

Your Name: _____

Your lab partner(s): _____

Lab 2: Special Theory of Relativity

A. Introduction to the Program

- In the Control Panel of your computer, change the security settings for *Java* to "medium".
- Insert the *Physlet* CD in the CD drive. Open the file called "*Start*" to begin the program.
- On the left-hand side of the screen, click on "*Special Relativity*".
- Some program features:
 - **Animations** are controlled by these buttons: *play*, *pause*, *step>>*, *<<step* and *reset*.
 - **Units** are given in boldface in the statement of the problem
 - **Data input** can be in the form of numbers or formulas
 - **Data output** can be in the form of on-screen numerical values, data tables or graphs; you can usually right-click on the graph to clone it and resize it for a better view.
- **Light Clock:** Many simulations will use a light clock, represented by a box with a mirror on its top wall and a light pulse emitter and detector at its bottom wall. After light is emitted from the bottom, reflects from the mirror, and then returns to the bottom, the detector triggers a tick of the clock and another light pulse is emitted. The total vertical distance traveled by the light for the stationary clock is L_0 , where $L_0/2$ is the distance between the bottom and top walls of the clock. The clock reads time in meters, which means that every click measures the time it takes for light to travel distance L_0 .

B. Space and Time in Special Relativity

1. After clicking on "*Special Relativity*", select "*Ch. 2: Space and Time in Special Relativity*". If asked, give your permission to run Java.
2. **Simultaneity:** When we are dealing with moving reference frames, we must keep in mind that events that are simultaneous in one reference frame are not simultaneous in another. To study this, click on "*Section 2.3: Simultaneity*". Read the explanations and follow the instructions for this section.
3. **Time Dilation:** Select "*Section 2.4*". In this animation, the red light clock is stationary. We will refer to a stationary observer as an observer that is at rest with respect to the red clock. Set the speed of the green clock to $\beta (= v/c) = 0.5$ and press the "set value and play" button. When the red clock registers 5 ticks, the reading on the green (moving) clock is:

Therefore, the moving clock runs _____ when observed from stationary frames.
(faster/slower)

This happens because the distances traveled by the two light pulses must be the same as viewed in the frame of the stationary clock. However, the distance traveled by the moving

clock, as seen from the point-of-view of an observer in the stationary clock's reference frame, involves both horizontal and vertical components, and it is only the vertical component of the light pulse's motion that contributes to the clock ticks (as seen from the reference frame of the clock at rest). Following the standard derivations (see the textbook), we get that:

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \beta^2}} = \gamma \Delta t'$$

$\Delta t' = L_0/c$ = the time interval seen by an observer that is stationary with respect to the red clock; this is the same time that an observer moving together with the green clock would observe on the green clock.

Δt = the time interval observed on the green clock by someone in the stationary frame (frame of the red clock in this animation).

4. **Length Contraction:** Still in "*Section 2.4*", go to the second exercise at the bottom of the screen. Set the speed of the green clock to $\beta (= v/c) = 0.5$ and press the "set value and play" button. In this animation, the light clocks are rotated 90° (position is given in meters by dragging the cross-haired cursor around the animation). The results we saw for the time dilation still occurs for the rotated clocks. Therefore, given the fact that in the stationary frame the moving clock still ticks slower, the only way for this to happen is for the distance between the walls of the moving clock to be contracted. In fact, the clock must appear to be contracted to $L' = L_0/\gamma$ where L_0 is the length of the clock as seen in its own (stationary) reference frame and L' is the length of the moving clock as seen from the stationary frame. Notice that length contraction occurs only in the direction of motion of the object and not in the direction perpendicular to it.

Do Physlet Problems listed below and answer the questions:

5. Problem 2.1: Which of the four Animations is not an inertial reference frame?

Answer: _____

6. Problem 2.3: What is the time difference between these light flashes as seen from

the beacon: _____

inside the space ship: _____

7. Problem 2.4: How much is the length of the pole in the reference frame of the barn?

Answer: _____

How much is the speed of the pole with respect to the barn?

Answer: _____

How much is the length of the pole in its own reference frame?

Answer: _____

8. Problem 2.7: Which event (1 or 2) occurs sooner for the following relative velocities:

v/c	Green observer Event #	Red observer Event #
0.2		
0.4		
0.6		
0.8		
0.99		

Which of the following is true for all relative velocities (i.e from 0.2c to 0.99c)?

- A) There is a unique speed when the events are simultaneous in both reference frames.
B) Event 1 always occurs before event 2.
C) At any speed, the red and green observers will always agree on which event occurred first.
_____ D) None of the above.

C. Relativistic Mechanics

For momentum \vec{p} to be conserved, it needs to be redefined as:

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

where \vec{v} is the velocity of the particle as measured in a given reference frame.

9. After clicking on "*Special Relativity*", select "*Ch. 3: Relativistic Mechanics*". If asked, give your permission to run *Java*.
10. Select "Section 3.2: Understanding Mass-Energy Equivalence". Play the animation and read the explanation of the mass-energy equivalence, $E = mc^2$.
11. Click on "Section 3.3: Understanding the Energy-Momentum Equation". The total energy, E , of a particle is:

$$E = \sqrt{(pc)^2 + (m_0c^2)^2}$$

where p is the relativistic momentum, c is the speed of light, and m_0 is the rest mass of the particle (the mass of the particle measured in its own reference frame).

The total energy of the particle can also be expressed through its kinetic energy:

$$E = m_0 c^2 + E_k$$

Set the values of m_0 and E as given in the table below and record your observations.

m_0 (MeV/c ²)	E (MeV)	Maximum β ($= v/c$)	For the max velocity, <u>visually</u> compare $m_0 c^2$ with E_k (you don't need to calculate anything)	For the max velocity, <u>visually</u> compare pc with E (you don't need to calculate anything)
200	2000			
1000	2000			
1200	100 (not a typo!)			
1999	2000			

12. Using the animation in "Section 3.4: Exploring Particle Decays", choose a mass and an initial speed of a particle that will spontaneously decay into two other particles. Pick the masses of the two decay products that you want to create. How does the momentum before the decay compare to the momentum after?

Now run the animation in reverse. Notice that you see two particles colliding and creating a third particle that is more massive than the sum of the two initial particles. How is this possible? Where does the additional mass come from?

13. In Section 3.5, study the relativistic Doppler effect - a phenomenon in which light from a moving source is shifted in wavelength. The Doppler shift is due to the relative motion of the two reference frames (the source and receiver) and is given by:

$$\frac{1}{\lambda} = \frac{1}{\lambda'} \cdot \sqrt{\frac{1 \mp \beta}{1 \pm \beta}}$$

We use the upper signs ($-$, $+$) for objects moving away from each other, and the lower signs ($+$, $-$) for objects moving towards each other. Here $\beta = v/c$, λ' is the proper wavelength measured in the frame of the light source, and λ is the wavelength as seen from the reference frame moving with respect to the light source.

Do Physlet Problems listed below and answer the questions.

14. Problem 3.1: Calculate the speed and kinetic energy of each particle from the yellow's point of view. Record in the table below.

	Red	Green	Pink	Blue
Speed				
Kinetic energy (MeV)				

From the yellow's point of view, which particle has the kinetic energy that is the

smallest: _____ largest: _____

15. Problem 3.2: Which graph correctly shows the total energy of an electron? Explain.

Answer: _____

16. Problem 3.3: Set the initial momentum of the Sigma particle (Σ) to 5000 MeV/c. Run the simulation in which the Σ particle decays into a pion (π) and a neutron (n). Use the conservation of the total energy before and after the decay to find the rest mass of the Σ particle. Show your calculations.

Answer: $m_{\Sigma} =$ _____

17. Problem 3.4: In this exercise, we will observe a collision of two protons, which results in the creation of three protons and one antiproton (same mass as proton, but oppositely charged). The rest mass of a proton (and anti-proton) is 938 MeV/c².

We will also need to consider the relativistic transformation of velocities of the colliding particles. Let a reference frame S' move with respect to S at velocity v . If u is the velocity of a particle as measured in S, then the velocity u' of the particle as measured in S' is given by the relativistic formula:

$$u' = \frac{u - v}{1 - \frac{vu}{c^2}}$$

In the above formula, u and v must be taken with the proper (+ or -) signs. Run the simulations to answer the following questions.

- a) In terms of c , how much is the speed of each colliding proton in the center of mass frame?

$$\frac{v_{\text{red}}}{c} =$$

$$\frac{v_{\text{blue}}}{c} =$$

- b) Calculate v/c for the red particle as measured in the laboratory frame. How much is v/c for the blue particle in this frame?

$$\frac{v_{\text{red}}}{c} = \underline{\hspace{2cm}}$$

$$\frac{v_{\text{blue}}}{c} = \underline{\hspace{2cm}}$$

- c) In the laboratory frame, the kinetic energy of the incoming particle (red) on the target (blue) is called the threshold kinetic energy to produce this reaction. How much is the kinetic energy of the red particle (the threshold energy) in the laboratory frame?

Answer: $E_{k, \text{red}} = \underline{\hspace{2cm}}$

About how many proton rest energies, mc^2 , is this kinetic energy equal to?

Answer: $E_{k, \text{red}} \approx$ _____

18. Problem 3.7:

How much is the frequency (in inverse years) of Pink's pulses in Pink's reference frame?

Answer: _____

How much is the frequency in Green's frame when Green is travelling away from Pink?

Answer: _____

How much is the frequency in Green's frame when Green is travelling toward Pink?

Answer: _____

19. Problem 3.8: Observe the two pairs of spectral lines from the car at rest (your car) and from the moving car. Left click on each line to read the corresponding wavelength. Calculate the average wavelength for each pair of spectral lines and record below.

For the car at rest: $\lambda_{\text{avg}} =$ _____ (nm)

For the moving car: $\lambda_{\text{avg}} =$ _____ (nm)

Is the car moving toward or away from you? Explain how you know this.

Answer: _____

Calculate the speed of the moving car and record below. Show your calculations.

Answer: _____