THE COLLEGE OF NEW JERSEY

ELC 321
Signal and Systems
Lecture 1

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Introduction

Signals

- **Signals** are modelled using **Mathematical Functions**.
- **Signals** can either be continuous-time or discrete-time.

Systems

- Physical systems are modelled using **Mathematical Equations**.
- Systems can either be continuous-time, discrete-time or hybrid

Signals and Systems

- Analysis involves computing the solutions of the equations (models of physical systems) when excited by the functions (signals).
- Mathematical tools for modelling and analysis include: Laplace Transform, Z-transform, Convolution integral and sums, Fourier Series and transforms, Matlab/Simulink.

Continuous-Time Signals: Definition

Signals

Signal can be defined as a set of information or data that can be modeled or represented as a function of one or more independent variables such as time, position, distance, temperature etc.

From example $y = f(t)\ ,\ t \epsilon \mathcal{R}$

We encounter signals virtually everywhere

- Speech
- Music
- Picture
- Video signals

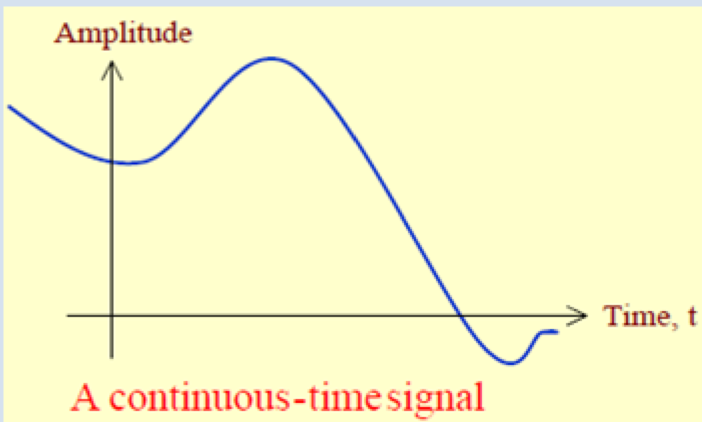
Signals carry information

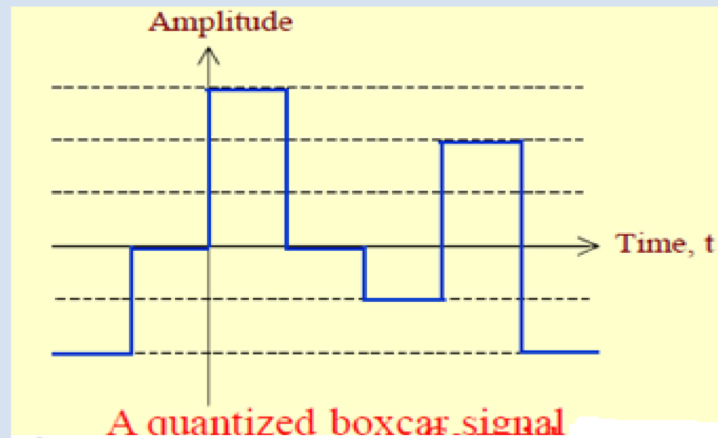
Our concern : **Independent variable of Time.**

Continuous-Time Signals: Definition

Continuous-Time Signals

A continuous-time signal is defined for **all values of time**.





A continuous-time signal $x(t)$ can be:

- Continuous-time with continuous-amplitude (**analog**) signal
  - **The time-varying amplitude can assume any value.**
- Continuous-time with discrete-amplitude (**quantized boxcar**) signal
  - **The time-varying amplitude can assume only certain defined amplitude.**

Continuous-Time Signals: Transformations

We consider two classes of the signal transformations:

The independent-variable transformations , and dependent variable transformations.

Transformation of the independent variable (Time)

We consider signal of the form $x(t)$ where $t\epsilon\mathcal{R}$ and $x(t)$ is real-valued, and $y(t)$ is the transformed signal.

- **Time Reversal**: The independent variable (t) is replaced by $(-t)$:

$$y(t) = x(-t).$$

- **Time Scaling**: The independent variable (t) is replaced by (at) where a$\epsilon\mathcal{R}$ is a constant.

$$y(t) = x(at).$$

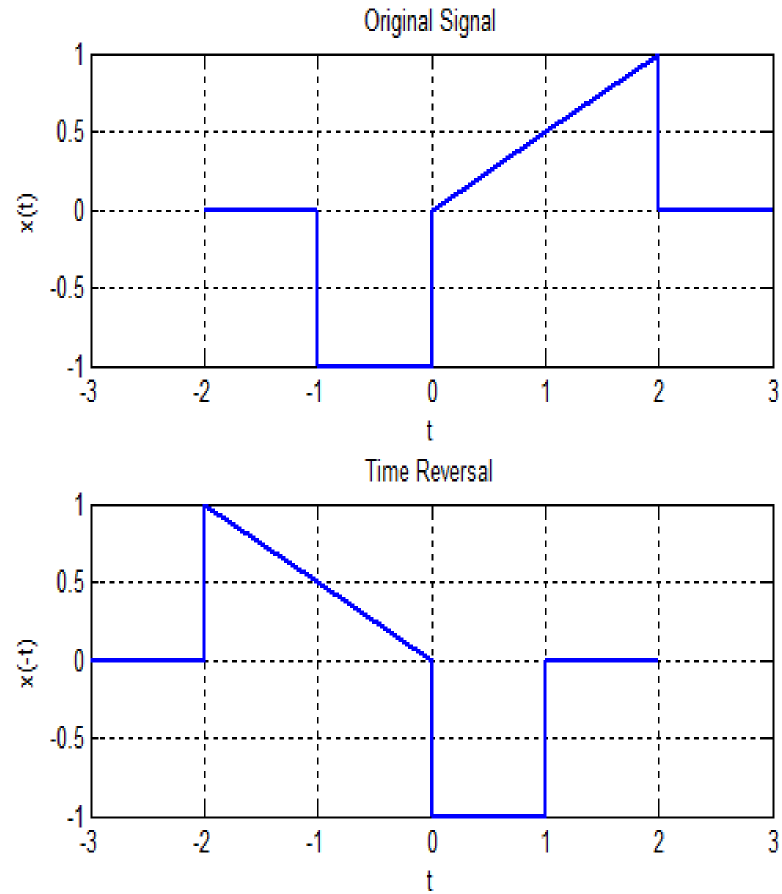
- **Time Shifting**: With t_o a constant, a time-shifted signal is

$$y(t) = x(t - t_o).$$

Continuous-Time Signals: Transformations

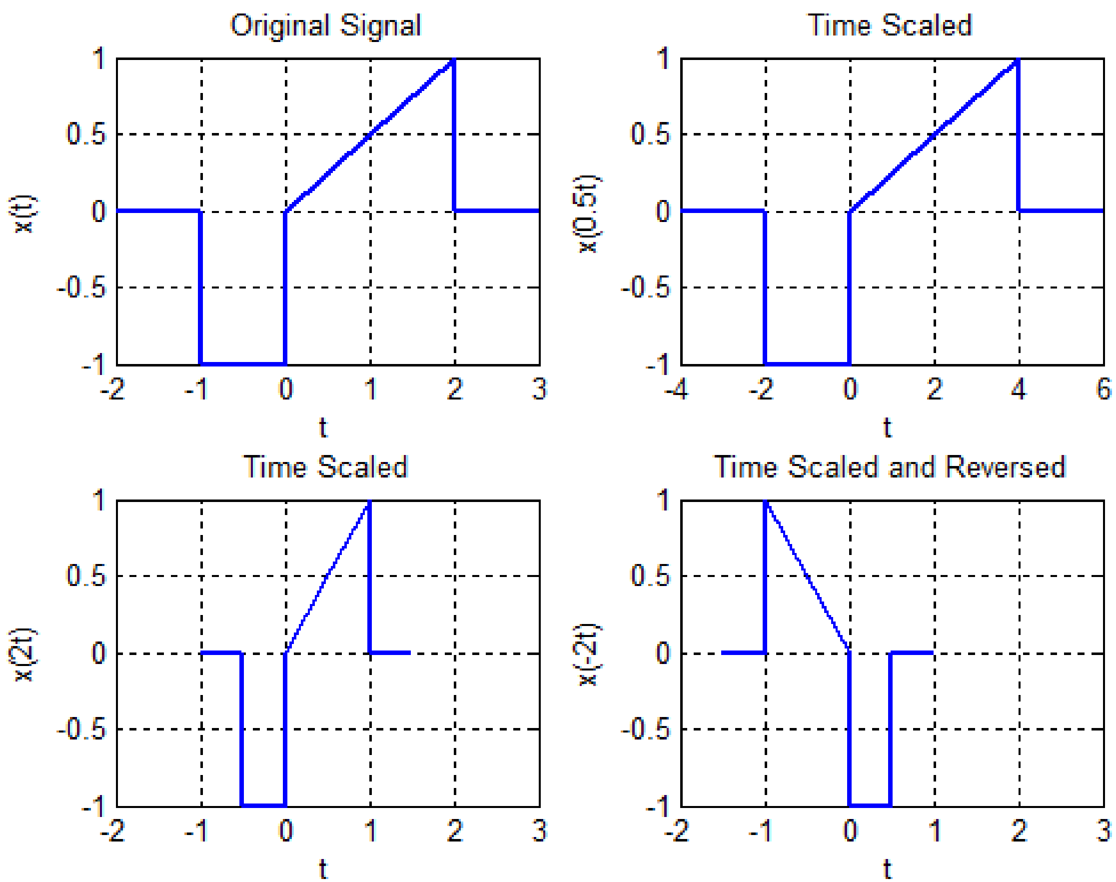
Matlab Code: Time-Reversal Transformation

```
%Original Signal
t=[-2 -1 -1  0  0   1    2   2   3]
x=[ 0  0 -1 -1  0 0.5  1    0   0]
subplot(211),plot(t,x)
plot(t,x,'linewidth',1.5)
xlabel('t')
ylabel('x(t)')
title('Original Signal'); axis([-3 3 -1 1])
grid
%Time reversed Signal
subplot(212)
plot(-t,x,'linewidth',1.5)
xlabel('t')
ylabel('x(-t)')
title('Time Reversal');axis([-3 3 -1 1])
grid
```



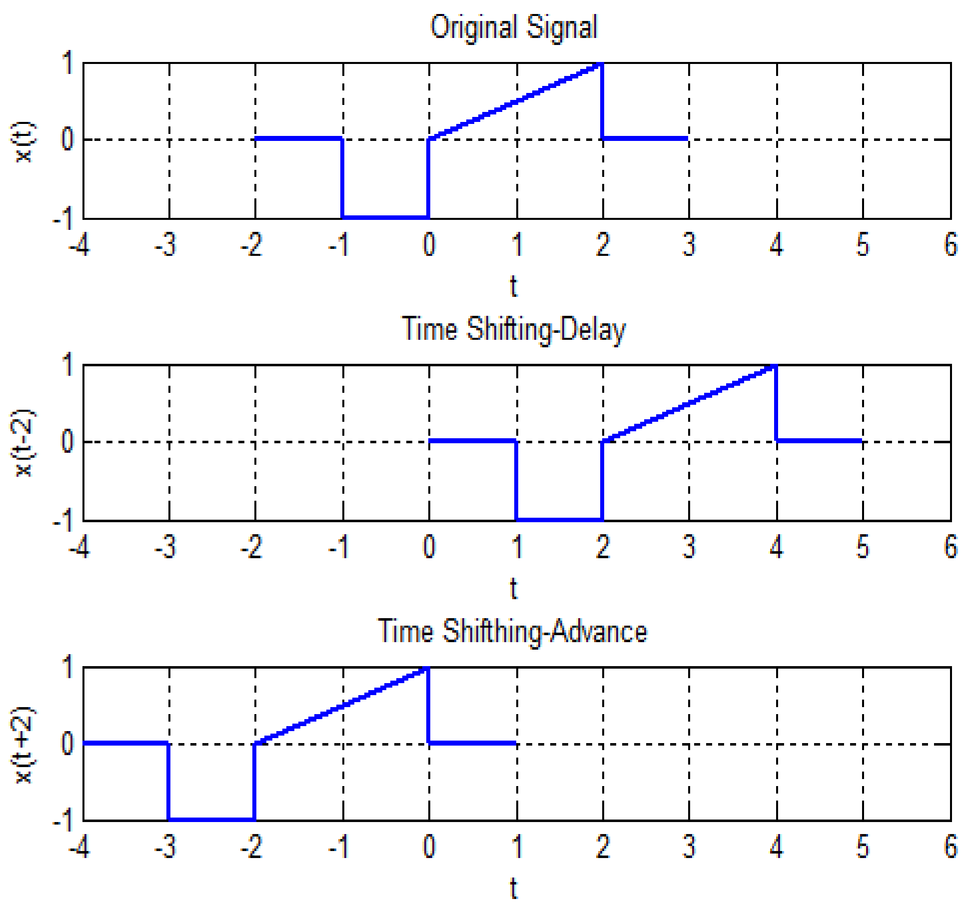
Continuous-Time Signals: Transformations

Time-Scaling and Time-Reversal Transformations



Continuous-Time Signals: Transformations

Time-Shifting Transformations



Continuous-Time Signals: Transformations

Transformation of the independent variable (Time)

In general, transformation of the form

$$y(t) = x(\alpha t + \beta)$$

where α and β are constants, preserves the shape of $x(t)$.

- If $|\alpha| < 1$: **Time Scaling** (Linear Stretching)
- If $|\alpha| > 1$: **Time Scaling** (Linear Compression)
- If $\alpha < 0$: **Time Reversal** and scaling; and
- If $\beta \neq 0$: **Time Shifting**.

Continuous-Time Signals: Transformations

Transformation of the dependent variable (Amplitude)

Amplitude transformations take the general form

$$y(t) = Ax(t) + B$$

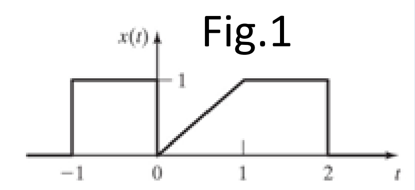
where A and B are constants, preserves the shape of $x(t)$.

- If $|A| \neq 0$: **Amplitude Scaling**
- If $|B| \neq 0$: **Amplitude Shifting**
- If $\mathrm{A} < 0$: **Amplitude Reversal** and scaling; and

Continuous-Time Signals: Transformations

Transformation of the independent variable

Example : Consider the signal $x(t)$ shown across. Obtain And plot the transformed signal



$$y(t) = 3x\left(1 - \frac{t}{2}\right) - 1$$

Solution:

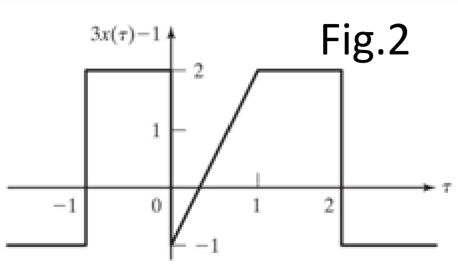
This example has 1) time reversal 2)time scaling 3) time shifting 4) amplitude scaling and 5) amplitude shifting.

First, we carry out the amplitude transformations.

Define $\tau = \left(1 - \frac{t}{2}\right)$ so that we

$$y(\tau) = 3x(\tau) - 1$$

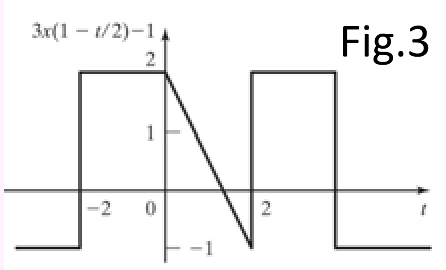
This is shown next in fig 2.



Now, we carry out the time transformations by solving for t in $\tau = \left(1 - \frac{t}{2}\right)$

$$t = 2 - 2\tau$$

The t-axis is then redrawn and the signal is plotted as shown in Fig 3.



Continuous-Time Signals: Characteristics

Signal Characteristics: Even and Odd Signals

- A signal $x(t)$ is an **even** signal if it is identical to its time-reversed counterpart.

$$\boldsymbol{x(-t) = x(t)}$$

For example: $cos(-\omega t) = cos(\omega t)$

  – An even function has symmetry with respect to the vertical axis.

- A signal is **odd** if

$$\boldsymbol{x(-t) = -x(t)}$$

For example: $sin(-\omega t) = -sin(\omega t)$

  – An odd function has symmetry with respect to the origin $x(0) = -x(0)$.

- Any signal can be expressed as the sum of an even part and an odd part.

$$x(t) = x_{even}(t) + x_{odd}(t) \quad \text{where}$$

$$x_{even}(t) = \frac{1}{2}[x(t) + x(-t)] \ \text{ and}$$

$$x_{odd}(t) = \frac{1}{2}[x(t) - x(-t)]$$

Continuous-Time Signals: Characteristics

Signal Characteristics: Even and Odd Signals

Example : Determine mathematically if the following signals are even, odd or neither

a) $x(t) = e^{-|t|}$

b) $x(t) = \sin\left(3t + \frac{3\pi}{2}\right)$

Solution:

a) $x(t) = e^{-|t|}$

$$x(-t) = e^{-|-t|} = e^{-|t|} = x(t)$$

b) $x(t) = \sin\left(3t + \frac{3\pi}{2}\right) = \sin\left(3\left(t + \frac{\pi}{2}\right)\right) = \cos 3t$

$$x(-t) = \cos(-3t) = \cos 3t = x(t)$$

Both signals are **even** functions

Continuous-Time Signals: Characteristics

Signal Characteristics: Periodic and Aperiodic Signals

- A continuous-time signals is **periodic** if there is a positive value T for which

$$x(t) = x(t + T)$$

  - Example of periodic signals are $cos\omega t$ and $sin\omega t$.

- A periodic signal also satisfies

$$x(t) = x(t + nT)$$

where n is any integer and T is the period.

- A signal that is not periodic is referred to as **aperiodic** signal.

Continuous-Time Signals: Characteristics

Signal Characteristics: Periodic and Aperiodic Signals

Example : Determine mathematically if each of the following signals is periodic or aperiodic.

a) $x(t) = e^{\sin t}$

b) $x(t) = te^{\sin t}$

Solution:

a) $x(t) = e^{\sin t}$

$$x(t+T) = e^{\sin(t+T)}$$

Since $\sin(t+T) = \sin t$ for $T = 2\pi$. We have

$$x(t+T) = e^{\sin(t+T)} = e^{\sin(t)} = x(t)$$

Hence $x(t) = e^{\sin t}$ is **periodic**.

b) $x(t) = te^{\sin t}$

$$x(t+T) = (t+T)e^{\sin(t+T)}$$

Since $\sin(t+T) = \sin t$ for $T = 2\pi$. We have

$$x(t+T) = (t+T)e^{\sin(t+T)} = (t+T)e^{\sin(t)}$$

$$= te^{\sin t} + Te^{\sin t} = x(t) + Te^{\sin t} \neq x(t)$$

Hence $x(t) = te^{\sin t}$ is **aperiodic**.