Lab 1. Measuring the Speed of Light

Purpose: - to become familiar with digital oscilloscopes
- to measure the speed of light in air and water and find the index of refraction of water

Equipment: 2-channel digital oscilloscope
Speed of Light Apparatus (base plate; emitting & receiving units; tube cell & stands; 2 lenses on magnetic holders; retroreflector)
laser distance meter
connecting wires
2 BNC cables
paper towels

Theory

Digital Oscilloscopes

Oscilloscopes are used to visualize time dependent waveforms, measure frequencies, and detect phase shifts between signals. Since they have a variety of controls and options, oscilloscopes may seem intimidating at first.

We will be using a two channel Tektronix digital oscilloscope. After powering on, the oscilloscope will run some internal tests. The screen will eventually display a bunch of data. We will disregard most of the information and focus on the readouts corresponding to the vertical scale factors of channel 1 (**CH1**) and channel 2 (**CH2**) as well as the main time base setting (M – followed by a time in seconds or fraction of seconds). The most relevant oscilloscope controls:

Vertical Controls

Position knob: positions a waveform vertically
1 & **2 Menu** button: displays the **Vertical** menu selections and toggles the display of the channel waveform on and off
Scale knob: selects vertical scale factors (volts/division)

Horizontal controls

Position knob: adjusts the horizontal position of all channel waveforms
Horiz. Menu button: displays the Horizontal Menu
Set to Zero button: sets the horizontal position to zero
Scale knob: selects the horizontal sec/div (scale) for the main window time base.

Menu and Control Buttons

Acquire: Displays the **Acquire Menu**
Display: Displays the **Display Menu**
AutoSet: Automatically sets the controls to make a usable display of the input signals

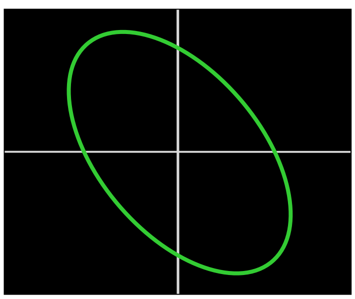
Many times, **Autoset** will be a lifesaver. Each time you push the **Autoset** button, the **Autoset** function obtains a stable waveform display for you. It automatically adjusts the vertical scale, horizontal scale and trigger settings. **Autoset** also displays several automatic measurements in the graticule area, depending on the signal type. (Please Note: If the oscilloscope seems to be scaling by a factor of 10 (e.g., it displays 0.1 V when it should be displaying 1 V) make sure that the switch <u>on the BNC cable</u> is set to 1x, and not to 10x.)

<u>Phase Relationships of Signals</u>

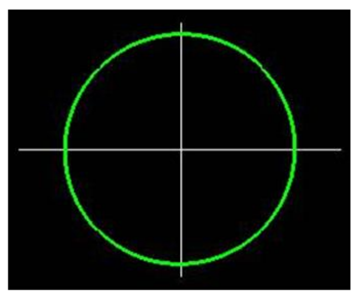
A phase difference between two signals implies that the signals do not reach their peak values at the same time. Phase measurements can be done using Lissajous figures. A Lissajous figure is obtained when two sine waves are displayed at right angles to each other.

The XY display on an oscilloscope can be used to display the variation of one signal with respect to a second signal, rather than with respect to time. One signal is applied to the horizontal (X) input, while the other signal is applied to the vertical (Y) input. The phase difference can be observed on the oscilloscope by pressing the **Display** button to see the **Display Menu**, then pushing **Format → XY**. The oscilloscope will display a Lissajous figure representing the phase difference between the two signals. You can use the vertical **Scale** knob to optimize the display.

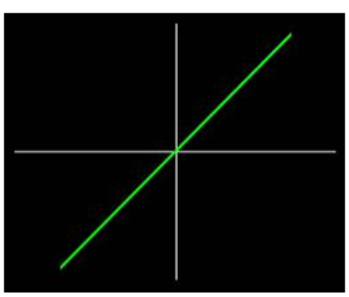
Fig. 1 illustrates Lissajous figures produced by the superposition of two sine waves of equal frequency and various phase differences and amplitude ratios.



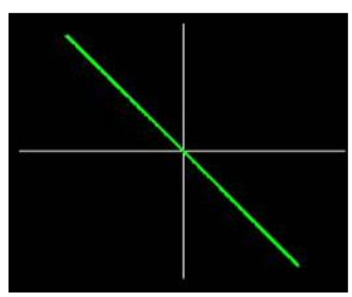
out-of-phase, different amplitudes



90^o out-of-phase, equal amplitudes



in-phase (0^o)



180^o out-of-phase

Figure 1

Measuring the Speed of Light

The speed of light measuring apparatus is shown in Fig. 2. A light-emitting diode is used as the light source. The diode is fed with a 50 MHz alternating voltage so that the emitted light is periodically intensity modulated. After travelling a certain distance, the light is directed by a retroreflector mirror toward the receiving photodiode that generates an alternating voltage of the same frequency. The generated voltage is phase-shifted with respect to the original signal. The phase shift depends on the length of the light path. If the length of a light path is known for which the phase shift between the emitted and received signal is 0^o, and the length of a second light path for which the phase shift is 180^o (= π) can be determined, then the speed of light can be calculated from the total difference in the lengths of the light paths, D.

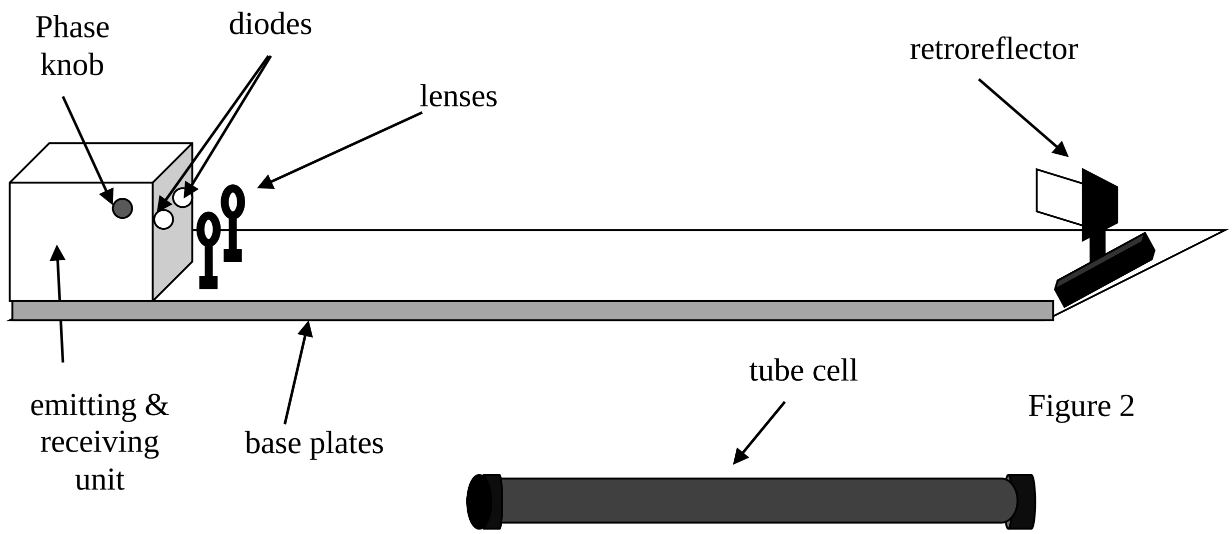


Figure 2

Light travels length D in a half-period, T/2, since the phase shift is π. Since T = 1/f, the time of travel is 1/(2f). In this experiment f = 50 MHz. Since the light beam is reflected back by the retroreflector, the light path must be doubled. Therefore, D = 2·[path travelled by the mirror]. Then, the speed of light is:

$$c = \frac{D}{\left({}^{1}/_{2f}\right)} = 2f \cdot D \qquad \text{Eq. 1}$$

Procedure

Part I - Setup

1. Assemble the base plates. Take care that the base is flat and that there are no gaps at the junctions of the three parts of the base plate.
2. Place the emitting and receiving unit at the very end of the base plate which is not scaled (truly at the very end and not a few centimeters away from the end!).
3. Place one of the lenses 2-3 cm in front of the emitter and the other lens about the same distance in front of the receiver, so that the plane lens sides face toward the respective diodes.
4. Place the retroreflector mirror at the outer end of the base plate. The mirror has been

correctly adjusted in the factory. The screws on the back of it should not be touched, except if the mirrors are obviously misaligned.

5. Switch on the emitting and receiving unit. With the room lights dimmed, follow the path of light from the emitter and through the first lens. Adjust the vertical and horizontal positions of the lens so that the light falls on the corresponding mirror. You can check this by holding a sheet of paper in front of the mirror. Then, follow the path of light from the second mirror through the second lens and finally to the receiver.
6. Using the BNC cables, connect the X and Y outputs of the emitting and receiving unit with the corresponding inputs of the oscilloscope.
7. Turn on the oscilloscope and set it to the Y-T mode. Set the vertical scale for each channel such that the signals occupy 4-5 vertical squares. Note that the frequency of the output signal is one thousandth of the true modulation frequency of the light source.

<u>Part II - Measuring the Speed of Light in Air</u>

8. Position the <u>notch</u> on the side of the retroreflector’s stand at 155 cm.
9. Set the oscilloscope display to the XY mode. Adjust the X and Y sensitivities on the oscilloscope so that the ellipse occupies about half the oscilloscope screen.

   <u>Troubleshooting</u>: If you cannot observe an ellipse, carefully check the alignment of every component (the base plates, lenses, retroreflector, & emitting/receiving unit). Usually, one or both of the lenses is a culprit. Try rotating the lens or adjusting it vertically or horizontally until you obtain an ellipse.
10. Using the knob labeled "Phase" <u>on the emitting and receiving unit</u> (not on the oscilloscope!), change the ellipse to an inclined straight line (or as close as possible to a line). This phase adjustment must remain unchanged during the measurements and it indicates that the X and Y inputs are in phase.

    <u>Troubleshooting</u>: If you have trouble deciding when the ellipse has become a straight line when moving the retroreflector, you can first bring the Lissajous figure to the middle of the oscilloscope screen and then increase the sensitivity of the oscilloscope until only the magnified middle of the screen is seen.
11. Slide the retroreflector towards the emitting and receiving unit. The straight line on the oscilloscope screen will open up to an ellipse. The main axis of the ellipse will change its slope, until a straight line is again formed. This straight line signifies that the phase difference between the signals changed by 180^o (see Fig. 1). You should verify the 180^o (π) phase shift by returning to the Y-T mode.
12. Calculate distance D from the length traveled by the retroreflector. Compute the speed of light in air using Eq. 1. Record in Table 1.
13. Repeat the necessary steps above so that three new measurements of the speed of light are obtained. Calculate the average speed and standard deviation. Record below Table 1.

<u>Part III - The Speed of Light and Index of Refraction of Water</u>

14. Carefully remove one of the end windows from the tube cell and measure its thickness. Measure the total length of the tube cell. Subtract from it the double thickness of the end window, to obtain the length of the water column, L_w. Record above Table 2.

15. Make sure that one of the end windows of the tube is tightly screwed on and that the rubber o-ring is properly placed. Otherwise, the water will leak. Completely fill the tube with water and tightly screw on the top window. Make sure that no air bubbles are trapped in the tube. If there is any leakage, readjust the o-ring position. You should have several paper towels handy, just in case of any leaks.
16. Mount the filled tube on its supports, so that the light beam passes through the cell, either on the emitter or the receiver side. Position the one end of the tube near the 0 cm mark. Start the measurements with the retroreflector close to the end of the tube that is facing away from the diodes.
17. Using the "Phase" knob on the emitting and receiving unit, change the ellipse to an inclined straight line. Leave this phase adjustment unchanged during the rest of the measurements.
18. Remove the tube cell. Slide the retroreflector away from the emitting and receiving unit until a straight line is again formed. Calculate the length of the path travelled by the mirror and record in Table 2.
19. Repeat the speed of light in water measurements three more times.
20. It can be shown that the speed of light in water, c_w, , the speed of light in air, c_{air}, length D, and the length of the water column, L_w, are related by the following formula:

$$\frac{c_{air}}{c_w} = \frac{D}{L_w} + 1 \qquad \text{Eq. 2}$$

    Use Eq. 2 and your measurements of c_{air}, D, and L_w to calculate the speed of light in water, c_w. Record in Table 2.
21. Calculate the average c_w and standard deviation. Record below Table 2.
22. Calculate the index of refraction of water, n_w, using your average values of c_{air} and c_w. Record below Table 2.

Part IV – Measuring the Speed of Light in Water Using a Laser Distance Meter

Laser distance meters work in a similar way to the method you have used: they compare a reference beam with a reflected beam and internally calculate the phase shift. The speed of light in vacuum is internally stored. The output is a distance to the object reflecting the light beam. The meter is calibrated to measure distances in air or vacuum.

To obtain the refractive indices of liquids using this apparatus, the length of a tube cell is first measured without liquid inside. The measurement is repeated after water fills the tube. Given that light slows through media such as water, and that the laser distance meter uses the speed of light in vacuum as its reference for calculating distance, the meter is “tricked” into measuring a longer apparent length of the tube.

We can derive the relationships between indices of refraction and the apparent distances reported by the laser meter. Let d_{air} be the distance measured with air in the tube and d_{liquid} the reported distance with a liquid inside. These distances are obtained from internally measured times t_{air} and t_{liquid}, respectively. Thus, $d_{air} = c \cdot t_{air}$ and $d_{liquid} = c \cdot t_{liquid}$. We use the speed of light in vacuum, c, in both formulas, because this is what the laser meter "thinks" it is measuring. Since light travels at a lower speed, v, in the liquid, t_{liquid} is longer than t_{air}. We have:

$$t_{liquid} = \frac{d_{liquid}}{c} \qquad \text{(Eq. 3)}$$

However, d_{air} is the true distance of the tube. When the tube is filled with a liquid, light would travel this distance at speed v in time t_{liquid}. Therefore, we can also write:

$$t_{liquid} = \frac{d_{air}}{v} \qquad \text{(Eq. 4)}$$

Combining Eq. 3 and Eq. 4, we get:

$$\frac{d_{air}}{v} = \frac{d_{liquid}}{c}$$

We then rearrange to get:

$$\frac{c}{v} = \frac{d_{liquid}}{d_{air}}$$

Since the index of refraction of the liquid is given by: $n_{liquid} = \frac{c}{v}$, we can write:

$$n_{liquid} = \frac{d_{liquid}}{d_{air}}$$

To determine the index of reflection of water, follow this procedure:

23. Switch on the laser meter and make sure it displays the appropriate units. The laser meter is actually measuring distances from its back end. Therefore, you need to "zero" it by pressing the gray button beneath the on/off switch. Once you do this, the distances will be measured from the front end. You need to re-zero if the laser meter switches off.
24. While the tube is still filled with water, position its far end on a reflecting surface (the floor, for example). Use the laser meter to measure the effective length of the water column. Make sure you subtract the thicknesses of the tube's windows and mounts from your measurements. Repeat 4 times and record d_{water} in Table 3.
25. Empty the tube and find its effective length. Repeat 4 times and record d_{air} in Table 3.
26. Calculate the index of refraction for each trial. Record in Table 3. Then calculate the average index of refraction of water, $\langle n_{water} \rangle$ and the standard deviation.
27. Using $\langle n_{water} \rangle$ and $c = 2.99792458 \cdot 10^8$ m/s find the speed of light in water, v.

Record your ***Results*** on the following two pages, which you may include in your Lab Notebook.

Results

Part II

Table 1 - Speed of Light in Air

Trial #	Path travelled by the mirror (m)	D (m)	c_{air} (m/s) $\cdot 10^8$
1			
2			
3			
4			

<c_{air} > = ____________________ $\sigma_{air} = \sqrt{\frac{\sum_{i=1}^{4}(\langle C_a \rangle - c_{a,i})^2}{3}} =$ ____________________

Final answer: $c_{air\_}$ =_______________________ (m/s)

Part III

L_w = ________________(m)

Table 2 - Speed of Light in Water

Trial #	Path travelled by the mirror (m)	D (m)	c_w (m/s) $\cdot 10^8$
1			
2			
3			
4			

<c_w > = _____________ (m/s) $\sigma_w = \sqrt{\frac{\sum_{i=1}^{4}(\langle C_w \rangle - c_{w,i})^2}{3}} =$ _____________

<n_w > = ____________________

Part IV

Table 3 – Index of Refraction Using the Laser Distance Meter

Trial #	d_{air} (m)	d_{water} (m)	n_{water}
1			
2			
3			
4			

$\langle n_{water} \rangle =$ ___________

The speed of light in water: v = __________________(m/s)

$$\sigma_n = \sqrt{\frac{\sum_{i=1}^{4}(\langle n_w\rangle - n_{w,i})^2}{3}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Final answer for the index of refraction: n_{water} = ____________ ± ____________

How do these values for n and v compare to those obtained in Part III?

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Sources of Error and Conclusions:

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