Name: Solutions

MAT 128
Quiz 9

1. Calculate the Maclaurin series of $f(x) = \cos x$

$f(0)=1,\ f'(0)=0,\ f''(0)=-1,\ f'''(0)=0,\ f^{(4)}(0)=1$

$$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}-\cdots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{2n!}$$

2. Calculate the Taylor series of $f(x) = \ln x$ at $a = 2$

$f'=\frac{1}{x}$, $f^{(n)}=\frac{(n-1)!}{2^n}$

$$\Rightarrow \ln x=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\,2^n}(x-2)^n+\ln 2$$

3. Use the binomial series to expand $f(x)=\dfrac{1}{(1+x)^2}$ as power series.

$$(1+x)^{-2}=1-2x+\frac{-2,-3}{2!}x^2+\frac{-2,-3,-4}{3!}x^3+\cdots$$

$$=\sum_{n=0}^{\infty}\binom{-2}{n}x^n$$

4. Use series to evaluate the limit: $\lim\limits_{x\to 0}\dfrac{\cos x-1}{x^2}$

$$\frac{\cos x-1}{x^2}=\frac{-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}-\cdots}{x^2}$$

$$=-\frac{1}{2}+\frac{x^2}{4!}-\frac{x^4}{6!}-\cdots$$

$$\lim_{x\to 0}\frac{\cos x-1}{x^2}=-\frac{1}{2}$$