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11/7

Eng 272 Matlab 4 Hand Solutions

- Section 4.1:

#15 Use the definition of a laplace transform to find the laplace transform $f(t) = e^{-t} \cdot \sin(t)$; $F(s) = \int_0^\infty e^{-st} \cdot f(t)\,dt$

* $\mathcal{L}\{f(t) = e^{-t} \cdot \sin(t)\} \Rightarrow F(s) = \int_0^\infty e^{-st} \cdot e^{-t} \cdot \sin(t)\,dt$

$F(s) = \int_0^\infty e^{t(-s-1)} \cdot \sin(t)\,dt$

using Table of laplace transforms #18

#18: $\int e^{at} \cdot \sin(kt) = \dfrac{k}{(s-a)^2+k^2}$

$\therefore \int e^{ax} \cdot \sin(bx)\,dx = \dfrac{e^{ax}}{a^2+b^2}\left[a\sin(bx) - b\cos(bx)\right]$

$$L\{f(t)\} = \left[\frac{e^{-(s+1)t}}{(s+1)^2+(1)^2} \cdot -\left((s+1)\cdot\sin(t) - (1)\cos(t)\right)\right]_0^\infty$$

$$= \frac{1}{(s+1)^2+(1)^2} = \frac{1}{s^2+2s+2}$$

#19 $\mathcal{L}\{2t^4\} = ?$

from table of laplace transforms #3 $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$

$$\mathcal{L}\{2t^4\} = 2 \cdot \mathcal{L}\{t^4\} = 2 \cdot \frac{4!}{s^{4+1}} = \frac{48}{s^5}$$

#25. $\mathcal{L}\{(T+1)^3\} = ?$

$(T+1)^3 = T^3 + 3T^2 + 3T + 1$

use $\mathcal{L}\{T^n\} = \dfrac{n!}{s^{n+1}}$

$$\mathcal{L}\{T^3 + 3T^2 + 3T + 1\} = \frac{3!}{s^4} + 3 \cdot \frac{2!}{s^3} + 3 \cdot \frac{1}{s^2} + \frac{1}{s}$$

$$= \frac{6}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s}$$

Section 4.2:

#9. find $\mathcal{L}^{-1}\left\{\frac{1}{4s+1}\right\} = ?$

$\mathcal{L}^{-1}\left\{\frac{1}{4s+1}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{4}}\right\}$

from table $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$ in this case $a = \frac{1}{4}$

$\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{4}}\right\} = e^{-\frac{1}{4}t}$ $\therefore \mathcal{L}^{-1}\left\{\frac{1}{4s+1}\right\} = \frac{1}{4}\cdot e^{-\frac{1}{4}t}$

#15 $\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = ?$

$\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = \mathcal{L}^{-1}\left\{\frac{2s}{s^2+9}\right\} - \mathcal{L}^{-1}\left\{\frac{6}{s^2+9}\right\} = 2\cdot\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - 6\cdot\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}$

for $\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}$ we use #8 in table: $\mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos(kt)$

in this case $k = 3$ so $\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} = \cos(3t)$

for $\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}$ we use #9 in table: $\mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} = \sin(kt)$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2+3^2}\right\} = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$ $\therefore \mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\} = \frac{1}{3}\cdot\sin(3t)$

Sub back in: $\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = 2\cdot\cos(3t) - 6\cdot\frac{1}{3}\sin(3t) = 2\cos(3t) - 2\sin(3t)$

#27 $\mathcal{L}^{-1}\left\{\frac{2s-4}{(s^2+s)(s^2+1)}\right\} = ?$

$$\frac{2s-4}{(s^2+s)(s^2+1)} = \frac{2s-4}{s(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{Cs+D}{(s^2+1)}$$

$A\cdot(s+1)(s^2+1) + B(s)(s^2+1) + ((Cs+D)(s)(s+1)) = 2s-4$

set $s=0$: $-4 = A(0+1)(0^2+1)$, $A=-4$

Set $s=-1$: $-6 = B(-1)(1+1)$; $-2B = -6$, $B=3$

$\therefore C = 1$

$\therefore \mathcal{L}^{-1}\left\{\frac{2s-4}{s(s+1)(s^2+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{-4}{s} + \frac{3}{s-1} + \frac{s+3}{s^2+1}\right\}$

$\mathcal{L}^{-1}\left\{\frac{2s-4}{s(s+1)(s^2+1)}\right\} = 4\cdot\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$, $\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$ use $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$ $\therefore a=-1$ $\therefore \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}$

$\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$ use $\mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos(kt)$ here $k=1$ $\therefore \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} = \cos(t)$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$ use $\mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} = \sin(kt)$ $\therefore \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin(t)$

Sub: $\mathcal{L}^{-1}\left\{\frac{2s-4}{s(s+1)(s^2+1)}\right\} = -4 + 3e^{-t} + \cos(t) + 3\sin(t)$