Integrating Factor Examples

p. 56 #16 :

$$y\,dx = (ye^y - 2x)\,dy$$

Solve in terms of dx/dy !

$$\frac{dx}{dy} = e^y - \frac{2x}{y}$$

$$\frac{dx}{dy} + \frac{2x}{y} = e^y$$

$$P(y) = \frac{2}{y}, \quad f(y) = e^y$$

Integrating factor is :

$$e^{\int P(y)dy} = e^{2\int \frac{dy}{y}} = e^{2\ln y} = [e^{\ln y}]^2 = y^2$$

$$\frac{d}{dy}\left[e^{\int P(y)dy}x\right] = e^{\int P(y)dy}f(y)$$

$$\frac{d}{dy}(xy^2) = y^2e^y$$

$$xy^2 = \int y^2e^y\,dy = y^2e^y - ye^y - ye^y + e^y + e^y + c$$

$$\boxed{x = e^y - \frac{2}{y}e^y + \frac{2}{y^2}e^y + \frac{c}{y^2}}$$

Interval: $0 < x < \infty$

Transient term is c/y^2

p. 56 # 26

$$y\frac{dx}{dy} - x = 2y^2, \quad y(1) = 5 \Big\}$$ Initial value Problem

$$\frac{dx}{dy} - \frac{x}{y} = 2y$$

$$P(y) = -\frac{1}{y}, \quad f(y) = 2y$$

$$\int P(y)\,dy = -\ln y, \quad e^{-\ln y} = \left[e^{\ln y}\right]^{-1} = \frac{1}{y}$$

$$\frac{d}{dy}\left[e^{\int P(y)dy} \cdot x\right] = e^{\int P(y)dy} f(y)$$

$$\frac{d}{dy}\left[\frac{1}{y} \cdot x\right] = \frac{2y}{y} = 2$$

$$\frac{x}{y} = \int 2\,dy = 2y + c$$

$$\boxed{x = 2y^2 + cy}$$ explicit solution for x

$$y(1) = 5, \quad (1, 5)$$

$$1 = 2(25) + c(5), \quad c = -\frac{49}{5}$$

$$\therefore \boxed{x = 2y^2 - \frac{49}{5}y}$$