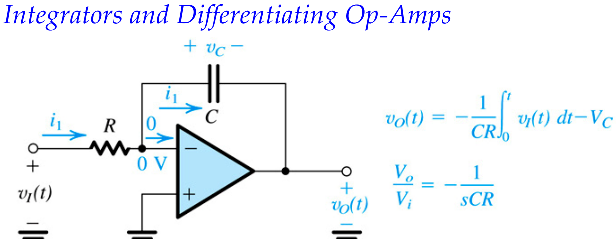


Figure 8: Integrating Amplifier Configuration without Feedback Resistance

$$\tau = RC\ and\ \omega_0 = \frac{1}{RC}$$

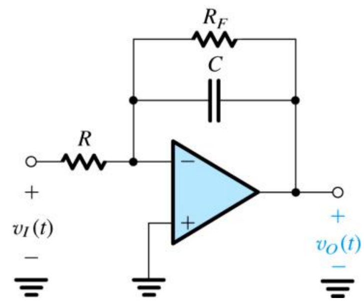


Figure 9: Integrating Amplifier with Feedback Resistance

$$steady\ state: \frac{v_O}{v_I} = -\frac{R_F/R_1}{1 + sR_FC_F}$$

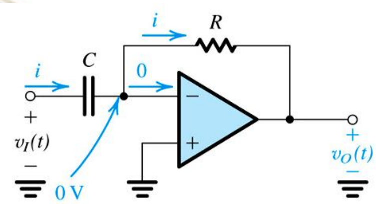


Figure 10: Differentiator Amplifier Configuration

$$transient: v_O(t) = -R_FC_1\frac{d\boldsymbol{v_I}(t)}{dt}$$
$$steady\ state: \frac{V_O(s)}{V_I(s)} = -sR_FC_1$$

Offset Voltage

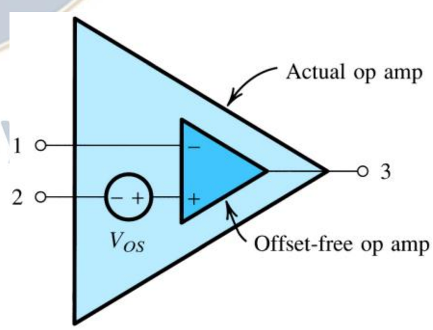


Figure 11: Example of Offset Voltage

$$v_O = V_{OS}\left(1 + \frac{R_F}{R_1}\right)$$

Logarithms and Decades

$$\textit{slope for 1st order low pass behavior} = -20dB/decade$$

$$\#\ \textit{decades btw frequencies}: n_{DEC} = \boldsymbol{log}\left(\frac{f_2}{f_1}\right)$$

$$f_2 = f_1(10^{n_{DEC}})$$

Introduction to Semiconductor Physics

$$n_i = BT^{3/2}\boldsymbol{exp}(-E_g/2kT)$$

$$B = 7.3 \times 10^{15} cm^{-3}K^{-3/2}$$
$$E_g = 1.12 eV \textit{ for silicon}$$
$$k = 8.62 \times 10^{-5}\frac{eV}{K} = 1.38 \times 10^{-23} J/K$$
$$q = 1.6 \times 10^{-19} C$$

$$p_p \times n_p = n_i^2 \textit{ and } p_n \times n_n = n_i^2$$

$$\mu_p = \frac{480cm^2}{Vs} \textit{ for silicon}$$
$$\mu_n = \frac{1350cm^2}{Vs} \textit{ for silicon}$$

$$v_{p-drift} = \mu_p E \textit{ and } v_{n-drift} = -\mu_n E$$
$$I_p = Aqpv_{p-drift} \textit{ and } I_n = -Aqnv_{n-drift}$$
$$I = I_p + I_n$$
$$J = I/A$$

$$\sigma = \frac{1}{\rho} = q(p\mu_p + n\mu_n)$$

$$J_p = -qD_p\frac{d\boldsymbol{p}(x)}{dx} \textit{ and } J_n = -qD_n\frac{d\boldsymbol{n}(x)}{dx}$$
$$D_p = \frac{12cm^2}{s} \textit{ and } D_p = \frac{35cm^2}{s} \textit{ for silicon}$$
$$J_{tot} = J_p + J_n$$

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T \approx 25.8mV$$

$$V_0 = V_T\boldsymbol{ln}\left(\frac{N_A N_D}{n_i^2}\right)$$

pn-Junction

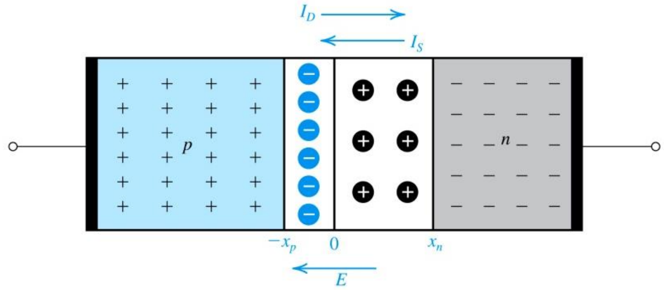


Figure 12: pn-Junction Example

$$Q_J = |Q_\pm| = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)W$$

$$Q_J = |Q_\pm| = A\sqrt{2\varepsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right)\underbrace{(V_0 - V_F)}_{or\ V_0 + V_R}}$$

$$\varepsilon_s = 11.7\varepsilon_0 = 1.04 \times 10^{-12} F/cm$$

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\underbrace{(V_0 - V_F)}_{or\ V_0 + V_R}}$$

$$i_D = I_s\left(\boldsymbol{exp}\left(\frac{v_D}{V_T}\right) - 1\right) \approx \underbrace{I_s\boldsymbol{exp}\left(\frac{v_D}{V_T}\right)}_{if\ v_D \gg V_T}$$

$$I_s = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$$

Diode Basics

$$i_D = I_S(e^{v_D/V_T} - 1) \approx I_S(e^{v_D/V_T})$$

$$V_T = \frac{kT}{q} = 25.8mV \textit{ (unless stated otherwise)}$$

$$v_D = V_T\boldsymbol{ln}\left(\frac{i_D}{I_S}\right)$$
$$\frac{\Delta V_D}{\Delta T} = \frac{-2mV}{1^o C} \textit{ approximately}$$

Iterative Analysis for Diode

Step #1: *Start with initial guess of* $v_D^{(0)}$.
Step #2: *Use nodal or mesh analysis to solve for* $i_D^{(0)}$.
Step #3: *Use exponential model to update* $v_D^{(1)}$.
Step #4: *Repeat Steps #1 through #3 until convergence.*

Small-Signal Model for Diode

$$i_D = I_D + i_d$$

$$\textbf{steady state}: \underbrace{I_D = I_S \boldsymbol{exp}(V_D/V_T)}_{V_D=700mV}$$

$$\textbf{total instanteous}: i_D = \underbrace{\left[I_S \boldsymbol{exp}\left(\frac{V_D}{V_T}\right)\right]}_{I_D} \boldsymbol{exp}\left(\frac{v_d}{V_T}\right)$$

$$\textbf{total instanteous}: i_D \approx I_D + \underbrace{\left(\frac{I_D}{V_T}\right) v_d}_{\substack{\text{small signal} \\ \text{approximation}}}$$

$$r_d = \frac{V_T}{I_D}$$

$$v_d < 5mV_{amp} \ for\ small\ signal\ approximation$$

Full-Wave Rectifier with Filter Capacitor

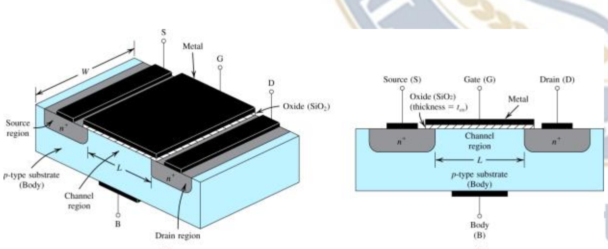
$$V_r(peak\ to\ peak) \approx V_{peak}\left(\frac{T}{RC}\right)$$

$$\Delta t = \frac{\sqrt{2V_r/V_{peak}}}{\omega}$$

Linearization

$$\boldsymbol{f}(x) \approx \left(\left[\frac{d\boldsymbol{f}}{dx}\right]_{x=x_0}\right)(x - x_0) + \boldsymbol{f}(x_0)$$

MOSFET-Basics



$$\varepsilon_0 = 3.45 \times 10^{-11}\ in\ \frac{F}{m}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}\ in\ \frac{F}{m^2}$$

$$v_{OV} = v_{GS} - V_t\ in\ V$$

$$v_{GD} = v_{GS} - v_{DS} = V_t + v_{OV} - v_{DS}\ in\ V$$

$$k_n = \underbrace{C_{ox}\mu_n}_{k_n'}\left(\frac{W}{L}\right)\ in\ \frac{A}{V^2}$$

Charge Stored in P/N-Channel

$$v_{DS} < 50mV: |Q| = C_{ox}(WL)v_{OV}\ (rectangle)$$

$$triode: |Q| = C_{ox}(WL)\left(v_{OV} - \frac{1}{2}v_{DS}\right)\ (trapezoid)$$

$$saturation\ v_{GS} > V_t: |Q| = \frac{1}{2}C_{ox}(WL)v_{OV}\ \ (tri)$$

iD vs. vDS Relationship

$$v_{DS} < 50mV: \quad i_D = (k_n v_{OV})v_{DS}\ in\ A$$

$$v_{DS} < 50mV: \quad r_{DS} = \frac{v_{DS}}{i_D} = \frac{1}{k_n v_{OV}}\ in\ \Omega$$

$$triode: \ i_D = k_n\left(v_{OV} - \frac{1}{2}v_{DS}\right)v_{DS}$$

$$saturation\ v_{GS} > V_t: i_D = \frac{1}{2}k_n v_{OV}^2$$

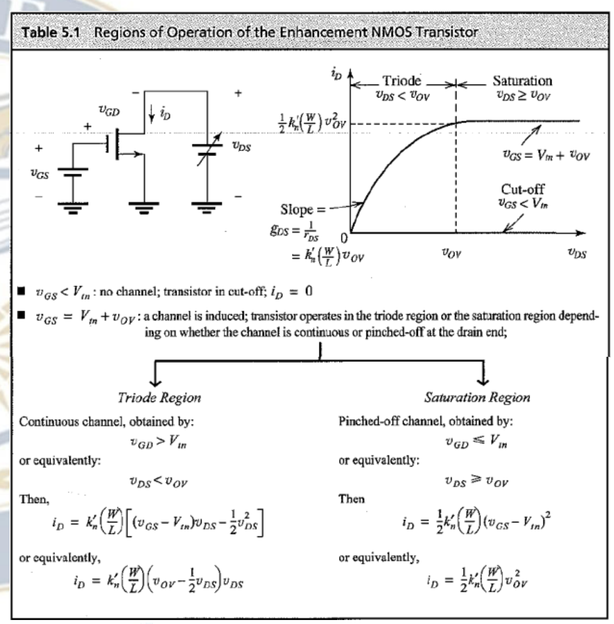


Table 5.1 Regions of Operation of the Enhancement NMOS Transistor

- $v_{GS} < V_{tn}$: no channel; transistor in cut-off; $i_D = 0$
- $v_{GS} = V_{tn} + v_{OV}$: a channel is induced; transistor operates in the triode region or the saturation region depending on whether the channel is continuous or pinched-off at the drain end;

Triode Region

Continuous channel, obtained by:

$$v_{GD} > V_{tn}$$

or equivalently:

$$v_{DS} < v_{OV}$$

Then,

$$i_D = k_n'\left(\frac{W}{L}\right)\left[(v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right]$$

or equivalently,

$$i_D = k_n'\left(\frac{W}{L}\right)\left(v_{OV} - \frac{1}{2}v_{DS}\right)v_{DS}$$

Saturation Region

Pinched-off channel, obtained by:

$$v_{GD} \leq V_{tn}$$

or equivalently:

$$v_{DS} \geq v_{OV}$$

Then

$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2$$

or equivalently,

$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)v_{OV}^2$$

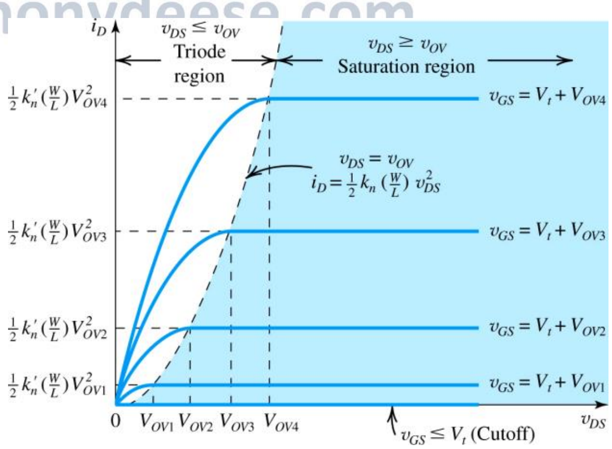


Figure 13: Voltage/Current Relationship for MOSFET

Channel Length Modulation

$$\textit{saturation } v_{GS} > V_t:\ i_D = \frac{1}{2}k_n v_{OV}^2(1+\lambda)$$

$$\textit{output resistance}:\ r_D = \left(\frac{1}{2}k_n v_{OV}^2\lambda\right)^{-1} = \frac{V_A}{i_D}$$

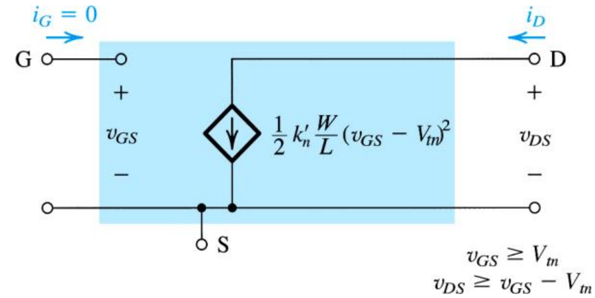


Figure 15: MOSFET Equivalent Circuit in Saturation Region

MOSFET-Based Amplifier

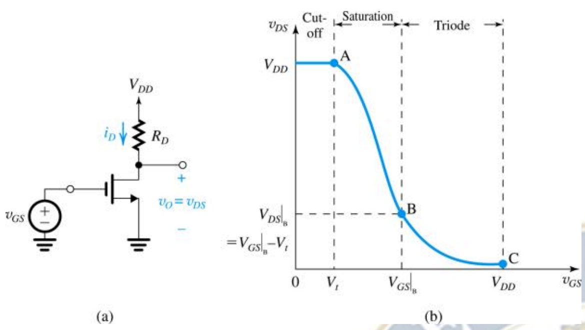


Figure 14: MOSFET Amplifier Configuration

$$v_{DS} = V_{DD} - i_D R_D \textit{ in } V$$

$$v_{DS} = V_{DD} - \underbrace{\left(\frac{1}{2}k_n v_{OV}^2\right)}_{\textit{assuming saturation}} R_D \textit{ in } V$$

$$V_{GS}|_A = V_t \textit{ in } V$$

$$V_{GS}|_B = V_t + \frac{\sqrt{2k_n R_D V_{DD} + 1} - 1}{k_n R_D} \textit{ in } V$$

$$\underbrace{A_v = -k_n V_{OV} R_D}_{\textit{use static/DC component of } v_{OV}}$$

$$\textit{amplifies TV only}:\ v_{ds} = A_v v_{gs}$$

$$max(A_v) = -10V_{DD}$$

SI Units

TABLE 1.5 Selected Prefixes Used in the Metric System

Prefix	Abbreviation	Meaning	Example
Giga	G	10^9	1 gigameter (Gm) $= 1 \times 10^9$ m
Mega	M	10^6	1 megameter (Mm) $= 1 \times 10^6$ m
Kilo	k	10^3	1 kilometer (km) $= 1 \times 10^3$ m
Deci	d	10^{-1}	1 decimeter (dm) $= 0.1$ m
Centi	c	10^{-2}	1 centimeter (cm) $= 0.01$ m
Milli	m	10^{-3}	1 millimeter (mm) $= 0.001$ m
Micro	μ[a]	10^{-6}	1 micrometer (μm) $= 1 \times 10^{-6}$ m
Nano	n	10^{-9}	1 nanometer (nm) $= 1 \times 10^{-9}$ m
Pico	p	10^{-12}	1 picometer (pm) $= 1 \times 10^{-12}$ m
Femto	f	10^{-15}	1 femtometer (fm) $= 1 \times 10^{-15}$ m

[a]This is the Greek letter mu (pronounced "mew").

$$\textit{temperature converion}: Kelvin = 273^O + Celsius = \frac{5}{9}(Farenheit + 459.67)$$