

Name: Solutions

MAT 128

Quiz 2

Evaluate the following integrals:

1. $\int \frac{x^2+2x-1}{x^3-x} dx$, $\frac{x^2+2x-1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$$\Rightarrow x^2+2x-1 = A(x^2-1) + Bx(x+1) + Cx(x-1)$$

$$x^2+2x-1 = (A+B+C)x^2 + (B-C)x - A$$

$$\Rightarrow \boxed{A=1} \quad B+C=0 \Rightarrow B=-C \Rightarrow 2B=2 \Rightarrow \boxed{B=1}, \boxed{C=-1}$$

$$\Rightarrow \int \frac{1}{x} dx + \int \frac{1}{x-1} dx - \int \frac{1}{x+1} dx = \ln|x| + \ln\left|\frac{x-1}{x+1}\right| + C$$

2. $\int \frac{10}{(x-1)(x^2+9)} dx = 10 \int \frac{1}{(x-1)(x^2+9)} dx =$

$$\Rightarrow \left[\frac{A}{x-1} + \frac{Bx+C}{x^2+9} \right] \Rightarrow A(x^2+9) + (Bx+C)(x-1) = 10$$

$$\Rightarrow 10 = (A+B)x^2 + (C-B)x - C + 9A$$

$$\Rightarrow A=-B=-C \Rightarrow A=1, B=C=-1 \Rightarrow$$

$$\Rightarrow \left[\int \frac{1}{x-1} dx - \int \frac{x+1}{x^2+9} dx \right] = \ln|x-1| - \frac{1}{2} \ln(x^2+9) - \frac{1}{3} \arctan\left(\frac{x}{3}\right)$$

3. $\int \frac{x^3-2x^2-4}{x^3-2x^2} dx$

$$\frac{x^3-2x^2-4}{x^3-2x^2} = 1 - \frac{4}{x^2(x-2)} \Rightarrow \frac{4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$\Rightarrow 4 = Ax(x-2) + B(x-2) + Cx^2$$

$$\text{If } x=0 \Rightarrow \boxed{B=-2}, \text{ If } x=2 \Rightarrow \boxed{C=1}$$

$$\text{If } x=1 \Rightarrow \boxed{A=-1}$$

$$\Rightarrow \int = \int 1 - \frac{1}{x} - \frac{2}{x^2} + \frac{1}{x-2} dx$$

$$= x - \left[-\ln|x| + \frac{2}{x} + \ln|x-2| \right] + C$$