Name: Solutions

MAT 128
Quiz 7

**Determine whether the following series converge absolutely, converge conditionally or diverge**

1. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

compare with $\sum \frac{1}{n}$ which div.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n}{n\sqrt{1+1/n^2}} = 1 > 0$

$\Rightarrow$ DIV

2. $\sum_{n=1}^{\infty} \left(\frac{-n}{3n+1}\right)^n$

$\sqrt[n]{|a_n|} = \frac{n}{3n+1}$, $\lim_{n\to\infty} \frac{n}{3n+1} = \frac{1}{3} < 1$

Conv. Abs.

3. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$

(1) $\left(\frac{x^2}{x^3+4}\right)' = \frac{4-x^4}{(x^3+4)^2} < 0$ dec.

(2) $\lim \frac{n^2}{n^3+4} = 0$

$\Rightarrow$ alt series converges

but $\int_1^{\infty} \frac{x^2}{x^3+4} = \lim_{b\to\infty} \frac{1}{3}\left[\ln(b^3+4) - \ln 5\right] = \infty$ Div

$\Rightarrow$ Conv. Cond

4. $\sum_{n=1}^{\infty} \frac{5^n}{n!}$

$\frac{a_{n+1}}{a_n} = \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n} = \frac{5}{n+1}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0 < 1$ Conv. Ab.