Final Semester Examination *

Signals and Systems (ELC 321-2)
Department of Electrical and Computer Engineering
The College of New Jersey.

Spring 2014

Last Name: **First Name:**

Instructions:

1. This is a closed-book examination
2. Attempt all questions. Total score obtainable is 100%

Problem 1 (25 Marks)**.** *Consider the system simulation diagram of Figure 1. This figure shows a simulation diagram form used in the area of automatic control*

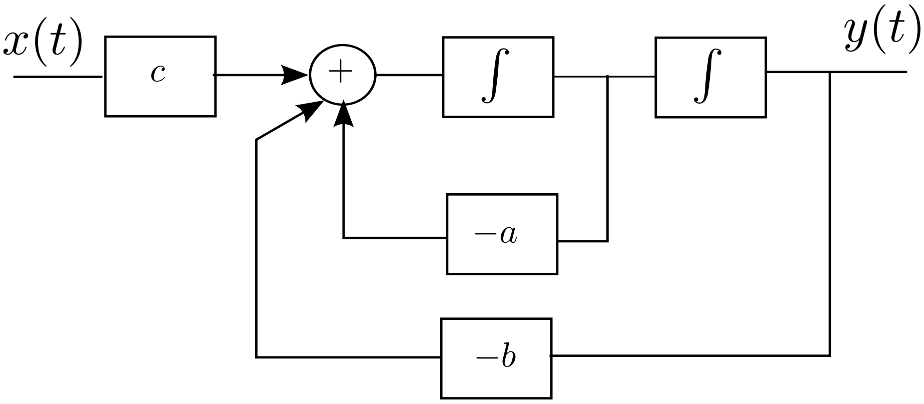

Figure 1: Simulation Diagram for Problem 1

a) Find the differential equation of the system.

b) Find the system transfer function $H(s)$. Assuming zero initial conditions.

c) Suppose that $a = 5$, $b = 6$ and $c = 10$, determine the impulse response $h(t)$ of the system.

d) Determine the stability and causality of the system for the values of a, b and c in (c)

□

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Solution 1 (25 Marks).

Problem 2 (25 Marks). *Consider the series RC circuit of Fig.2. The input is the applied voltage $v(t)$ and the output is the voltage $v_c(t)$ across the capacitor.*

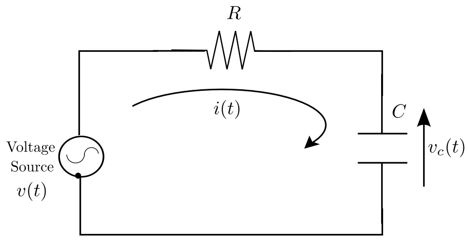

Figure 2: Sequence for Problem 1

a) *Show that the differential equation describing the RC circuit is given by;*

$$RC\frac{dv_c(t)}{dt} + v_c(t) = v(t). \tag{1}$$

b) *Determine if the system is a causal linear time-invariant.*

c) *Suppose that $R = 0.5\Omega$ and $C = 0.25F$, determine an output expression for the voltage across the capacitor $v_c(t)$ for a unit step input ($v(t) = \mu(t)$) and the initial condition $v_c(0) = 0$.*

d) *Suppose that we would like to recover the original signal $v(t)$ from the output $v_c(t)$ by using a system with transfer function $W(s)$(with input $v_c(t)$ and output $v(t)$), determine $W(s)$.*

□

Solution 2 (25 Marks)**.**

Problem 3 (25 Marks)**.** *A simple way to smooth data is to take a weighted average of a number of samples. Consider the following moving average filter:*

$$y[n] = (1-\alpha)x[n] + \alpha x[n-1] \tag{2}$$

where $x[n]$ and $y[n]$ represent the input and output sequences respectively. The coefficients α is the smoothing factor.

a) Draw the simulation diagram for the filter.

b) Determine the impulse response $h[n]$ for the filter.

c) Find the Discrete-time Fourier transform $H(\Omega)$ of $h[n]$

d) Determine the system transfer function $H(z)$. Is it possible for the filter to be unstable?

□

Solution 3 (25 Marks).

Problem 4 (25 Marks)**.** *The block diagram of Figure 4 is an electronic oscillator for generating pure sinusoidal signal of a particular frequency, say ω_o. The block comprises of a square wave generator and a filter.*

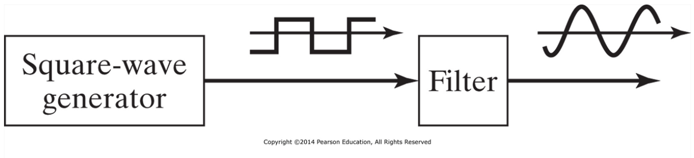

Figure 3: Electronic Oscillator

Let the output of the square wave generator be as shown in Fig. 2 and the final sinusoidal output be $V_o(t) = Asin(\omega_o t)$.

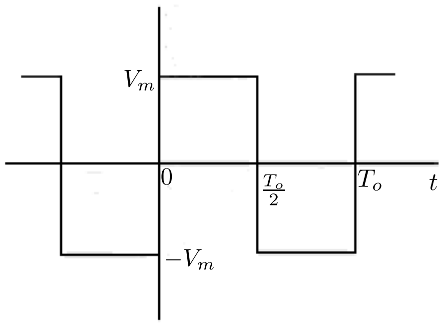

Figure 4: Square Wave

a) *Express the square wave as an exponential Fourier series.*

b) *Calculate the average value of the square wave signal.*

c) *What frequencies or harmonics must be filtered out by the filter circuit to obtain the final sinusoidal output $V_o(t)$ and what type of filter would you deploy for this purpose?*

Solution 4 (25 Marks)**.**

1 Reference

The Fourier series of a continuous-time signal $x(t)$ is defined as:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t} \tag{3}$$

$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt \tag{4}$$

The continuous-time Fourier transform (inverse Fourier transform)of $x(t)$ is defined as

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt \tag{5}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega. \tag{6}$$

The magnitude and phase spectra of $X(\omega)$ are given by $|X(\omega)|$ and $\angle X(\omega)$ respectively.
The Laplace transform (two-sided or bilateral) of signal $x(t)$ is defined as

$$X(s) = \int_{t=-\infty}^{\infty} x(t)e^{-st}dt \tag{7}$$

For example $\mathcal{L}[\mu(t)] = \frac{1}{s}$ and $\mathcal{L}[e^{-at}\mu(t)] = \frac{1}{s+a}$
Given a discrete-time signal $x[n]$, its discrete-time Fourier transform (DTFT) (inverse DTFT)

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \tag{8}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}d\Omega \tag{9}$$

$X(\Omega)$ is said to be periodic with respect Ω if $X(\Omega + kT) = X(\Omega)$ where T is the period and k is any integer.
The z-transform (two-sided or bilateral) of signal $x[n]$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{10}$$

For example $\mathcal{Z}(\mu[n]) = \frac{z}{z-1}$, $\mathcal{Z}(a^n\mu[n]) = \frac{z}{z+a}$ and $\mathcal{Z}(x[n-1]) = z^{-1}X(z)$.
Given that an LTI system has an impulse response $h[n]$, the output response of the system $y[n]$ for an input $x[n]$ is given by

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{k=\infty} h[n-k]x[k] = \sum_{k=-\infty}^{k=\infty} h[k]x[n-k] \tag{11}$$

The system function is given by

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=-\infty}^{k=\infty} h[n]z^{-n} \tag{12}$$