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Tutoring Notes

10/15

#7. $y' + y = xy^5$, $y' + y = xy^5$

$u = y^{-4} = y^{-4}$

$\frac{dy}{dx} = -\frac{1}{4} u^{-5/4} \frac{du}{dx}$

$u^{5/4} \left[-\frac{1}{4} u^{-5/4} \frac{du}{dx} + u^{-1/4} \right] = xu^{-5/4} \Rightarrow -\frac{1}{4} \frac{du}{dx} + u = x$

$\frac{du}{dx} - 4u = -4x$

$P(x) = -4$

$e^{\int -4 dx} = e^{-4x} = e^{-4x}$

$e^{-4x} \frac{du}{dx} - 4ue^{-4x} = -4xe^{-4x}$

$\frac{d}{dx}(ue^{-4x}) = -4xe^{-4x}$

$\int \frac{d}{dx}(ue^{-4x}) = \int -4xe^{-4x}$

$ue^{-4x} = -4 \int xe^{-4x} dx$

$u = x \quad dv = e^{-4x} dx, \quad du = dx, \quad v = -\frac{1}{4}e^{-4x}$

$ue^{-4x} = -4 \left[-\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x} dx \right] = xe^{-4x} + \frac{1}{4}e^{-4x} + C$

$ue^{-4x} = xe^{-4x} + \frac{1}{4}e^{-4x} + C, \quad u = x + \frac{1}{4} + Ce^{4x}$

$y^{-4} = x + \frac{1}{4} + Ce^{4x}, \quad y = (x + \frac{1}{4} + Ce^{4x})^{-1/4}$

• Undetermined Coefficients: ex.) $y'' - 10y' + 25y = 30x + 3$

$y = y_c + y_p$, for y_c : $y'' - 10y' + 25y = 0$

$m^2 - 10m + 25 = 0, (m-5)^2 = 0, m = 5 \text{ twice}$

$y_c = C_1 e^{5x} + C_2 x e^{5x}$

for y_p : guess: $y_p = Ax + B, y_p' = A, y_p'' = 0$

$0 - 10(A) + 25(Ax + B) = 30x + 3$

$25Ax + 25B - 10A = 30x + 3, 25A = 30, A = \frac{6}{5}, 25B - 10A = 3, B = \frac{3}{5}$

$y_p = \frac{6}{5}x + \frac{3}{5}$

Combined

$y = C_1 e^{5x} + C_2 x e^{5x} + 1.2x + 0.6$