Lab 7: Wave-Particle Duality of Electrons

(Partially based on CENCO's and TELTRON's experimental guides)

Purpose:

- to determine the specific charge of an electron, e/m_e
- to study the wave properties of electrons through electron diffraction
- to study the diffraction pattern formed when light passes through an aperture

Equipment:

Part I - Specific Charge of an Electron

Specific Charge of Electron Apparatus (electron tube with Helmholtz coils)
High-voltage DC power supplies (2) High-current power supply
0 – 6 VAC power supply 4 multimeters
connecting wires (shorter, to reduce magnetic fields) compass or Gaussmeter

Part II - Electron Diffraction

electron diffraction tube 5 kV voltage control
6-inch see-through ruler optical caliper leads

Introduction:

Sir J. J. Thomson is generally credited for the discovery of electrons. At the end of the 19th century, Thomson did a series of experiments that showed that cathode rays consisted of negatively charged particles, later named electrons. Electrons are emitted by the negative electrode (cathode). Since the electrons have a negative charge, they are attracted to the positive electrode (anode). The beam of electrons can be deflected by applying an electromagnetic field. This demonstrates particle properties of electrons.

The first part of this lab is a modification of Thomson's original work. You will find a value for the specific charge (e/m) of electrons by measuring the deflection that a magnetic field produces on a beam of electrons with a known energy. The accepted value for e/m of an electron is $\mathbf{(1.7588196 \pm 0.0000005) \times 10^{11}}$ **C/kg**.

In the years following their discovery, cathode rays were found to show properties of both waves and particles. This caused heated scientific discussions on their nature. In 1924, de Broglie showed that electrons act as both waves and particles, just as it was found out earlier for light. In 1927, the wave-like behavior of electrons was directly demonstrated by observing the electron diffraction phenomena. In the second part of this lab, you will study electron diffraction phenomena.

In the third part of the lab, you will observe the diffraction pattern formed when light passes through a small circular aperture and verify that the positions of the minima in the diffraction pattern match the positions predicted by theory.

PART I - SPECIFIC CHARGE OF AN ELECTRON

Theory:

The magnitude of the force acting on an electron moving with velocity $\vec{v}$ perpendicular to the direction of the magnetic field $\vec{B}$ is given by the following formula:

$$F = evB$$

where the electron's charge is e = $1.602176565\times10^{-19}$ C.

Since the magnetic force is perpendicular to the velocity, the electron will be moving in a circle of radius r. Therefore, the magnetic force will be equal to the centripetal force:

$$m_e\frac{v^2}{r} = evB \qquad \text{Eq. 1}$$

where m_e is the electron's mass.

The velocity can be eliminated from the above equation using conservation of energy. If an initially stationary electron is accelerated through an electric potential difference V, then its kinetic energy after acceleration is equal to the change of the potential energy, eV:

$$eV = \frac{1}{2}m_e v^2 \qquad Eq. 2$$

From Eq. 2, we can write:

$$v^2 = \frac{2eV}{m_e}$$

We now substitute v^2 in Eq. 1, cancel, and rearrange to get:

$$v = \frac{2V}{Br}$$

Substituting v back into Eq. 1, we get that the specific charge of the electron is:

$$\frac{e}{m_e} = \frac{2V}{B^2r^2} \qquad \text{Eq. 3}$$

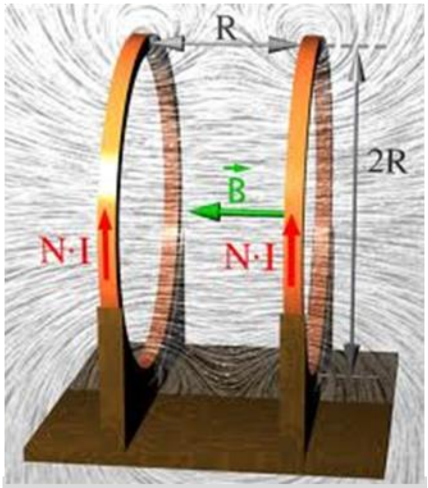
Apparatus used in this lab allows us to measure V, B, and r and therefore determine e/m_e.

Apparatus:

Helmholtz coils (Fig. 1) produce a nearly uniform magnetic field perpendicular to the electron beam. When the distance between the coils is equal to radius, R, of either coil, an approximately uniform filed is created at the midway point (R/2). The magnitude of the filed produced at the midpoint is:

$$B = \frac{8\mu_o NI}{R\sqrt{125}} \qquad \text{Eq. 4}$$

where $\mu_o = 4\pi\times10^{-7}$ W/A-m is the permeability of vacuum, N is the number of turns per coil, I the current in the coils, and R the radius of the coil (notice that R is not the same as r). Field B can be calculated by knowing the characteristics of the coils (N and R) and measuring the current I.



From http://www.hlooser.ch

Figure 1

The magnetic fields needed to produce the curved paths for the electrons in this experiment are of the order of 10^{-3} T. While the Earth's magnetic field (~ 10^{-5} T) may possibly be neglected, there may be other stray magnetic fields around the apparatus (due to the power supplies, meters, or other sources) that need to be taken into account. One way to minimize effects of these fields is to rotate the apparatus so the local field is parallel to the motion of the electron beam, and therefore exerting zero force on it. The significance of the ambient fields can be qualitatively gauged by setting the current in the Helmholtz coils to zero and observing whether the electron beam is somewhat curved. By rotating the e/m apparatus, the orientation which gives a minimum deflected can usually be found.

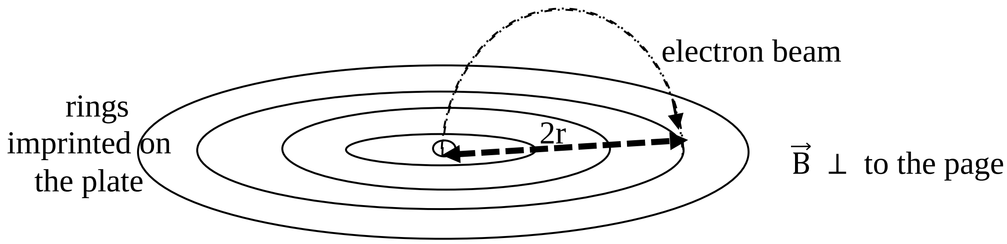
The apparatus is shown in Fig. 2. An evacuated glass tube contains a wire loop (filament). The filament heats a metal plate (cathode), which releases electrons. A grid at a positive potential with respect to the cathode focuses the electron beam. The electrons are accelerated by a second plate (anode), which is at a high positive potential relative to the cathode. The accelerated beam then emerges through a small hole in the plate into the vaulted region of the tube. The top surface of the plate has concentric rings of radii 0.5 cm, 1.0 cm, 1.5 cm, and 2.0 cm imprinted on it (Fig. 2).

The electron beam is invisible. However, the glass wall of the tube and the plate rings are coated with a material that glows when its atoms are struck by electrons. The tube also has a small amount of an inert gas that produces a visible trace of the beam.

Voltage V between the filament and plate can be changed. The voltage divider (Fig. 2) allows that a variable portion of V be applied between the filament and grid. The grid voltage, which influences the electric field lines between the filament and plate, is used to focus the electron beam. The tube is mounted between a pair of Helmholtz coils. The coils supply a uniform magnetic field, which exerts a force perpendicular to the direction of motion of the electrons in the beam.

CAUTION:

- Make sure voltage is off before touching any connections.
- Currents in the Helmholtz coils over 5 A can cause overheating.
- The apparatus has limited total lifetime, so both the accelerating voltage V and the filament voltage should be kept off except when measurements are being made.
- The electron tube is expensive. Please do not damage it by too high filament current.
- The filament voltage should always be kept at the lowest setting that will produce a well-focused, visible beam. After you obtained a well-focused beam, try reducing the filament current, but make sure you can still see the beam.



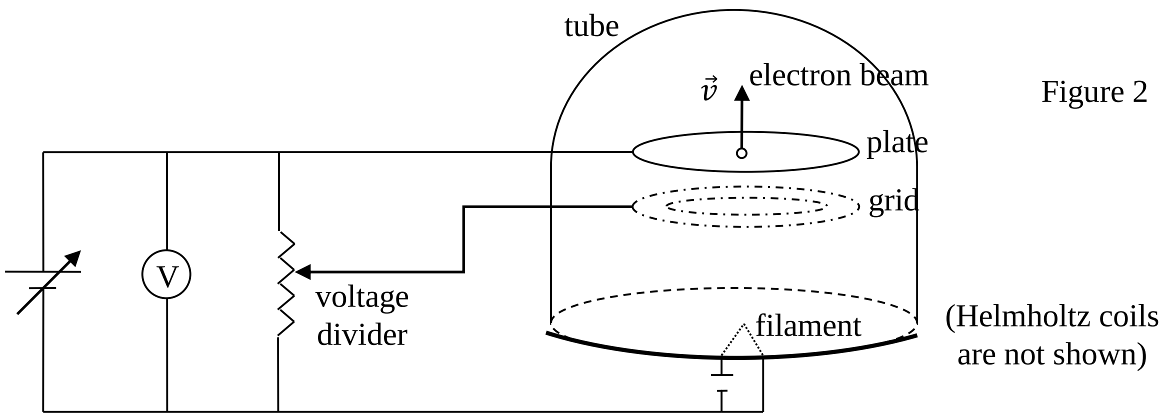


Figure 2

DO NOT INCREASE THE FILAMENT CURRENT OVER **0.9 AMPS**. DO NOT INCREASE THE CURRENT IN HELMHOLTZ COILS OVER **5 A**. SWITCH OFF THE VOLTAGES WHEN NO MEASUREMENTS ARE MADE.

1. Measure the outer and inner diameters of one of the Helmholtz coils. Use these measurements to calculate the radius, R, of the middle of the coil.

   The "middle" radius of the Helmholtz coil R = ____________ (m)

2. Record the number of turns per coil, N. This number can differ for different sets of coils, so make sure to get the value for the coils you use.

   N = _________

3. Compare Fig. 2 and Fig. 3 with the actual setup. Identify the four circuits: filament, grid, plate, and magnetic field. Connect the circuits. Before switching on the power supplies, make sure all the knobs are turned to zero.
4. Currents and voltages should be measured using separate multimeters. An ammeter should be connected in series within the circuit and a voltmeter in parallel. A voltmeter (not shown in Fig. 3) should be connected in parallel to measure the plate voltage. Make a note which multimeter measures which current or voltage.

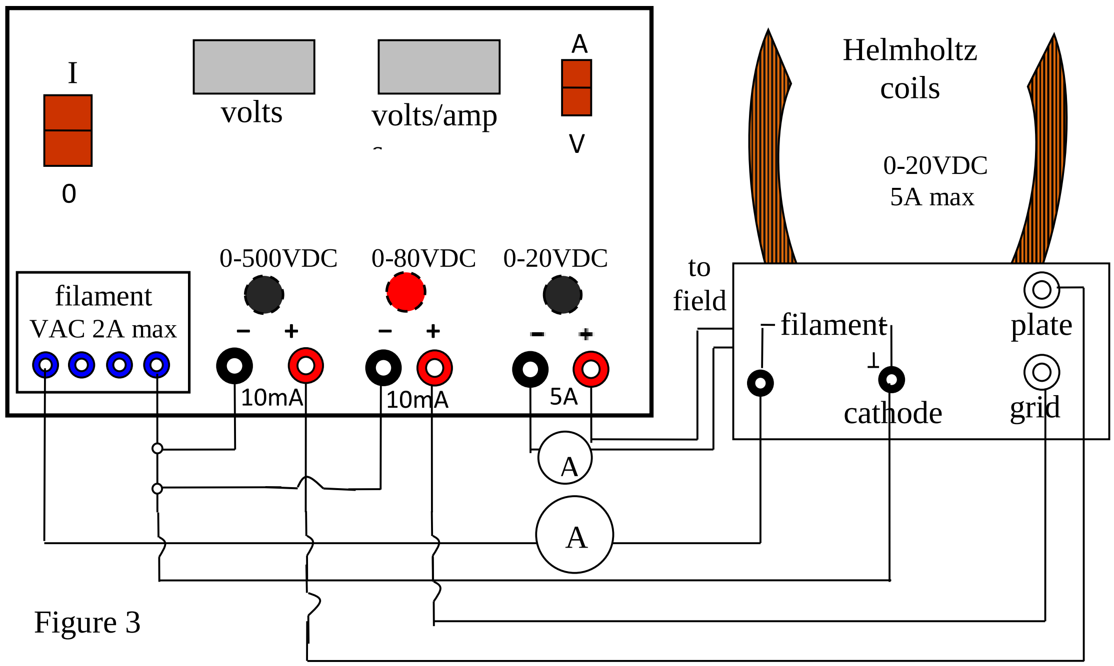


Figure 3

5. Dim the lights in the room and switch on the tube voltages. Set the filament voltage to 6 VAC, making sure that the filament current does not go over 0.9 A (as measured with an ammeter). After allowing the filament to warm up for 1-2 minutes, apply both the grid and plate voltages and notice the beam of electrons rising from the central hole.
6. Reduce the filament current as much as possible so that you still have a visible beam. Adjust the plate voltage, V, to 80 – 100 V. Change the grid potential to get a focused beam. (The grid voltage will change proportionately as V is changed, so the beam will likely remain focused. However, if needed, you can try refocusing the beam by using the voltage divider in the grid control circuit).
7. Before applying current to the Helmholtz coils, check if the electron beam is slightly curved from the straight path. If so, try to orient the e/m apparatus so that the effects of any ambient magnetic fields are minimized. **Be careful not to touch any high-voltage connections!**
8. Slowly increase the current through the Helmholtz coils to see the beam bend and eventually hit the top of the plate. Vary the plate potential and the field current to make the beam hit the outermost ring. You may need to adjust the grid potential to keep the beam focused.
9. In Table 1, record radius *b* of the outermost ring, I in the coils, and V on the plate. Calculate magnetic field B from Eq. 4 and then the specific charge of the electron, e/m_e,

from Eq. 3. Record in Table 1. In the space below, show complete calculations for this ring (e.g. formulas, substitutions of numbers, etc.).

Calculations of B and e/m_e:

10. Repeat this procedure, each time starting from zero voltages and currents, and bending the beam each time to the outmost ring. Record in Table 1.
11. Keeping the plate voltage, V, constant, change current I to bend the electron beam on each of the 3 other rings. MAKE SURE YOU DO NOT EXCEED 5 A in the field coils. Repeat two times for each ring. Calculate B and e/m_e. Record everything in Table 1.

    NOTE: *It may be very hard to bend the beam onto the two smallest rings (#1 and #2). If so, reduce V and see if you can bend the beam. If not, bend the beam on "ring" 3.5 (middle between rings 3 and 4) and on "ring" 2.5 (middle between rings 2 and 3). Make appropriate changes in Table 1 and record your measurements for rings 3.5 and 2.5 instead of for rings 2 and 1.*

12. Once done, switch everything off, including the multimeters.

Table 1

ring #	**2r** (m)	Trial #	I_{coils} (A)	V_{plate} (V)	B (T)	e/m_e (C/kg)
4 (outer)	0.020	1				
		2				
3	0.015	1				
		2				
2	0.010	1				
		2				
1	0.0075	1				
		2				

13. Use your e/m_e values from Table 1 to calculate the average e/m_e. Find the standard deviation and compare your experimental value with the accepted value of e/m_e (see the Introduction section). Record below.

Measured <e/m> = ________________ (C/kg) Accepted e/m_e = ______________

$$\sigma_{e/m} = \sqrt{\frac{\sum_{i=1}^{12} [\langle e/m \rangle - (e/m)_i]^2}{11}} = \text{_______________ (C/kg)}$$

Final answer: <e/m> =_____________ (C/kg)

Within the experimental error, does your average measured value agree with the accepted value of e/m?

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PART II - ELECTRON DIFFRACTION

Theory

In 1924 de Broglie proposed that every particle with momentum $m\upsilon$ has a wavelength associated with it and given by the same formula as for photons:

$$\lambda = \frac{h}{m \cdot \upsilon}$$

When an electron is accelerated in potential V, it will gain kinetic energy. If V is not too big (less than 10 kV in this experiment) relativistic corrections are not needed. The kinetic energy is related to V by the following formula:

$$eV = \frac{1}{2} m_e \upsilon^2$$

Here, m_e is the mass of the electron and e is its charge. We can substitute υ into de Broglie equation for an electron to get:

$$\lambda = \frac{h}{\sqrt{2em_eV}}$$

Using the known values for h, m_e, and e, we get λ in the unit of meter:

$$\lambda = \frac{1.23 \cdot 10^{-9}}{\sqrt{V}} \quad (m) \qquad \text{Eq. 1}$$

In 1927, Davisson and Germer obtained the first experimental evidence of the existence of matter waves. They observed that electrons, much like photons, can be diffracted by the crystal lattice of a solid. A crystal lattice is an arrangement of atoms that keeps repeating itself. These atoms define many sets of planes that are parallel to one another. Adjacent planes of a given set are separated by certain distance d. A beam of monoenergetic electrons will be diffracted by the crystal planes according to Bragg's law:

$$2\, d \sin\theta = n\lambda \qquad \text{Eq. 2}$$

where θ is the angle at which the maximum scattering occurs, d is the separation between the planes, λ the electron's wavelength given by de Broglie formula, and n = 1, 2, 3, ... is the order of the diffraction.

In this experiment we will study diffraction of monoenergetic electrons by a very thin film of polycrystalline carbon. The apparatus is shown in Fig. 4. An evacuated glass bulb has an electron gun in its neck. The electrons are accelerated and pass through a grid with a thin polycrystalline carbon film on it. The film has many small crystals which are randomly oriented. The electron scattering from crystals that satisfy the Bragg condition is strong at the Bragg angle. Since the crystals are randomly orientated, the strongly scattered electrons form a cone. When they are projected on a phosphor surface deposited on the inside of the bulb, the scattered electrons create a circle of light (Fig. 4 and Fig. 6). More than one ring can be observed if there is more than one set of planes. In this experiment, only the first order (n = 1) diffraction is observed. However, two sets of planes are observed, with Bragg plane separations of d_1 = 0.213 nm (inner ring) d_2 = 0.123 nm (outer ring). The two sets of planes are shown in Fig. 5.

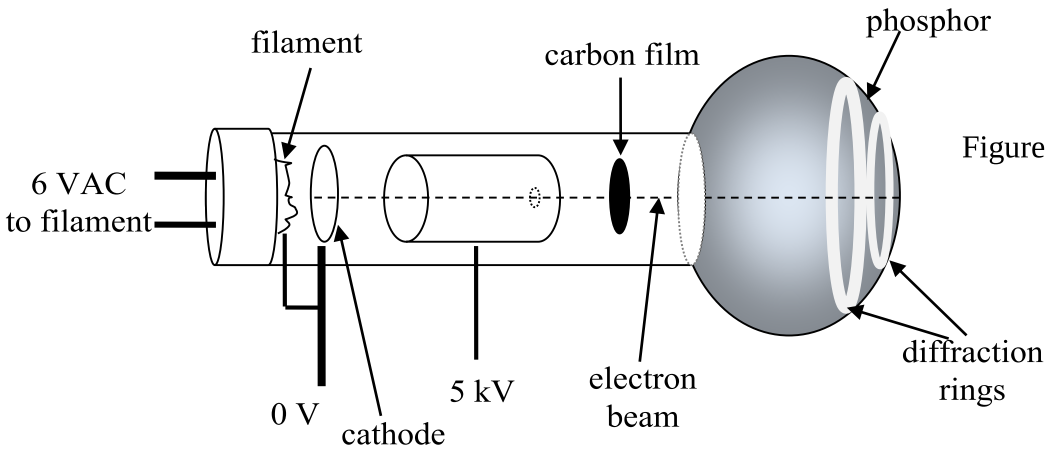

Figure 4

The electron diffraction tube

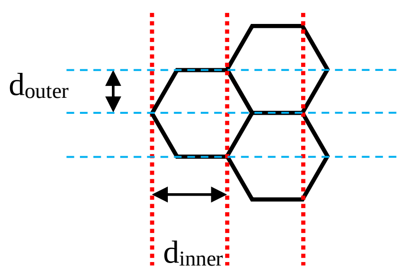

Figure 5

Graphite crystal structure

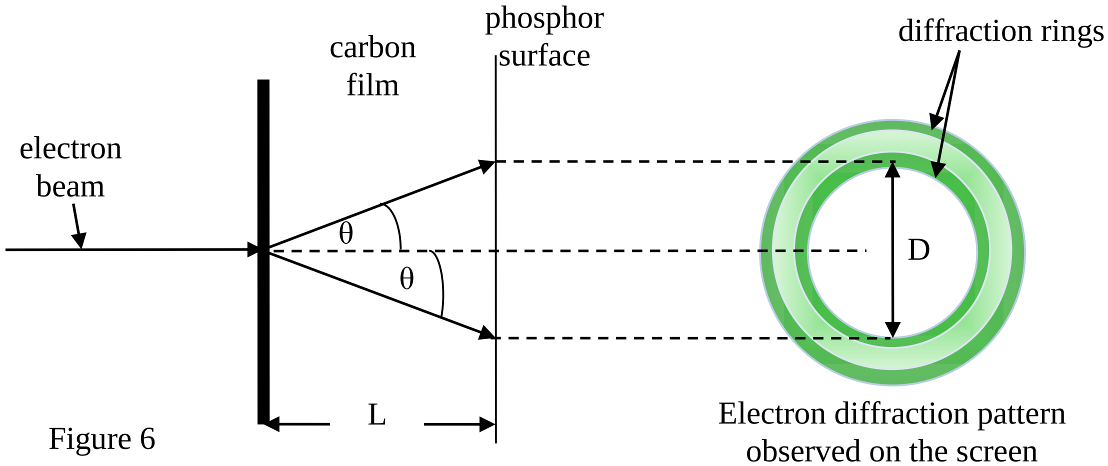

Figure 6

Fig. 6 is a schematic representation of the electron diffraction apparatus. In the figure, L is the distance between the carbon film and the phosphor surface and D is the diameter of the observed ring. The geometry of the setup gives:

$$\sin\theta = \frac{D/2}{L}$$

Substituting this into Eq. 2 for n = 1 and expressing λ from Eq. 1, we get:

$$\frac{1}{\sqrt{V}} = \left(\frac{d}{1.23 \cdot 10^{-9}\ L}\right) \cdot D \qquad\qquad Eq. 3$$

If we plot $\frac{1}{\sqrt{V}}$ vs D, we get a straight line. The slope of the line and d are related as:

$$d = (1.23 \cdot 10^{-9}) \cdot L \cdot (\text{slope}) \qquad\qquad \text{Eq. 4}$$

By knowing L and the slope, we can calculate d, the spacing between adjacent Bragg planes.

Procedure

CAUTION:

The voltage in this experiment is ~ 5000 V, WHICH IS a **LETHAL VOLTAGE**! DO NOT TOUCH THE LEADS WHEN THE HIGH VOLTAGE IS ON!

- The cathode can be damaged if high voltage is applied to the tube before the cathode is fully warmed up. BEFORE turning on the high voltage, make sure the voltage control knob is turned **fully counterclockwise** so the high voltage is zero. AFTER turning on the supply, wait 1 minute before turning the knob clockwise and applying the high voltage.
- Turn the high voltage to zero before turning off the power supply.
- The tube is quite expensive. To prolong its life, turn the high voltage to zero when not actually observing or measuring.

1. The wiring schematic is shown in Fig. 7. With the high voltage off connect the circuit.
2. With the high voltage knob turned fully counterclockwise, turn on the supply and wait 1 minute. You should be able to see the glow of the filament.
3. Set the voltage to about 2.5 kV. Observe the two rings and measure their diameters, D_{inner} and D_{outer}. Record in Table 2.
4. Repeat the previous step for voltages up to 4.5 kV in steps of 0.5 kV. (A cool fact: As you change the voltage, you are changing the wavelength of the electron!) At each voltage measure the diameter (D) of the two rings and record in Table 2.

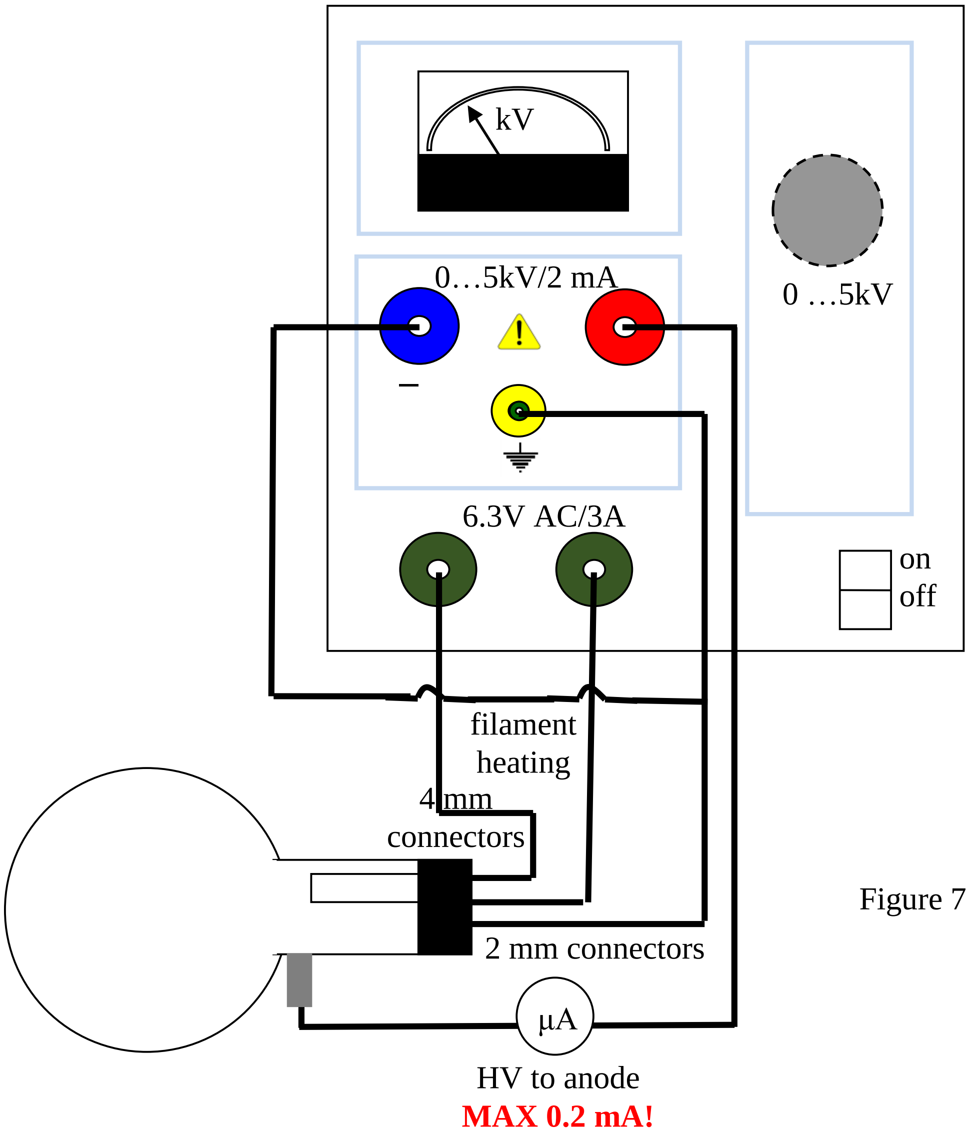

Figure 7

5. Plot $\frac{1}{\sqrt{V}}$ versus D for the inner and outer ring measurements, respectively. Find the slope of each line. Use the slope, Eq. 4, and the value L = 0.13 m to calculate the Bragg plane spacing, d, in the carbon film for each set of measurements. Compare with the accepted values. Record in Table 3.

Table 2

V	$\frac{1}{\sqrt{V}}$	D (m)	
(kV)	$1/\sqrt{\text{volt}}$)	inner	outer

Table 3

	for the inner ring	for the outer ring
slope (nm)		
d_{exp} (nm)		
$d_{accepted}$ (nm)	0.213	0.123
% error		

Questions

Q1 Make a rough estimate of the spacing between carbon atoms using Avogadro's number ($6.022 \cdot 10^{23}$ mol^{-1}), molar mass (12 g) and the density of carbon (~ 2.1 g/cm^3). Assume a cubic lattice. Show your calculations below:

d_{carbon} ~ ____________________ (nm)

Q2 How does the order of magnitude of the value estimated in Q1 compare to the carbon spacings you obtained in the electron diffraction experiment?

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