

Answers:

Bryan Gunter

9/14

e 1. (e) second order nonlinear

D 2. the value^{of} for which $y = e^{mx}$ is a solution of $y'' - 5y' + 6y = 0$
 $y = e^{mx}$, $y' = me^{mx}$, $y'' = m^2 e^{mx}$

$$y'' - 5y' + 6y = 0, \quad m^2 e^{mx} - 5(me^{mx}) + 6(e^{mx}) \stackrel{?}{=} 0$$

$$0 = (m^2 - 5m + 6)e^{mx}, \quad (m-3)(m-2)e^{mx} = 0$$

$$m = 2, 3$$

A 3. Values of m for which $y = x^m$ is a solution of $x^2 y'' - 5xy' + 8y = 0$

$$y = x^m, \quad y' = mx^{m-1}, \quad y'' = (m-1)m \cdot x^{m-2} = (m^2 - m)x^{m-2}$$

$$x^2 y'' - 5xy' + 8y = 0$$

$$x^2((m^2 - m)x^{m-2}) - 5x(mx^{m-1}) + 8(x^m) = 0$$

$$(m^2 - 6m + 8)x^m = 0, \quad (m-4)(m-2)x^m = 0$$

$$m = 2, 4 \quad \text{if } x \neq 0$$

C 4. The values of C for which $y = C$ is a constant solution of

$$y' = y^2 + 3y - 4$$

$$y' = C^2 + 3C - 4, \quad (C-1)(C+4) = 0$$

$$C = 1, -4$$

e 5. The values of m for which $y = e^{mx}$ is a solution of $y'' - 4y' - 5y = 0$

$$y = e^{mx}, \quad y' = me^{mx}, \quad y'' = m^2 e^{mx}$$

$$m^2 e^{mx} - 4me^{mx} - 5e^{mx} = 0, \quad (m^2 - 4m - 5)e^{mx} = 0$$

$$(m-5)(m+1)e^{mx} = 0, \quad m = 5, -1$$

A 6. $\frac{dP}{dt} = kP$, $P(0) = 1000$ A 7. $y = Ce^{3x}$ is solution to IVP $y' = 3y$, $y(0) = 2$ where $C = ?$

$$y' = 3Ce^{3x} = 3y, \quad C(3e^0) = 6, \quad 3C = 6, \quad C = 2$$

Continued ↓

B 8. $y' = 2y + x$, $x(1) = \frac{1}{4}$, Solution: $y = -\frac{x}{2} - \frac{1}{4} + Ce^{2x}$ where $C = ?$
 $y' = 2Ce^{2x} - \frac{1}{2}$, $2Ce^{2x} - \frac{1}{2} = 2y + x$, $C(2e^2) - \frac{1}{2} = \frac{1}{2} + 1$
 $C \cdot 2e^2 = 2$, $C \cdot e^2 = 1$, $C = \frac{1}{e^2}$

C 9. $\frac{dT}{dt} = k(T - 70)$

B 10. $\frac{dA}{dt} = 8 - \frac{1}{100}A$, $R_{in} = 2 \cdot 4 = 8$, $R_{out} = \frac{A(t)}{100} \cdot 4$