Technical breakdown of position based fluid

CHANGLIN SU* and QINGYUAN QIE*, University of Toronto, Canada

This paper will implement a parelle version of Position Based Fluids (PBF) method [Miles Macklin et al. 2013] for real-time fluid simluation and disscuss the effects of different confinement in the original paper. Compare to BPF, the widly used Lagrangian methods often secrafies the incompressablity of fluids for better performance, which cause the reuslt to be inaccurate compare to the behavior of real fluids. To addess this isssue, PBF method numerically integrate quatities from the neibouring particles with a smoothing kernel to calculate displacement while enforcing the incompressable contraint of fluid for the Navier-Strokes equations. This paper uses CUDA framework in our implementation to achieve real-time, accurate result.

CCS Concepts: • **Computer methodologies** → **Pysical simulation.**.

Additional Key Words and Phrases: Fluids, Physically Based Animation

ACM Reference Format:
Changlin Su and Qingyuan Qie. 2018. Technical breakdown of position based fluid. *ACM Trans. Graph.* 37, 4, Article 111 (August 2018), **??** pages. https://doi.org/10.1145/1122445.1122456

1 INTRODUCTION

Fluids such as water or gases, has played a important role in the field of computer based physical animations. Although fluid may look simple, but they are much more complicate to simulate than solids. The motion of fluids are governed by the Navier–Stokes equations. The solutions of the Navier–Stokes equations often include turbulence which makes finding a analytical solution exteremly hard in three dimensions. Although countless efforts are put into this field, the progress is still limited. Currently numerical simluation remains the most effective way to approximate a complex Navier–Stokes systems. In this report, we'll breakdown the Position based method and disscuss the significans and effects of all the constrant it proposed. In positioned based fluid, fluids are divided into particles and then compute the interactions between the fluid particles such as density and viscosity confinement, combine with external forces, we then update the velocities and positions of the fluid particle. To optimize for better performance, we divided the simulation spaces into grids so that particles only interact with other particles in the neibouring grid.

*Both authors contributed equally to this research.

Authors' address: Changlin Su, sheldon.su@mail.utoronto.ca; Qingyuan Qie, placeholder, University of Toronto, Toronto, Ontario, Canada.

0730-0301/2018/8-ART111 $15.00
https://doi.org/10.1145/1122445.1122456

2 RELATE WORK

According to simulation method, algorithm can generally be divide to two different catgories: Euler and Lagrangian methods. Euler methods split the space into grids and uses fintie differential mehtod to evualate the model. One of the example is Stam's stable fluid [Stam 1999]. The Lagrangian methods divide the fluid into free particles under constrains. One of the most known method of this kind is the SPH method [Smoothed Particle Hydrodynamics]. Both category has its own drawbacks: Euler methods easyily effected by numerical dissapation during simulation and Lagrangian methods needs smaller steps each iteration to ensure the stability of the algorithm. Current fluid simulation methods such as Position Based Fluid mehtods uses a hybrid of both method to achieve high level of accuracy while having reasonable performance.

3 PBF METHOD

For any Abitary fluid, its motion is governed by the Navier–Stokes equation:

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = \rho \boldsymbol{g} - \nabla p + \mu \nabla^2 \boldsymbol{v} \tag{1}$$

where ρ is the density of the fluid, μ is the viscosity constant of the fluid, p is the pressure and v denote the velocity. However, the above equation does not satisfy the incompressable property of fluids, so an additional constrains needs to be added:

$$\nabla \cdot \boldsymbol{v} = 0 \tag{2}$$

In PBF method, we represent the fluid with N particels, and their postions is denote as $\boldsymbol{P} = \{\boldsymbol{p}_1, \boldsymbol{p}_2, ..., \boldsymbol{p}_2\}$.

3.1 Density confinement

Since we are simulating a fluid, we have to satisfy the incompressable property of the fluid. Instead of statisfying equation (2) directly, BPF uses desity constraint to satisfy the reqirement:

$$C_i(\boldsymbol{p}_1, \boldsymbol{p}_2, ..., \boldsymbol{p}_2) = \frac{\rho_\text{i}}{\rho_0} - 1 \tag{3}$$

Where ρ_0 is the rest density of the fluid and ρ_i is given by the standard SPH estimator:

$$\rho_\text{i} = \sum_j m_j W(\boldsymbol{p}_i - \boldsymbol{p}_j, h) \tag{4}$$

where m_j is the mass of the neiboring particles and W is the poly6 smoothing kernel as gievn in [Müller et al. 2003]. In this paper, we assume all the particles have a mass of 1. Since a fluid is incompressable, intuitively this means that the density for any given part of the fluid should equal to the rest density of the fluid, which means the result of (3) should be equal to zero at all time:

$$C_i(\boldsymbol{p}_1, \boldsymbol{p}_2, ..., \boldsymbol{p}_2) = 0 \tag{5}$$

Furthurmore, when there is an displacement in the fluid particle, the density should also reamins the same, this property is statisfied through the following constraint:

$$C(\boldsymbol{p} + \Delta\boldsymbol{p}) = 0 \tag{6}$$

We can estimate a proper $d\Delta p$ equation (6) with Newton steps along the gradent of equation(5):

$$\Delta p = \nabla C(\boldsymbol{p})\lambda \tag{7}$$

And we can approximate the value of equation (6) with linear approximation:

$$C(\boldsymbol{p} + \Delta\boldsymbol{p}) \approx C(\boldsymbol{p}) + \nabla C^T \nabla C \lambda \tag{8}$$

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User-defined keywords are a comma-separated list of words and phrases of the authors' choosing, providing a more flexible way of describing the research being presented.

Table 1. Frequency of Special Characters

Non-English or Math	Frequency	Comments
Ø	1 in 1,000	For Swedish names
π	1 in 5	Common in math
$	4 in 5	Used in business
Ψ_1^2	1 in 40,000	Unexplained usage

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6 SECTIONING COMMANDS

Your work should use standard LaTeX sectioning commands: `section`, `subsection`, `subsubsection`, and `paragraph`. They should be numbered; do not remove the numbering from the commands.

Simulating a sectioning command by setting the first word or words of a paragraph in boldface or italicized text is **not allowed.**

7 TABLES

The "`acmart`" document class includes the "`booktabs`" package — https://ctan.org/pkg/booktabs — for preparing high-quality tables.

Table captions are placed *above* the table.

Because tables cannot be split across pages, the best placement for them is typically the top of the page nearest their initial cite. To ensure this proper "floating" placement of tables, use the environment **table** to enclose the table's contents and the table caption. The contents of the table itself must go in the **tabular** environment, to be aligned properly in rows and columns, with the desired horizontal and vertical rules. Again, detailed instructions on **tabular** material are found in the *LaTeX User's Guide*.

Immediately following this sentence is the point at which Table **??** is included in the input file; compare the placement of the table here with the table in the printed output of this document.

To set a wider table, which takes up the whole width of the page's live area, use the environment **table*** to enclose the table's contents and the table caption. As with a single-column table, this wide table will "float" to a location deemed more desirable. Immediately following this sentence is the point at which Table **??** is included in the input file; again, it is instructive to compare the placement of the table here with the table in the printed output of this document.

Always use midrule to separate table header rows from data rows, and use it only for this purpose. This enables assistive technologies to recognise table headers and support their users in navigating tables more easily.

Table 2. Some Typical Commands

Command	A Number	Comments
`\author`	100	Author
`\table`	300	For tables
`\table*`	400	For wider tables

8 MATH EQUATIONS

You may want to display math equations in three distinct styles: inline, numbered or non-numbered display. Each of the three are discussed in the next sections.

8.1 Inline (In-text) Equations

A formula that appears in the running text is called an inline or in-text formula. It is produced by the **math** environment, which can be invoked with the usual `\begin . . . \end` construction or with the short form `$ . . . $`. You can use any of the symbols and structures, from α to ω, available in LaTeX [?]; this section will simply show a few examples of in-text equations in context. Notice how this equation: $\lim_{n\rightarrow \infty} x = 0$, set here in in-line math style, looks slightly different when set in display style. (See next section).

8.2 Display Equations

A numbered display equation—one set off by vertical space from the text and centered horizontally—is produced by the **equation** environment. An unnumbered display equation is produced by the **displaymath** environment.

Again, in either environment, you can use any of the symbols and structures available in LaTeX; this section will just give a couple of examples of display equations in context. First, consider the equation, shown as an inline equation above:

$$\lim_{n\rightarrow \infty} x = 0 \tag{9}$$

Notice how it is formatted somewhat differently in the **displaymath** environment. Now, we'll enter an unnumbered equation:

$$\sum_{i=0}^{\infty} x + 1$$

and follow it with another numbered equation:

$$\sum_{i=0}^{\infty} x_i = \int_{0}^{\pi+2} f \tag{10}$$

just to demonstrate LaTeX's able handling of numbering.

9 FIGURES

The "`figure`" environment should be used for figures. One or more images can be placed within a figure. If your figure contains third-party material, you must clearly identify it as such, as shown in the example below.

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A figure description must be unformatted plain text less than 2000 characters long (including spaces). **Figure descriptions should not repeat the figure caption – their purpose is to capture important information that is not already provided in the caption or the main text of the paper.** For figures that convey important and complex new information, a short text description may not be adequate. More complex alternative descriptions can be placed in an appendix and referenced in a short figure description. For example, provide a data table capturing the information in a bar chart, or a structured list representing a graph. For additional information regarding how best to write figure descriptions and why doing this is so important, please see https://www.acm.org/publications/taps/describing-figures/.

9.1 The "Teaser Figure"

A "teaser figure" is an image, or set of images in one figure, that are placed after all author and affiliation information, and before the body of the article, spanning the page. If you wish to have such a figure in your article, place the command immediately before the `\maketitle` command:

```
\begin{teaserfigure}
  \includegraphics[width=\textwidth]{sampleteaser}
  \caption{figure caption}
  \Description{figure description}
\end{teaserfigure}
```

10 CITATIONS AND BIBLIOGRAPHIES

The use of BibTeX for the preparation and formatting of one's references is strongly recommended. Authors' names should be complete — use full first names ("Donald E. Knuth") not initials ("D. E. Knuth") — and the salient identifying features of a reference should be included: title, year, volume, number, pages, article DOI, etc.

The bibliography is included in your source document with these two commands, placed just before the `\end{document}` command:

```
\bibliographystyle{ACM-Reference-Format}
\bibliography{bibfile}
```

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```
\citestyle{acmauthoryear}
```

Some examples. A paginated journal article [?], an enumerated journal article [?], a reference to an entire issue [?], a monograph (whole book) [?], a monograph/whole book in a series (see 2a in spec. document) [?], a divisible-book such as an anthology or compilation [?] followed by the same example, however we only output the series if the volume number is given [?] (so Editor00a's series should NOT be present since it has no vol. no.), a chapter in a divisible book [?], a chapter in a divisible book in a series [?], a multi-volume work as book [?], a couple of articles in a proceedings (of a conference, symposium, workshop for example) (paginated proceedings article) [??], a proceedings article with all possible elements [?], an example of an enumerated proceedings article [?], an informally published work [?], a couple of preprints [??], a doctoral dissertation [?], a master's thesis: [?], an online document / world wide web resource [???], a video game (Case 1) [?] and (Case 2) [?] and [?] and (Case 3) a patent [?], work accepted for publication [?], 'YYYYb'-test for prolific author [?] and [?]. Other cites might contain 'duplicate' DOI and URLs (some SIAM articles) [?]. Boris / Barbara Beeton: multi-volume works as books [?] and [?]. A couple of citations with DOIs: [??]. Online citations: [???]. Artifacts: [?] and [?].

11 ACKNOWLEDGMENTS

Identification of funding sources and other support, and thanks to individuals and groups that assisted in the research and the preparation of the work should be included in an acknowledgment section, which is placed just before the reference section in your document.

This section has a special environment:

```
\begin{acks}
...
\end{acks}
```

so that the information contained therein can be more easily collected during the article metadata extraction phase, and to ensure consistency in the spelling of the section heading.

Authors should not prepare this section as a numbered or unnumbered `\section`; please use the "`acks`" environment.

12 APPENDICES

If your work needs an appendix, add it before the "`\end{document}`" command at the conclusion of your source document.

Start the appendix with the "`appendix`" command:

```
\appendix
```

and note that in the appendix, sections are lettered, not numbered. This document has two appendices, demonstrating the section and subsection identification method.

13 SIGCHI EXTENDED ABSTRACTS

The "`sigchi-a`" template style (available only in LaTeX and not in Word) produces a landscape-orientation formatted article, with a wide left margin. Three environments are available for use with the "`sigchi-a`" template style, and produce formatted output in the margin:

- `sidebar`: Place formatted text in the margin.
- `marginfigure`: Place a figure in the margin.
- `margintable`: Place a table in the margin.

ACKNOWLEDGMENTS

To Robert, for the bagels and explaining CMYK and color spaces.

A RESEARCH METHODS

A.1 Part One

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A.2 Part Two

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