

CUDA Implementation of Position Based Fluids

CSC417 Course Project

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Enforcing incompressibility

For particle i at position p_i , we compute the density of the fluid around particle i using the estimator:

$$\rho_{F(i)} = \sum_{j \in F(i)} m_j W_{poly6}(\mathbf{p}_i - \mathbf{p}_j, h)$$

Enforcing incompressibility

For particle i at position \mathbf{p}_i , we compute the density of the fluid around particle i using the estimator:

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Then we have the constant density constraint:

$$C_i(\mathbf{p}) = \frac{\rho_i}{\rho_0} - 1$$

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And we want to compute a position correction $\Delta\mathbf{p}$, such that:

$$C(\mathbf{p} + \Delta\mathbf{p}) = 0$$

Position Update from Solving Incompressibility

For particle i at position \mathbf{p}_i , we have,

$$\lambda_i = -\frac{C_i(\mathbf{p})}{\sum_k |\nabla_{\mathbf{p}_k} C_i|^2}$$

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And the position update is,

$$\mathbf{p}_i^* = \mathbf{p}_i + \Delta\mathbf{p}_i$$

Simulation Step

Algorithm 1 simulation step

```
1: for all particles  $i$  do:                                ▶ Fluid advection
2:   apply forces  $\mathbf{v}_i = \mathbf{v}_i + \Delta t \mathbf{f}_{ext}$ 
3:   predict position  $\mathbf{x}_i^* = \mathbf{x}_i + \Delta t \mathbf{v}_i$ 
4: end for
5: for all particles  $i$  do:
6:   find neighboring particles  $F(\mathbf{x}_i^*)$ 
7: end for
8: while  $iter < solverIterations$  do:                      ▶ Iteratively solves
   incompressibility constraints
9:   for all particles  $i$  do:
10:    calculate  $\lambda_i$ 
11:   end for
12:   for all particles  $i$  do:
13:    calculate  $\Delta \mathbf{p}_i$ 
14:    perform collision detection and response
15:   end for
16:   for all particles  $i$  do:
17:    update position  $\mathbf{x}_i^* = \mathbf{x}_i^* + \Delta \mathbf{p}_i$ 
18:   end for
19: end while
20: for all particles  $i$  do:
21:   update velocity  $\mathbf{v}_i = \frac{1}{\Delta t} (\mathbf{x}_i^* - \mathbf{x}_i)$ 
22:   apply viscosity
23:   update position  $\mathbf{x}_i = \mathbf{x}_i^*$ 
24: end for
```

Performance of Implementation

References

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