Quiz 7

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quiz7

ENG 342: Advanced Engineering Math II

Quiz #7

December 7, 2016

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Before you begin the quiz, please clear off everything from your desk except for (i) a pencil, (ii) your book opened to the probability tables, (iii) a calculator, and/or (iv) your computer opened to Matlab, Excel, or another pre-approved statistical program. The last page is a formula sheet for your reference.

Please note that you are NOT permitted to (i) open your book to any page besides the tables, and (ii) go anywhere on your computer besides a statistical program.

For each problem, you must show your work to receive credit (if code was used, write out the code you typed in to the computer). If you need more room than what is provided, please continue on the back of the page.

You have 30 minutes. Good luck!

Problem 1 [3 pts]

Consider three normally distributed random variables $A \sim N(15, 2)$, $B \sim N(10, 3)$, and $C \sim N(3, 1)$ in the format $N(\mu, \sigma^2)$. Let $D = A - B + 2C$.

(a) What is the distribution of D?

$N(15,2) - N(10,3) + 2(N(3,1))$

$= N(15-10+6, \underline{2+3+2^2 \cdot 1})$

$\boxed{= N(11, 9)}$

(b) What is $P(\underset{a}{7} < D < \underset{b}{10})$?

$\frac{7-11}{\sqrt{9}} = Z_a$ $\quad \frac{10-11}{\sqrt{9}} = Z_b$ $\quad \frac{a-\mu}{\sigma} = Z_a$

$P(Z_b - Z_a)$

(c) For which interval $(\mu - d, \mu + d)$ around the mean is $P(\mu - d < D < \mu + d) = \underline{0.80}$?

$\mu = 11$

$Z_b - Z_a = 0.80$

$\Delta Z = 0.80$

$\mu - \frac{\Delta Z}{2}$

$d = \frac{\Delta Z}{2}$

$\frac{b-\mu}{\sigma} - \frac{-b-\mu}{\sigma}$

$\frac{b-\mu+b+\mu}{\sigma}$

$\frac{2b}{\sqrt{9}} = 0.8$

$b = \frac{0.8 \cdot 3}{2}$

Problem 2 [3.5 pts]

Each bit sent over a noisy communication channel has a chance of being received incorrectly. Suppose we send 50 bits over a channel, and find that 10 of them are improperly received. We want to model the outcome of sending another n bits.

(a) What is the estimated probability $\hat{p}$ of a bit being received incorrectly on this channel? What is the uncertainty in this estimate?

$\frac{10}{50}$ $\quad \sigma_{\hat{p}} = \sqrt{\frac{.2(1-.2)}{50}}$ $\frac{0.16}{50}$

(b) Use $\hat{p}$ from (a) to express the number of incorrect bits out of n as a Binomial random variable X.

$\frac{n!}{x!(n-x)!} p^x(1-p)^{n-x} \quad x = 0, 1, \ldots$

$n = 50$

$p = 0.2$

(c) If $n = 5$, find the probability mass function of X.

$n=5$

(d) If $n = 5$, what is the probability that at least 2 bits are received incorrectly?

$p(X \geq 2) = p(1 - p(1) - p(0))$

Problem 3 [3.5 pts]

The datapoints $20, 22, 23, 25, 27$ are drawn from a population. We want to construct a confidence interval for the population mean.

(a) Do we need to know anything more about the population? Why or why not?

(b) Find a 95% two-sided confidence interval for μ.

(c) Find a 95% lower confidence interval for μ.

(d) Roughly how confident can we be that μ lies in $(21.2, 25.6)$?

Binomial Random Variable $X \sim \mathrm{Bin}(n, p)$

$$p_X(x) = p(X = x) = \begin{cases} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} & x = 0, 1, ..., n \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_X = np \qquad \sigma_X^2 = np(1-p)$$

$$\hat{p} = \frac{\text{number of successes}}{\text{number of trials}} = \frac{X}{n} \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Poisson Random Variable $X \sim \mathrm{Poisson}(\lambda)$

$$p(x) = \begin{cases} e^{-\lambda} \frac{\lambda^x}{x!} & \text{if } x \text{ is a non-negative integer} \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_X = \lambda \qquad \sigma_X^2 = \lambda$$

$$\hat{\lambda} = \frac{\text{number of occurrences}}{\text{number of units}} = \frac{X}{t} \qquad \sigma_{\hat{\lambda}} = \sqrt{\frac{\lambda}{t}}$$

Normal Random Variable $X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

$$Z \sim N(0,1) \rightarrow z = \frac{x - \mu}{\sigma}$$

$$\bar{X} = \text{mean of sample} \qquad \sigma_{\bar{X}} = \frac{s}{\sqrt{n}}$$

$$aX + b \sim N(a\mu + b, a^2\sigma^2)$$

$$c_1X_1 + c_2X_2 + \cdots + c_nX_n \sim N\left(c_1\mu_1 + c_2\mu_2 + \cdots + c_n\mu_n,\ c_1^2\sigma_1^2 + c_2^2\sigma_2^2 + \cdots + c_n^2\sigma_n^2\right)$$

Central Limit Theorem: For n large ($n \geq 30$),

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \qquad \text{approximately}$$

$$S_n \sim N\left(n\mu, n\sigma^2\right) \qquad \text{approximately}$$

Large Sample 100(1 − α)% Confidence Intervals

$$\bar{X} \pm z_{\alpha/2}\sigma_{\bar{X}} \qquad (\bar{X} - z_\alpha\sigma_{\bar{X}}, +\infty) \qquad (-\infty, \bar{X} + z_\alpha\sigma_{\bar{X}}) \qquad \sigma_{\bar{X}} = \sigma/\sqrt{n}$$

Small Sample 100(1 − α)% Confidence Intervals

$$\bar{X} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} \qquad \left(-\infty,\ \bar{X} + t_{n-1,\alpha}\frac{s}{\sqrt{n}}\right) \qquad \left(\bar{X} - t_{n-1,\alpha}\frac{s}{\sqrt{n}},\ +\infty\right)$$

100(1 − α)% Confidence Intervals for Proportions

$$\tilde{p} \pm z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} \qquad \left(\tilde{p} - z_\alpha\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}},\ 1\right) \qquad \left(0,\ \tilde{p} + z_\alpha\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}\right)$$

$$\tilde{n} = n + 4 \qquad \tilde{p} = (X + 2)/\tilde{n}$$