Equivalent Mathematical Models

Consider the ODE:

$$\frac{d^2y}{dx^2} + C_1 \frac{dy}{dx} + C_2 = F(t)$$

This is a non-homogeneous, linear second order ODE

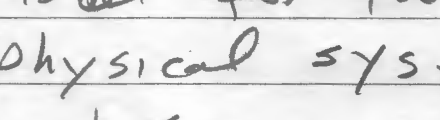
Now consider this ODE as a mathematical Model for two very famous physical systems:

1. Damped-Forced Spring system
2. LRC electric circuit

Damped-Forced Spring System:

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t)$$

OR $$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = F(t)$$



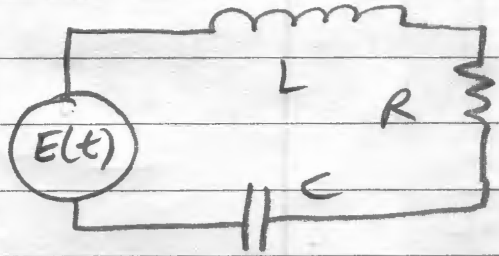
Where

$m \rightarrow$ mass (kg)

$c \rightarrow$ damping constant

$k \rightarrow$ spring constant (N/m^2)

LRC Electric Circuit:

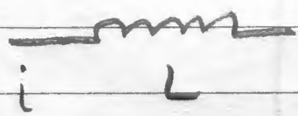


$L \rightarrow$ Inductor
$R \rightarrow$ Resistor
$C \rightarrow$ Capacitor

From Kirchhaff's Laws
(ie Conservation of Energy):

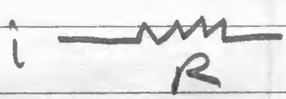
$$E(t) = V_L(t) + V_R(t) + V_C(t)$$

Inductor:



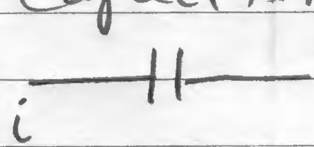
$$V_L(t) = L\frac{di}{dt} = L\frac{d^2q}{dt^2}$$

Resistor:



$$V_R(t) = iR = R\frac{dq}{dt}$$

Capacitor:



$$V_C(t) = \frac{1}{C}q$$

L (inductance in Henrys)
R (Resistance in Ohms)
C (Capacitance in Farads)

In terms of charge, the voltage equation becomes

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = E(t)$$

OR

$$\ddot{q} + \frac{R}{L} \dot{q} + \frac{1}{LC} q = E(t)$$

Terms	Spring	Circuit
Forcing	$F(t)$	$E(t)$
First Derivative	c/m	R/L
dependent	k/m	$1/LC$

There are three different solutions to these two ODEs depending on the values of the constants. Only one of the solutions is oscillatory!