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2.3.3

a.) find a general solution to first order ode:

$\frac{dY}{dt} - y = e^{2t}$, $y' - y = e^{2t}$ linear

Complement:

$y' - y = 0$ $\frac{dy}{dt} = y$, $\frac{dy}{y} = dt$

$\int \frac{dy}{y} = \int dt$, $\ln(y) = t + c$, $y = e^{t} \cdot c$,

Paticular: $y' - y = e^{2t}$ $P(x) = -1$ $e^{\int P(x)dx} = e^{\int -1 dt} = e^{-t}$

$y' \cdot e^{-t} - y \cdot e^{-t} = e^{2t} \cdot e^{-t} = e^{t}$

$y' e^{-t} - y e^{-t} = e^{t}$

$\frac{d}{dx}(e^{-t} \cdot y) = e^{t}$, $\int \frac{d}{dt}(e^{-t} \cdot y) = \int e^{t} dt$

$y e^{-t} = e^{t} + c$

$y = \frac{e^{t+c}}{e^{-t}} = e^{2t} \cdot C_2$

$y = e^{t} \cdot C_1 + e^{2t} \cdot C_2$

b. find general solution to 2nd order ODE

$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 16y = 0$

$y'' + 8y' + 16y = 0$, $m^2 + 8m + 16 = 0$

$a = 1$ $b = 8$ $c = 16$

$m = \underset{\alpha}{-3} \pm i\underset{\beta}{\sqrt{7}}$

$y = e^{-3}(C_1 \cdot \cos(\sqrt{7}x) + C_2 \cdot \sin(\sqrt{7}x))$

C. find paticular solution to a given $y(0) = 5$

$5 = e^{0} \cdot C_1 + e^{0} + C_2$, $C_1 + 1 + C_2 = 5$

$y = (4 - C_2)e^{t} + C_2 \cdot e^{2t}$ $C_1 + C_2 = 4$

$= 4e^{t} - C_2 e^{t} + C_2 \cdot e^{2t}$ $C_1 = 4 - C_2$

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2.2 #51 : a.) find an implicit solution of IVP
$(2y+2)dy - (4x^3+6x)dx = 0$ $y(0) = -3$
b.) use part a. to find an explicit solution $y = \phi x$ of the IVP
c.) consider part b's answer as a function only, graph this function and use the graph to estimate domains.

$$(2y+2)dy - (4x^3+6x)dx = 0$$

$\frac{\partial}{\partial x}(2y+2) = 0$ $\frac{\partial}{\partial y}(4x^3+6x) = 0$

$y = -\sqrt{-x^4-3x^2+1}$

$y = \sqrt{-x^4-3x^2+1}$

$$\int(2y+2)dy = y^2+2y+C$$

$$-\int(4x^3+6x) = -x^4+-3x^2+C$$

implicit $f = -x^4+-3x^2+y^2+2y+C = 0$

$C = -x^4-3x^2+y^2+2y$ if $x=0$ & $y=-3$

$$C = 0+0+9-6 = 3$$

$$\therefore -x^4-3x^2+y^2+2y = 3$$

$$y^2+2y = x^4+3x^2+3$$

- Take $\frac{1}{2}$ the coefficient of y and square it, then add it to both sides.

$$y^2+2y+1 = +x^4+3x^2+4$$

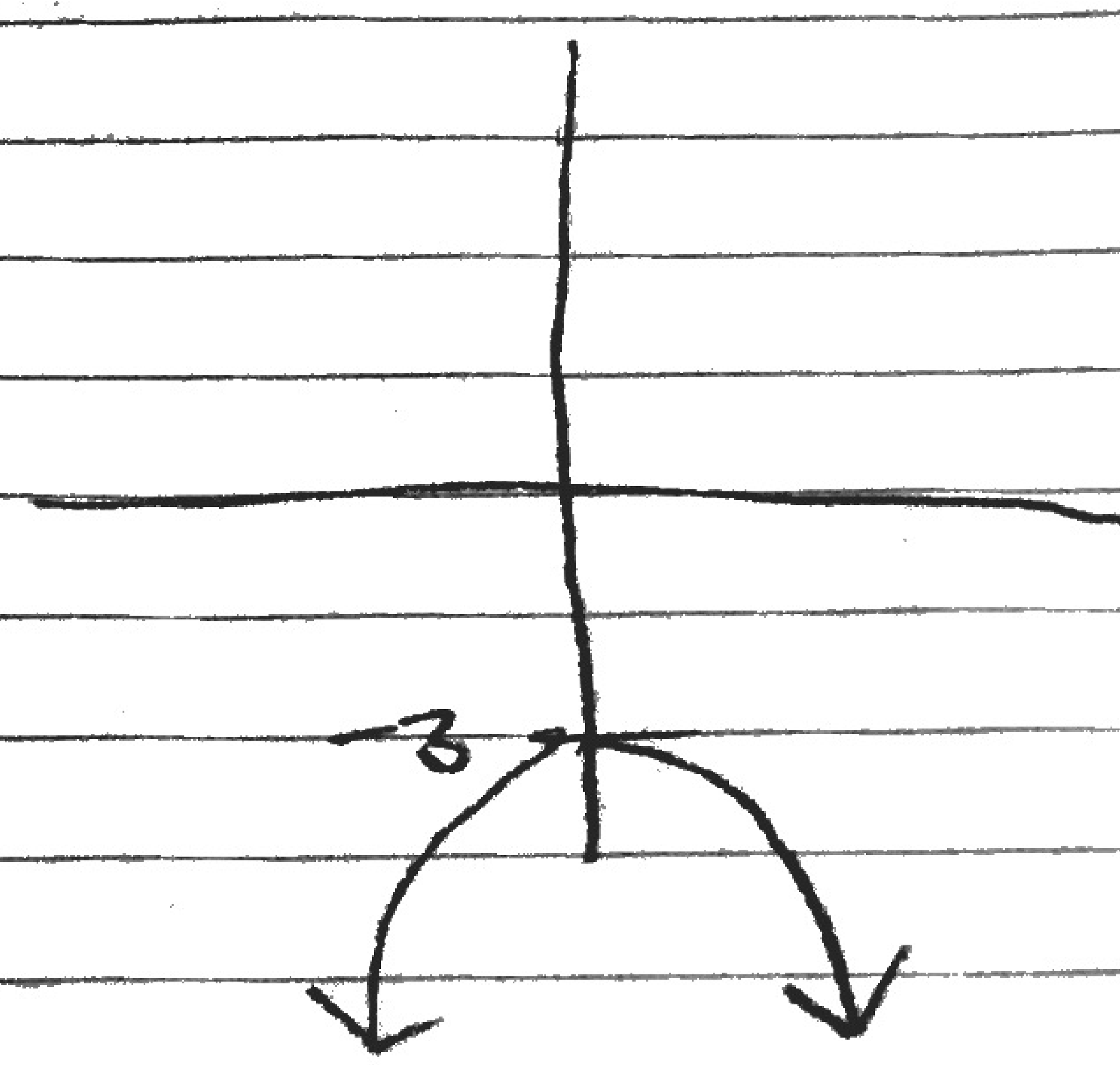
$$(y+1)^2 = +x^4+3x^2+4$$

explicit
$$y = -\sqrt{+x^4+3x^2+4} - 1$$
$$y = \sqrt{+x^4+3x^2+4} - 1$$

C.

$y+1 = x^4+3x^2+4$

domain $= \mathbb{R}$



domain $= \mathbb{R}$

2.6 Euler's method to obtain 4 decimal approximation

$h = 0.5$

$y_{n+1} = y_n + h \cdot f(x_n, y_n)$

$x_{n+1} = x_n + h$

#5. $y' = e^{-y}$, $y(0) = 0$; $y(0.5)$

x_n	y_n
0	0
0.5	0.5

$y_{n+1} = 0 + .5 \cdot e^{-0}$

#7. $y' = (x-y)^2$, $y(0) = .5$, $y(0.5)$

x	y
0	.5
0.5	.625

$y_{n+1} = .5 + .5 \cdot (0 - 0.5)^2$

$= .5 + 0.125$

$= .625$

#9. $y' = xy^2 - \frac{y}{x}$ $\quad y(1) = 1$, $y(1.5)$

x	y
1	1
1.5	1

$y_{n+1} = 1 + .5(1 - 1) = 1$