Final Semester Examination *

Signals and Systems (ELC 321)
Department of Electrical and Computer Engineering
The College of New Jersey.

Spring 2015

Last Name: **First Name**:

Instructions:

1. This is a closed-book examination
2. Attempt all questions. Total score obtainable is 100%

Problem 1 (25 Marks)**.** *Consider the R-C circuit of Fig.1 where the voltage $v_c(t)$ across the capacitor is the output and applied voltage $v(t)$ is the input.*

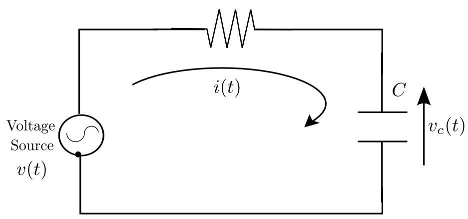

Figure 1: Sequence for Problem 1

Assuming zero initial conditions;

a) Derive the differential equation which relates the capacitor voltage $v_c(t)$ to the applied voltage $v(t)$.

b) Determine the impulse response [denoted as $h(t)$] of the R-C circuit.

c) Determine the step response of the R-C circuit.

d) By approximating the derivative using the backward difference method

$$\frac{dv_c(t)}{dt} = \frac{v_c(t) - v_c(t - T_s)}{T_s}, \tag{1}$$

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it is possible to describe the R-C circuit at times $t = nT_s$ *by the following difference equation*

$$v_c[n] = \alpha v_c[n-1] + \beta v[n] \tag{2}$$

where $v_c[n] = v_c(nT_s) = v_c(t)\,|_{t=nT_s}$ *and* T_s *is the sampling period. Write the expressions for* α *and* β *in terms of* R, C *and* T_s*.*

□

Solution 1 (25 Marks)

Solution 1 (25 Marks)**.**

Problem 2 (25 Marks). *Consider a discrete-time Infinite Impulse Response (IIR) system represented by the following difference equation:*

$$y[n] = \epsilon y[n-1] + \eta x[n] + \epsilon x[n-1] \tag{3}$$

where $x[n]$ and $y[n]$ represent the input and the output sequences respectively. The coefficients ϵ and η are some constants.

a) *Draw the simulation diagram using the Direct-Form II for the filter.*

b) *Determine the filter's system function.*

c) *Is it possible for the filter to be unstable? For what range of values of ϵ is the filter guaranteed to be stable.*

d) *Setting $\epsilon = 0.6$, $\eta = 1$ and assuming zero initial conditions, determine the system's response to an input $x[n] = 0.8^n \mu[n]$.*

□

Solution 2 (25 Marks)

Solution 2 (25 Marks)**.**

Problem 3 (25 Marks). *Consider the system simulation diagram of Figure 2. This figure shows a simulation diagram form used in the area of automatic control*

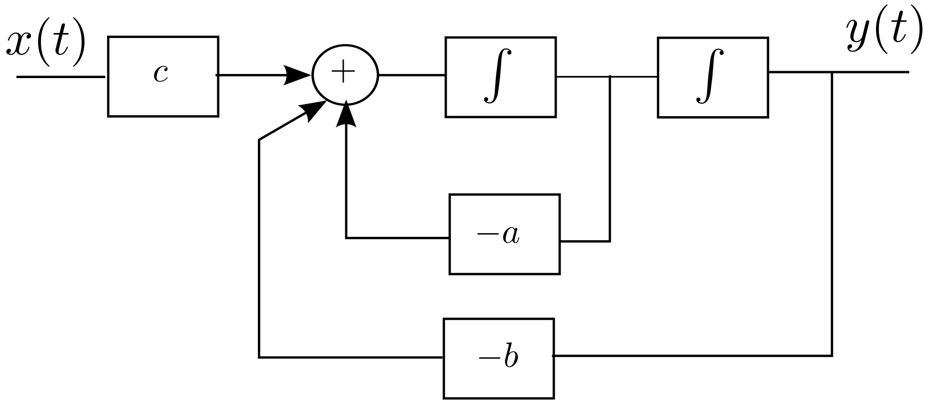

Figure 2: Simulation Diagram for Problem 1

a) *Derive the differential equation relating $y(t)$ to $x(t)$.*

b) *Find the system transfer function $H(s)$. Assuming zero initial conditions.*

c) *Assuming the system has an impulse response given by $h(t) = 2[e^{-2t} - e^{-3t}]$. Determine appropriate values for a, b and c.*

d) *Using the values of a, b and c in* (c)*, determine the system's response to a unit step input [i.e. $x(t) = u(t)$].*

□

Solution 3 (25 Marks)

Solution 3 (25 Marks).

Problem 4 (25 Marks)**.** *The block diagram of Figure 4 is an electronic oscillator for generating pure sinusoidal signal of a particular frequency, say ω_o. The block comprises of a square wave generator and a filter.*

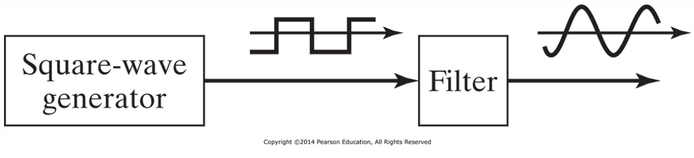

Figure 3: Electronic Oscillator

Let the output of the square wave generator be as shown in Fig. 1 and the final sinusoidal output be $V_o(t) = Asin(\omega_o t)$.

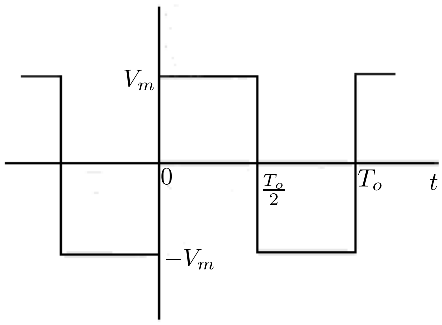

Figure 4: Square Wave

a) *Calculate the average value of the square wave signal.*

b) *Express the square wave as an exponential Fourier series.*

c) *What frequencies or harmonics must be filtered out by the filter circuit to obtain the final sinusoidal output $V_o(t)$ and what type of filter would you deploy for this purpose?*

d) *Compute the power of the square-wave signal. Hint: The power of a periodic signal $x(t)$ of fundamental period T_o is given by*

$$P = \frac{1}{T_o} \int_{t_0}^{t_0+T_o} |x(t)|^2 dt, \text{ for any } t_0 \tag{4}$$

□

Solution 4 (25 Marks)

Solution 4 (25 Marks).

1 Reference

The Fourier series of a continuous-time signal $x(t)$ is defined as:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t} \tag{5}$$

$$c_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_o t}dt \tag{6}$$

The continuous-time Fourier transform (inverse Fourier transform)of $x(t)$ is defined as

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt \tag{7}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega. \tag{8}$$

The magnitude and phase spectra of $X(\omega)$ are given by $|X(\omega)|$ and $\angle X(\omega)$ respectively.
The Laplace transform (two-sided or bilateral) of signal $x(t)$ is defined as

$$X(s) = \int_{t=-\infty}^{\infty} x(t)e^{-st}dt \tag{9}$$

For example $\mathcal{L}[\mu(t)] = \frac{1}{s}$ and $\mathcal{L}[e^{-at}\mu(t)] = \frac{1}{s+a}$
Given a discrete-time signal $x[n]$, its discrete-time Fourier transform (DTFT) (inverse DTFT)

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \tag{10}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}d\Omega \tag{11}$$

$X(\Omega)$ is said to be periodic with respect Ω if $X(\Omega + kT) = X(\Omega)$ where T is the period and k is any integer.
The z-transform (two-sided or bilateral) of signal $x[n]$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{12}$$

For example $\mathcal{Z}(\mu[n]) = \frac{z}{z-1}$, $\mathcal{Z}(a^n\mu[n]) = \frac{z}{z+a}$ and $\mathcal{Z}(x[n-1]) = z^{-1}X(z)$.
Given that an LTI system has an impulse response $h[n]$, the output response of the system $y[n]$ for an input $x[n]$ is given by

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{k=\infty} h[n-k]x[k] = \sum_{k=-\infty}^{k=\infty} h[k]x[n-k] \tag{13}$$

The system function is given by

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=-\infty}^{k=\infty} h[n]z^{-n} \tag{14}$$