Homework 1 - Solution Guide

Assignment on Continuous-Time and Discrete-time Signals *

Signals and Systems (ELC 321)
Department of Electrical and Computer Engineering
The College of New Jersey.

Instructions:

1. The assignment questions are extracted from the Text (Signals, Systems, and Transforms, Fifth edition)
2. Figure reference in this assignment refers to the Text's Figure label.
3. When using MATLAB to plot signals, scale your time axis such as to allow sufficient amount of the signal to be plotted. Use subplot to give 3 or 4 plots per page; label the axes of your plots accordingly e.g Time (sec) on the x-axis and x(t) on the y-axis; the title should be the problem number, for example 2a).
4. You do not need to submit the Matlab codes for this assignment but the generated figures must be printed out and submitted alongside your detailed solutions.
5. **Due Date: February 14, 2014.**

Problem 1 (10 Marks). *For the signal $x(t)$ of Figure P2.1a plot Use Matlab to plot the following functions*

a) $x(3t-6)$, *and*

b) $-4x(t)+2$.

□

Problem 2 (10 Marks). *For the general case of transformations of discrete signals, given signals $x[n]$ and $x_t[n]$ can be expressed as*

$$x_t[n] = Ax[an+n_o] + B \tag{1}$$

where a is rational and n_o is an integer.

a) Solve this expression for $x[n]$.

b) Suppose that for the signal of Figure p9.7

$$x_1[n] = 0.5x_3[-n+1] + 2 \tag{2}$$

sketch $x_3[n]$.

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□

Problem 3 (20 Marks). *For each of the signals*

1. $x[n] = 2\mu[n]$, *and*
2. $x[n] = cos[0.1n]$.
   (a) Determine mathematically whether the signal is **even, odd,** or **neither**, *and*
   (b) Find the even part and the odd part of each of the signals.
   (c) Use Matlab to plot the signals, their even and odd parts.

□

Problem 4 (20 Marks). *For each of the signals*

1. $x(t) = -4t$, *and*
2. $x(t) = -\mu(t-1) + \mu(-t-1)$.
   (a) Determine mathematically whether the signal is **even, odd,** or **neither**, *and*
   (b) Find the even part and the odd part of each of the signals.
   (c) Use Matlab to plot the signals, their even and odd parts.

□

Problem 5 (10 Marks). *Consider the signal*

$$x(t) = cos(\pi t) + 5e^{-j15t} + \sin(7t) \tag{3}$$

If the signal is periodic, find its

a) fundamental period T_o, and its

b) fundamental frequency ω_o.

Otherwise, prove that the signal is not periodic.

□

Problem 6 (20 Marks). *A continuous-time signal $x(t)$ =cos(πt) is sampled every T seconds resulting in the discrete-time signal $x[n] = x(nT)$. Determine whether the sampled signal is periodic for*

1. $T = 0.125s$,
2. $T = 0.130s$.

For the sampled signal that is periodic, find

a) the number of periods of $x(t)$ in one period of $x[n]$,

b) the number of samples in one period of $x[n]$.

□

Problem 7 (10 Marks). *Prove the time-scaling property of the Dirac delta function*

$$\int_{-\infty}^{\infty} \delta(at - t_o)dt = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta(t - \frac{t_o}{a})dt \tag{4}$$

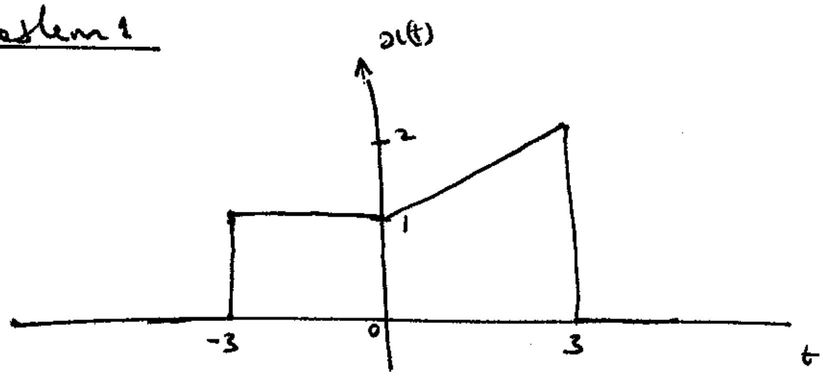
Hence or otherwise, evaluate the following integral

$$\int_{-\infty}^{\infty} sin[t - \frac{\pi}{6}]\delta(2t - \frac{2\pi}{3})dt \tag{5}$$

□

ELC 321 Assignment 1
Solution Guide & Marking Scheme

Problem 1

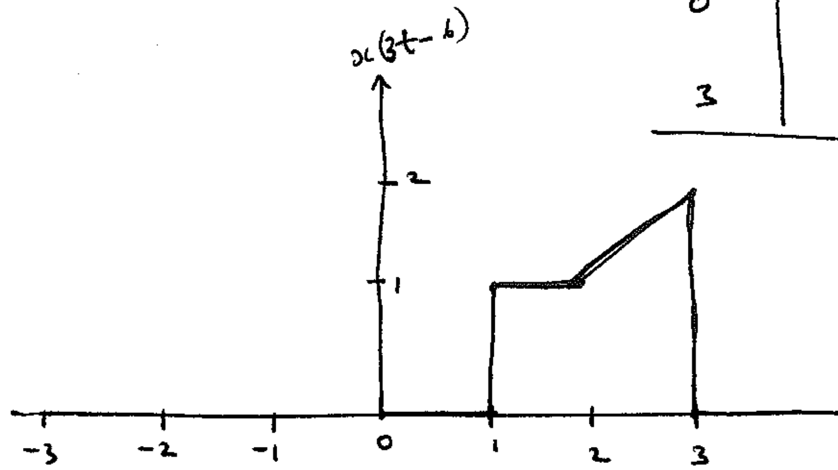


(a) $x(3t-6)$ — This is time-transformation

Let $\tau = 3t - 6$

Then $t = (\tau+6)/3$

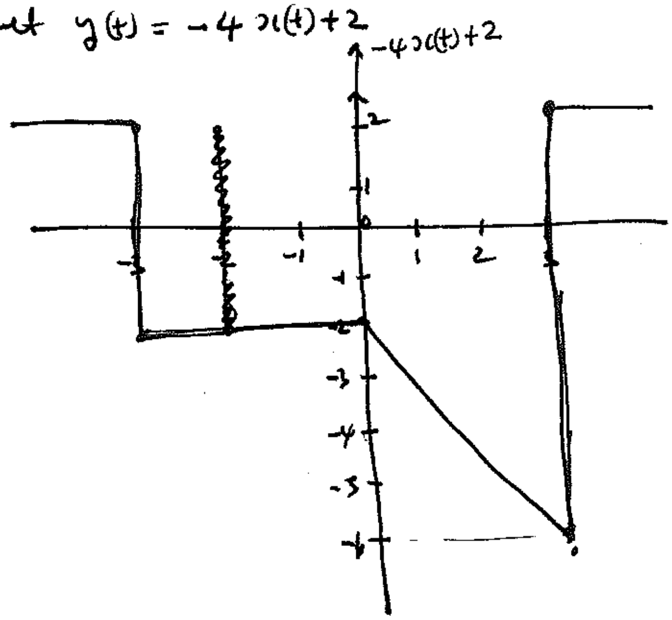
τ	t
-3	1
0	2
3	3



(b) $-4x(t)+2$ — This is amplitude-transformation.

Let $y(t) = -4x(t)+2$

$x(t)$	$y(t)$
0	2
1	-2
2	-6



Problem 2

(a) $x_t[n] = A x[an + n_0] + B$ ——— (1)

Let $m = an + n_0$ such that

$n = \frac{1}{a}(m - n_0)$ ——— (2)

substituting into (1) gives

$x_t\left[\frac{1}{a}(m-n_0)\right] = A x[m] + B$

which yields

$x[m] = \frac{1}{A}\left[x_t\left[\frac{1}{a}(m-n_0)\right] - B\right]$

Replacing m with n gives the final result

$x[n] = \frac{1}{A}\left(x_t\left[\frac{1}{a}(n-n_0)\right] - B\right).$

(b)

$x_1[n] = 0.5 x_3[-n+1] + 2$

let $m = -n+1 \Rightarrow n = -m+1$

$\therefore x_1[-m+1] = 0.5 x_3[m] + 2$

$x_3[m] = 2(x_1[-m+1] - 2)$

or

$x_3[n] = 2(x_1[-n+1] - 2)$

— This involves both time- and amplitude transformations

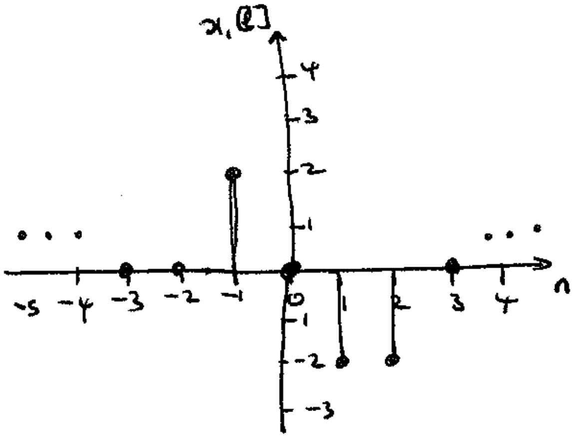
we do amplitude transformation first i.e

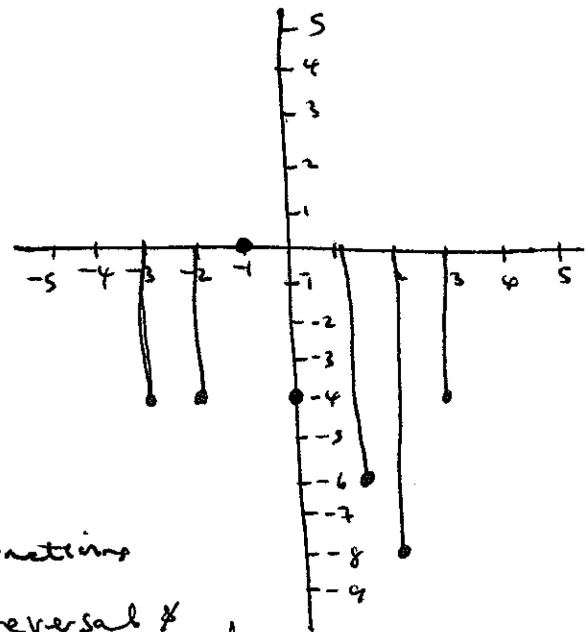
$y[\ell] = 2 * (x_1[\ell] - 2)$

when $\ell = -n+1$

ℓ	$x_1[\ell]$	$y[\ell]$
−3	0	−4
−2	0	−4
−1	2	0
0	0	−4
1	−1	−6
2	−2	−8
3	0	−4

Problem 2 continued



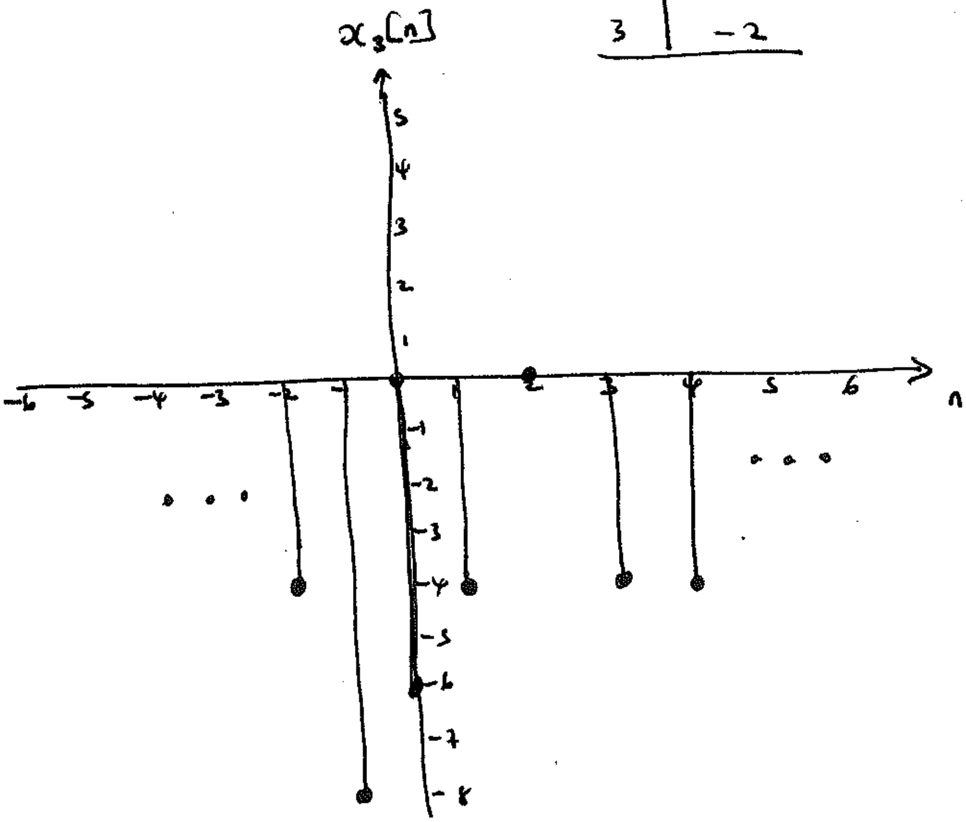


Finally we do the time transformations

from $l = -n+1$ —— Time reversal & Time shifting.

we have $n = -l+1$

l	n
-3	4
-2	3
-1	2
0	1
1	0
2	-1
3	-2



Problem 3

a, $$x[n] = 2u[n] = \begin{cases} 2 & ; \ n \geq 0 \\ 0 & ; \ n < 0 \end{cases}$$

$$x[-n] = 2u[-n] = \begin{cases} 2 & ; \ -n \geq 0 \\ 0 & ; \ -n < 0 \end{cases} = \begin{cases} 2 & ; \ n \leq 0 \\ 0 & ; \ n > 0 \end{cases}$$

Since $x[n] \neq x[-n]$ and
$x[n] \neq -x[-n]$, the signal is neither even or odd.

b, The even part is

$$x_{even}[n] = \frac{1}{2}\left[x[n] + x[-n]\right] = \frac{1}{2}\left(2u[n] + 2u[-n]\right) = u[n] + u[-n]$$

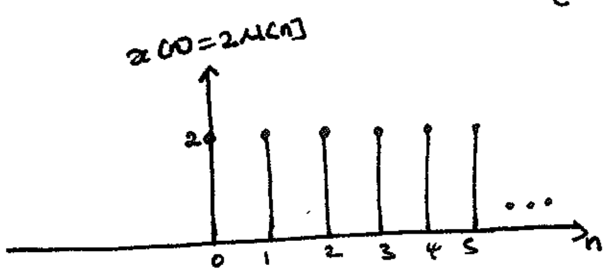
$$= \begin{cases} 2 & ; \ n = 0 \\ 1 & ; \ n \neq 0 \end{cases}$$

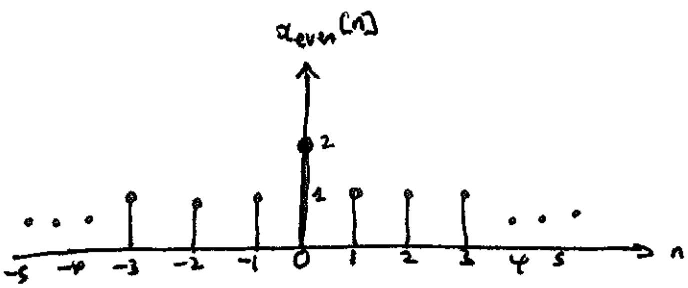
The odd part is

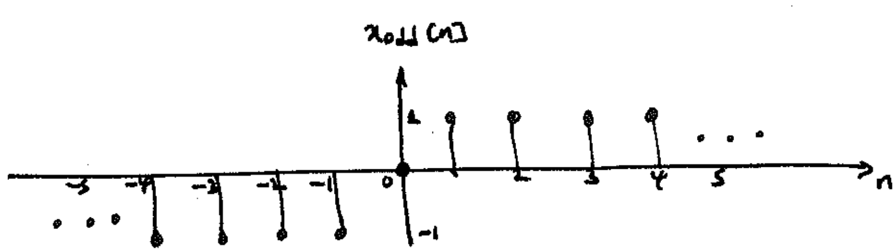
$$x_{odd}[n] = \frac{1}{2}\left[2u[n] - 2u[-n]\right] = u[n] - u[-n]$$

$$= \begin{cases} 1 & ; \ n > 0 \\ 0 & ; \ n = 0 \\ -1 & \ \ n < 0 \end{cases}$$

c,







problem 3 continued

a_{II} $x[n] = \cos[0.1n]$

$x[-n] = \cos[-0.1n] = \cos[0.1n]$

Since $x[-n] = x[n]$, the signal is an <u>even signal</u>

b_{II} Even part

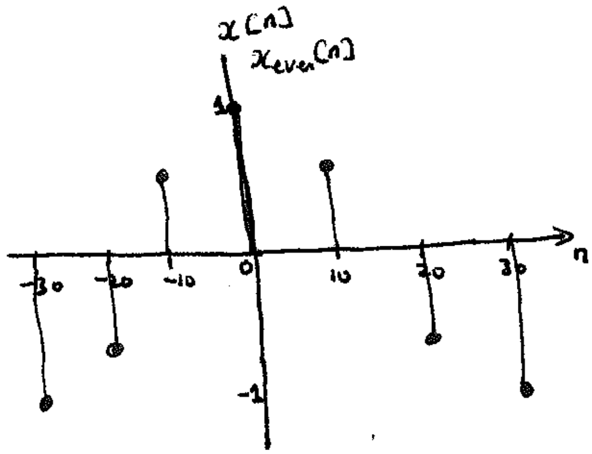
$x_{even} = \frac{1}{2}\left[\cos[0.1n] + \cos[-0.1n]\right] = \frac{1}{2}\left[\cos[0.1n] + \cos[0.1n]\right]$

$= \cos[0.1n] = x[n]$

odd part

$x_{odd} = \frac{1}{2}\left[\cos[0.1n] - \cos[-0.1n]\right] = \frac{1}{2}\left[\cos[0.1n] - \cos[0.1n]\right] = 0$

c_{II}



problem 4

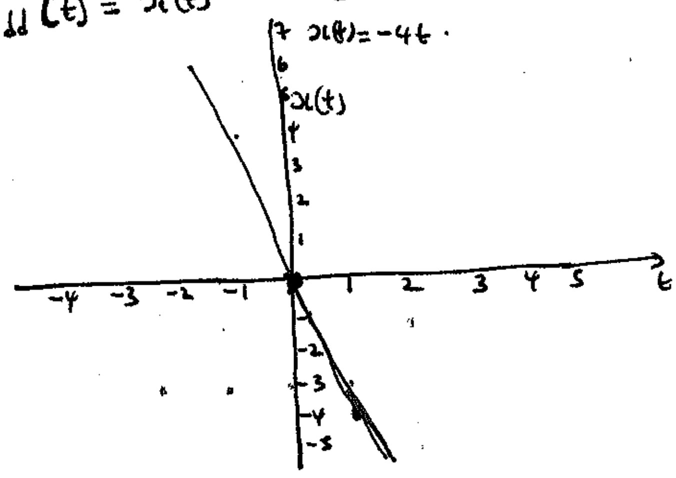
a_I $x(t) = -4t$

$x(-t) = 4t$

Since $x(t) = -x(-t)$ ∴ $x(t)$ is <u>odd</u>

b_I ∴ $x_{odd}(t) = x(t)$ and $x_{even}(t) = 0$

c_I



Problem 4 continued

a II $x(t) = -u(t-1) + u(-t-1)$

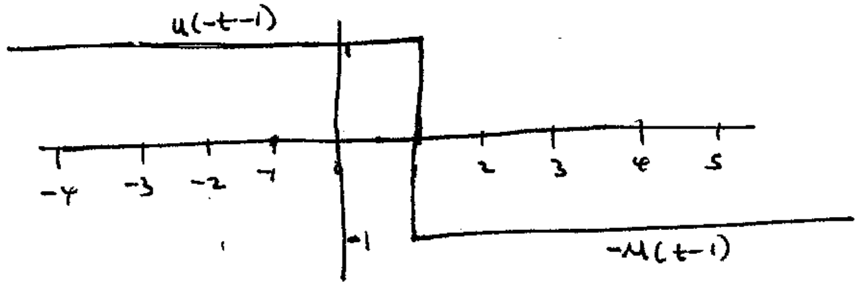
$x(-t) = -u(-t-1) + u(t-1) = -(-u(t-1) + u(-t-1)) = -x(t)$

Since $x(t) = -x(-t)$

The signal $x(t)$ is odd

b II $x_{even}(t) = 0 \qquad x_{odd}(t) = x(t)$

C II



Problem 5

$x(t) = \cos(\pi t) + 5e^{-j15t} + \sin(7t)$

$x(t) = x_1(t) + x_2(t) + x_3(t)$

$x_1(t) = \cos(\pi t) \equiv \cos(\omega t)$ is periodic with $\omega_1 = \pi$ and $T_1 = 2$

$x_2(t) = 5e^{-j15t} \equiv 5e^{-j\omega t}$ is periodic with $w_2 = 15$ and $T_2 = 2\pi/15$

$x_3(t) = \sin(7t) \equiv \sin(\omega t)$ is periodic with $w_3 = 7$ and $T_3 = \frac{2\pi}{7}$

checking for the condition for $x(t)$ to be periodic

$\frac{T_1}{T_2} = \frac{15}{\pi}, \quad \frac{T_1}{T_3} = \frac{7}{\pi}$

Since π is an irrational number, the ratio of the periods are not rational. Therefore signal $x(t)$ is not periodic i.e aperiodic.

Problem 6 $\quad x(t) = \cos(\pi t)$

$$x(t)\Big|_{t=nT} = \cos(\pi n T) \equiv \cos(\omega_0 n T) = \cos\left(2\pi\left(\frac{T}{T_0}\right)n\right)$$

The period T_0 of $x(t)$ is obtained from

$$\omega_0 t = \pi t$$

$$\Rightarrow \frac{2\pi}{T_0} = \pi \quad \Rightarrow \quad T_0 = 2$$

For periodic signal

$$\frac{T}{T_0} = \frac{K}{N} \text{ — ratio of integers}$$

So for $T = 0.125$

we have $\frac{K}{N} = \frac{0.125}{2} = \frac{1}{16}$

Hence $x[n]$ is periodic with $N = 16$

a_I Since $NT = KT_0$ $\qquad K = 1.$

$$\Rightarrow 16T = T_0$$

a_{II} $\qquad N = 16$

b_I for $T = 0.130$

$$\frac{T}{T_0} = \frac{K}{N} \Rightarrow \frac{K}{N} = \frac{0.130}{2} = \frac{13}{200} \text{ — periodic with } N = 200$$

$K = 13$

b_{II} $\qquad N = 200.$