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#2

a.) the expression $X_t[n]$ in terms of $X[n]$ is

$$X_t[n] = A \cdot X[an + n_0] + B$$

let $an + n_0 = k$, $an = k - n_0$

$n = \frac{k - n_0}{a}$

$$X_t\left[\frac{k-n_0}{a}\right] = A \cdot X[k] + B$$

$$A \cdot X[k] = X_t\left[\frac{k-n_0}{a}\right] - B$$

$$X[k] = \frac{X_t\left[\frac{k-n_0}{a}\right] - B}{A}$$

Replace the variable k with n (time invari

$$X[n] = \frac{X_t\left[\frac{n-n_0}{a}\right] - B}{A}$$

b. suppose that for a signal of

$$X_1[n] = 0.5 X_3[-n+1] + 2$$

$A = 0.5$
$B = 2$
$a = -1$
$n_0 = 1$

$$X[n] = \frac{X_t\left[\frac{n-n_0}{a}\right] - B}{A}$$

$$X_3[n] = 2 X_1[-n+1] - 4$$

Continued on back

↓

$X_3[n] = 2X_1[-n+1] - 4$

$X_1[n]$ Values ...

n	$X_1[n]$
-3	0
-2	0
-1	2
0	0
1	-2
2	-2
3	0

n	$X_3[n]$
-3	invalid
-2	$2 \cdot X_1[3] - 4 = -4$
-1	$2 \cdot X_1[2] - 4 = -8$
0	$2 \cdot X_1[1] - 4 = -8$
1	$2 \cdot X_1[0] - 4 = -4$
2	$2 \cdot X_1[-1] - 4 = 0$
3	$2 \cdot X_1[-2] - 4 = -4$
4	$2 \cdot X_1[-3] - 4 = -4$

```
n = [-2 -1 0 1 2 3 4]
X3 = [-4 -8 -8 -4 0 -4 -4]
Plot (n, X3, 'linewidth', 1.5)
Xlabel ('n')
ylabel ('X3')
title ('2.b')
axis([-3 5 -9 1]);
grid;
```

$-x[-n] = x[n]$

Bryan Guner 2/12

#3. For each of the signals

1. $X[n] = 2\,u[n]$ (step function) and
2. $X[n] = \cos[0.1n]$

a.) Determine mathematically whether the signal is even, odd or neither
b) Find the even part & the odd part of each signal
c) Use matlab to plot the signals

3.1

$X[n] = 2u[n]$ → even $X[n] = -X[-n]$ → odd

$X[n] = x[-n]$

$X[-n] = 2u[-n] \neq X[n]$ ∴ not even

$-X[-n] = -2u[-n] \neq X[n]$

$X_{even} = \frac{1}{2}[x[n] + X[-n]]$
$= \frac{1}{2}[2u[n] + 2u[-n]]$
$= u[n] + u[-n]$

$X_{odd} = \frac{1}{2}[x[n] - x[-n]]$
$= \frac{1}{2}[2u[n] - 2u[-n]]$
$= u[n] - u[-n]$

$X = u[n] + u[-n] + u[n] - u[-n]$
$X[n] = 2u[n]$

3.2

$X[n] = \cos[0.1n]$

$X[n] = X[-n]$ → even

$X[-n] = \cos[-.1n] = \cos[0.1n] = x[n]$ ✓

$X_{even} = \frac{1}{2}[X[n] + X[-n]] = \frac{1}{2}[\cos[.1n] + \cos[-.1n]]$
$= \cos[.1n]$

Bryan Guner

#4. For each of the signals

1. $x(t) = -4t$
2. $x(t) = -u(t-1) + u(-t-1)$

a.) Determine mathematically whether the signal is even, odd or neither and

b.) Find the even part & odd part of the Signal

C.), Use matlab to plot the signals / their even & odd parts.

4.1

$X(t) = -4t$ odd

$X(t) = -X(-t)$

$-X(-t) = -(-4(-t)) = -4t$ odd

$X_{odd} = \frac{1}{2}(X(t) - X(-t)) = \frac{1}{2}(-4t - 4t) = -4t$ ✓

4.2 $X(t) = -u(t-1) + u(-t-1)$

Check even

$X(-t) = X(t)$

$X(-t) = -u(-t-1) + u(t-1) \neq X(t)$

Check odd

$X(t) = -X(-t)$

$-X(-t) = -[-u(-t-1) + u(t-1)]$

$-X(-t) = u(-t-1) - u(t-1)$

$-X(-t) = -u(t-1) + u(-t-1) = X(t)$ is odd ✓

$X_{even} = 0$

Bryan Guner 2/13

#5. Consider the signal $X(t) = 5\sin(15t - \pi/3) + 2\sin(7t)$

a.) determine if the signal is perodic, if so, find it's fundamental period T_0

b.) use matlab to plot the signal.

a.) for $5\sin(15t - \pi/3)$; $T_{0_1} = \frac{2\pi}{15}$

for $2\sin(7t)$; $T_{0_2} = \frac{2\pi}{7}$

for $X(t) = 5\sin(15t - \pi/3) + 2\sin(15t - \pi/3)$

$(T_{0_1} / T_{0_2}) = \frac{2\pi}{15} \div \frac{2\pi}{7} = \frac{7}{15}$ = ratio of integers, $\therefore$ signal is perodic

Fundamental period of $X(t) = \frac{\text{LCM (of numerators)}}{\text{HCF (of denominators)}}$

$= \frac{2\pi}{1} = 2\pi$

#6. Consider the following continuous - time signal
$x(t) = \cos(\pi t)$.
Sampled sequence is
$x[n] = x(nT) = \cos(\pi n T)$
let N_0 be the fundamental period
let k = # of periods of $x(t)$ in one period of $x[n]$

1 * Sampling period: $T = 0.125$ sec
$x[n] = \cos(0.125 \pi n)$ sub $n + N_0$ for n
$x[n + N_0] = \cos(0.125\pi(n + N_0)) = \cos(0.125\pi n + 0.125\pi N_0)$

Criteria: signal to be periodic: $x[n + N_0] = x[n]$
$0.125 \pi N_0 = 2\pi k$ let $k=1$
$N_0 = 8$ → the sampled signal is periodic

2 * Sampling period: $T = 0.13$ s
$x[n] = \cos(0.13\pi n)$ sub $n + N_0$ for n
$x[n + N_0] = \cos(0.13\pi(n + N_0)) = \cos(0.13\pi n + 0.13\pi N_0)$
for signal to be periodic $0.13\pi N_0 = 2\pi k$
$N_0 = (100/13)k = 7.692k$ → let $k = 13$
$N_0 = 100$ periodic

a) # of periods of $x(t)$ in one period of $x[n]$ is k

1. 1
2. 13

# of samples in one per $x[n] = N_0$

1. 8
2. 100

Bryan Guner 2/13

#7. Prove the time-scaling property of the dirac delta function:

$$\int_{-\infty}^{\infty} \delta(at-t_0)\,dt = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta\left(t-\frac{t_0}{a}\right)dt$$

hence evaluate the following integral

$$\int_{-\infty}^{\infty} \sin(t-\pi/6)\cdot\delta\left(2t-\frac{2\pi}{3}\right)dt$$

Proof

(Property) $\delta(an) = \frac{1}{a}\,\delta(n)$

$$\int_{-\infty}^{\infty} \delta(at-t_0)\,dt = \int_{-\infty}^{\infty} \delta\left(a\left(t-\frac{t_0}{a}\right)\right)dt$$

Change of Variables: let $t-\frac{t_0}{a} = u$, $dt = du$ (because $\frac{t_0}{a}$ is a constant)

$$\int_{-\infty}^{\infty} \delta(au)\,du = \frac{1}{a}\int_{-\infty}^{\infty} \delta(u)\,du = \frac{1}{a}\int_{-\infty}^{\infty}\delta\left(t-\frac{t_0}{a}\right)dt$$

Solution

$$\int_{-\infty}^{\infty} \sin\left(t-\frac{\pi}{6}\right)\cdot\delta\left(2t-\frac{2\pi}{3}\right)dt = \int_{-\infty}^{\infty} \sin\left(\frac{2\pi}{3}-\frac{\pi}{6}\right)\cdot\delta\left(2t-\frac{2\pi}{3}\right)$$

$$= \sin\left(\frac{\pi}{2}\right)\int_{-\infty}^{\infty} \delta\left(2t-\frac{2\pi}{3}\right)d\left(2t-\frac{2\pi}{3}\right) = 1\cdot 1 = 1$$