Name: Solutions

MAT 128
Quiz 10

1. (a) Find the Cartesian coordinates of P if
P=$(r, \theta) = (2, \pi/3)$

$x = 2\cos\pi/3 = 2 \cdot \frac{1}{2} = 1$

$y = 2\sin\pi/3 = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

(b) Find the Polar coordinates of Q if
Q = $(x, y) = (-2, 2)$

$r = \sqrt{x^2+y^2} = \sqrt{4+4} = 2\sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{2}{-2} = -1 \Rightarrow \theta = \frac{3\pi}{4}$

2. Find the points on the curve $r = 3\cos\theta$ where the tangent is horizontal

$x = 3\cos^2\theta \Rightarrow \frac{dx}{d\theta} = -6\cos\theta\sin\theta$

$y = 3\cos\theta\sin\theta \Rightarrow \frac{dy}{d\theta} = -3\sin^2\theta + 3\cos^2\theta = 0$

$\Rightarrow \sin^2\theta = \cos^2\theta \Rightarrow \sin\theta = \pm\sin\theta$

$\Rightarrow \theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

3. Find the area of the region enclosed by $r = 1 + \cos\theta$, with $0 \le \theta \le \pi$

$A = \frac{1}{2}\int_0^\pi 1 + 2\cos\theta + \cos^2\theta \, d\theta =$

$= \frac{1}{2}\int_0^\pi 1 + 2\cos\theta + \frac{1+\cos2\theta}{2} \, d\theta =$

$= \frac{1}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{\sin2\theta}{4}\right]_0^\pi = \frac{3\pi}{4}$