12/7

Problem 1 [3 pts]

Consider three normally distributed random variables $A \sim N(15, 2)$, $B \sim N(10, 3)$, and $C \sim N(3, 1)$ in the format $N(\mu, \sigma^2)$. Let $D = A - B + 2C$.

(a) What is the distribution of D?

$N(15,2) - N(10,3) + 2(N(3,1)) = N(1 \cdot \mu_A - 1 \cdot \mu_B + 2\mu_C,\ 1^2 \cdot \sigma_A^2 - 1^2 \cdot \sigma_B^2 + 2^2 \sigma_C^2)$

$= N(15 - 10 + 6,\ 2 - 3 + 2^2 \cdot 1) = N(11, 3)$

(b) What is $P(7 < D < 10)$?

$Z_7 = \frac{7-11}{\sqrt{3}} = -2.3094$

$Z_{10} = \frac{10-11}{\sqrt{3}}$

-0.57735

$P(Z < -0.57) = 0.2843$

$P(Z < -2.3094) = 0.0107$

$P(-2.3 < Z < -0.57) = P(Z < -0.57) - P(Z < -2.3) = 0.2736$

(c) For which interval $(\mu - d, \mu + d)$ around the mean is $P(\mu - d < D < \mu + d) = 0.80$?

$\mu = 11$, $P(11 - d < D < 11 + d)$ $\Rightarrow$ look for z scores w $\frac{100-80}{2}$ % of area

10% above & below

$Z \simeq \pm 1.28$

$\therefore P(-1.28 < Z < 1.28) = 0.8$

Problem 2 [3.5 pts]

Each bit sent over a noisy communication channel has a chance of being received incorrectly. Suppose we send 50 bits over a channel, and find that 10 of them are improperly received. We want to model the outcome of sending another n bits.

(a) What is the estimated probability $\hat{p}$ of a bit being received incorrectly on this channel? What is the uncertainty in this estimate?

$\hat{p} = \frac{10}{50} = 0.2$ $\quad\quad$ $\sigma_{\hat{p}} = \sqrt{\frac{.2(1-.2)}{50}} = 0.056569$

(b) Use $\hat{p}$ from (a) to express the number of incorrect bits out of n as a Binomial random variable X.

$\hat{p} - \frac{X}{n}$ $\quad\quad$ $X = n\hat{p} = .2n$

$p=0.2$

(c) If $n = 5$, find the probability mass function of X.

$$P_X(x) = P(X=x) = \begin{cases} \frac{5!}{x!(5-x)!}(0.2)^x(0.8)^{5-x} & x=0,1,2,\ldots \\ 0 & \text{otherwise} \end{cases}$$

(d) If $n = 5$, what is the probability that at least 2 bits are received incorrectly?

$P(x \geq 2) = 1 - (P(1) - P(0))$

$P(1) = 0.4096$

$P(0) = 0.32768$

$P(x \geq 2) = 1 - 0.4096 - 0.32768$

$= 0.26272$

Problem 3 [3.5 pts]

The datapoints 20, 22, 23, 25, 27 are drawn from a population. We want to construct a confidence interval for the population mean.

(a) Do we need to know anything more about the population? Why or why not?

We need to know if the population is normal because $n = 5 < 30$

(b) Find a 95% two-sided confidence interval for μ.

$$\overset{23.4}{\bar{X}} \pm 1.96 \frac{\overset{2.70185}{s}}{\sqrt{5}}$$

25.7683

21.0265

(21.0265, 25.7683)

(c) Find a 95% lower confidence interval for μ.

$(\bar{X} - z_\alpha \sigma_{\bar{X}}, +\infty) \Rightarrow (21.4123, \infty)$

(d) Roughly how confident can we be that μ lies in (21.2, 25.6)?

$$21.2 = 23.4 + z_{\alpha/2} \cdot \frac{2.702}{\sqrt{5}}$$

$$z_{\alpha/2} = -1.82063$$

$$\Rightarrow 0.0344 = \alpha/2$$

$$\alpha = 0.0688$$

6.9 % confident