Evaluate the following integrals:

1. $\int \arcsin x \, dx$

$$\begin{bmatrix} u = \arcsin x & du = \frac{1}{\sqrt{1-x^2}} dx \\ dv = dx & v = x \end{bmatrix}$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$= x \arcsin x + \sqrt{1-x^2} + C$$

for $\int \frac{x}{\sqrt{1-x^2}} dx$ Let $w = 1-x^2$
$dw = -2x \, dx$
$$= -\frac{1}{2} \int w^{-1/2} dw = -(1-x^2)^{1/2}$$

2. $\int e^x \sin x \, dx$

$$\begin{bmatrix} u = e^x & du = e^x dx \\ dv = \sin x \, dx & v = -\cos x \end{bmatrix}$$

$$= -e^x \cos x + \int e^x \cos x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x$$

$$\Rightarrow \int e^x \sin x = \frac{1}{2}\left[-e^x \cos x + e^x \sin x\right] + C$$

3. $\int \tan x \sec^3 x \, dx$

Let $u = \sec x \Rightarrow du = \tan x \sec x \, dx$

$$\Rightarrow \int u^2 du = \frac{1}{3} \sec^3 x + C$$

4. $\int \sqrt{1-9x^2} \, dx$

$3x = \sin\theta \Rightarrow \sqrt{1-9x^2} = \cos\theta$
$3dx = \cos\theta \, d\theta \Rightarrow \frac{1}{3} \int \cos^2\theta \, d\theta$

$$= \frac{1}{3} \int \frac{1+\cos 2\theta}{2} = \frac{1}{6}\left[\theta + \frac{\sin 2\theta}{2}\right] = \frac{1}{6}\left[\theta + \sin\theta\cos\theta\right]$$

$$= \frac{1}{6}\left[\arcsin(3x) + 3x \cdot \sqrt{1-9x^2}\right] + C$$