Name: solutions

MAT 128
Quiz 3

1. Determine whether the improper integral converges or diverges

$$\int_{-\infty}^{\infty} xe^{-x^2}\,dx = \lim_{a\to-\infty}\int_a^0 xe^{-x^2}dx + \lim_{b\to\infty}\int_0^b xe^{-x^2}dx$$

$$= \lim_{a\to-\infty} -\frac{1}{2}\left[1-\frac{1}{e^{a^2}}\right] + \lim_{b\to\infty}\frac{-1}{2}\left[\frac{1}{e^{b^2}}-1\right] = -\frac{1}{2}+\frac{1}{2} = 0$$

Converges

2. Determine whether the improper integral converges or diverges

$$\int_e^{\infty}\frac{1}{x(\ln x)^3}dx = \lim_{b\to\infty}\int_e^b \frac{1}{u^3}du$$

where $u=\ln x$, $du=\frac{1}{x}dx$

$$= \lim_{b\to\infty} -\frac{1}{2}\frac{1}{(\ln x)^2}\Big]_e^b = \lim_{b\to\infty}\frac{-1}{2}\left[\frac{1}{(\ln b)^2}-1\right] = \frac{1}{2}$$

converges

3. Write the Sum to approximate $\int_2^3 \frac{1}{x}dx$ using the Trapezoidal Rule with n = 10

$$\Delta x = \frac{3-2}{10} = 0.1$$

$$\text{Sum App.} = \frac{0.1}{2}\left[\frac{1}{2}+\frac{2}{2.1}+\frac{2}{2.2}+\frac{2}{2.3}+\frac{2}{2.4}+\frac{2}{2.5}+\frac{2}{2.6}+\frac{2}{2.7}+\frac{2}{2.8}+\frac{2}{2.9}+\frac{1}{3}\right]$$