Bryan Guner 11/30

7.1:

#1.) In problems 1-8 find (a) $3a$, (b) $a+b$, (c) $a-b$, (d) $\|a+b\|$, (e) $\|a-b\|$: $a = 2\hat{i} + 4\hat{j}$ $b = -1\hat{i} + 4\hat{j}$

a.) $3a = 6\hat{i} + 12\hat{j}$ b.) $a+b = 1\hat{i} + 8\hat{j}$ c.) $a-b = 3\hat{i} + 0\hat{j}$

d.) $\|a+b\| = \|1\hat{i} + 8\hat{j}\| = \sqrt{1^2+8^2} = \sqrt{65}$ e.) $\|a-b\| = \|3\hat{i}\| = \sqrt{3^2} = 3$

#5.) $a = -3\hat{i} + 2\hat{j}$ $b = 7\hat{j}$:

a.) $3a = -9\hat{i} + 6\hat{j}$ b.) $a+b = -3\hat{i} + 9\hat{j}$ c.) $a-b = -3\hat{i} - 5\hat{j}$

d.) $\|a+b\| = \|-3\hat{i} + 9\hat{j}\| = \sqrt{(-3)^2+9^2} = \sqrt{90} = 3\sqrt{10}$

e.) $\|a-b\| = \|-3\hat{i} - 5\hat{j}\| = \sqrt{(-3)^2+(-5)^2} = \sqrt{36} = 6$

#9. Find (a) $4a-2b$ (b) $-3a-5b$:

$a = \langle 1, -3 \rangle$ $b = \langle -1, 1 \rangle$

$4a - 2b = 4\hat{i} - 12\hat{j} - (-2\hat{i} + 2\hat{j}) = 6\hat{i} - 14\hat{j} = \langle 6, -14 \rangle$

7.2:

In problems #41-48, $a = \langle 1, -3, 2 \rangle$, $b = \langle -1, 1, 1 \rangle$ and $c = \langle 2, 6, 9 \rangle$ Find the indicated vector or scalar.

#41.) $a + (b+c)$

$$b + c = -\hat{\imath} + \hat{\jmath} + \hat{k} + 2\hat{\imath} + 6\hat{\jmath} + 9\hat{k} = \hat{\imath} + 7\hat{\jmath} + 10\hat{k}$$

$$a + (b+c) = \hat{\imath} - 3\hat{\jmath} + 2\hat{k} + \hat{\imath} + 7\hat{\jmath} + 10\hat{k} = 2\hat{\imath} + 4\hat{\jmath} + 12\hat{k} = \langle 2, 4, 12 \rangle$$

#43.) $b + 2(a - 3c)$

$$a - 3c = \hat{\imath} - 3\hat{\jmath} + 2\hat{k} - 3(2\hat{\imath} + 6\hat{\jmath} + 9\hat{k}) = -5\hat{\imath} - 21\hat{\jmath} - 25\hat{k}$$

$$2(a - 3c) = 2(-5\hat{\imath} - 21\hat{\jmath} - 25\hat{k}) = -10\hat{\imath} - 42\hat{\jmath} - 50\hat{k}$$

$$b + 2(a - 3c) = -\hat{\imath} + \hat{\jmath} + \hat{k} - 10\hat{\imath} - 42\hat{\jmath} - 50\hat{k} = \langle -11, 41, -49 \rangle$$

#47.) $\left\| \frac{a}{\|a\|} \right\| + 5 \left\| \frac{b}{\|b\|} \right\|$

$$\|a\| = \|\hat{\imath} - 3\hat{\jmath} + 2\hat{k}\| = \sqrt{1^2 + (-3)^2 + 2^2} = \sqrt{14} \; ; \; \left\| \frac{a}{\|a\|} \right\| = \left\| \frac{\hat{\imath} - 3\hat{\jmath} + 2\hat{k}}{\sqrt{14}} \right\| = \frac{1}{\sqrt{14}} \cdot \sqrt{1^2 + (-3)^2 + 2^2} = 1$$

$$\|b\| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3} \; ; \; \left\| \frac{b}{\|b\|} \right\| = \frac{1}{\sqrt{3}} \cdot \sqrt{(-1)^2 + 1^2 + 1^2} = 1$$

$$\therefore \left\| \frac{a}{\|a\|} \right\| + 5 \left\| \frac{b}{\|b\|} \right\| = 1 + 5(1) = 6$$

scalar

Bryan Gumer

11/30

$\boxed{7.3:}$

#3.) In problems 1-12, $a = \langle 2, -3, 4 \rangle$, $b = \langle -1, 2, 5 \rangle$ & $c = \langle 3, 6, -1 \rangle$ find the indeccated Vector or Scalar.

find $a \cdot c$:

$$a \cdot c = (2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} + 6\hat{j} - \hat{k}) = (2 \cdot 3) + (-3 \cdot 6) + (4 \cdot -1) = -16$$

#9.) $a \cdot (a+b+c)$:

$$a+b+c = 2\hat{i} - 3\hat{j} + 4\hat{k} - \hat{i} + 2\hat{j} + 5\hat{k} + 3\hat{i} + 6\hat{j} - \hat{k} = 4\hat{i} + 5\hat{j} + 8\hat{k}$$

$$a \cdot (a+b+c) = (2\hat{i} - 3\hat{j} + 4\hat{k})(4\hat{i} + 5\hat{j} + 8\hat{k}) = (2 \cdot 4) + (-3 \cdot 5) + (4 \cdot 8) = 25$$

#11.) $\left(\frac{a \cdot b}{b \cdot b}\right) \cdot b$

$$a \cdot b = (2\hat{i} - 3\hat{j} + 4\hat{k})(-\hat{i} + 2\hat{j} + 5\hat{k}) = (2 \cdot -1) + (3 \cdot -2) + (4 \cdot 5) = 12$$

$$b \cdot b = (-\hat{i} + 2\hat{j} + 5\hat{k})(-\hat{i} + 2\hat{j} + 5\hat{k}) = (-1 \cdot -1) + (2 \cdot 2) + (5 \cdot 5) = 30$$

$$\left(\frac{a \cdot b}{b \cdot b}\right) \cdot b = \frac{4}{10}(-\hat{i} + 2\hat{j} + 5\hat{k}) = \langle \frac{-2}{5}, \frac{4}{5}, 2 \rangle = \langle -0.4, 0.8, 2 \rangle$$

7.4: In problems #1-10 find $a \times b$:

#3.) $a = \langle 1, -3, 1 \rangle \quad b = \langle 2, 0, 4 \rangle$

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 2 & 0 & 4 \end{vmatrix} = \begin{vmatrix} -3 & 1 \\ 0 & 4 \end{vmatrix}\hat{i} - \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix}\hat{j} + \begin{vmatrix} 1 & -3 \\ 2 & 0 \end{vmatrix}\hat{k}$$

$$= (-12-0)\hat{i} - (4-2)\hat{j} + (0+6)\hat{k}$$

$$a \times b = -12\hat{i} - 2\hat{j} + 6\hat{k}$$

#5.) $a = 2\hat{i} - \hat{j} + 2\hat{k} \qquad b = -\hat{i} + 3\hat{j} - \hat{k}$

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ -1 & 3 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix}\hat{i} - \begin{vmatrix} 2 & 2 \\ -1 & -1 \end{vmatrix}\hat{j} + \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}\hat{k}$$

$$= (1-6)\hat{i} - (-2+2)\hat{j} + (-1+6)\hat{k} = -5\hat{i} + 5\hat{k}$$

#7.) $a = \langle \frac{1}{2}, 0, \frac{1}{2} \rangle \quad b = \langle 4, 6, 0 \rangle$

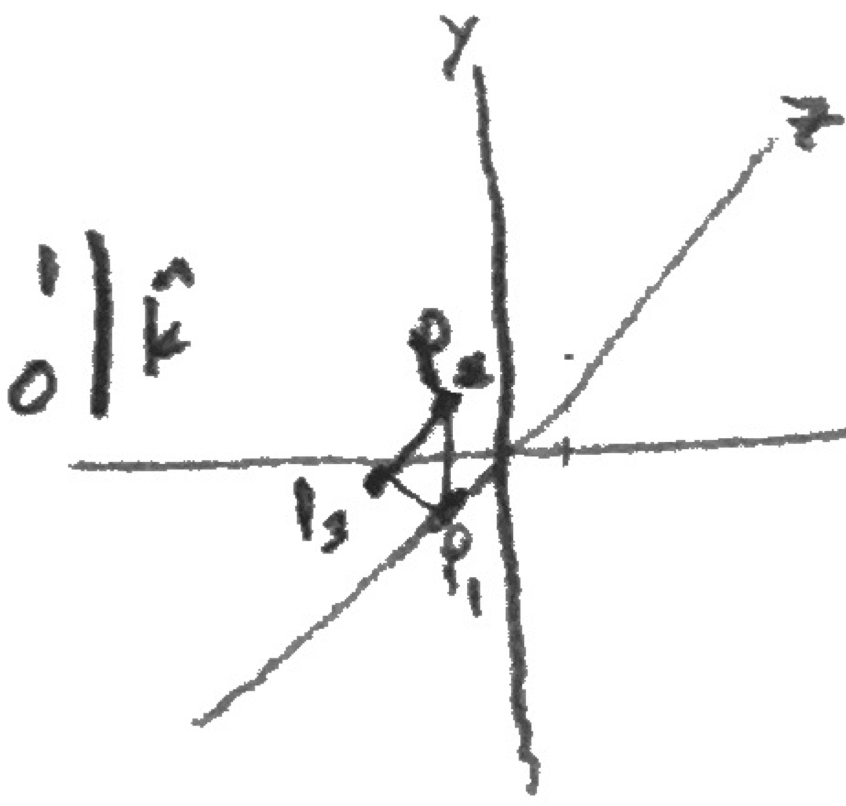
$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 4 & 6 & 0 \end{vmatrix} = \begin{vmatrix} 0 & \frac{1}{2} \\ 6 & 0 \end{vmatrix}\hat{i} - \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ 4 & 0 \end{vmatrix}\hat{j} + \begin{vmatrix} \frac{1}{2} & 0 \\ 4 & 6 \end{vmatrix}\hat{k}$$

$$= (0-3)\hat{i} - (0-2)\hat{j} + (3-0)\hat{k} = -3\hat{i} + 2\hat{j} + 3\hat{k}$$

#47. Find area of triangle formed by points $P_1(1,1,1)$ $P_2(1,2,1)$ $P_3(1,1,2)$

$\overrightarrow{P_1P_2}$ & $\overrightarrow{P_1P_3}$ give us width & length

$\overrightarrow{P_1P_2} = 0\hat{i} + \hat{j} + 0\hat{k}$; $\overrightarrow{P_1P_3} = \hat{k}$



$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\hat{i} - \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}\hat{j} + \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}\hat{k}$$

$$= \hat{i}$$

Area $= \frac{1}{2}\left\|\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}\right\| = \frac{1}{2}\|\hat{i}\| = \frac{1}{2}\sqrt{1^2} = \frac{1}{2}$