Lab4: Stefan-Boltzmann Law

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Abstract:

The objective of this lab was to learn about thermal radiation, the Stefan-Boltzmann Law and to study emission rates of thermal radiation from different surfaces. This procedure consisted of two different sets of apparatus, one to examine thermal radiation using the Stefan-Boltzmann Law, and the other to compare thermal radiation emission rates through surfaces made of different materials exposed to the same source of radiation. Our results overall were very accurate, but each data point was not particularly precise in relation to it's surrounding data points. Rather than sum to a large overall error, our errors canceled each other out because they were made with no overall direction. Our final result was an exponent of 4.0012 compared to the exponent of 4 in the Stefan-Boltzmann Law with 0.03% error between these values. Our results in the radiation cube setup were not meant to be compared to standard values but rather relative to one and other and did confirm the emissivity results we expected.

Theoretical Basis:

1. The Stephan –Boltzmann Law: states that total radiant heat energy emitted from a surface is proportional to the fourth power of its absolute temperature. Furthermore, thermal energy radiated by a black body radiator per second per unit area is proportional to the 4th power of the absolute temperature.
   $\frac{P}{A} = \sigma T^4$ j/m^(2)s

   $\sigma = 5.6704 * 10^{-8}$ watt/m^(2)*k^(4)

   For objects that are very hot (not ideal radiators), the law can be expressed in the form:
   $$\frac{P}{A} = e\sigma T^4$$
   Where "e" is the emissivity constant which happens to equal 1 for an ideal radiator.

2. Thermal Radiation is energy transfer by the emission of electromagnetic waves, which carry energy away from the emitting object. For ordinary temperatures (less than glowing red hot), the radiation is in the inferred region of the Electro Magnetic Spectrum.

   The relationship governing net radiation from hot objects is the Stephan Boltzmann law:
   $$P = e\sigma A(T^4 - T_c^4)$$
   Where "P" is net radiated power, "e" the emissivity, "σ" is the SB constant, "A" is radiating surface area, "T" is the absolute temperature of the radiator and "T_c" is the temperature of the surroundings.

3. Another way of examining the Stephen Boltzmann Law is mathematically in terms of total energy flux "Φ" emitted from a blackbody at temperature "T".

$$\Phi = \pi \int_0^\infty B_\lambda \, d\lambda \qquad \pi \int_0^\infty \frac{2c^2h}{\lambda^5 (e^{hc/(\lambda kT)} - 1)} \, d\lambda$$

$$2\pi c^2 h \int_0^\infty \frac{1}{\lambda^5 [e^{hc/(\lambda kT)} - 1]} \, d\lambda$$

Where" B_λ "is planks law brightness, "c" is the speed of light, "h" is planks constant, "k" is Boltzmann's constant, and π converts from a brightness per solid angle to total brightness, and comes from the equation $\Phi = \pi B$. This allows us to calculate flux"Φ" at a surface of uniform brightness "B". We then proceed to integrate by substitution.

$$u \equiv \frac{hc}{\lambda kT}$$

$$\lambda = \frac{hc}{ukT}$$

$$d\lambda = -\frac{hc}{u^2 kT} du,$$

Now we substitute:

$$\Phi = 2\pi c^2 h \int_\infty^0 \frac{-\frac{hc}{u^2 kT}}{\left(\frac{hc}{ukT}\right)^5 (e^u - 1)} \, du$$

$$= 2\pi c^2 h \left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{u^3}{e^u - 1} \, du$$

By application of the definition of the Riemann Zeta function is equal to:

$$\Phi = \frac{2\pi c^2 h k^4 T^4}{h^4 c^4} \zeta(4)\Gamma(4) = \frac{2\pi k^4 T^4}{h^3 c^2} \frac{\pi^4}{90} 3!$$

$$= \frac{2\pi^5 k^4}{15 h^3 c^2} T^4$$

$$\equiv \sigma T^4,$$

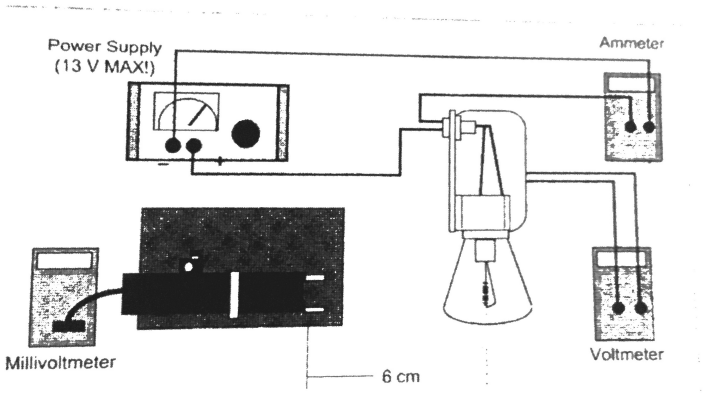
4. Radiation: is the emission of energy as electromagnetic waves or as moving subatomic particles, especially high-energy particles that cause ionization.

5. Apparatus:

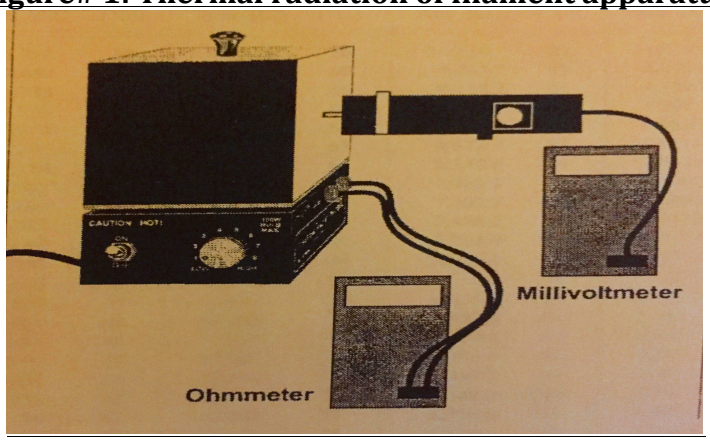
    a. **TD-8554A Radiation Cube** has four surfaces each made of a different material that can all be heated to 120°C. The cube is heated by a 100-watt light bulb. Temperature can be measured by connecting an ohmmeter to the terminal labeled Thermistor using banaplugs. After measuring resistance, temperature can be derived by referencing Table 4 in the lab.
    b. **TD-8555 Stefan-Boltzmann Lamp** is a high temperature source of thermal radiation. The high temperature simplifies analysis because the 4th power of the ambient temperature is negligible compared to the 4th power of the temperature of the lamp filament. By carefully controlling the voltage and current into the lamp, temperature of the filament can be obtained.
    c. **The PASCO TD-Radiation Sensor** measures the relative intensities of incident thermal radiation. The sensing element is a miniature thermopile that produces a voltage in proportion with the intensity of the radiation via the photoelectric effect.
    d. **Voltmeter:** A measurement device placed in parallel with a circuit element that possesses relatively infinite resistance in order to measure the electric potential difference between two points.
    e. **Ammeter:** A measurement device placed in series with a circuit element, of nearly zero resistance in order to measure the current flowing through it.

Experimental Design and Procedure:

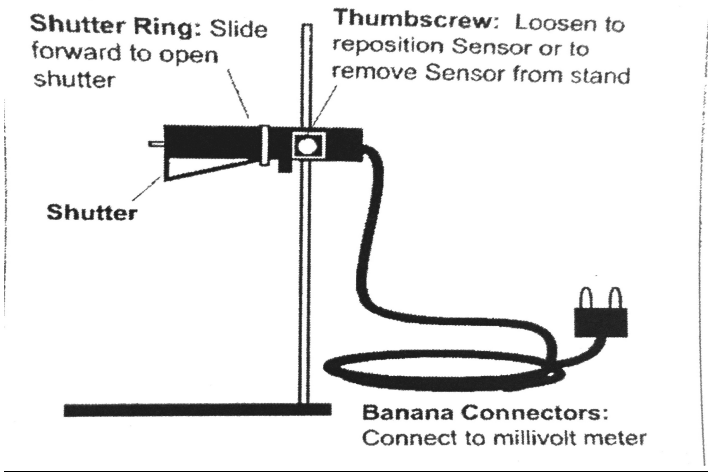
Set Up: In order to confirm the Stefan-Boltzmann Law we used an incandescent light bulb that's filament temperature could be controlled by monitoring the voltage across it and the current through it. Therefore by measuring the radiation emitted (with radiation sensor and heat shield) we can experimentally confirm or deny the Stefan-Boltzmann Law. We preformed an additional experiment, in order to investigate the emissivity of different materials. Thermal radiation produced by a 100w incandescent bulb, located inside a cube, with 4 different vertical faces each made of a different material; was recorded by a radiation sensor set at a fixed distance away from each vertical face of the cube. It was then possible to compare the emissivity of each cube face relative to each other by comparing the cube temperature to the intensity of thermal ration being emitted by each of the vertical faces.



(Figure# 1: Thermal radiation of filament apparatus)



(Figure# 2: radiation cube apparatus)



(Figure# 3: Radiation Detector)

Table 4 - Use for the Radiation Cube

Resistance versus Temperature for the Thermal Radiation Cube

Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)
207,850	10	66,356	34	24,415	58	10,110	82	4,615.1	106	2,281.0	130
197,560	11	63,480	35	23,483	59	9,767.2	83	4,475.0	107	2,218.3	131
187,840	12	60,743	36	22,590	60	9,437.7	84	4,339.7	108	2,157.6	132
178,650	13	58,138	37	21,736	61	9,120.8	85	4,209.1	109	2,098.7	133
169,950	14	55,658	38	20,919	62	8,816.0	86	4,082.9	110	2,041.7	134
161,730	15	53,297	39	20,136	63	8,522.7	87	3,961.1	111	1,986.4	135
153,950	16	51,048	40	19,386	64	8,240.6	88	3,843.4	112	1,932.8	136
146,580	17	48,905	41	18,668	65	7,969.1	89	3,729.7	113	1,880.9	137
139,610	18	46,863	42	17,980	66	7,707.7	90	3,619.8	114	1,830.5	138
133,000	19	44,917	43	17,321	67	7,456.2	91	3,513.6	115	1,781.7	139
126,740	20	43,062	44	16,689	68	7,214.0	92	3,411.0	116	1,734.3	140
120,810	21	41,292	45	16,083	69	6,980.6	93	3,311.8	117	1,688.4	141
115,190	22	39,605	46	15,502	70	6,755.9	94	3,215.8	118	1,643.9	142
109,850	23	37,995	47	14,945	71	6,539.4	95	3,123.0	119	1,600.6	143
104,800	24	36,458	48	14,410	72	6,330.8	96	3,033.3	120	1,558.7	144
100,000	25	34,991	49	13,897	73	6,129.8	97	2,946.5	121	1,518.0	145
95,447	26	33,591	50	13,405	74	5,936.1	98	2,862.5	122	1,478.6	146
91,126	27	32,253	51	12,932	75	5,749.3	99	2,781.3	123	1,440.2	147
87,022	28	30,976	52	12,479	76	5,569.3	100	2,702.7	124	1,403.0	148
83,124	29	29,756	53	12,043	77	5,395.6	101	2,626.6	125	1,366.9	149
79,422	30	28,590	54	11,625	78	5,228.1	102	2,553.0	126	1,331.9	150
75,903	31	27,475	55	11,223	79	5,066.6	103	2,481.7	127		
72,560	32	26,409	56	10,837	80	4,910.7	104	2,412.6	128		
69,380	33	25,390	57	10,467	81	4,760.3	105	2,345.8	129		

(Figure# 4: Table 4 For radiation cube)

Use for the Filament Temperature

Table 5 Temperature and Resistivity for Tungsten

R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm
1.0	300	5.65	5.48	1200	30.98	10.63	2100	60.06	16.29	3000	92.04
1.43	400	8.06	6.03	1300	34.08	11.24	2200	63.48	16.95	3100	95.76
1.87	500	10.56	6.58	1400	37.19	11.84	2300	66.91	17.62	3200	99.54
2.34	600	13.23	7.14	1500	40.36	12.46	2400	70.39	18.28	3300	103.3
2.85	700	16.09	7.71	1600	43.55	13.08	2500	73.91	18.97	3400	107.2
3.36	800	19.00	8.28	1700	46.78	13.72	2600	77.49	19.66	3500	111.1
3.88	900	21.94	8.86	1800	50.05	14.34	2700	81.04	26.35	3600	115.0
4.41	1000	24.93	9.44	1900	53.35	14.99	2800	84.70			
4.95	1100	27.94	10.03	2000	56.67	15.63	2900	88.33			

(Figure# 5: Table 5 for filament temperature)

Steps taken to obtain Data:

1. Before beginning with the filament apparatus, we were instructed to turn on the thermal cube to high and allow it to reach thermal equilibrium while conducting the rest of the lab.
2. We used appendix B provided in the lab manuel to measure T_{ref} (the room temperature in kelvin) and R_{ref} (the resistance of the filament of the lamp at T_{ref}).
3. Then we set up equipment as shown in figure 1, the front face of the sensor should be about 6 cm away from the filament.
4. Finally we turn on power supply, set Voltage to 1V(measure by voltmeter not source) record ammeter current, and Rad (in the millivolt meter). All while making sure to use heat shield between radiation detector and filament when not recording.
5. Repeat this procedure 11 times while incrementing the voltage reading by +1Volt for each trial.
6. It was then possible to calculate R, the resistance of the filament by using the table in figure 5. Calculate T^4, record.
7. In order to look at our results as a whole we made graphs of Rad vs. T and Rad vs. T^4, preform a power regression on the plots.
8. Once calculations from the first section were complete we connected the ohmmeter and and voltmeter as shown in figure2.
9. When the ohmmeter reading went down to 40kOhms we set the power switch to 5.
10. When the cube reaches thermal equilibrium, the ohmmeter readings will fluctuate around a relatively fixed value. We then used the radiation sensor to measure the radiation emitted by each of the 4 different vertical faces of the cube.
11. Repeat step 9 with the power set to 8.

Table 1

Data		
V (V)	I (A)	*Rad* (mV)
1.05	0.97	1.5
2.01	1.23	4.8
3.0	1.50	12.9
4.04	1.70	14.7
5.04	1.89	28.3
6.0	2.06	31.2
7.06	2.24	45.8
8.02	2.38	60.1
9.01	2.53	77.1
10.01	2.67	95.4
11.02	2.81	99.5
12.00	2.94	111.8

Table 2

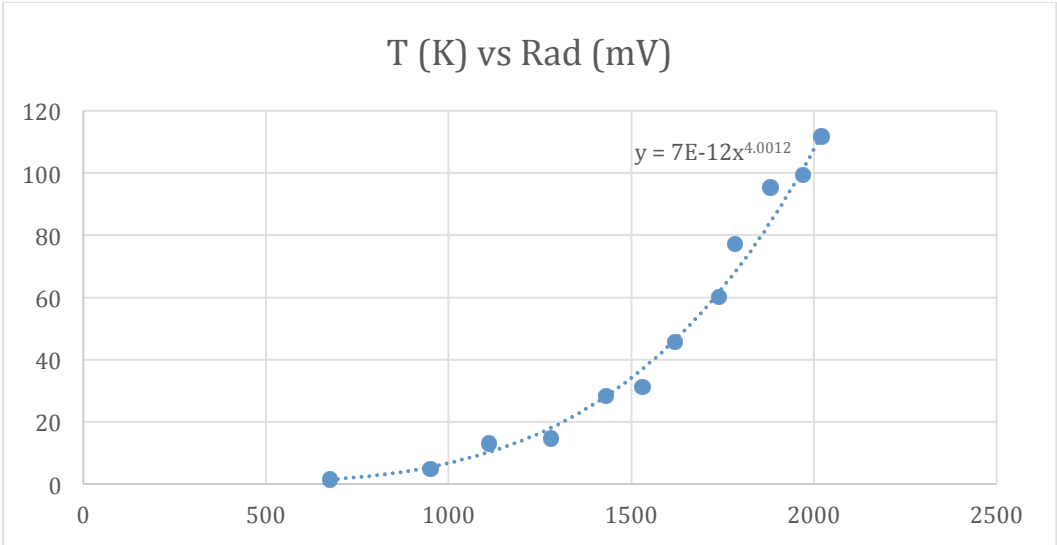
Calculations	and	Analysis
R (=V/I) (Ω)	T (K)	T^4 (K^4)
1.08	675	$2.07 \cdot 10^{11}$
1.634	950	$8.145 \cdot 10^{11}$
2	1110	$1.518 \cdot 10^{12}$
2.376	1280	$2.63 \cdot 10^{12}$
2.67	1430	$4.18 \cdot 10^{12}$
2.912	1530	$5.48 \cdot 10^{12}$
3.152	1620	$6.88 \cdot 10^{12}$
3.369	1740	$9.116 \cdot 10^{12}$
3.557	1785	$1 \cdot 10^{13}$
3.749	1880	$1.25 \cdot 10^{13}$
3.922	1970	$1.5 \cdot 10^{13}$
4.08	2020	$1.66 \cdot 10^{13}$

(Figure# 6: Light bulb filament Data)

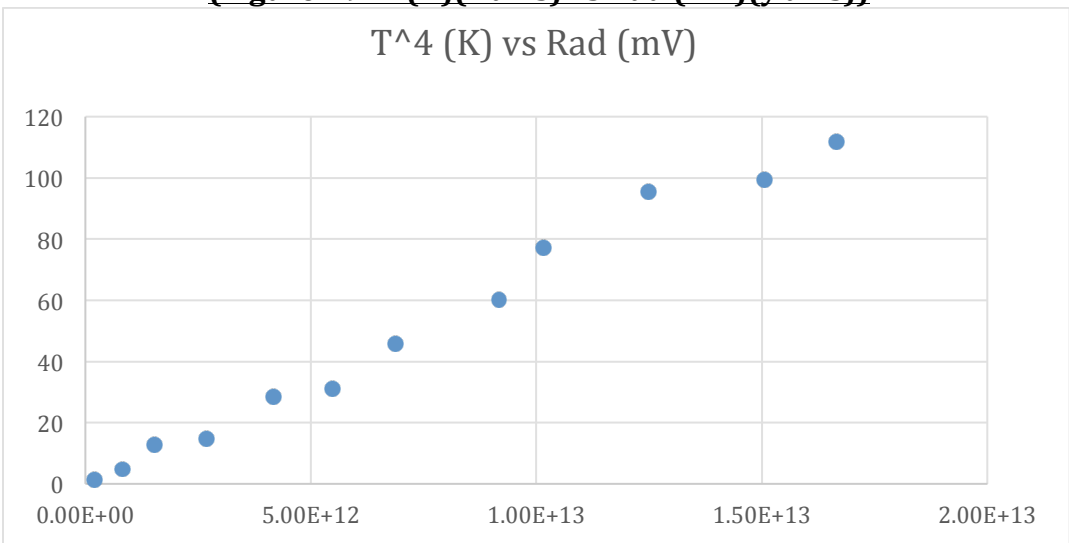
Table 3

Power Setting	5.0	8.0
Thermistor Resistance (kΩ)	19.3	5.5
Temperature (°C)	64	100
Surface	Sensor Reading (mV)	
Polished Aluminum	0.5	0.5
White	6.6	14.5
Black	6.8	14.9
Dull Aluminum	1.9	3.9

(Figure# 7: Thermal Radiation Cube Data)



(Figure# 9: T (K)(x axis) vs Rad (mV)(y axis))



(Figure# 10: T(k)^4(x axis) vs. Rad(mV)(y axis))

Results and Analysis:

Table 1&2 combined:

T(k)	Rad(mV)	T(k)^4
675	1.5	2.08E+11
950	4.8	8.15E+11
1110	12.9	1.52E+12
1280	14.7	2.68E+12

1430	28.3	4.18E+12
1530	31.2	5.48E+12
1620	45.8	6.89E+12
1740	60.1	9.17E+12
1785	77.1	1.02E+13
1880	95.4	1.25E+13
1970	99.5	1.51E+13
2020	111.8	1.66E+13

Table 3:

Power setting:	5.0	8.0
Thermistor Resistance (KΩ)	19.3	5.5
Temperature(°C)	64	100
Surface:	Sensor Reading (mV)	
Polished aluminum	0.5	0.5
White	6.6	14.5
Black	6.8	14.9
Dull aluminum	1.9	3.9

For auxiliary data, reference tables and pictures please reference the figures section of the **Experiment Design and Procedure** section.

This lab produced 0.03% error for the exponent of absolute temperature in the Stefan-Boltzmann equation which establishes the exponent as "4" whereas our analysis produced an exponent of "4.0012". That being said there were sources of error, for example the radiation detector would absorb heat from any source if not properly shielded and therefore it is difficult to take nearly instantaneous measurements. Furthermore in the radiation cube apparatus, we treated the cube as a perfect black body, when in reality there are none.

Conclusions and Summary:

The high experimental accuracy that was produced in this procedure allowed for many conclusions to be drawn. For one, thermal radiation is a product of heat, which is constantly emitted by the objects that surround us in daily life; yet is simply not visible to the human eye. We can support this conclusion by the observation that; no matter what direction you point it in, and even if you shield it, the thermal radiation sensor is always picking up at least a small radiation reading. It was also surprising that some variables, like the temperature of the filament, were better represented by indirect measurements of voltage and current. Another observation that defied expectation was that poor thermal absorbers are also poor

emitters of thermal radiation. Perhaps the most interesting lesson learned however, was that error in data is not necessarily cumulative. In most labs, error serves as the biggest obstacle, because it generally obeys the principle of superposition and leads to a result that is off by the summation of each individual error. In this lab, because measurements were taken in increments of time that were subject to human interpretation, the errors were made with no specific trend and served to cancel each other out rather than to sum together. When calculation the exponent of T in the Stefan-Boltzmann Law we arrived at a value of only 0.03% error compared to the known value of four.

Suggestions to improve the experimental accuracy of this procedure:

- To take a larger number of smaller increments of voltage
- Finding a way of "zeroing" the radiation sensor to the ambient levels of thermal radiation in the laboratory.
- Waiting longer for the thermal cube to equalize.
- Using an object of directly measurable temperature and accounting for the background temperature in calculations rather than indirectly measuring the temperature of an incandescent filament.
- Comparing results against other data produced by following a different procedure designed to derive the Stefan-Boltzmann Law.
- Finding temperature of the radiation cube by a function of resistance rather than approximating with an incremented table.

References:

1. Hyperphysics Online Textbook, http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/stefan.html
   last accessed on 10/5/15

2. Encyclopedia Britannica, http://www.britannica.com/science/thermal-radiation
   last accessed on 10/5/15

3. Sienceworld.Wolfram.com, http://scienceworld.wolfram.com/physics/Stefan-BoltzmannLaw.html
   last accessed 10/5/15

4. Google definitions

5. Appendix A from the lab Manuel.