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9/22

B 1. The DE $(x^2+y^2)y' = xy$ is homogeneous by process of elimination

D 2. Solution of $\frac{dy}{dx} = xy$

$dy = xy\,dx$, $\frac{dy}{y} = x\,dx$, $\int \frac{dy}{y} = \int x\,dx$

$\ln(y) = \frac{x^2}{2} + C$, $e^{\ln(y)} = e^{\frac{x^2}{2}+C}$, $y = C_1 \cdot e^{x^2/2}$

e 3. The Solution of $y' - y = x$: linear first order

step 1.) complementary : $\frac{dy}{dx} - y = 0$, $\frac{dy}{dx} = y$, $dx = \frac{dy}{y}$, $\int dx = \int \frac{dy}{y}$

$\ln(y) = x + C_1$, $e^{\ln(y)} = e^{x+C_1}$, $y = C_1 \cdot e^x$

step 2.) particular : $\frac{dy}{dx} - y = x$, $P(x) = -1$, $e^{\int P(x)dx} = e^{\int -1 dx} = e^{-x}$

$e^{-x}\left(\frac{dy}{dx}\right) - e^{-x}(y) = e^{-x}(x)$, $\frac{d}{dx}(e^{-x} \cdot (y)) = xe^{-x}$, $\int \frac{d}{dx}(e^{-x} \cdot (y)) = \int xe^{-x}dx$

$ye^{-x} = (-x-1)e^{-x} + C$, $y = -x - 1 + C_1$

step 3.) $y = -x - 1 + C_1 e^x$

4. Solve DE $(x+2y)dx + y\,dy = 0$ using substitution :

D

$M(tx, ty) = Tx + 2Ty = Tf(x,y)$ ✓ first order homogeneous

$N(tx, ty) = Ty = Tf(x,y)$ ✓

Substitute $y = ux$, $dy = u\,dx + x\,du$, $u = \frac{y}{x}$

$(x + 2(ux))dx + (ux)(u\,dx + x\,du) = 0$

$(x + 2ux)dx + u^2 x\,dx + ux^2\,du = 0$

$(x + 2ux + u^2 x)dx + (ux^2)du = 0$

$x(1 + 2u + u^2)dx + x^2(u)\,du = 0$, $\frac{dx}{x} + \frac{u\,du}{(u^2+2u+1)} = 0$

$\int \frac{dx}{x} + \int \frac{u\,du}{(u^2+2u+1)} = 0$

$\ln|x| + \ln(u+1) + \frac{1}{u+1} + C = 0$, $C = \ln(x) + \ln(u+1) + \frac{1}{u+1}$

$\ln(x) = C - \ln\left(\frac{y}{x}+1\right) + \frac{1}{\frac{y}{x}+1}$, $e^{\ln(x)} = e^{C - \ln(\frac{y}{x}+1) + \frac{x}{x+y}} = e^C \cdot \frac{x}{x+y} \cdot e^{\frac{x}{x+y}}$

$x = C_1 \cdot \frac{x}{x+y} \cdot e^{\frac{x}{x+y}}$

5. find the integrating factor for $x^2 y' + xy = 1$:

C

$y' + \frac{xy}{x^2} = \frac{1}{x^2}$, $e^{\int \frac{1}{x}dx} = e^{\ln(x)} = x$

D 6. Solution of DE: $y' + \frac{y}{x} = y^2$:

Bernoulli equation: $\frac{dy}{dx} + P(x)Y = f(x) \cdot y^n$

$P(x) = \frac{1}{x}$, $n = 2$

Substitute: $Y = u^{-1}$, $Y' = -u^{-2}\frac{du}{dx}$: $-\frac{1}{u^2}\frac{du}{dx} + \frac{u^{-1}}{x} = u^{-2}$

$-u^2\left[-\frac{1}{u^2}\frac{du}{dx} + \frac{1}{ux} = \frac{1}{u^2}\right]$ gives $\frac{du}{dx} - \frac{u}{x} = -1$

integrating factor $e^{\int P(x)dx} = e^{-\int \frac{1}{x}dx} = e^{-\ln(x)} = e^{\ln(\frac{1}{x})} = \frac{1}{x}$

$\frac{1}{x}\cdot\frac{du}{dx} - \frac{1}{x}\cdot\frac{u}{x} = -\frac{1}{x}$, $\frac{d}{dx}\left(\frac{1}{x}\cdot u\right) = -\frac{1}{x}$, $\int \frac{d}{dx}\left(\frac{1}{x}\cdot u\right) = -\int \frac{1}{x}dx$

$\frac{u}{x} = -\ln(x) + C$, $u = -x\ln(x) + Cx = \frac{1}{Y}$, $Y = \frac{1}{(C_1x - x\ln(x))}$

C 7. The DE $y' = (xe^y) \div (Y)$ is ? :

$\frac{dy}{dx} = \frac{xe^y}{Y}$, $\frac{dy}{dx}(Y) = xe^y$, $\frac{dy}{dx}\cdot Y \cdot \frac{1}{e^y} = x$, $\left(\frac{Y}{e^y}\right)dy = x\,dx$] - separable

D 8. The DE $(Y^3 + 6xY^4)dx + (3xY^2 + 12x^2Y^3)dy = 0$ is ? :

$\frac{\partial}{\partial Y}(Y^3 + 6xY^4) = 3Y^2 + 24xY^3$ $\frac{\partial}{\partial x}(3xY^2 + 12x^2Y^3) = 3Y^2 + 24Y^3x$

exact $(3Y^2 + 24xY^3 = 3Y^2 + 24Y^3x)$

$\int (Y^3 + 6xY^4)dx = 3Y^4x^2 + xY^3 + C$
$= Y^3x(3Yx + 1) + C$

$\int (3xY^2 + 12x^2Y^3)dy = 3x^2Y^4 + xY^3 + C$
$= xY^3(3xY + 1) + C$

$f = Y^3x(3xY + 1) + C$

9 & 10. B & A DE $(x - 2y)dx + (Y)\,dy = 0$ Solve using substitution:

$u = Y/x$, $Y = ux$, $dy = u\,dx + x\,du$

Substitute: $(x - 2ux + u^2x)dx + (x^2u)du = 0$, $x(1 - 2u + u^2)dx + x^2(u)du = 0$

$\frac{dx}{x} + \frac{u\,du}{(1 - 2u + u^2)} = 0$, $\int \frac{dx}{x} + \int \frac{u\,du}{(1 - 2u + u^2)} = 0$

$\ln(x) + \ln(u - 1) - \frac{1}{u-1} + C = 0$, $\ln(x) + \ln\left(\frac{Y}{x} - 1\right) - \frac{1}{\frac{Y}{x} - 1} = C$

$C = \ln(x) + \ln\left(\frac{Y}{x} - 1\right) + \frac{x}{x - Y}$, assuming x & Y are positive $\ln(x) + \ln\left(\frac{Y}{x} - 1\right)$
$= \ln(Y - x)$

$C = \ln(Y - x) + \frac{x}{x - y} = \ln(Y - x) - \frac{x}{Y - x}$