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THE COLLEGE OF NEW JERSEY
ENG 272 – ADVANCED ENGINEERING MATH I
EXAM #2, FALL 2015

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Each problem part is worth 10 points. YOU MUST SHOW ALL WORK FOR CREDIT.

1. Show that $y_1 = e^{5x}$ and $y_2 = e^{-7x}$ form a fundamental set of solutions of the differential equation $y''+2y'-35y=0$ on the interval $(-\infty,\infty)$.

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{5x} & e^{-7x} \\ 5e^{5x} & -7e^{-7x} \end{vmatrix} = (e^{5x} \cdot -7e^{-7x}) - (5e^{5x} \cdot e^{-7x})$$

$$= -7e^{-2x} - (5e^{-2x}) = -12e^{-2x} \neq 0$$

Y_1 & Y_2 are lineally independant.

$$y'' + 2y' - 35y = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m^2 + 2m - 35 = 0$$

$a = 1 \quad b = 2 \quad c = -35$

$$m = \frac{-2 \pm \sqrt{4 + 140}}{2} = \frac{-2 \pm 12}{2}$$

$b^2 = 4 > 4ac = 4 \cdot 1 \cdot (-35) = -140$

$= -1 \pm 6$

real distinct roots

$m_1 = -7$
$m_2 = 5$

$$Y_c = C_1 e^{5x} + C_2 e^{-7x}$$

$\frac{d}{dx}\cos(x)$ $-\sin(x)$

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2. If $y(x) = \cos(4x)$ is a solution to the differential equation $y'' + 16y = 0$, use the method of reduction of order to find a second solution to this differential equation. You must use the substitution $y_2 = uy_1$ for this problem.

Sub: $Y_2(x) = U(x) \cdot Y_1(x)$ $\therefore$ $Y_2 = U \cdot \cos(4x)$

$Y_2' = [u'] \cos(4x) + u \cdot [-\sin(4x) \cdot 4]$

$Y_2'' = ([u''] \cdot \cos(4x) + u' \cdot [-4\sin(4x)]) - (4u \cdot [\cos(4x) \cdot 4] + [u'] \cdot 4\sin(4x))$

Plug into DE $Y'' + 16Y = 0$;

$0 = Y'' + 16Y = [u'' \cdot \cos(4x) - \frac{1}{2} u' \cdot \sin(4x) - 16u \cdot \cos(4x)] + 16[u \cdot \cos(4x)]$

$0 = u'' \cdot \cos(4x) - \frac{1}{2} u' \cdot \sin(4x)$

Sub $w = u'$; $w' \cdot \cos(4x) - \frac{1}{2} w \cdot \sin(4x) = 0$ linear first order ODE

$P(x) = -\frac{1}{2}\sin(4x)$ integrating factor: $e^{\int P(x)dx}$

$\int -\frac{1}{2}\sin(4x)dx = \frac{1}{2}\int \sin(4x)dx = -\frac{1}{2} \cdot (-\cos(4x)) \cdot \frac{1}{4} + C$

$= \frac{1}{8}\cos(4x) + C$

$e^{\int P(x)dx} = e^{\frac{1}{8}\cdot\cos(4x)}$ $\therefore$ $w' \cdot \cos(4x) \cdot e^{\frac{1}{8}\cos(4x)} - \frac{1}{2} w \cdot \sin(4x) \cdot e^{\frac{1}{8}\cos(4x)} = 0$

$\int \frac{d}{dx}(w \cdot e^{\frac{1}{8}\cos(4x)}) = \int 0$; $C_1 = w \cdot e^{\frac{1}{8}\cos(4x)}$

$w = u' = \frac{C_1}{e^{\frac{1}{8}\cos(4x)}}$

$u = \int \frac{C_1}{e^{\frac{1}{8}\cos(4x)}} dx = C_1 \cancel{\int e^{-\frac{1}{8}\cos(4x)} dx}$ $=$

$\cancel{u = C_1 \, 8e^{-\frac{1}{8}\cos(4x)} + C_2}$

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3. A.) Use the method of undetermined coefficients to solve the initial value problem
$y''-14y'+49y=5e^{7x}+1,\ y(0)=2,\ y'(0)=11.$
B.) Use the reduction of order formula to confirm a second solution to the homogeneous equation.

$-3\ y(x)$

$-7 = 11$

find Y_c: $m^2-14m+49$

$a=1 \quad b=-14 \quad c=49$

$b^2=196$

$4ac=196$

$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{14\pm\sqrt{196-196}}{2} = 7$ twice

$Y_c = C_1e^{7x}+C_2xe^{7x}$

guess $Y_p = Ae^{7x}\cdot x + B$

$Y_p' = 7xAe^{7x}+Ae^{7x}$

$Y_p'' = 7Ae^{7x}+(7Ae^{7x})+x(49Ae^{7x}) = 14Ae^{7x}+49xAe$

~~Plug in:~~

$\cancel{14Ae^{7x}+49xAe^{7x}-14[Ae^{7x}+7xAe^{7x}]+49[xAe^{7x}+B]=5e^{7x}+1}$

$\cancel{49B=5e^{7x}+1}$

$\cancel{B=5e^{7x}+1}$

$B=5e^{7\cdot 0}+1$

$B=6$

B. $\cancel{Y_2=Y_1\int\frac{e^{-\int P dx}}{Y_1^2}dx}$

for

$\text{ok} \sim$

4. Use variation of parameters to solve the initial value problem $y''-14y'+49y=5e^{7x}+1$, $y(0)=2$, $y'(0)=11$.

from #3: $y_c = c_1 e^{7x} + c_2 x e^{7x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{7x} & xe^{7x} \\ 7e^{7x} & e^{7x}+7xe^{7x} \end{vmatrix} = e^{7x}\cdot(e^{7x}+7xe^{7x}) - 7xe^{7x}$$

$$= e^{14x}+7xe^{14x}-7xe^{14x} = e^{14x}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & xe^{7x} \\ 5e^{7x}+1 & e^{7x}+7xe^{7x} \end{vmatrix} = -xe^{7x}(5e^{7x}+1)$$

$$= 5xe^{14x} - xe^{7x}$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} e^{7x} & 0 \\ 7e^{7x} & 5e^{7x}+1 \end{vmatrix} = 5e^{14x}+e^{7x}$$

$$u_1' = \frac{W_1}{W} = \frac{-5xe^{14x} - xe^{7x}}{e^{14x}} = 5x - xe^{-7x}$$

$$u_2' = \frac{W_2}{W} = \frac{5e^{14x}+e^{7x}}{e^{14x}} = 5 + e^{-7x}$$

ok

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$$u_1 = \int -5x - xe^{-7x}\,dx = -\int 5x\,dx - \int x\,dx \cdot \int e^{7x}\,dx = -5x^2 \cdot \frac{x^2}{2} \cdot \frac{1}{7}$$

$$u_2 = \int 5 + e^{-7x}\,dx = \int 5\,dx + \int e^{-7x}\,dx = 5x - \frac{1}{7}e^{-7x}$$

$$y(x) = ?$$

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5. A.) Find the inverse Laplace transform of $F(s)=\dfrac{3}{9s^2+4}$.

~~B.) Then take the Laplace transform of the solution y(t) in part A to confirm the result.~~ for extra Credit

$$\mathcal{L}^{-1}\left\{F(s)=\frac{3}{9s^2+4}\right\} = \cancel{\frac{2}{(3s-(-2))}}$$

using #16 $\left(f(t)=e^{at}\cdot t\right.$, $F(s)=\dfrac{1}{(s-a)^2}$

$$\mathcal{L}^{-1}\left\{\frac{3}{9s^2+4}\right\} = \cancel{e^{-2t}}\cdot t$$

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6. **Use the Laplace transform technique to solve the initial-value problem:**
$y''+16y=e^{5t}$, $y(0)=0$, $y'(0)=0$. **This is a mathematical model for what kind of spring system?**

$$\mathcal{L}\{y''+16y=e^{5t}\} = s^2Y(s)-sY(0)-Y'(0)+16[Y(s)]=\frac{1}{s-5}$$

$$s^2Y(s)-0-0+16Y(s)=\frac{1}{s-5}$$

$$Y(s)[s^2+16]=\frac{1}{s-5}$$

$$Y(s)=\frac{1}{s-5}\cdot\frac{1}{s^2+16}$$

Need Partial Fractions

$$Y(t)=\mathcal{L}\left\{\frac{1}{s-5}\right\}\cdot\mathcal{L}\left\{\frac{1}{s^2+16}\right\}$$

$$Y(t)=e^{5t}\cdot e^{-4t}\cdot t = Te^{T}$$

This is an undamped spring system

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