Assignment 2: Linear Time-Invariant Systems*

Signals and Systems (ELC 321)
Department of Electrical and Computer Engineering
The College of New Jersey.

Instructions:

1. Some of the assignment questions are adapted from the Text (Signals, Systems, and Transforms, Fifth edition)
2. When using MATLAB to plot signals, scale your time axis such as to allow sufficient amount of the signal to be plotted. Use subplot to give 3 or 4 plots per page; label the axes of your plots accordingly e.g Time (sec) on the x-axis and x(t) on the y-axis; the title should be the problem number, for example 2a).
3. Type-set your solutions for the assignment using microsoft office or any other word processor and include your Matlab codes in an appendix section. Matlab generated figures must be included as part of the solution.
4. Upload your final solution to canvas using appropriate naming convention e.g.

   `Firstame_Lastname_Assignment_1`

5. **Due Date: February 29, 2016**.

Problem 1 (25 Marks)**.** *Consider the series RC circuit of Fig.1. The input is the applied voltage $v(t)$ and the output is the voltage $v_c(t)$ across the capacitor.*

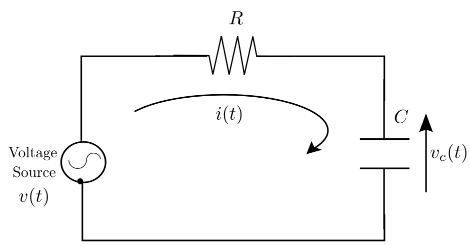

Figure 1: Sequence for Problem 1

a) Show that the differential equation describing the RC circuit is given by;

$$RC\frac{dv_c(t)}{dt} + v_c(t) = v(t). \tag{1}$$

*Dr. Ambrose A. Adegbege

b) Determine if the system is a causal linear time-invariant.

c) Suppose that $R = 0.5\Omega$ and $C = 0.25F$, determine an output expression for the voltage across the capacitor $v_c(t)$ for a unit step input ($v(t) = \mu(t)$) and the initial condition $v_c(0) = 0$.

□

Problem 2 (25 Marks)**.** *A system with impulse response $h(t)$, input $x(t)$ and output $y(t)$ can be modeled using the convolution integral*

$$y(t) = h(t) * x(t) = \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \tag{2}$$

Consider the system with the input-output relationship

$$y(t) = x(t-1) \tag{3}$$

a) Determine whether this system is 1) linear, 2) causal , and 3) time-invariant

b) Determine the impulse response $h(t)$ of the system.

c) Obtain the system response to the input $x(t) = e^{jt}$.

□

Problem 3 (25 Marks)**.** *Consider the simulation diagram of Fig. 2*

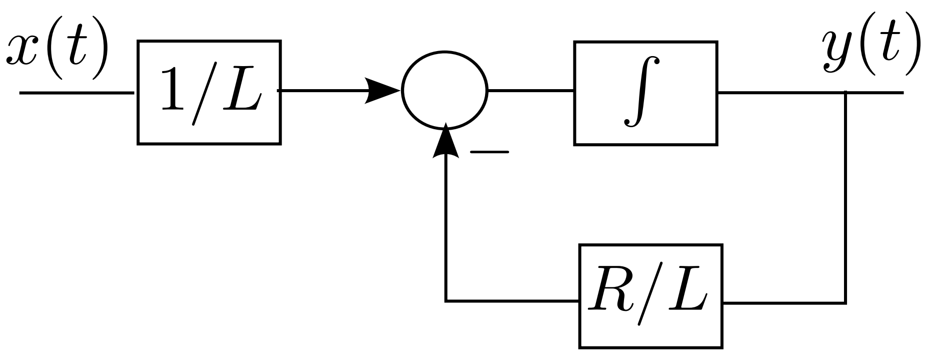

Figure 2: Simulation Diagram

a) Find the differential equation description of the system

The integrator in the simulation diagram of Fig. 2 can be implemented in analog computer by the operational amplifier circuit of Fig. 3

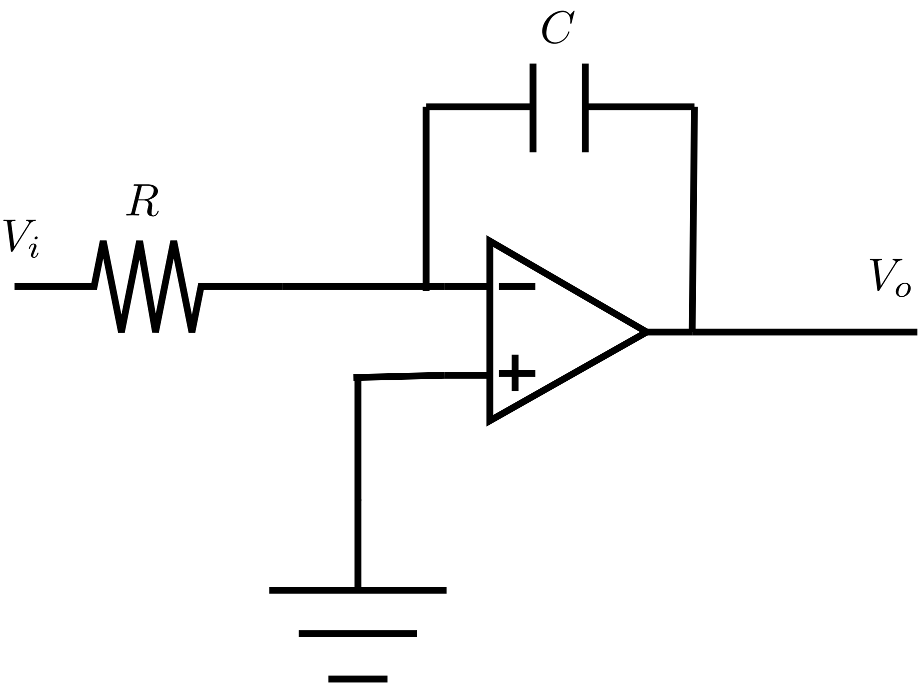

Figure 3: Operational Amplifier Circuit

b) Derive the differential equation model for the operational amplifier circuit

c) Modify the circuit such that it models an ideal integrator i.e

$$V_o(t) = \int_{\tau=-\infty}^{t} V_i d\tau \tag{4}$$

□

Problem 4 (25 Marks)**.** *Consider the system simulation diagram of Figure 4. This figure shows a simulation diagram form used in the area of automatic control*

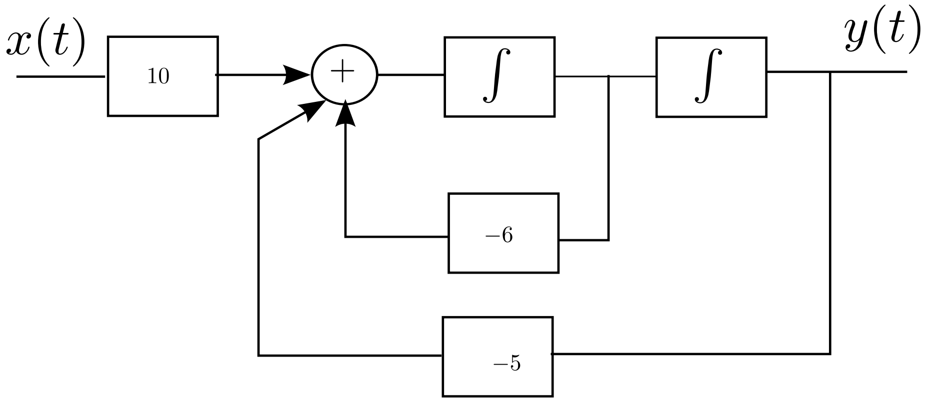

Figure 4: Simulation Diagram for Problem 1

a) Find the differential equation of the system.

b) Find the response $y(t)$ of the system if the input is a unit step i.e. $x(t) = u(t)$. Assuming the following initial condition $y(0) = -1$ and $\frac{dy(t)}{dt}|_{t=0} = 0$.

c) Use Matlab to plot the trajectories of $y(t)$, obtained in (b), *and $x(t)$ on the same figure.*

d) Use Matlab command "Dsolve" to solve the differential equation obtained in (a) *and compare the plot to that obtained in* (c) *above.(Hint: Use Matlab help command to explore the functionality of "dsolve" command.)*

e) Design an analog computer (using op-amps, resistors and capacitor) to solve the differential equation of (a)

□