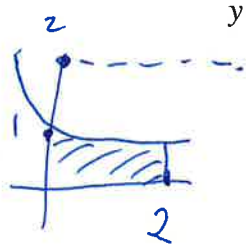


Name: Solutions

MAT 128
Quiz 4

1. Find the volume of the solid obtained by rotating the region bounded by $y = e^{-x}$, $x = 0$, $x = 2$ around the line $y = 2$



$$V = \pi \int_0^2 4 - [4 - 4e^{-x} + e^{-2x}] dx = \pi \int_0^2 4e^{-x} - e^{-2x} dx$$

$$= \pi \left[-4e^{-x} + \frac{1}{2}e^{-2x} \right]_0^2 = \pi \left[-4e^{-2} + \frac{1}{2}e^{-4} \right] - \pi \left[-4 + \frac{1}{2} \right]$$

2. Find the length of the curve $y = \ln(1-x^2)$, $0 \leq x \leq \frac{1}{2}$

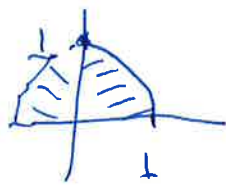
$$1 + (y')^2 = 1 + \left(\frac{-2x}{1-x^2} \right)^2 = \left(\frac{x^2+1}{1-x^2} \right)^2 \Rightarrow \sqrt{1+y'^2} = \frac{x^2+1}{1-x^2} = -1 + \frac{2}{1-x^2}$$

$$= -1 + \frac{1}{1-x} + \frac{1}{1+x} \Rightarrow \int \sqrt{1+y'^2} dx =$$

$$= -x - \ln|1-x| + \ln|1+x| \Big|_0^{1/2} = -\frac{1}{2} + \ln(3)$$

3. The given curve is rotated about the y-axis. Find the area of the resulting surface.

$$y = 1 - x^2, 0 \leq x \leq 1$$



$$R = x \quad S = \int_0^1 2\pi x \sqrt{1+y'^2} dx$$

$$(y')^2 = (-2x)^2 \Rightarrow S = \int_0^1 2\pi x \sqrt{1+4x^2} dx$$

$$\text{Let } u = 1+4x^2 \Rightarrow du = 8x dx$$

$$\Rightarrow S = \frac{2\pi}{8} \int_0^1 u^{1/2} du = \frac{\pi}{4} \cdot \frac{2}{3} u^{3/2} = \frac{\pi}{6} (1+4x^2)^{3/2} \Big|_0^1$$

$$= \frac{\pi}{6} \left[5^{3/2} - 1 \right]$$