Bryan Gunter

8.1

12/1

#15.) If $A = \begin{pmatrix} 4 & 5 \\ -6 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 6 \\ 8 & -10 \end{pmatrix}$ find

(a) $A + B$ & (b) $B - A$ & (c) $2A + 3B$

* a.)

$$A + B = \begin{pmatrix} 4 & 5 \\ -6 & 9 \end{pmatrix} + \begin{pmatrix} -2 & 6 \\ 8 & -10 \end{pmatrix} = \begin{pmatrix} 2 & 11 \\ 2 & -1 \end{pmatrix}$$

b.)

$$B - A = \begin{pmatrix} -2 & 6 \\ 8 & -10 \end{pmatrix} - \begin{pmatrix} 4 & 5 \\ -6 & 9 \end{pmatrix} = \begin{pmatrix} -6 & 1 \\ 14 & -19 \end{pmatrix}$$

c.)

$2A + 3B$

$$2A = 2 \cdot \begin{pmatrix} 4 & 5 \\ -6 & 9 \end{pmatrix} = \begin{pmatrix} 8 & 10 \\ -12 & 18 \end{pmatrix}$$

$$3B = 3 \cdot \begin{pmatrix} -2 & 6 \\ 8 & -10 \end{pmatrix} = \begin{pmatrix} -6 & 18 \\ 24 & -30 \end{pmatrix}$$

$$2A + 3B = \begin{pmatrix} 8 & 10 \\ -12 & 18 \end{pmatrix} + \begin{pmatrix} -6 & 18 \\ 24 & -30 \end{pmatrix} = \begin{pmatrix} 2 & 28 \\ 12 & -12 \end{pmatrix}$$

#23.) If $A = \begin{pmatrix} 3 & 4 \\ 8 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 10 \\ -2 & -5 \end{pmatrix}$ find

(a) $(AB)^T$ (b) $B^T A^T$

* a.) $(AB)^T$; $AB = \begin{pmatrix} 3 & 4 \\ 8 & 1 \end{pmatrix} \begin{pmatrix} 5 & 10 \\ -2 & -5 \end{pmatrix} = \begin{pmatrix} 3 \cdot 5 + 4(-2) & 3 \cdot 10 + 4(-5) \\ 8 \cdot 5 + 1(-2) & 8 \cdot 10 + 1(-5) \end{pmatrix}$

$= \begin{pmatrix} 7 & 10 \\ 38 & 75 \end{pmatrix}$; $(AB)^T = \begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

8.2 In problems # 1-20, use either Gaussian elimination or Gauss-Jordan elimination to solve the given system or show that no solution exists.

#11.)
$$\begin{aligned} 2x_1 + 2x_2 \quad &= 0 \\ -2x_1 + x_2 + x_3 &= 0 \\ 3x_1 + \quad x_3 &= 0 \end{aligned}$$

$$\begin{bmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & 2 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right) \xrightarrow{\frac{1}{2}R_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right) \xrightarrow{R_2 - R_3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ -5 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right) \xrightarrow{R_1 - R_2}$$

$$\left(\begin{array}{ccc|c} 6 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right) \xrightarrow{\frac{1}{6}R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right) \xrightarrow{R_2 + 5R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}\right) \xrightarrow{R_3 - 3R_1}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \quad \therefore \quad x_1 = 0, \quad x_2 = 0, \quad x_3 = 0$$

#15.)
$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ x_1 - x_2 - x_3 &= -1 \\ 3x_1 + x_2 + x_3 &= 5 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & -1 & -1 \\ 3 & 1 & 1 & 5 \end{array}\right) \xrightarrow[R_3 - 3R_1]{R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -2 & -4 \\ 0 & -2 & -2 & -4 \end{array}\right) \xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -2 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{-\frac{1}{2}R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \therefore \quad \begin{aligned} x_1 &= 1 \\ x_2 + x_3 &= 2 \end{aligned}$$

an infinite # of solutions exist because we have 2 equations for 3 variables

Bryan Gunter 12/1

8.4 In Problems 15-28 evaluate the determinant of the given matrix by cofactor expansion.

#15.) $A = \begin{pmatrix} 0 & 2 & 0 \\ 3 & 0 & 1 \\ 0 & 5 & 8 \end{pmatrix}$

$\det A = 0C_{11} + 2C_{12} + 0C_{13}$

$C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 5 & 8 \end{vmatrix} = (-1)^2 \cdot (0-5) = -5$

$C_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 1 \\ 0 & 8 \end{vmatrix} = (-1)(24-0) = -24$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 0 & 5 \end{vmatrix} = (-1)^4 (15-0) = 15$

$\det A = 0C_{11} + 2C_{12} + 0C_{13} = 2(-24) = -48$

#17. $A = \begin{pmatrix} 3 & 0 & 2 \\ 2 & 7 & 1 \\ 2 & 6 & 4 \end{pmatrix}$

$\det A = 3C_{11} + 0C_{12} + 2C_{13}$

$C_{11} = (-1)^{1+1} \begin{vmatrix} 7 & 1 \\ 6 & 4 \end{vmatrix} = 7(4) - 6(1) = 22$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 7 \\ 2 & 6 \end{vmatrix} = 12 - 14 = -2$

$\det A = 3(22) + 2(-2) = 62$

8.5. In 15 - 18 evaluate determinant without expanding cofactors

#15.)

$$A = \begin{pmatrix} 6 & 1 & 8 & 10 \\ 0 & \frac{2}{3} & 7 & 2 \\ 0 & 0 & -4 & 9 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

* A is a $n \times n$ triangular matrix

$\det A = a_{11}, a_{22} \ldots a_{nn}$

$\det A = (6)(\frac{2}{3})(-4)(-5) = 80$

#17.)

$$C = \begin{pmatrix} -5 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

* $\det C = (-5)(7)(3) = -105$

→ for #A, verify $\det A = \det A^T$ for given matrix

#A.)

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 1 & -1 \\ 1 & 2 & -1 \end{pmatrix}$$

* $\det A = 1(-1+2) - 2(-4+1) + 1(8-1) = 14$

$$A^T = \begin{pmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & -1 \end{pmatrix}$$

$\det A^T = 1(-1+2) - 4(-2-2) + 1(-2-1) = 14$ ✓

Continued.

12/1

Bryan Gunner

8.5 (continued) # 21.) Consider the matricies

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -1 \\ 0 & 2 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 1 & 5 \\ 4 & 3 & 8 \\ 0 & -1 & 0 \end{pmatrix}$$

Verify that $\det(AB) = \det A \cdot \det B$

$$A \cdot B = \begin{pmatrix} 2\cdot 2 - 1\cdot 4 + 1\cdot 0 & 2\cdot 1 + 3(-1) + 1(-1) & 2\cdot 5 - 1\cdot 8 + 1\cdot 0 \\ 3\cdot 2 + 1\cdot 4 - 1\cdot 0 & 3\cdot 1 + 1\cdot 3 + (-1)(-1) & 3\cdot 5 + 1\cdot 8 + 0\cdot(-1) \\ 0\cdot 2 + 2\cdot 4 + 2\cdot 0 & 0\cdot 1 + 2\cdot 3 + 2(-1) & 0\cdot 5 + 2\cdot 8 + 2\cdot 0 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 2 \\ 10 & 7 & 23 \\ 8 & 4 & 16 \end{pmatrix}$$

$$\det(A\cdot B) = 0(7\cdot 16 - 4\cdot 23) - (-2)(10\cdot 16 - 8\cdot 23) + 2(10\cdot 4 - 7\cdot 8) = -80$$

$$\det A = 2(2+2) + 1(6-0) + 1(6-0) = 20$$

$$\det B = 2(0+8) - 1(0) + 5(-4-0) = -4$$

$$\det B \cdot \det A = -4 \cdot 20 = -80 = \det(A\cdot B)$$

✓

finding the inverse Pg 397

8.6 In problems 15-26 use theorem 8.6.4 to find the inverse of the matrix or show that no inverse exists.

#15.) $A = \begin{pmatrix} 6 & -2 \\ 0 & 4 \end{pmatrix}$

* $\det A = 6(4) - 0(-2) = 24 \neq 0$ inverse exists

$$\left(\begin{array}{cc|cc} 6 & -2 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{array}\right) \xrightarrow[\frac{1}{4}R_2]{\frac{1}{6}R_1} \left(\begin{array}{cc|cc} 1 & -\frac{1}{3} & 1/6 & 0 \\ 0 & 1 & 0 & 1/4 \end{array}\right) \xrightarrow{R_1 + \frac{1}{3}R_2} \left(\begin{array}{cc|cc} 1 & 0 & 1/6 & 1/12 \\ 0 & 1 & 0 & 1/4 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 1/6 & 1/12 \\ 0 & 1/4 \end{pmatrix}$$

#23. $A = \begin{pmatrix} -1 & 3 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

* $\det A = -1(-4-1) - 3(2-0) + 0(1-0) = -1 \neq 0$ inverse exists

$$\left(\begin{array}{ccc|ccc} -1 & 3 & 0 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \Leftrightarrow (R_2+R_1)} \left(\begin{array}{ccc|ccc} -1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \Leftrightarrow (R_3-R_2)} \left(\begin{array}{ccc|ccc} -1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right) \xrightarrow{R_2 - R_3} \left(\begin{array}{ccc|ccc} -1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 - 3R_2} \left(\begin{array}{ccc|ccc} -1 & 0 & 0 & -5 & -6 & 3 \\ 0 & 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right) \xrightarrow{-R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 6 & -3 \\ 0 & 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 5 & 6 & -3 \\ 2 & 2 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

Bryan Guner 12/1

8.6 (continued) #25.)

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & 2 & 1 \\ 2 & 1 & -3 & 0 \\ 1 & 1 & 2 & 1 \end{pmatrix}$$

There is not enough time in the day!

#27.) Use the given matricies to find $(AB)^{-1}$

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{5}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} \frac{2}{3} & \frac{4}{3} \\ -\frac{1}{3} & \frac{5}{2} \end{pmatrix}$$

$(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$

$$(AB)^{-1} = \begin{pmatrix} \frac{2}{3} & \frac{4}{3} \\ -1/3 & \frac{5}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{5}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{3}-\frac{2}{3} & -\frac{5}{3}+2 \\ -\frac{1}{6}-\frac{5}{4} & \frac{5}{6}+\frac{15}{4} \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ -17/12 & 55/12 \end{pmatrix}$$

A is orthogonal if $A^{-1} = A^T$

#33. (a) Verify the matrix in # 32 is orthogonal

(b) verify that $A = \begin{pmatrix} 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{pmatrix}$ is orthogonal.

a.) $A = \begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$; $A^{-1} = \frac{\text{adj } A}{\det A}$

$\det A = \sin^2\theta + \cos^2\theta = 1$

$$A^{-1} = 1 \cdot \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}$$

$$A^T = \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}$$

Since $A^{-1} = A^T$ the matrix is orthogonal.

b.) $$A^T = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 1/\sqrt{6} \end{pmatrix}$$

$A \cdot A^{-1} = I = A \cdot A^T$

$$A \cdot A^T = \begin{pmatrix} 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{pmatrix}\begin{pmatrix} 1/\sqrt{3} & 0 & 1/\sqrt{3} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ -2/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore A^T = A^{-1}$

Bryan Gumer 12/1 12/1

8.10 In #s 11–18 construct an orthogonal matrix from the eigenvectors of the given symmetric matrix

15.) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$\det(A - \lambda I) = 0$

$\det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix} = 0$

$(1-\lambda)\left[(1-\lambda)^2 - 0\right] + 1\left[0 - (1-\lambda)\right] = 0$

$(1-\lambda)\left[1+\lambda^2-2\lambda\right] - (1-\lambda) = 0$; $(1-\lambda)(\lambda^2 - 2\lambda) = 0 = (1-\lambda)(\lambda)(\lambda-2)$ eigenvalues of matrix A

$\lambda = 1, 0$ or 2

→ for $\lambda = 1$: $(A - I \mid 0) = \left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$

$k_3 = 0$, $k_1 = 0$

Create non-zero vector $k_2 = 1$

$k_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

for $\lambda = 0$: $(A - 0I \mid 0) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right)$

$\xrightarrow{R_3 - R_1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ ∴ $k_1 + k_3 = 0$, $k_2 = 0$

∴ let $k_3 = -1$ ∴ $k_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

for $\lambda = 2$: $(A - 2I \mid 0) = \left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right)$

$\xrightarrow{R_3 + R_1} \left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $-k_1 + k_3 = 0$

$-k_2 = 0$

let $k_3 = +1$ → $k_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

because $k_1 \cdot k_2 = k_2 \cdot k_3 = k_3 \cdot k_1 = 0$

$k_{1,2,3}$ are orthagonal

Continued ↓

8.10
#15
Continued

let $V_1 = k_1$ & $V_2 = k_2 - \left(\frac{k_2 V_1}{V_1 \cdot V_1}\right) V_1$

$$V_3 = k_3 - \left(\frac{k_3 V_1}{V_1 \cdot V_1}\right) V_1 - \left(\frac{k_3 V_2}{V_2 \cdot V_2}\right) V_2$$

$$k_1 = V_1 = (0 \quad 1 \quad 0)$$

$$V_2 = (0 \quad 1 \quad 0) - \left(\frac{(1 \quad 0 \quad -1)(0 \quad 1 \quad 0)}{(0 \quad 1 \quad 0)(0 \quad 1 \quad 0)}\right)(0 \quad 1 \quad 0)$$

$$= (1 \quad 0 \quad -1) - (0)(0 \quad 1 \quad 0) = (1 \quad 0 \quad -1)$$

$\|V_1\| = \sqrt{1}$ $\quad \|V_2\| \ \sqrt{2}$ $\quad \|k_3\| = \sqrt{2}$

$$P = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Bryan Gunner 12/1

8.10 #17.

$$A = \begin{pmatrix} -8 & 5 & 4 \\ 5 & 3 & 1 \\ 4 & 1 & 0 \end{pmatrix}$$

$$\det(A - \lambda I) = 0 \Rightarrow \det \begin{pmatrix} -8-\lambda & 5 & 4 \\ 5 & 3-\lambda & 1 \\ 4 & 1 & -\lambda \end{pmatrix} = 0$$

$$\lambda^3 + 5\lambda^2 - 66\lambda = 0$$

$$\lambda(\lambda^2 + 5\lambda - 66) = 0 \qquad \lambda = 0, 6, -11$$

→ for $\lambda = -11$:

$$\begin{pmatrix} 3 & 5 & 4 \\ 5 & 14 & 1 \\ 4 & 1 & 11 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} = X_1$$

→ for $\lambda = 0$:

$$\begin{pmatrix} -8 & 5 & 4 \\ 5 & 3 & 1 \\ 4 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix} = X_2$$

→ for $\lambda = 6$

$$\begin{pmatrix} -14 & 5 & 4 \\ 5 & -3 & 1 \\ 4 & 1 & -6 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = X_3$$

eigenVectors of matrix A

$$\|X_1\| = \sqrt{X_1^T \cdot X_1} = \sqrt{11}$$

$$\|X_2\| = \sqrt{X_2^T \cdot X_2} = \sqrt{66}$$

$$\|X_3\| = \sqrt{X_3^T \cdot X_3} = \sqrt{6}$$

→ multiply eigenvectors by normalized coeificients

$$P = \begin{pmatrix} -3/\sqrt{11} & 1/\sqrt{66} & 1/\sqrt{6} \\ 1/\sqrt{11} & -4/\sqrt{66} & 2/\sqrt{6} \\ 1/\sqrt{11} & 7/\sqrt{66} & 1/\sqrt{6} \end{pmatrix}$$

2.) (a) Verify that the indicated column vectors are eigenvectors of given symmetric matrix & (b) identify corresponding eigenvalues. (c) Use Gram-Schmidt process to construct an orthogonal matrix P.

$$A = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix} \qquad k_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \qquad k_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad k_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

a.)

$$A k_1 = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} = -2\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

is an eigenvector of A

$$A \cdot k_2 = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} = -2\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$A \cdot k_3 \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

b.) $k_1 \Rightarrow -2$ $\qquad k_2 \Rightarrow -2$ $\qquad k_3 \Rightarrow 4$

c.) $k_1^T \cdot k_2 = (1 \ -1 \ 0)\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 1 \neq 0$

$k_1^T \cdot k_3 = (1 \ -1 \ 0)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$ $\qquad 0 = k_2^T \cdot k_3 = (1 \ 0 \ -1)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

let $V_1 = k_1^T = (1 \ -1 \ 0)$ $\qquad V_2 = k_2^T - \left(\frac{k_2^T \cdot V_1}{V_1 \cdot V_1}\right) V_1 = (\frac{1}{2} \ -\frac{1}{2} \ 0)$

$V_3 = k_3^T - \left(\frac{k_3^T \cdot V_1}{V_1 \cdot V_1}\right) V_1 - \left(\frac{k_3^T \cdot V_2}{V_2 \cdot V_2}\right) V_2 \qquad k_3 = (1 \ 1 \ 1)$

Continued ↓

Bryan Gunner

8.10

21 (Continued):

$\|v_1\| = \sqrt{2}$ $\quad \|v_2\| = 1/\sqrt{2}$ $\quad \|k_3\| = \sqrt{3}$

$$P = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{3} \\ -1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{3} \\ 0 & 0 & 1/\sqrt{3} \end{pmatrix}$$