Name: Solutions

MAT 128
Quiz 6

Determine whether the following series converge or diverge:

1. $\sum_{k=1}^{\infty} \frac{k(k+2)}{(k+3)^2}$

$a_k = \frac{k^2+2k}{k^2+6k+9}$

$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{1+2/k}{1+6/k+9/k^2} = 1 \neq 0$

$\Rightarrow$ series $\boxed{\text{diverge}}$

2. $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum\left(\frac{1}{3}\right)^n + \sum\left(\frac{2}{3}\right)^n$, $\left|\frac{1}{3}\right| < 1$, $\left|\frac{2}{3}\right| < 1$

both converge

$\Rightarrow$ series $\boxed{\text{converge}}$

3. $\sum_{n=e}^{\infty} \frac{1}{n\ln n}$

$f(x) = \frac{1}{x\ln x} > 0$ when $x > 1$

$f' = -\frac{(\ln x + 1)}{(x\ln x)^2} < 0 \quad x > 1$

$\Rightarrow \int_e^{\infty} \frac{1}{x\ln x}\,dx$, let $u = \ln x$, $du = \frac{1}{x}dx$, $\Rightarrow \int \frac{1}{u}\,du = \ln(\ln x)$

$\Rightarrow \lim_{b\to\infty} \ln(\ln b) = \infty \Rightarrow \boxed{\text{diverge}}$

4. $\sum_{n=1}^{\infty} \frac{1}{n^2+n^3} = \sum \frac{1}{n^2(1+n)}$

$f' = \frac{-1}{2n+3n^2} < 0$ dec

$\int^{\infty} \frac{1}{x^2+x^3}\,dx = \lim_{b\to\infty} \int -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x+1}\,dx$

$= \lim_{b\to\infty} \left[\ln\left(\frac{x+1}{x}\right) - \frac{1}{x}\right]_1^b = \lim_{b\to\infty} \ln\left(\frac{b+1}{b}\right) - \frac{1}{b} - \ln 2 + 1$

$= -\ln 2 + 1$

$\Rightarrow \boxed{\text{Converges}}$