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SPECIAL ISSUE

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Charging management and pricing strategy of electric vehicle charging station based on mean field game theory

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Abstract

As an intermediary between the power grid and the electric vehicles (EVs) in the charging station, the charging pile promotes the exchange of electric energy between the power grid and EV group and also brings benefits to the charging station. It is difficult to accurately analyze the detailed energy trading behavior of a large number of charging piles with the power grid and EV group. Based on the theoretical framework of mean field game (MFG), this paper considers the battery degradation and charging efficiency taking into account the charging demand of EVs, the charging control problem of energy storage charging piles is proposed to achieve the goal of minimizing the cost of the charging station. In addition to modeling the interaction between the charging station and power grid and EVs as a finite-time dynamic game problem, optimal decentralized energy scheduling control strategies are formulated for charging piles, and by introducing the mean field term, the optimal pricing strategy for power trading between the charging station and EVs is obtained under the mean field equilibrium condition. An iterative algorithm is proposed to solve the charging control problem, and the rationality of the problem and the effectiveness of the algorithm are verified by numerical simulation.

KEYWORDS

energy storage charging pile, mean field game, optimal decentralized charging strategy, optimal pricing strategy

1 | INTRODUCTION

With the increasingly serious world energy crisis and ecological environment problems, the development of renewable resources has become the most urgent task. As a clean energy vehicle, electric vehicle (EV) can solve the environmental burden of the automotive industry [1–3]. The rapid development of the EV industry has led to the rapid growth of EV ownership, and the development of EVs requires investment in the construction of supporting charging infrastructure, including charging piles. The installation of distributed energy storage system inside and around charging piles can alleviate the impact of random charging mode of EVs on the power grid. Charging pile system with integrated charging and storage will play an important role in the design of charging piles and be widely used in the future.

The development of vehicle-to-grid (V2G) technology will further accelerate the development of a new energy industry; the on-board battery of EV is used as a mobile energy storage unit for bidirectional electric energy conversion with the power grid and charging station [4, 5]. Energy storage charging piles participate in the demand-side response service of the power grid, which can not only play the advantages of rapid power regulation of the energy storage system but also obtain benefits. In addition, by controlling the power of the energy storage charging pile, the combination configuration of the charging pile and the power grid can be adjusted, thus improving the benefit of the energy storage charging pile and reducing the penalty caused by the power prediction error.

The effective construction of charging model and the reasonable formulation of charging and discharging strategies of group behavior determine the optimal operation of aggregator and EVs. Most studies are carried out on the basis of the assumed unidirectional power flow model. However, bidirectional charging model can carry out bidirectional energy transmission in the power grid, so that charging stations and EVs can gain benefits from energy exchanging, which is more flexible than the unidirectional model [6]. As for the charging control strategy, centralized control is widely used because all the information is available and the optimal scheduling strategy can be calculated; however, the centralized control algorithm has computational difficulties when applied to large-scale EVs [7]. Due to its scalability, decentralized charging control allows a large number of EVs to participate in the scheduling process [8]. In addition, as an optimization problem, most of the research aim at balancing the load on the grid [9], or maximize the benefits of aggregators [10], or reducing the charging cost for EV owners [11, 12].

To cope with a large number of EVs, the combination of decentralized control and mean field game (MFG) theory to simulate the interactions among numerous particles is a suitable method to solve the charging coordination problem [13, 14], in which the game theory provides a theoretical framework for the balance of EV interactions in the study of coordination systems. Tajeddini and Kebriaei [15] considered charging cost and battery degradation cost; the mean field approximation is used to coordinate EV charging balance. The MFG theory is proposed to simulate the selfish charging behavior of EV and hybrid EV owners in Cournot market in [16]. Mohammed et al. [17] proposed a scheduling charging model to minimize the charging cost of EVs in commercial and residential charging stations. Zhang et al. [18] constructed a coordinated game charging problem, in which the objectives of each EV aims to minimize the charging cost and maximize the charging revenue. Zhu et al. [19] proposed to minimize the total cost of charging by controlling the dynamic charging process of each EV and completing the process within an appropriate time. In view of optimal operation of energy storage system and EV, the current research mainly focuses on how to achieve the minimum cost by controlling the charging strategy; however, it lacks the constraint of state of charge (SoC) after charging, the consideration of battery degradation, and the discussion of power pricing, and there is the problem that the mean field control problem is difficult to solve.

In this paper, the power is unidirectional exchanged between the power grid and the charging station, and bidirectionally exchange between the charging station and the EV group, so as to maximize the interests of the charging station. This competitive interaction resulting from energy trade can be analyzed using the tools of dynamic game theory. In addition, the number of charging piles and EVs is very large, and traditional game theory models and finite-dimensional control are neither easy to handle nor computationally efficient. Therefore, MFG theory is an appropriate method to solve this charging problem. The main contributions of this paper are summarized as follows:

- (i) On the basis of MFG framework, the charging control problem of energy-storage charging pile is proposed, and the coordination game between charging station, power grid, and EVs is established to realize the purpose of minimizing the cost of charging station.
- (ii) The corresponding theoretical framework is given, and the optimal decentralized charging strategy of charging pile is realized. Under the condition of mean field equilibrium, the optimal pricing strategy for the transaction between charging station and EVs is obtained.
- (iii) The corresponding solving algorithm of the charging control problem is proposed.

The rest of this article is organized as follows. In Section 2, we formulate charging control problem among the charging station, the grid, and EVs. Section 3 designs the optimal decentralized charging strategies for charging piles and obtains the optimal pricing strategy under the condition of mean field equilibrium. In addition, an iterative algorithm is proposed to solve the mean field control problem. Section 4 presents the corresponding numerical simulations.

2 | CHARGING CONTROL PROBLEM

We consider a finite set $i \in \{1, \ldots, N\}$ of grid-connected charging piles and EVs participating in energy transactions with the power system over horizon $[0, T]$, where T is the terminal charging time. As shown in Figure 1, the charging station with charging piles integrated into the power grid is equipped with an energy storage device, which can be used to bidirectional transmission of energy with EVs, and unidirectional transmission with geographic areas, which can improve the reliability of energy supply. On the other hand, EVs are parked in the parking position, buying and selling

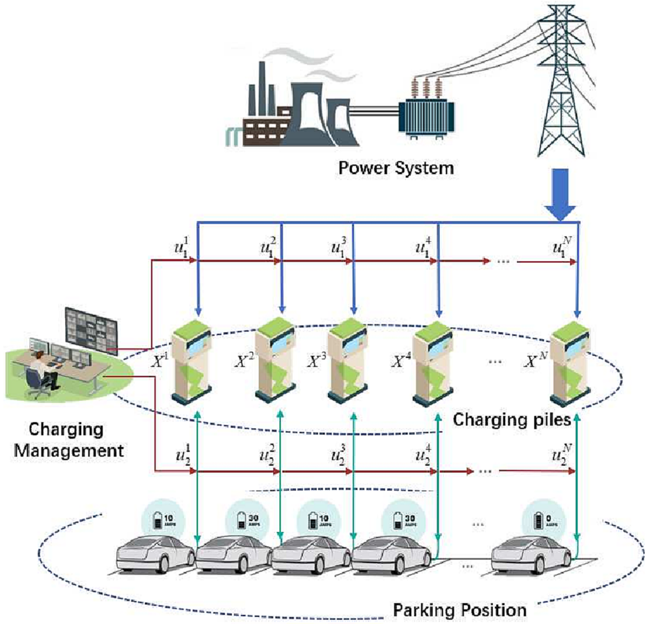

FIGURE 1 The trade of electricity among the power system, charging piles and EVs.

energy through charging piles. This competitive interaction drives the optimal design of charging and discharging strategies for charging piles. We hope to minimize the cost of the charging station through charging management in electricity trading.

The charging station trades electricity with the power grid and EVs, and the SoC of the energy storage device of the ith charging pile is expressed by $X^i(t) \in [0, 1]$, which depends on its initial state and the power of the electricity trading with the power system and the ith EV. In that case, the SoC of the energy storage device of the ith charging pile satisfies the following SDE:

$$\begin{cases} dX^i(t) = \left(u_1^i(t) + u_2^i(t)\right) dt + \sigma(t) dW^i(t) \\ X^i(0) = X_0^i \end{cases} \tag{1}$$

where $X_0^i \in R$ is the initial SoC state. $u_1^i(t) > 0$ indicates the energy purchase rate of unidirectional energy transmission between the power grid and the charging pile. $u_2^i(t)$ indicates the energy transaction rate of bidirectional energy transmission between the ith charging pile and the ith EV, $u_2^i(t) < 0$ represents that the EV charges from the charging pile, and $(u_2^i(t) > 0$ represents that the EV discharges to the charging pile. The standard Brownian motion $W^i(t)$ is taken to be independent of each other and of the initial conditions X_0^i.

The goal of the charging station is to minimize the total cost, taking into account battery degradation cost and the benefits of trading with EVs, so in this competitive game, the objective function of the ith charging pile is defined as

$$\begin{aligned} J^i(u_1, u_2) = \mathbb{E}\Bigg[\int_0^T \left(\frac{1}{2}Q_1(u_1^i(t))^2 + \frac{1}{2}Q_2(u_2^i(t))^2\right. \\ \left. + P^\mu(t) u_2^i(t)\right) dt + f(X)\left(\eta - X^i(T)\right)\Bigg] \end{aligned} \tag{2}$$

The optimization goal consists of four terms. The first term considers the battery degradation cost during the transaction between charging pile and power grid; the form is assumed to be quadratic where $Q_1 > 0$ depends on voltage and energy capacity loss parameters (for details on this form, see [20]); moreover, since Q_1 is positive definite, this term is strictly convex in controls. Then, by limiting the energy purchase rate $u_1^i(t)$, the loss caused by battery degradation can be constrained, so as to reduce the cost of charging station.

The second term represents the cost of battery degradation in the process of electricity trading between the charging pile and the ith EV. Similarly, $Q_2 > 0$ depends on the parameters of voltage and capacity loss; also, this term is strictly convex in controls. Then, part of the cost decrease of the charging station is achieved by limiting the energy transaction rate $u_2^i(t)$ to reduce the loss caused by battery degradation.

The third term corresponds to the cost (revenue) generated by the purchase (sale) of electricity at charging station and EV exchanges. $P^\mu(t)$ is a function of electricity supply and demand, representing the cost (income) generated by the transaction with EVs, the power price under equilibrium condition, and μ is the empirical distribution of the SoC of the charging pile expressed as $\mu(t) = \frac{1}{N}\sum_{i=1}^N \delta_{X^i}(t)$, where $\delta_{X^i}(t)$ represents the Dirac measure (unit point mass). This term is linear in controls and therefore convex. Note that the price $P^\mu(t)$ depends on the distribution of states of all participants and is a response to interactions between participants. At present, the form of $P^\mu(t)$ is not defined, but it is obtained later.

The fourth term ensures that the energy storage device of the charging pile is fully charged at the end of the charging cycle, where $X^i(T)$ represents the SoC of the energy storage device at the terminal time T and η represents the expected terminal SoC. We note that $(\eta - X^i(T))_+$ is used to constrain the terminal state, but it is not differentiable; $f(X)$ is introduced to ensure that the term is convex to maintain the convexity of the optimization problem, and the term is once differentiable and twice differentiable to ensure that the functional is differentiable everywhere.

We assume that a single charging pile and a single EV participating in the energy trading are indistinguishable, and each charging pile has similar energy storage device performance and the same optimization objectives, so that these participants have similar charging behaviors and actions in the equilibrium state. However, all charging piles seek to minimize their own costs, so the strategies of each charging pile to trade energy with the grid and EV are adjusted according to its own state. Then, the problem of optimal charging control strategies for the parking pile is now summarized as finding an optimal control set $\left(u_1^{i,\star}, u_2^{i,\star}\right)$ that minimizes the cost function Equation (2), which is denoted by

$$\left(u_1^{i,\star}, u_2^{i,\star}\right) = \arg\min J^i\left(u_1^i, u_2^i, u_1^{-i,\star}, u_2^{-i,\star}\right) \tag{3}$$

where u_1^{-i} and u_2^{-i} represent the actions of all other charging piles. In this case, each participant responds optimally to the behavior of the other players or empirical distribution μ, and the system reaches equilibrium because no participant can benefit unilaterally by deviating from this equilibrium, which is the Nash equilibrium.

Definition 1 (Nash equilibrium)**.** A set of controls $(u_1^{i,\star}, u_2^{i,\star})$ forms a Nash equilibrium for a fixed time $t \in [0, T]$ if and only if for every player i

$$J^i\left(u_1^{i,\star}, u_2^{i,\star}\right) \le J^i\left(u_1^i, u_2^i, u_1^{-i,\star}, u_2^{-i,\star}\right) \tag{4}$$

In addition, we consider the charging demand of the EV group and hope to obtain the trading pricing strategy $P^{\mu}(t)$ under the optimal control law $u_2^{i,\star}$, which is expressed as

$$\int_0^T \sum_{i=1}^N u_2^{i,\star}(t)dt + D = 0 \tag{5}$$

where D is the total charging demand for N EVs during the charging cycle, which is related to the initial SoC distribution and the expected terminal SoC of EVs. That is, in the electric power transaction, the transaction between the charging piles and the EVs is equal to the total power demand for EVs to be fully charged, so based on this condition and the optimal control, $u_2^{i,*}$ endogenously defines an equilibrium price of $P^{\mu}(t)$.

It should be noted that empirical distribution μ is both the input and output of the charging management problem in the MFG framework, because it determines the equilibrium price $P^{\mu}(t)$, and the optimal control set $\left(u_1^{i,\star}, u_2^{i,\star}\right)$ determines the empirical distribution μ through Equation (1), thus forming a fixed point problem.

3 | SOLUTION OF THE CHARGING CONTROL PROBLEM

We aim to capture the group behavior of a large number of charging piles by describing the statistical behavior of the participants. Therefore, individual decisions at each charging station do not affect the mean field, whereas all decisions are coupled through a mean field term μ that depends on the statistics of the charging piles throughout the charging station. Under our problem formulation, the profit of charging station management is ultimately affected by the battery loss during the charging process and the charging rate of parked EVs in the charging station. We seek a set of solution $\left(u_1^{i,\star}, u_2^{i,\star}\right)$ that minimize the cost function Equation (2) by formulating the optimal decentralized control strategies and obtaining the optimal electricity transaction price $P^{\mu}(t)$.

3.1 | Solution for charging station management

System dynamics Equation (1) and cost function Equation (2) constitute a stochastic optimal control problem; we first prove the existence of the derivative of the cost function and then derive the form of the optimal control.

To begin with, we assume $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge 0}, \mathbb{P}\right)$ to be a standard filtered probability space, and $u_j \in \mathcal{A}_i, j = 1, 2$, is the admissible control set for the ith charging pile. Moreover, with the previous statement, the cost function is strictly convex in $\mathcal{A}_i$ for any given mean field distribution μ.

Definition 2 (Gâteaux derivative [21])**.** The function J^i defined on a neighbourhood of $u_j \in \mathcal{A}_i$ with values in R is Gâteaux differentiable at u_j in the direction of $\omega^{u_j} \in \mathcal{A}_i$ if there exists $\mathcal{D}J^i(u_j) \in \mathcal{A}_i^*$, such that

$$\left\langle \mathcal{D}J^i\left(u_j^i; \mu\right), \omega^{u_j}\right\rangle : = \lim_{\epsilon\to 0} \frac{J^i\left(u_j^i + \epsilon\omega^{u_j}; \mu\right) - J^i\left(u_j^i; \mu\right)}{\epsilon} \tag{6}$$

The function $\mathcal{D}J^i\left(u_j^i; \mu\right)$ is called the Gâteaux derivative of J^i at u_j.

Theorem 1. *Given a mean field distribution μ, the cost function Equation (2) of the ith charging pile is Gâteaux differentiable in $\mathcal{A}_i$, in all directions, then the derivative at $u_1 \in \mathcal{A}_i$ in the direction of $\omega^{u_1} \in \mathcal{A}_i$ is*

$$\left\langle \mathcal{D}J^i, \omega^{u_1}\right\rangle = \mathbb{E}\left[\int_0^T \omega^{u_1}(t)\left(Q_1\left(u_1^i(t)\right) - Y^i(t)\right)dt\right] \tag{7}$$

where $Y^i(t)$ is the adjoint equation for the control problem, which represents the probability that the charging pile does not reach the expected SoC at time t, denoted by

$$Y^i(t) := \mathbb{E}\left[f'\left(\eta - X^i(T)\right) | \mathcal{F}^i(t)\right] \tag{8}$$

Proof. Through Definition 2, take the derivation by taking the limit in the direction of ω^{u_1}, for the direction of ω^{u_1}

$$\left\langle \mathcal{D}J^i\left(u_1^i, u_2^i; \mu\right), \omega^{u_1}\right\rangle : = \lim_{\epsilon\to 0} \frac{J^i\left(u_1^i + \epsilon\omega^{u_1}, u_2^i; \mu\right) - J^i\left(u_1^i, u_2^i; \mu\right)}{\epsilon} \tag{9}$$

where

$$\begin{aligned} &J^i\left(u_1^i + \epsilon\omega^{u_1}, u_2^i; \mu\right) - J^i\left(u_1^i, u_2^i; \mu\right) \\ =&\mathbb{E}\left[\int_0^T \left\{\frac{1}{2}Q_1\left(u_1^i(t) + \epsilon\omega^{u_1}(t)\right)^2 - \frac{1}{2}Q_1\left(u_1^i(t)\right)^2\right\} dt \right.\\ &\left. + f\left(\eta - X^i(T) - \epsilon\int_0^T \omega^{u_1}(t)dt\right) - f\left(\eta - X^i(T)\right)\right] \end{aligned} \tag{10}$$

From the Taylor expansion of f, we obtain

$$\begin{aligned} &f\left(\eta - X^i(T) - \epsilon\int_0^T \omega^{u_1}(t)dt\right) = f\left(\eta - X^i(T)\right) \\ &- \epsilon\int_0^T \omega^{u_1}(t) f'\left(\eta - X^i(t)\right) dt + O\left(\epsilon^2\right) \end{aligned} \tag{11}$$

Substituting Equation (11) into Equation (10), we obtain

$$\begin{aligned} &J^i\left(u_1^i + \epsilon\omega^{u_1}, u_2^i; \mu\right) - J^i\left(u_1^i, u_2^i; \mu\right) \\ &= \mathbb{E}\left[\epsilon\int_0^T Q_1\left(u_1^i(t)\right)\omega^{u_1}(t)dt \right.\\ &\left. -\epsilon\int_0^T \omega^{u_1}(t) f'\left(\eta - X^i(t)\right) dt + O\left(\epsilon^2\right)\right] \end{aligned} \tag{12}$$

Dividing the above equation by ϵ and taking the limit on both sides as $\epsilon \to 0$,

$$\begin{aligned} &\left\langle \mathcal{D}J^i, \omega^{u_1}\right\rangle \\ &= \mathbb{E}\left[\int_0^T Q_1\left(u_1^i(t)\right)\omega^{u_1}(t)dt - \int_0^T \omega^{u_1}(t) f'\left(\eta - X^i(T)\right) dt\right] \end{aligned} \tag{13}$$

Applying iterated expectations implies

$$\left\langle \mathcal{D}J^i, \omega^{u_1}\right\rangle = \mathbb{E}\left[\int_0^T \omega^{u_1}(t)\left(Q_1\left(u_1^i(t)\right) - Y^i(t)\right) dt\right]$$

where $Y^i(t) := \mathbb{E}\left[f'\left(\eta - X^i(T)\right) | \mathcal{F}^i(t)\right]$ □

The existence of derivative of cost function J^iin the direction of ω^{u_1} has been proved, and its form has also been obtained. Next, a similar method is used to obtain the derivative of the cost function J^i in the direction of ω^{u_2}.

Theorem 2. *Given a mean field distribution μ, the cost function Equation (2) of the ith charging pile is Gâteaux differentiable in $\mathcal{A}_i$, in all directions, then the derivative at $u_2 \in \mathcal{A}_i$ in the direction of $\omega^{u_2} \in \mathcal{A}_i$ is*

$$\begin{aligned} &\left\langle \mathcal{D}J^i, \omega^{u_2}\right\rangle \\ &= \mathbb{E}\left[\int_0^T \omega^{u_2}(t)\left(Q_2\left(u_2^i(t)\right) + P^\mu(t) - Y^i(t)\right) dt\right] \end{aligned} \tag{14}$$

Proof. Applying the same method as the proof in the direction of ω^{u_1}, we obtain the derivative form in the direction of ω^{u_2}; the detailed proof process is omitted. □

At this point, a set of derivative forms are obtained, and based on this result, the forms of the optimal control strategies for the mean field limit of the ith charging pile trading with the grid and the $i$the EV are determined by the following theorem.

Theorem 3. *Given a mean field distribution μ, the Gâteaux derivative vanishes for all $\omega \in \mathcal{A}_i$, and $\left(u_1^{i,\star}, u_2^{i,\star}\right)$ is an optimal control set for the cost function Equation (2) expressed as*

$$\begin{aligned} u_1^{i,\star}(t) &= \frac{1}{Q_1} Y^i(t) \\ u_2^{i,\star}(t) &= \frac{1}{Q_2}\left(Y^i(t) - P^\mu(t)\right) \end{aligned} \tag{15}$$

Proof. Based on the previous statement of Equation (2), through linearity of integrals, and the sum of strictly convex and strictly convex functions is strictly convex, then J^i is strictly convex function of controls and differentiable everywhere in all directions. Therefore, a necessary condition for $\left(u_1^{i,\star}, u_2^{i,\star}\right)$ to be the optimal control in all directions is $\left\langle \mathcal{D}J^i, \omega^{u_j}\right\rangle = 0$. Therefore, according to the form of the directional derivative of Equation (7) and Equation (14), we obtain

$$\begin{aligned} u_1^i(t) - \frac{1}{Q_1} Y^i(t) &= 0 \\ u_2^i(t) - \frac{1}{Q_2}\left(Y^i(t) - P^\mu(t)\right) &= 0 \end{aligned} \tag{16}$$

Since Y^i and P^μ are non-negative, $\mathcal{F}$-adapted and bounded, the optimal response set $\left(u_1^{i,\star}, u_2^{i,\star}\right)$ of the ith charging pile is admissible, which is expressed as

$$\begin{aligned} u_1^{i,\star}(t) &= \frac{1}{Q_1} Y^i(t) \\ u_2^{i,\star}(t) &= \frac{1}{Q_2}\left(Y^i(t) - P^\mu(t)\right) \end{aligned}$$

□

Through the proof of the above theorem, the optimal control laws of the charging pile trading with power grid and the ith EV is derived. As mentioned above in the establishment of the charging problem, in addition to obtaining the optimal control strategies, the optimal equilibrium price P^{μ} needs to be further established. However, the introduced equilibrium price P^{μ} has not yet been defined and needs to be constructed.

3.2 | Optimal electricity transaction price

Charging piles purchase and sell electricity with EVs based on optimal control Equation (15) and obtain the transaction price in the form of a mean field equilibrium by satisfying the condition of the total charging power demand of EVs. Then, under the above conditions, the equilibrium price condition Equation (17) is rewritten as

$$\int_0^T \sum_{i=1}^{N} \frac{1}{Q_2} Y^i(t)dt - \int_0^T \frac{N}{Q_2} P^{\mu}(t)dt + D = 0 \tag{17}$$

so as to obtain

$$P^{\mu}(t) = \frac{\sum_{i=1}^{N} \frac{1}{Q_2} Y^i(t) + \frac{D}{T}}{\frac{N}{Q_2}} \tag{18}$$

Therefore, when μ is given, a unique solution to the associated control problem can be obtained so as to achieve the equilibrium price $P^{\mu}(t)$ defined by the optimal charging control and the SoC statistics of EV group.

Under the assumption that μ is given, the optimal control forms of charging pile management and pricing strategy of charging pile transaction are obtained; however, the solution problem has not been solved yet. In addition, it is necessary to determine the mean field distribution μ that minimizes the cost of each charging pile, which means that the game only includes the interaction between each charging pile and μ. Next, we propose the algorithm to solve the above problem.

3.3 | Solution algorithm of the charging problem

In the constructed charging control problem, the optimal control of the i charging pile minimize the cost function Equation (2), which is determined by $Y^i(t)$ and $P^{\mu}(t)$, as shown in Equation (15), constrained by the dynamic equation of the system. It can be seen that the problem is an optimal solution with fixed initial and terminal states, and $dY^i(t)$is introduced as the adjoint equation, so that the forward state equation and the backward state equation are coupled to each other, forming a new differential equation system, that is, combining the dynamic Equation (1) and the optimal control strategies Equation (15),

$$\begin{cases} dX^i(t) = \left(\frac{1}{Q_1} + \frac{1}{Q_2}\right) Y^i(t)dt \\ -\sum_{i=1}^{N} \frac{1}{NQ_2} Y^i(t)dt + \frac{D}{TN} + \sigma(t)dW^i(t) \\ X^i(0) = X_0^i \\ dY^i(t) = Z^i(t)dW^i(t) \\ Y(T) = f'\left(\eta - X^i(T)\right) \end{cases} \tag{19}$$

We aim to obtain a progressively measurable triple $(X^i(t), Y^i(t), Z^i(t))$ and a mean field distribution μ as solutions to Equation (19), thus enabling the optimal control laws and pricing strategy; correspondingly, an iterative solution method is proposed to solve the charging problem; the algorithm is shown in Algorithm 1. First, an initial value of μ^0 is given and solve the system equation (19). When all energy storage devices are charged and discharged in accordance with the obtained mean field strategies, distribution of $X^i(t)$ is obtained, which is represented by μ. This process is repeated until μ^{k+1} and μ^k are infinitely close, indicating convergence of the mean field distribution. At this time, a set of control low $(u_1^{i,*}, u_2^{i,*})$ is obtained through the form of the solution of the optimal control of the charging pile in Equation (15), and the optimal pricing strategy is obtained by Equation (18).

Algorithm 1 Solution algorithm.

Initialization. Set $k = 0$ and $\epsilon_{stop} = 1 \times 10^{-4}$.
Choose an initial condition μ^0
while $\left(\epsilon \geq \epsilon_{stop}\right)$ **do**
for $t = T, \ldots, 0$.
Calculate $X^i(t)$ from Equation (19) using the previous $Y^i(t)$ and μ^{k-1}, calculate $Y^i(t)$ from Equation (19);
end
for $t = 0, \ldots, T$.
Update the distribution to obtain μ^k using the updated $Y^i(t)$ and Equation (19) ;
end
$\epsilon = \left\|\mu^{k+1} - \mu^k\right\|_1$;
$k = k + 1$;
end
Output
Obtain optimal control laws $u_1^{i,\star}(t), u_2^{i,\star}(t)$ from Equation (15) and pricing strategy $P^{\mu}(t)$ from Equation (18)

4 | SIMULATION

In the numerical simulation, we considered the electricity trading behavior among 200 charging piles and the power grid and EVs during the period of 200 EVs parking and charging in the charging station from 8 p.m. to 6 a.m., which is in line with the travel rules of most EV owners. The established charging and discharging model of the *i*th charging pile in the charging station is shown as Equation (1). The goal is to obtain the optimal management of maximizing the profit of the charging station and to be fully charged before the end of charging, as shown in Equation (2), and achieve the optimal pricing strategy under the premise of mean field equilibrium. Accordingly, the optimal control strategies of *i*th charging pile is Equation (15), and the optimal pricing strategy is Equation (18). In addition, the demands for full charging of EVs D are also considered in Equation (5), setting the average initial SoC of EVs at 0.4, and they all expect the SoC to reach 0.8 by the end of charging. Relevant simulation parameters are shown in Table 1.

The optimal charging strategies of the charging station trading with the grid and the EVs, respectively, are shown in Figures 2 and 3, where the sum of the energy providing rate u_1 and the energy exchanging rate u_2 is the SoC changing rate of the energy storage device in charging pile as shown in Equation (1). Moreover, the energy providing rate u_1 is always greater than or equal to 0 in Figure 2, which is in line with the unidirectional energy transmission set in our original problem. In Figure 3, the energy exchanging rate u_2 is positive and negative, which means the bidirectional transmission of energy; most of the EVs are buying electricity to meet the charging demand, while a small number of EVs are selling electricity. Combined with Figures 2 and 3, it can be analyzed that the initial distribution is concentrated in the initial set that charging piles not only provide electric energy for EVs to obtain profits but also meet their own terminal SoC requirements by controlling the energy providing rate and energy exchanging rate, which is consistent with our initial construction of the charging problem Equation (2).

TABLE 1 Associated parameters.

Parameters	Value
N	200
T	10 h
Q_1	0.8
Q_2	1
η	0.8
$\mu(0)$	0.6
$\sigma(0)$	0.1

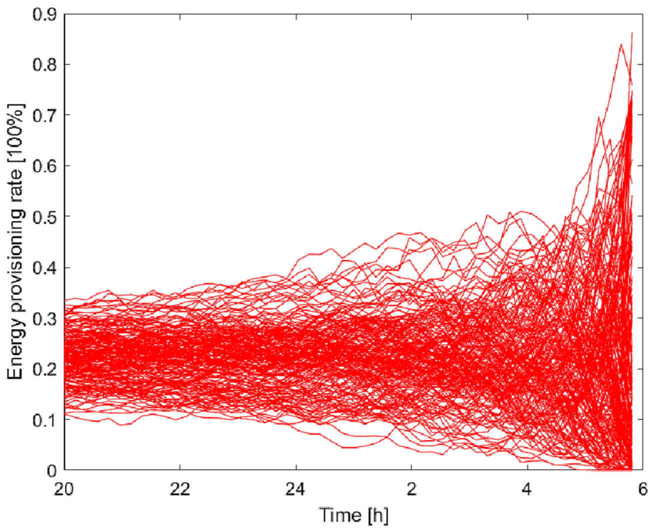

FIGURE 2 Optimal charging strategy for transactions between charging piles and power grid.

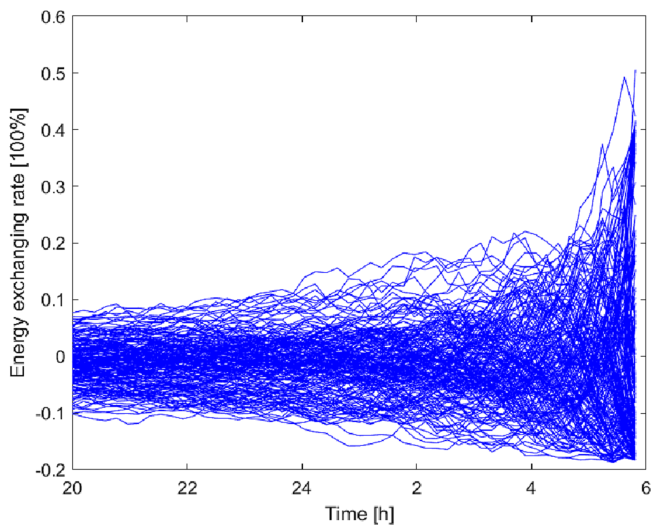

FIGURE 3 Optimal charging strategy for transactions between charging piles and EVs.

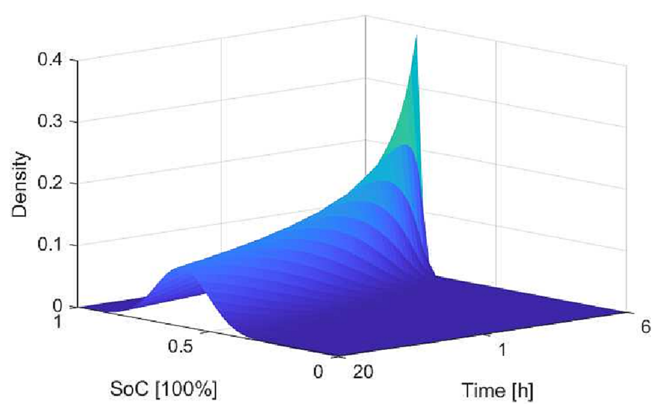

FIGURE 4 Optimal pricing strategy for electricity trading with EVs.

The pricing strategy of charging station and EV transactions under the condition of mean field equilibrium is shown in Figure 4; it shows that the transaction price gradually decreases with time, because the charging pile energy storage device is about to be fully charged as the time gets closer to the end of the charging cycle. Therefore, under the premise that the charging station is close to meeting its SoC terminal state, the charging station appropriately relaxed the price restriction, and as described in Eq. (18), the closer the charging pile is to full charge, the smaller the Y^i, resulting in a smaller P^μ

The empirical distribution of the mean field is shown in Figure 5; it can be analyzed that the initial distribution is concentrated near the initial set $\mu(0) = 0.6$. As time goes by, the charging pile accumulates electric energy, and the mass transfers upward. When t is close to T, the mass is concentrated around 0.8, reaching the peak; namely, the charging piles are all fully charged, which also conforms to the setting of parameter $\eta = 0.8$ above. On the other hand, the price in Figure 4 is relatively smooth, which is because the pricing strategy is determined by the state of the mean field, and the mean field transformation is relatively gentle, so the price fluctuation is not large.

In addition, we study the influence of some parameter settings on the simulation results. The initial SoC distribution of the energy storage devices will directly affect the charging rate; Figures 6 and 7 discuss the strategies

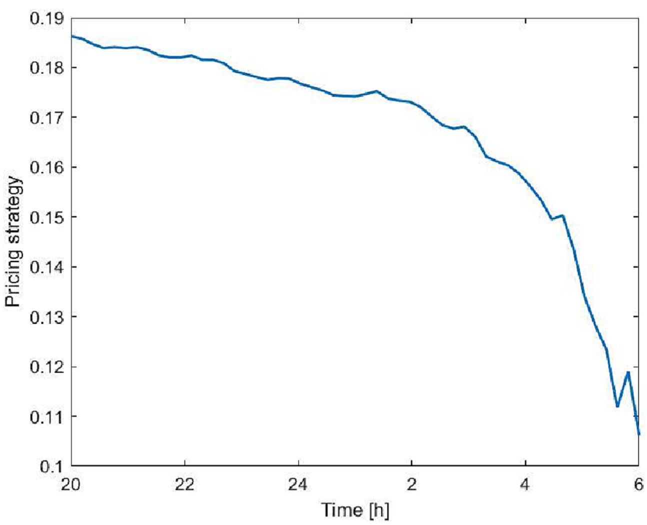

FIGURE 5 Empirical distribution of charging pile state.

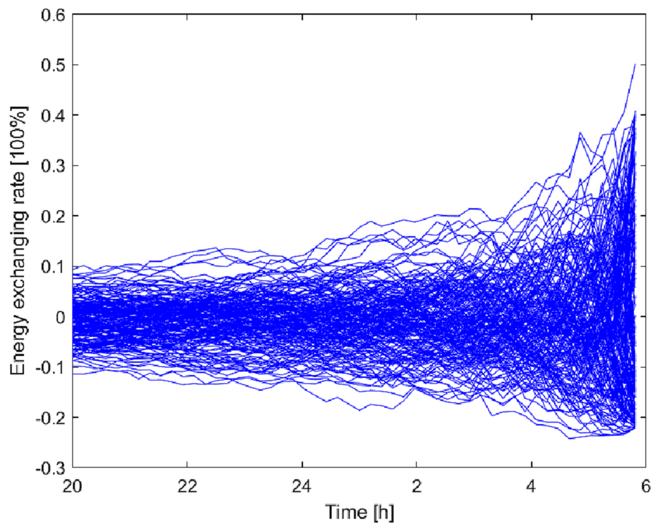

FIGURE 7 Optimal charging strategy for transactions between charging piles and EVs $\mu(0) = 0.5$.

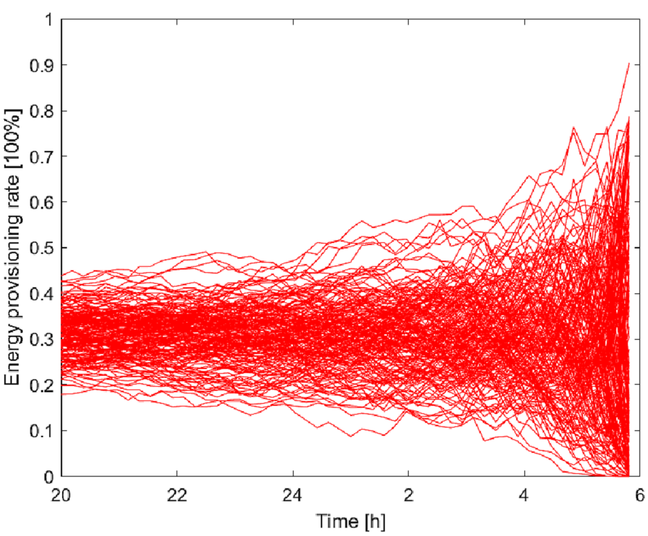

FIGURE 6 Optimal charging strategy for transactions between charging piles and power grid with $\mu(0) = 0.5$.

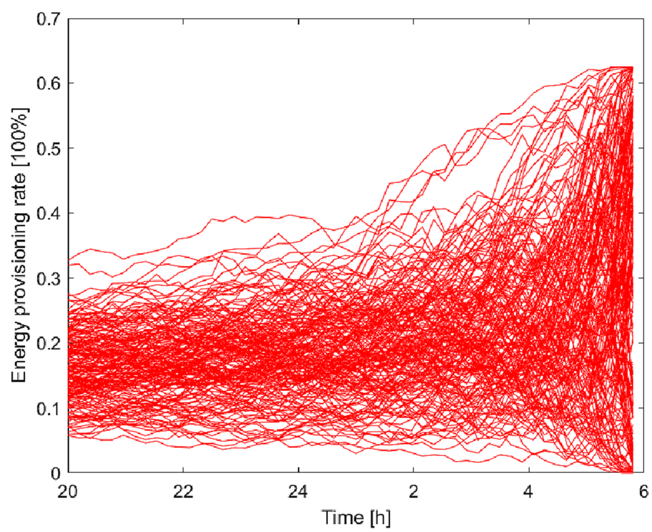

FIGURE 8 Optimal charging strategy for transactions between charging piles and power grid with $Q_1 = 1.6$ and $Q_2 = 2.5$.

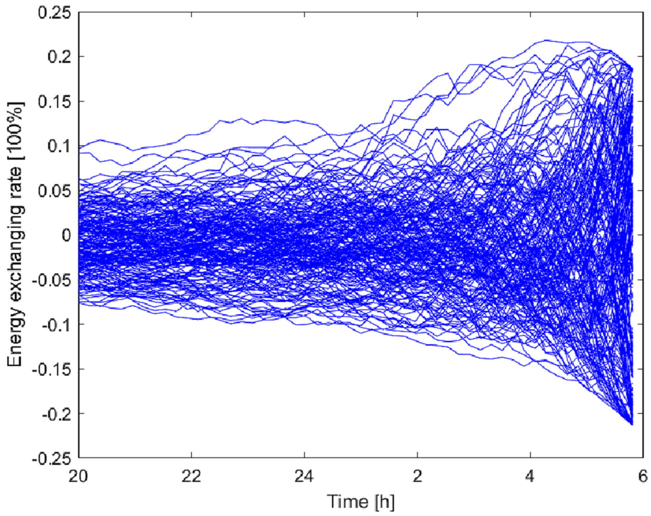

FIGURE 9 Optimal charging strategy for transactions between charging piles and EVs with $Q_1 = 1.6$ and $Q_2 = 2.5$.

of energy exchange between the charging piles and the power grid and the EV group, respectively, when the initial SoC distribution $\mu(0)$ is 0.5. Obviously, compared with result of the initial SoC distribution $\mu(0) = 0.6$ in Figure 2, for a smaller initial SoC, the storage device requires more electricity to be fully charged and to provide power to the EV, which will then trade power with the power grid at a higher charging rate. However, it is not difficult to see that there is no significant difference in the results of the energy exchanging rate between the energy storage device and the EV group under the two initial SoC distributions as shown in Figures 3 and 7, because the EV group has specific charging demand D and expects the battery to be fully charged at a relatively smooth charging rate. Furthermore, in the optimization objective Equation (2), in order to ensure the health of the battery, Q_1 and Q_2 are used as the weight coefficients to adjust the power proving rate u_1 and the exchanging rate u_2; that is, if the values of Q_1 and Q_2 are larger, then the absolute values u_1 and u_2 will be smaller, so as to reduce the battery loss caused by energy exchanging rate fluctuations. Figures 8 and 9 are the results of resetting the weight coefficient; it can be seen that when $Q_1 = 1.6$ and $Q_2 = 2.5$, the energy trading rates of the charging piles charging from the power grid and the energy exchanging rates of the charging piles trading with the EV group are generally reduced, which is consistent with the initial charging control problem we constructed. However, while the Q_2 setting changes greatly, and u_2 does not decrease significantly, because the EVs hopes to be fully charged with a stable power before the end of the charging cycle. And the reason why some of the EVs increase the discharge rate with energy storage devices at the end of the charging cycle is that the energy transaction rate between energy storage devices and the power grid decreases, resulting in the energy storage devices not sufficient to be fully charged at the end of the charging, and cannot meet the requirements of the terminal SoC; therefore, this small amount of electricity is supplied through the transaction between energy storage devices and EVs.

5 | CONCLUSION

In this paper, a three-layer charging model of power grid, charging station and EV group is constructed to study the optimal energy trading of the charging piles and pricing strategies between the energy storage charging piles and EVs in the charging station, which minimizes the cost of charging station and considers the charging demand of EVs. Due to the huge number of charging piles and EVs, the traditional finite dimension control has difficulties in solving problems. By introducing the MFG theory, the communication complexity and computational complexity are reduced, thus simplifying the difficulty of problem analysis. Through the application of mathematical tools such as variational analysis, not only the optimal charging strategies for charging piles to purchase electricity from the grid and the optimal energy exchange strategies between charging piles and EVs are proposed but also the optimal pricing problem for electricity transactions with EVs is solved under the condition of mean field equilibrium. In addition, numerical experiments verify the effectiveness of the proposed algorithm, which can minimize the cost of charging station and illustrate the rationality of the proposed charging strategies and pricing strategy.

AUTHOR CONTRIBUTIONS

Runzi Lin: methodology; software; writing—original draft; writing—review and editing. **Hongqing Chu:** data curation; investigation; validation. **Jinwu Gao:** conceptualization; project administration; writing—original draft. **Hong Chen:** conceptualization; supervision; writing—review and editing.

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CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflict of interests.

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