1.

First find $\vec{B}\times\vec{C}$:

$$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 4 & 1 & -6 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 1 & 0 \\ 4 & 1 \end{matrix}$$

$$= \hat{i}\left[(0)(-6) - (-3)(1)\right] + \hat{j}\left[(-3)(4) - (-6)(1)\right]$$

$$+ \hat{k}\left[(1)(1) - (0)(4)\right]$$

$$= 3\hat{i} - 6\hat{j} + \hat{k}$$

So $\vec{A}\cdot(\vec{B}\times\vec{C}) = (2\hat{i} + 5\hat{j})\cdot(3\hat{i} - 6\hat{j} + \hat{k})$

$$= (2)(3) + (5)(-6) + (0)(1)$$

$$= -24$$

Then $5\vec{A}\cdot(2\vec{B}\times 3\vec{C}) = (5)(2)(3)\,\vec{A}\cdot(\vec{B}\times\vec{C})$

$$= -720$$

2.

$$\vec{r}(t) = \left[(-5t)\hat{i} + (3t^2 + 2t - 7)\hat{j}\right]\text{m}$$

a) $\vec{v}(t) = \left[-5\hat{i} + (6t + 2)\hat{j}\right]$ m/s

$$\vec{v}(2) = \boxed{(-5\hat{i} + 14\hat{j})\ \text{m/s}}$$

b) $\boxed{\vec{a}(t) = 6\hat{j} \ m/s^2 = \vec{a}(2)}$

c) $\vec{v}_{avg} = \dfrac{\vec{r}(2) - \vec{r}(1)}{(2-1)s} = \dfrac{[(-10\hat{i} + 9\hat{j}) - (-5\hat{i} - 2\hat{j})] \ m}{1 \ s}$

$\boxed{= (-5\hat{i} + 11\hat{j}) \ m/s}$

d) $v(2) = \sqrt{5^2 + 14^2} = \boxed{15 \ m/s}$

3.

$\theta = 60°$

$V_0 = 120 \ m/s$

a) $V_y = V_{oy} - gt = 0$

$t = \dfrac{V_{oy}}{g} = \dfrac{V_0 \sin\theta}{g} = \boxed{10.6 \ s}$

b) $V_y = \pm V_x = V_{ox} = V_0 \cos 60° = 60 \ m/s$

$\pm V_{ox} = V_{oy} - gt \qquad V_{oy} > 0$

$t = \dfrac{V_{oy} \mp V_{ox}}{g} = \dfrac{V_0 (\sin\theta \mp \cos\theta)}{g}$

$\boxed{= 4.5 \ s, \ 16.7 \ s}$

4.

(a)

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$y - 1000\,m = 5\,m/s\,(2\,s) - \frac{1}{2}(9.8\,m/s^2)(2\,s)^2$$

$$\boxed{y = 990\,m}$$

(b)

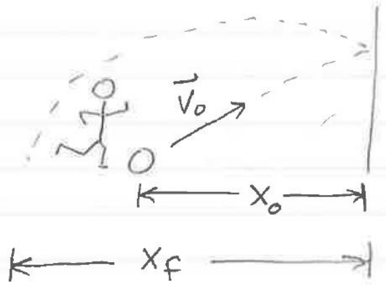
$\uparrow$ 500N

$\downarrow$ $mg = 49N$

$$500N - mg = ma$$

$$\boxed{a = \frac{500N}{m} - g = -0.709\,m/s^2}$$

5.



$$\vec{V}_0 = (10\hat{i} + 10\hat{j})\,m/s$$

$$x_0 = 4.0\,m$$

If we knew the velocity and position of the ball when it hits the wall, then we can calculate x_f.

How high does it travel before hitting the wall?

$$y - y_0 = V_{oy} t - \frac{1}{2} g t^2$$

where $t = \dfrac{x_0}{V_{0x}} = \dfrac{4.0\,m}{10\,m/s} = .40\,s.$

$$y = (10\,m/s)(.40\,s) - \frac{1}{2} g (.40\,s)^2$$

$$= 3.2\,m$$

$$V_y = V_{oy} - g t = 10\,m/s - (9.8\,m/s^2)(.40\,s) = 6.1\,m/s$$

So the ball hits the wall at a height of 3.2 m above the ground.

Since only its horizontal velocity changes direction, it leaves the wall with a velocity of:

$$\vec{v} = (-10\hat{i} + 6.1\hat{j})\,m/s.$$

How far will it travel now?

$$x_f = V_{ox} t'$$

What is t'? The time it takes to fall back to the ground:

$$y - y_0 = V_{oy} t' - \frac{1}{2} g t'^2$$

$$0 - 3.2\,m = (6.1\,m/s) t' - \frac{1}{2} g t'^2$$

$$t' = \frac{-6.1 \pm \sqrt{(6.1)^2 + 2g(3.2)}}{(-g)}\,s = -.40\,s,\ 1.64\,s$$

$x_f = (-10\,m/s)(1.64\,s) = 16\,m$

Thus the ball lands $\boxed{12\,m \text{ behind the boy.}}$

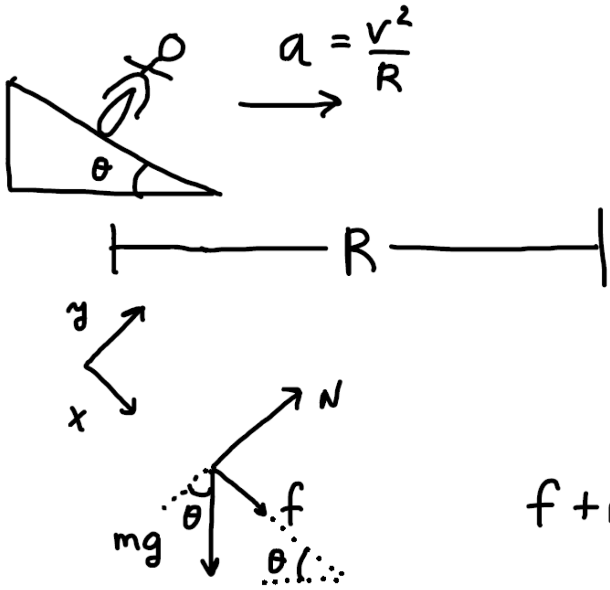
A quick way to do this problem is to see that since only the direction of v_x changed, the horizontal distance traveled by the ball is the same as if there were no wall there. Thus, $R = v_{0x} t$

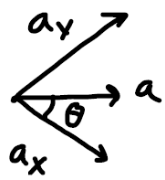
$y - y_0 = v_{0y}t - \frac{1}{2} g t^2 = 0$

$t = 2.04\,s$

$R = 20.4\,m \Rightarrow R = x_0 + x_f \Rightarrow \boxed{x_f = 16\,m}$

6.





$$\sum F_x = ma_x$$

$$f + mg\sin\theta = m\frac{v^2}{R}\cos\theta$$

$$f = \frac{mv^2}{R}\cos\theta - mg\sin\theta$$

$$\sum F_y = ma_y$$

$$N - mg\cos\theta = \frac{mv^2}{R}\sin\theta \Rightarrow N = \frac{mv^2}{R}\sin\theta + mg\cos\theta$$

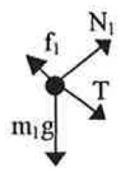
$$\Rightarrow \quad a) \quad \boxed{N = m\left(\frac{v^2}{R}\sin\theta + g\cos\theta\right)}$$

$$b) \quad \boxed{f = m\left(\frac{v^2}{R}\cos\theta - g\sin\theta\right)}$$

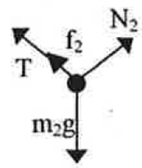
7.

4) Define motion down the plane as positive. Draw free body diagrams for each mass

For mass 1



For mass 2



Summing the forces for each of the masses perpendicular to the plane gives

$$\sum F_{1,\perp} = N_1 - m_1 g\cos\theta = m_1 a_{1,\perp}$$
$$\sum F_{2,\perp} = N_2 - m_2 g\cos\theta = m_2 a_{1,\perp}$$

Since there is no motion perpendicular to the plane we have

$$N_1 - m_1 g\cos\theta = 0$$
$$N_1 = m_1 g\cos\theta$$

Summing the forces parallel to the plane gives

$$\sum F_{1,\parallel} = T + m_1 g\sin\theta - f_1 = m_1 a_{1,\parallel}$$
$$\sum F_{2,\parallel} = -T + m_2 g\sin\theta - f_2 = m_2 a_{2,\parallel}$$

where

$$f_1 = \mu_1 N = \mu_1 m_1 g\cos\theta \quad = 24.0\,N$$
$$f_2 = \mu_2 N = \mu_2 m_2 g\cos\theta \quad \approx 6.00\,N$$

Since the blocks are tied together they are accelerating at the same rate so

$$a_{1,\parallel} = a_{2,\parallel} = a$$

Plugging this equation back in to the force equations and adding the two equations gives

$$g(m_1 + m_2)\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta = a(m_1 + m_2)$$

Solving for a gives

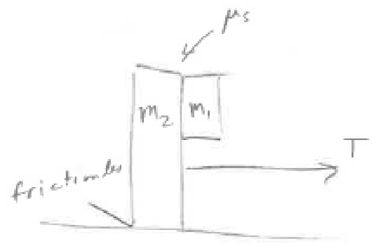
$$T = m_1\left[a + g\left(\mu_1\cos\theta - \sin\theta\right)\right]$$
$$\boxed{= 4.03\,N}$$

$$a = g\sin\theta - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2} g\cos\theta$$

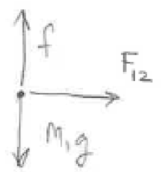
$$\boxed{a = \left(9.8\,m/s^2\right)\sin 40^\circ - \frac{(0.20)(16.0\,kg) + (0.10)(8.0\,kg)}{16.0\,kg + 8.0\,kg}\left(9.8\,m/s^2\right)\cos 40^\circ = 5.05\,m/s^2}$$

8.

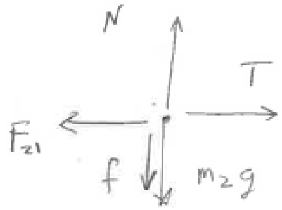
a)



m_1 :



m_2 :



$$F_{net,x} = F_{12} = m_1 a$$

$$F_{net,y} = f - m_1 g = 0$$

$$f = f_s = \mu_s F_{12}$$

$$F_{net,x} = T - F_{21} = m_2 a$$

$$F_{net,y} = N - f - m_2 g = 0$$

$$\mu_s F_{12} = m_1 g$$

$$\Rightarrow \mu_s \cancel{m_1} a = \cancel{m_1} g$$

$$\boxed{a = \frac{g}{\mu_s}}$$

$$F_{12} = F_{21}$$

$$T = F_{21} + m_2 a$$

$$= m_1 a + m_2 a$$

$$\boxed{= (m_1 + m_2)\, g / \mu_s}$$

9.

2. Since the block is sitting on a turntable the block is moving in circular motion. We know that in order for an object to maintain circular motion there must be a force to supply the block with the centripetal acceleration. In this case that force is friction. The maximum force friction can supply is given by

$$f_{Max} = \mu N$$

Where N is the normal force and μ is the coefficient of static friction. Since the block is resting on a horizontal plane the normal force must be equal to the weight of the object (otherwise it would be accelerating in the vertical direction)

$$N = mg.$$

Where m is the mass of the object and g is the acceleration due to gravity. The force required to provide centripetal motion is

$$F_C = \frac{mv^2}{R},$$

where R is the radius of the circular path and v is the velocity of the particle. As soon as the velocity increases to a point where the force required to sustain circular motion exceeds the maximum force fiction can exert, the block will begin to move. This gives the relation

$$F_{C\,Max} = \frac{mv_{Maz}^2}{R} = f_{Max} = \mu m g$$

Solving for v_{Max} yields

$$v_{Max} = \sqrt{\mu g R}$$

In the problem, we are asked to find the frequency at which this occurs. So, we need to relate the velocity of the block to the frequency that the turntable is spinning. When the block goes around the turntable once it travels the circumference of the circle on which it is sitting. Thus the total distance the particle travels in one period is 2πR. Where the period is the time it takes to make one revolution. The speed of the particle is then given by

$$v = \frac{2\pi R}{T},$$

where T is the period. The frequency of the turntable is the number of revolutions that occur in a given time which is simply the inverse of the period

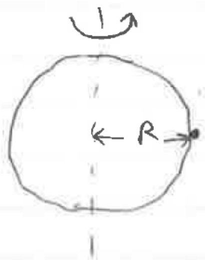
$$freq = \frac{1}{T},$$

where freq is the frequency. Thus we can relate the velocity of the particle to the frequency of the turntable

10.

$R = 6.37 \times 10^6 \, m$

$T = 24 \, h$



a) $a_c = \dfrac{v^2}{R} = \dfrac{(2\pi R/T)^2}{R}$

$= \dfrac{4\pi^2 R}{T^2}$

$\boxed{= .03 \, m/s^2}$

b) $g = \dfrac{4\pi^2 R}{T^2} \Rightarrow T = 2\pi\sqrt{\dfrac{R}{g}}$

$\boxed{= 1.4 \, h}$