A Comprehensive Game-Theoretic Model for Electric Vehicle Charging Station Competition

Efstathios Zavvos, Enrico H. Gerding, and Markus Brede

***Abstract*— En-route charging stations are essential to ensure the adoption of electric vehicles. However, careful planning is necessary due to high cost in infrastructure and potentially long waiting queues. Existing literature on the placement of charging stations largely disregards competition, sets prices to cover costs and/or disregards queues. In contrast, this work models competing station investors who aim to maximise expected profit, while electric vehicle drivers aim to minimise expected travel costs including queues. Following a game-theoretic approach, investors strategically decide station capacities, locations and charging unit power outputs as well as fees, taking into consideration building and operational costs. Given the complexity of the problem, the solution involves a combination of theoretical and algorithmic techniques to obtain subgame-perfect equilibria of investor and driver choices. Subgame-perfect equilibria are found to be at least 92.85% efficient, for reasonable fluctuations of problem parameters. Furthermore, it is found that charging prices can be up to approximately 5 times higher than marginal cost due to long charging times, and also that better charging technology may not necessarily benefit drivers in the near future. Finally, subsidies towards the purchase of charging units are shown to be beneficial for both drivers and investors, being able to generate up to 14.3% additional value than the cost of the subsidy. In contrast, subsidies on the energy price for stations are found to have small effect and can be abused by investors.**

***Index Terms*— Electric vehicle, game theory, charging station, firm, competition, subsidies.**

I. Introduction

ELECTRIC Vehicles (EVs) are not yet widely popular, which is largely attributed to their limitations in range combined with long charging times [1]. In addition, charging infrastructure is expensive to build [2], [3] something that can potentially lead to congestion at en-route charging stations, with long waiting times for EV drivers [4], [5]. In this paper, we address the problem by simultaneously modelling charging station investors' decisions about the *locations*, *capacities* (i.e. number of available charging units), *the charging units' power outputs and the prices of charging* (or *LCOP* for short) for their stations. We do so by considering multiple competing charging stations, as well as the EV driver behaviour.

In modelling EV driver behaviour, existing work utilises game theory to model the choice of charging stations by EV drivers (see Section II). For example, [6] consider stochastic choices by EV drivers, taking into account potential queues. The model optimises charging station capacities over a given urban area with fixed station locations. However, that type of work assumes that (1) there is a monopoly in charging stations and (2) the recharging fee is equal to the cost of electricity. In contrast, this paper investigates the setting where several self-interested investors (i.e. station owners) compete, which affects charging prices, the choice of locations and capacities, and the power outputs of charging units.

Competition between firms has received extensive attention in the literature, but existing research (in Section II) does not consider the multiple choices made by station investors. For example in [7], charging stations first announce prices and then drivers select stations based on prices and capacities, and the authors prove that an equilibrium in prices exists. However, this results in somewhat arbitrary prices, and further insight into pricing and competition is needed. To address these limitations, this work presents a novel sequential, game-theoretic, model for firm competition and algorithms for solving it. In doing so, the work makes the following contributions.

Firstly, we produce the first model where competing investors can own several heterogeneous charging stations, and can decide on locations, capacities, charging unit power outputs and prices for their stations. Based on investor decisions, EV drivers then choose stochastically among stations so as to minimise the expected cost of travelling, by trading off between travel time, expected queuing time, and charging fees. We also include an outside option where drivers can use a different mode of transport instead. As we will show, this helps fine tune the model to obtain well-scaled prices.

Second, the model is solved by combining theoretical and algorithmic techniques to locate subgame-perfect equilibria (SPEs). This helps improve computational complexity and also provides important insight into pricing competition. SPEs are highly efficient compared to centralised (e.g. monopolistic) station allocations that optimise station profit. In particular, worst-case social welfare (i.e. the utilities of both drivers and investors combined) in SPE is found to be within 92.85%

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of optimal utility for reasonable fluctuations of the problem's parameters, and in many cases within 98% of the optimal.

Third, we show that equilibrium prices are significantly higher than the marginal cost (up to approximately 5 times), This is because EVs need non-negligible time to recharge. In addition, the travel time to reach stations is a significant product differentiation factor and we find that investors would prefer to compete on the same rather than different routes. Further, better charging units improve charging time, but we find that this does not necessarily translate to improved travel costs—or even queuing—in the foreseeable future.

Fourth, this is the first work to compare the effectiveness of subsidies to competing station investors. Our empirical results show that subsidising the purchase cost of charging units is beneficial for both drivers and investors, and can generate value in excess of the cost of the subsidy. In the examples studied here, they generate up to 14.3% extra value. On the other hand, subsidising the price of electricity can be highly ineffective or even detrimental to drivers.

The remainder of this paper is structured as follows. Section II discusses additional relevant academic literature. Next, Section III presents the formal model for this work and Section IV presents the solution to the model, together with a theoretical analysis on equilibrium prices. An empirical evaluation then follows in Section V, and Section VI concludes this paper and discusses future directions for this work.

II. Related Work

Apart from research discussed in Section I, there is further work which is of interest. In [8], the authors present a model for nearly-optimal control of EV charge and discharge with the goal of utilising renewable resources effectively during times when charging prices are low. This is a very interesting problem but it does not include charging stations nor the competitive business aspect of recharging EVs. A game-theoretic model for coordinating the day-ahead charging problem for EVs is presented in [9]. However, two key differences with our work are that (1) it does not address the need for en-route charging and (2) it looks at the power grid as a whole and is therefore indifferent to charging stations and their competition.

Regarding competition, Bertrand's model expects that firms will set prices to the marginal cost (e.g. the cost of recharging one more EV). However, this assumes that all customers choose the cheapest firm, firms will satisfy all demand, and goods are homogeneous [10]. Based on this, [11] present an oligopoly where two charging stations in fixed locations decide charging prices. They relax goods homogeneity by assuming different travel times for each station, and find empirically that prices will differ from marginal cost. In contrast, we argue that peak charging demand may not be satisfied all at once due to limitations in charging, and in conjunction with high investment costs this may lead to queuing. In addition to prices, firms in [12] also decide production capacities, and then customers choose firms based on these. A randomly selected proportion of customers are not served if the customers arriving at the firm are more than its production capacity (called rationing). However, rationing excludes queuing (i.e. waiting) as an option, and is not realistic for en-route charging. Notable is the work in [13], where stations select charging prices and drivers choose charging stations based on these. The authors present a more realistic driver model than the one we consider in this paper, but due to the increased complexity charging station competition is simpler. We have deliberately chosen the other route of simplifying the driver model to gain insights into charging station competition. Hence [13] is different from our work in that (1) charging stations can only choose charging prices, (2) station utility only considers gross earnings, (3) the authors find the ϵ-Nash equilibrium approximation in prices and (4) there is no insight in the actual scale of prices. In contrast, charging stations in our work make several decisions including prices, we include the building and operational costs for stations, our work finds the true equilibrium in prices theoretically which we use to gain several insights, and our work takes into account the value of time for drivers and extraneous competition to produce more realistically-scaled prices and station capacities.

This work further improves significantly upon Network Pricing Games (NPGs). These model network providers who select prices for their service in order to maximise profit, and users who optimise over the price and quality of the service, which is conceptually similar to the LCOP. In [14], deterministic concave demand is assumed for modelling services where users will switch to a comparable alternative if prices are too high. More advanced NPGs such as [15] use a dedicated customer model. This research significantly extends these by introducing locations, service rates (capacity), and the speed of service (power output at firms. Moreover, an alternate option for customers allows for *stochastic* demand satisfaction.

Related is spatial competition with homogeneous [16], [17] or heterogeneous [18] firms. This is involved with the location choice of firms in a uniform area, like a marketplace, where those that are closer to each other may compete more intensely for customers. Although the location choice presented here is a form of spatial competition, it is not a typical example. Typical models consider a homogeneous product and do not consider queuing. Spatial competition here is more abstract, with different travel times to reach different stations. This induces product differentiation and—although this may as well be in uniform space—we focus on non-uniform space (different routes). Spatial competition within the same route requires too many new parameters (e.g. distance from power substations, further costs), which would hinder analysis and raise questions on where their valuations come from.

This work resembles a Stackelberg game where players compete by moving sequentially. However, it is different from typical Stackelberg competition [19], in that firms compete on several levels, including service rate, service speed, locations and prices, and in that firms move simultaneously in making certain decisions. The price competition with queues in [20] may resemble our model, but (1) customer flows to firms are deterministic, (2) service rate is always greater than the arrival rate, and (3) each firm has only one server. In contrast, in our work driver flows to stations are stochastic, we are especially interested in the situation where queues are over-saturated, and stations can serve multiple drivers synchronously.

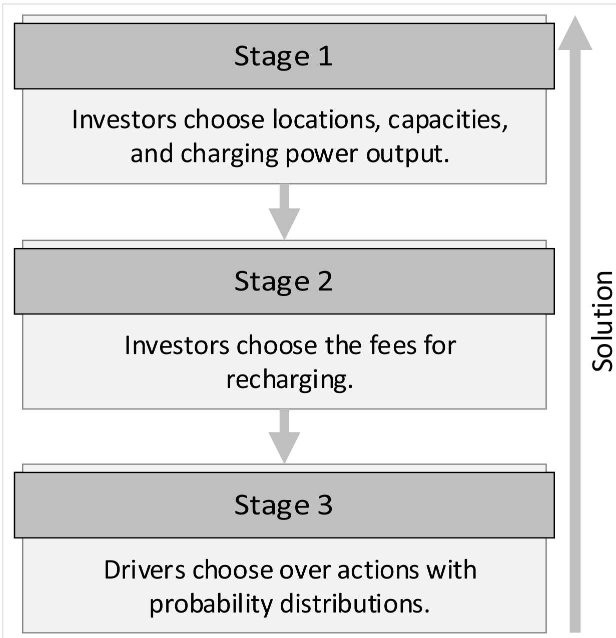


Fig. 1. The three stages of the extensive-form game. Players at each stage can observe the outcome of the previous stages and initialisation (perfect information). The solution follows backward induction, hence stage 1 corresponds to Section III-C and stages 2 and 3 to sections III-B and III-A.

Situations where the utility of using a resource decreases as the number of players that choose it increases, often call for using congestion games [21]. This is similar to our setting where queuing time increases with the number of EVs that travel to the same station. However, congestion games assume that players have incentive to follow a pure-strategy Nash equilibrium, which requires some coordination mechanism, as in [22], but in general it is more realistic to assume a mixed strategy for drivers, as is the case in other work [4], [6], [7].

Last, other research considers *monopolistic* charging station infrastructure optimisation. For example, [23] and [24] focus on reducing building costs, and [25]–[28] additionally consider the EV routing problem, but station capacity does not affect driver choices. Further research is concerned with maximum local population coverage [28]–[30], or optimising power distribution [31]–[34], and therefore does not consider parameters such as prices, queues and driver behaviour. Driver behaviour is considered in [35], where the power grid decides its price in order to maximise revenue, and then drivers make charging decisions based on that, but that again refers to monopoly whereas this work examines competition.

III. MODEL

The LCOP problem is formalised as an extensive-form game comprising the following three stages/subgames: (1) first, investors announce locations, capacities and charging unit power outputs; (2) then, investors announce charging prices given the previous stage; (3) finally, drivers announce probabilities over actions given the previous stages, where an action is choosing either one of the stations or an outside option. Since solutions are obtained using backward induction, in what follows we present these stages in reverse order.

A. Stage 3: Drivers Choose Stations

In this subgame, a finite number of EV drivers simultaneously choose among several en-route charging stations. This models a particular time of day when demand is high and potential queues can form. Stations can be at different locations with different prices, capacities, and charging unit power output. EVs are required to recharge and have to choose one of the stations. To prevent stations from setting arbitrarily high prices, we introduce an outside option for drivers, which can be considered an alternative means of transport. Furthermore, for simplicity, we assume that all EVs are identical, they have the same start and destination, and will recharge fully. These assumptions do not affect the issues this paper negotiates, and are commonly made to simplify the complexity of mixed strategy Nash equilibria and to promote game-theoretic analysis (e.g. [11], [14], [16], [17], [20]).

In more detail, this subgame is defined as the tuple $\langle N, A, u(\cdot)\rangle$ where N is the finite set of n *drivers*. Let $A_i = \{1, \ldots, m\}$ be the finite set of m *actions* (station choices) available to driver i, where the m^{th} action refers to the outside alternative. Then, $A = A_1 \times \ldots \times A_n$ is the set of action profiles and each vector $a = \langle a_1, \ldots, a_n\rangle \in A$ is an action profile. Last, $u(\cdot) = \langle u_1(\cdot), \ldots, u_n(\cdot)\rangle$ is the n-tuple of driver *utilities*, where $u_i(\cdot) : A \mapsto \mathbb{R}$ is a real-valued utility function for driver i. We assume that, in equilibrium, drivers play a *mixed* strategy, i.e. drivers randomise their choices each day. Given this, let $\Pi\,(A_i)$ be the set of all probability distributions over all actions in A_i. Then $S_i = \Pi\,(A_i)$ is the set of mixed strategies for driver i and $S = S_1 \times \ldots \times S_n$ is the set of mixed strategy profiles. Each vector $s = \langle s_1, \ldots, s_n\rangle \in S$ is a mixed strategy profile which contains the mixed strategies $s_i = \{s_i^1, \ldots, s_i^m\}$ played by each driver in that mixed strategy profile. By $s_i^j \in [0, 1]$ we denote the probability that driver i chooses action $j \in A_i$, and it must be $\forall i : \sum_{j\in A_i} s_i^j = 1$.

We now introduce the utility function of a driver. Given that drivers start simultaneously, driver i will arrive at any place in the queue of station j with the same probability. Then i will experience an average delay due to congestion at station j: $Q\,(x) = \sum_{k=0}^{x} \lfloor k/c_j \rfloor R_j/(x+1)$, where x is the number of other drivers that choose station j, $c_j \in \mathbb{N}^+$ is the number of charging units (capacity) of station j and R_j is the time it takes to recharge a single EV at j. However, this does not provide a closed-form solution to work with. It is thus approximated as $Q\,(x) = \frac{x}{2c_j} R_j$, which is an overestimate especially when capacity is small. In order for drivers to trade-off between time and monetary costs, we use the *value of time* parameter, $v_d \in \mathbb{R}^+$. This represents how much a driver is willing to pay in order to save time [36]. The utility for driver i, for choosing station j given that x other drivers choose the same station is:

$$u_i^j\,(x) = -v_d(t_j + \frac{x}{2c_j} R_j + R_j) - f_j \tag{1}$$

where f_j is the price set by station j and $t_j \in \mathbb{R}^+$ is the travel time needed to get to the destination, if the driver chooses station j. In addition, the utility for the outside option is:

$$u_i^m\,(x) = -v_m \cdot t_m - x \cdot D - f_m \tag{2}$$

where $\upsilon_m \in \mathbb{R}^+$ is the value of time for the means of transport the outside option represents, $t_m \in \mathbb{R}^+$ is the travel time and $f_m \in \mathbb{R}$ is the fee for using the outside option. Parameter D is a model calibration parameter used to set a level of satisfactory service and is explained in Section V-A.

The assumptions made so far mean that all drivers have the same utility function, which guarantees a symmetric mixed strategy Nash equilibrium (NE) where drivers choose over available actions with the same probability distribution [21]. To compute it, we need to determine the *expected* utility for driver i of choosing action j. After trivial binomial transformations on (1), the expected utility for i of choosing station j in mixed strategy profile s, where x out of $n-1$ other drivers also choose j with probability s_{-i}^{j} is:

$$\mathbb{E}[u_i^j(x)|s_{-i}] = -\upsilon_d\left(t_j + \frac{s_{-i}^j(n-1)}{2c_j}R_j + R_j\right) - f_j \quad (3)$$

where $j \leq m-1$. The expected utility of choosing the outside option from (2) is:

$$\mathbb{E}[u_i^m(x)|s_{-i}] = -\upsilon_m t_m - s_{-i}^m(n-1)D - f_m \quad (4)$$

B. Stage 2: Investors Choose Prices

In this subgame, station investors compete with each other in order to maximise net profit, by selecting prices for each of their stations. Each investor may own multiple charging stations, but at most one at each available location. It is further assumed that peak congestion at stations can occur a given number of times per day, and that queues are empty when peak traffic arrives at stations. It is unlikely that the number of competitors in reality can be so large as to render observation of the opponents' prices the previous day difficult or partial. Hence it was chosen that investors will play pure strategies, that is pick a single price for each station and play it. To avoid more confusing notation, j will be used here to denote the station a single investor may own in a particular location.

Given these, this sub-game is defined as a tuple $\langle I, F, r(\cdot)\rangle$, where I is the finite set of z *charging station investors*. Let $L = \{l_1, \ldots, l_\mu\}$ be the finite set of locations available to investors and $F_k^L = (-\infty, +\infty)$ be the infinite set of price options available to investor k. Then, $F_k = (l_1 \times F_k^0) \times (l_2 \times F_k^0) \times \ldots \times (l_\mu \times F_k^0)$ is the set of *actions* available to investor k. $F = F_1 \times \ldots \times F_z$ is the set of pure strategy profiles and $f = \langle f_L^1, \ldots, f_L^z\rangle \in F$ is a pure strategy profile. Thus pure strategy f_L^k contains the prices f_j^k investor k chose for each location $j \in L$ in pure strategy profile f. Finally, $r(\cdot) = \langle r^1(\cdot), \ldots, r^z(\cdot)\rangle$ is the z-tuple of *utilities* for the investors.

Station utility is introduced as the net profit of a station, i.e. the normalised earnings minus the costs. If s_i^{jk} is the probability of n drivers choosing station j owned by investor k, then expected traffic flow toward it is $s_i^{jk}n$, which is in fact a function of the prices of all stations. Then expected utility for station j setting price f_j^k in pure strategy profile f is:

$$\mathbb{E}[r_j^k(f)|s] = \begin{cases} s_i^{jk}(f)n(f_j^k - h_j^k)w - b_j^k c_j^k - o_j^k, & c_j^k > 0 \\ 0, & c_j^k = 0 \end{cases} \quad (5)$$

where h_j^k is the cost for the station to recharge each EV (i.e. the price paid to the energy grid), b_j^k is the building cost for each charging unit, c_j^k (capacity) is the number of charging units in the station and o_j^k is an one-time building cost for station j. The parameter w normalises earnings for a given time frame. Maintenance costs can also be weighed like this, or they can be integrated directly into the cost of building a charging unit b_j^k for the whole time-frame under examination. Note that when $c_j^k = 0$, the driver's expected utility for that station in (3) is not defined, hence due to backward induction s_i^{jk} is not defined. In that case, expected utility is explicitly set to 0 to reflect a state where investor k does not operate a station in that location. Then, the expected utility for investor k of playing in pure strategy profile f is the sum of expected utilities of all potential stations investor k can own across locations.

$$\mathbb{E}[r^k(f)|s] = \sum_{j \in L} \mathbb{E}[r_j^k(f)|s] \quad (6)$$

C. Stage 1: Investors Choose Locations, Capacities and Charging Unit Power Outputs

Here, station investors compete with each other to maximise net profit by deciding locations, capacities and the power output of charging units for their stations. In addition to the assumptions from Section III-B, we will assume that an investor will choose one power output to be used across all the investor's stations. This will keep the model tractable, and does not affect the outcomes of this work. An investor's behaviour is expected to show long-term commitment when it comes to deciding the magnitude of investment, therefore, pure strategies will again be used. The potential existence of many symmetric or asymmetric pure strategy NE in this game is something that will offer better insight into firm competition.

This sub-game is defined as a tuple $\langle I, C, r(\cdot)\rangle$, where I is the set of z investors. Let $C_k^0 = [0, \Theta] \subsetneq \mathbb{N}$ be the finite set of capacity choices available to investor k and G be the finite set of charging unit power output options. If $L = \{l_1, \ldots, l_\mu\}$ is the finite set of locations available to investors, then $C_k = ((l_1 \times C_k^0) \times (l_2 \times C_k^0) \times \ldots \times (l_\mu \times C_k^0)) \times G$ is the set of *actions* available to investor k. Then, $C = C_1 \times \ldots \times C_z$ is the set of pure strategy profiles and each vector $c = \langle c_L^1, \ldots, c_L^z\rangle \in C$ is a pure strategy profile. So pure strategy c_L^k contains the capacities c_j^k investor k chose for each location $j \in L$, and the power output option g^k in pure strategy profile c. Finally, $r(\cdot) = \langle r^1(\cdot), \ldots, r^z(\cdot)\rangle$ is the z-tuple of investor *utilities*.

The utility and expected utility for investors and their stations are the same as in the price game (Section III-B), only now they are functions of pure strategy profile c.

IV. Solution and Theoretical Analysis

Solving the model employs backward induction to locate Subgame-Perfect Equilibria (SPEs), which is common in solving extensive-form games. Because players have perfect information, any combination of Nash equilibria s^*, f^*, c^* of the subgames is also a SPE of the extensive-form game [21]. First we solve the driver and price equilibria theoretically, and last the equilibrium in locations, capacities, and power output with an algorithm. To maintain intelligibility, a two-station example (stations 1 and 2) with no outside option, and simple notation as in Section III-A will be used.

A. Stage 3: Drivers' Equilibrium & Boundary Conditions

In the mixed NE s^*, drivers must have no incentive to deviate from their chosen mixed strategy [21]. Intuitively, it must be that for driver i, choosing an action j yields the same expected utility as choosing any other action. In addition, we seek a symmetric equilibrium s^* where driver i chooses j with the same probability as other drivers do. That is $s_i^{*j} = s_{-i}^{*j}$. Therefore, to find the symmetric mixed NE we solve the following $m \times m$ system of linear equations:

$$\begin{aligned} \mathbb{E}[u_i^1(x)\,|s_{-i}^*] &= \mathbb{E}[u_i^2(x)\,|s_{-i}^*] \\ &\cdots \\ \mathbb{E}[u_i^{m-1}(x)\,|s_{-i}^*] &= \mathbb{E}[u_i^m(x)\,|s_{-i}^*] \\ s_{-i}^{*j} + \ldots + s_{-i}^{*m} &= 1 \end{aligned} \tag{7}$$

Solving the NE for two stations from (3) and (7) is straightforward. Assuming recharge time R_j is the same ($R_1 = R_2 = R$) at both stations for simplicity, drivers' NE probabilities are:

$$\begin{aligned} s_i^{1*} &= \frac{c_1 v_d R(n-1) + 2c_1c_2v_d(t_2 - t_1) + 2c_1c_2(f_2 - f_1)}{v_d(n-1)(c_1+c_2)R} \\ s_i^{2*} &= \frac{c_2 v_d R(n-1) + 2c_1c_2v_d(t_1 - t_2) + 2c_1c_2(f_1 - f_2)}{v_d(n-1)(c_1+c_2)R} \\ &\quad v_d, c_1, c_2 > 0 \quad n > 1 \end{aligned} \tag{8}$$

Let us now examine boundary conditions for the probabilities. In order to have $s_i^{1*} < 0$, it must be that the numerator of s_i^{1*} in (8) is negative:

$$\begin{aligned} & c_1 v_d R(n-1) + 2c_1c_2v_d(t_2-t_1) + 2c_1c_2(f_2-f_1) < 0 \\ \Leftrightarrow\ & v_d\frac{n-1}{2c_2}R + v_d t_2 - v_d t_1 + f_2 - f_1 < 0 \\ \Leftrightarrow\ & -v_d t_1 - f_1 < -v_d t_2 - v_d\frac{n-1}{2c_2}R - f_2 \\ \Leftrightarrow\ & -v_d t_1 - v_d R - f_1 < -v_d t_2 - v_d\frac{n-1}{2c_2}R - v_d R - f_2 \\ \Leftrightarrow\ & \textit{Replacing from } (2) \qquad u_i^1(0) < u_i^2(n-1). \end{aligned} \tag{9}$$

This means that $s_i^{1*} < 0$ when the utility for driver i of going to station 1 with no queue, is less than the utility of going to station 2, even if all $n-1$ other drivers went to 2 as well. In that case, station 1 is conceptually undesirable and drivers should play the pure strategy of going to station 2. Alternatively, from (9), station 1 is not desirable when:

$$t_1 > t_2 + \frac{n-1}{2c_2}R + \frac{f_2 - f_1}{v_d}$$

More generally for m stations, if $-j$ is a station other than j, station j is not desirable by driver i when:

$$t_j > \sum_{-j\in A_i} t_{-j} + R\left(\sum_{-j\in A_i}\frac{n-1}{2c_{-j}} + m - 2\right) + \frac{\sum_{-j\in A_i} f_{-j} - f_j}{v_d}$$

Being that for driver i the probabilities for all choices add up to 1, a probability greater than 1 simply means that the probability for some other station is negative.

B. Stage 2: Equilibrium in Prices

Concerning the pure strategy NE f^*, investors should not have incentive to deviate from the equilibrium strategy. More intuitively, each investor's prices are a best response to the other investors' prices; that is each investor will maximise utility, given the other investors also maximise [37]. This translates to solving the following system of partial derivatives:

$$\forall k \in I:\ \forall j \in L:\ \frac{\partial \mathbb{E}[r^k(f^*)|s^*]}{\partial f_j^{k*}} = 0 \tag{10}$$

Theorem 1: Let a two-investor instance of the investors' price game in Section III-B, where investors have already chosen locations, capacities and the speed of service. Without loss of generality, it is assumed for simplicity that each investor has only one station, that the cost of charging EVs is the same for both stations ($h_1 = h_2 = h$), and that charging speed is the same for both stations ($R_1 = R_2 = R$). Last, it is assumed that there is no outside option for drivers.

This game has a unique Nash equilibrium in prices $f^* = (f_1^*, f_2^*)$, in which charging prices will deviate from the marginal charging cost (i.e. Bertrand equilibrium) h due to the inability to satisfy charging demand immediately, and due to goods heterogeneity different travel times impose.

Proof: For proving Theorem 1, the two-station example that was shown in Section IV-A is useful. From there, the next step is to substitute the probabilities of (8) into investor expected utilities in (5) and (6). Then we solve the simple system of two partial derivatives from (10). This part is long and has been omitted, as it is trivial for two stations. It results, however, in the equilibrium prices being:

$$\begin{aligned} f_1^* &= h - \frac{1}{3}v_d(t_1 - t_2) + Rv_d(n-1)\frac{2c_1 + c_2}{6c_1c_2} \\ f_2^* &= h - \frac{1}{3}v_d(t_2 - t_1) + Rv_d(n-1)\frac{c_1 + 2c_2}{6c_1c_2} \\ &\quad v_d, c_1, c_2 > 0 \quad n > 1 \end{aligned} \tag{11}$$

It is evident from (11) that prices in equilibrium deviate from the marginal cost h because of the $Rv_d(n-1)\frac{2c_1+c_2}{6c_1c_2}$ and $Rv_d(n-1)\frac{c_1+2c_2}{6c_1c_2}$ terms. There is also some fluctuation in prices due to different travel times to reach stations, through the $vd(t_1 - t_2)$ and $vd(t_2 - t_1)$ terms. This makes the product stations sell heterogeneous even though charging times are the same. A station with a lower travel time has an advantage in the ability to ask for a higher fee, whereas one with higher travel time has to reduce price to remain competitive. Finally, from (11) we have that:

$$\lim_{R\to 0^+} f_1^* = h - \frac{1}{3}\upsilon d(t_1 - t_2)$$
$$\lim_{R\to 0^+} f_2^* = h - \frac{1}{3}\upsilon d(t_2 - t_1)$$

That is when both stations take the same time to reach ($t_1 = t_2$), equilibrium prices converge asymptotically to the marginal cost h with a decreasing charging time R, something which is in line with Bertrand competition [10]. It must be noted that in the case where travel times are different, price cannot be lower than h. In the greater picture of the extensive form game this would mean losses, and a losing strategy will always be strictly dominated in the first stage by the strategy of not opening the station at all. ■

C. Stage 1: Equilibrium in Locations, Capacities & Charging Unit Power Outputs

Unlike stages 2 and 3, stage 1 is solved with an algorithm which will now be explained. First, the EV drivers' NE is solved symbolically (line 3) (i.e. without replacing parameters). Next, this is used to solve the equilibrium in prices symbolically (line 4). Then, the pure strategy NE in locations, capacities and speed of service is solved and subgame-perfect equilibria are obtained (lines 5-19). Instead of calculating all the utilities, we employ an Iterated Best Response algorithm, where investors are initialised in a random state (lines 6, 7) and take turns playing their best strategy given the other players' strategies (lines 10-15). Then, if a full round (lines 9-16) passes without change in the strategy profile investors started the round with, that profile is a pure strategy NE.

1: **procedure** FIND SPEs
2: $X \leftarrow \{\}$ ▷ Set of all SPEs found
3: $s^* \leftarrow$ solve system (7) symbolically
4: $f^* \leftarrow$ solve system (10) symbolically given s^*
5: **for** *threshold*$= 1 \to K$ **do**
6: $curCapState \leftarrow rand\{c_1^1, \dots, c_\mu^1, \dots, c_1^z, \dots, c_\mu^z\}$
7: $curSpeedState \leftarrow randomise\ \{g^1, \dots, g^k\}$
8: $O \leftarrow shuffle(I)$ ▷ Randomise investor order
9: **repeat**
10: *prevCapState* $\leftarrow$ *curCapState*
11: *prevSpeedState* $\leftarrow$ *curSpeedState*
12: **for** $k = 1 \to z$ **do**
13: $player \leftarrow O(k)$
14: *curCapState, curSpeedState* $\leftarrow$ $\arg\max_{c_L^k \in C_k} \mathbb{E}[r^k(c_L^k, c_L^{-k}) | s^*, f^*]$
15: **end for**
16: **until** *prevCapState*=*curCapState* **and**
17: *prevSpeedState*=*curSpeedState* ▷ SPE found
18: $c^* \leftarrow$ *(curCapState, curSpeedState)*
19: $X \leftarrow X \cup \{\{s^*, f^*, c^*\}\}$
20: **end for**
21: **end procedure**

Note that in maximising the investor's utility (line 14), prices are actually calculated numerically first. Then drivers' probabilities are calculated numerically. At this point, boundary conditions are checked and enforced. If the probabilities for some stations are negative, they are set to zero and probabilities and prices are recalculated as these stations are not desirable (see Section IV-A). In case all stations have zero capacity and there is no outside option, station utilities are set to $-\infty$ to ensure that at least one station is there to serve the drivers. Last, the IBR algorithm can locate only one SPE with a given initialisation and playing order. To locate all possible equilibria, the IBR is repeated several times (line 5), adding the resulting SPE to the set of SPEs if not present (line 19). It has been determined empirically that if there are λ SPEs, a number of repetitions $K = 10\lambda$ will find all equilibria.

V. EMPIRICAL ANALYSIS

As is common in game-theoretic analysis, we utilise duopoly examples—where allowed—for tractability in evaluating qualitative behavioural characteristics. The presented work, however, can be utilised for larger numbers of investors and locations. In some cases there are many SPEs for particular parameter settings. Only the SPE with the worst utility for investors will be shown. Furthermore, because capacities are discrete, it is possible for investors to play different strategies (asymmetric equilibria) even where all their parameters are the same. In that case, all permutations of these strategies across stations are also SPEs. The experiments that follow present a sensitivity analysis which utilises reference settings. In each experiment one parameter is varied to observe the outcome.

Reference parameter settings will now be explained. Drivers drive the Nissan Leaf with a $24kW$ battery and charging efficiency of 85% [2]. This results in a requirement $E = 28.24kW$ to fully charge the battery. Given a cost of £0.1 per kWh, which was a realistic price at the time of the experiments, recharging each EV costs $h_1 = h_2 = \$2.824$. Charging unit power output is set to $50kW$ [2]. For simplicity, it is assumed that the output of charging units is linear over time, hence charging time is roughly 33 minutes and 40 seconds. In the model, however, time will be represented in half-hours which makes charging time $R = 1.1294$ half-hours.

For a more realistic setting, experiments correspond to the inter-city trip Central Southampton→Central London, with a length of 80 miles, where EV drivers can also opt to use the train. At an average speed of 60*mph* the trip takes $10/3$ half-hours which is a realistic value without traffic. The value of time for driving and taking the train have been set to $\upsilon_d = 12.56$ *£/half-hour* and $\upsilon_t = 18.1$ *£/half-hour* respectively, based on the tables of the UK Department of Transport [36]. Trip length for going through charging stations is set to $t_1 = t_2 = 10/3$. To normalise profits, it was considered that a peak traffic of $n = 30$ drivers occurs three times a day. The station's daily income consists of the income during peak hours, plus income from the rest of the day which is assumed to be equal to the income in peak hours. The game is played with a horizon of 1 year. This makes profit normalisation $w = 365 * 6 = 2190$. One-time building costs are set to $o_1, o_2 = \$30000$, and costs for adding charging units b_1, b_2

are set to $36000. Rapid charging unit installation costs can vary, but these values were realistic to consider for $50kW$ rapid DC chargers at the time of the experiments, including a cycle of yearly maintenance [3]. The train ticket costs $f_t =$ £21.9, and the trip with the train lasts is $t_t = 4$ half-hours, including 20 minutes to commute to and from train stations. Last, the calibration parameter D is set to $D = 0.95$, and this is explained in Section V-A below.

A. Calibrating the Model

The parameter D is used in Section III-A to model the drivers' disappointment at not being able to use their EV. An increasing D will effectively set a worse benchmark for stations, and a decreasing D a better one, because stations can take action whereas the train cannot. Indeed, using the reference settings, investors reduce capacity for an increasing D (Fig. 2a top) and increase price (Fig. 2a bottom), and an increasing portion of drivers opt to use the train (Fig. 2b). Looking at (3), (4) and (7), an increasing D biases drivers toward using their EVs more, by setting lower utility for the train. This gives investors headroom to increase prices and decrease capacity, which investors take advantage of. In turn, this causes more drivers to use the train as services deteriorate. This behaviour can be observed in Fig. 2b, where the probability of taking the train climbs every time investors either decrease capacity or increase prices significantly. After each climb, it slowly reduces due to the bias D induces, even though prices keep increasing.

It is still a question, however, what value should be set to D. With good service by stations, no EV driver should have to use the train. However, setting D exactly on the margin where $s_i^m > 0$ will mean that a small increase in traffic will cause some drivers to use the train. Therefore D has to be a little lower, to allow for fluctuation in EV traffic. For the reference settings, a value of $D = 0.95$ will be used for $n = 30$ drivers.

B. SPE Efficiency and Robustness

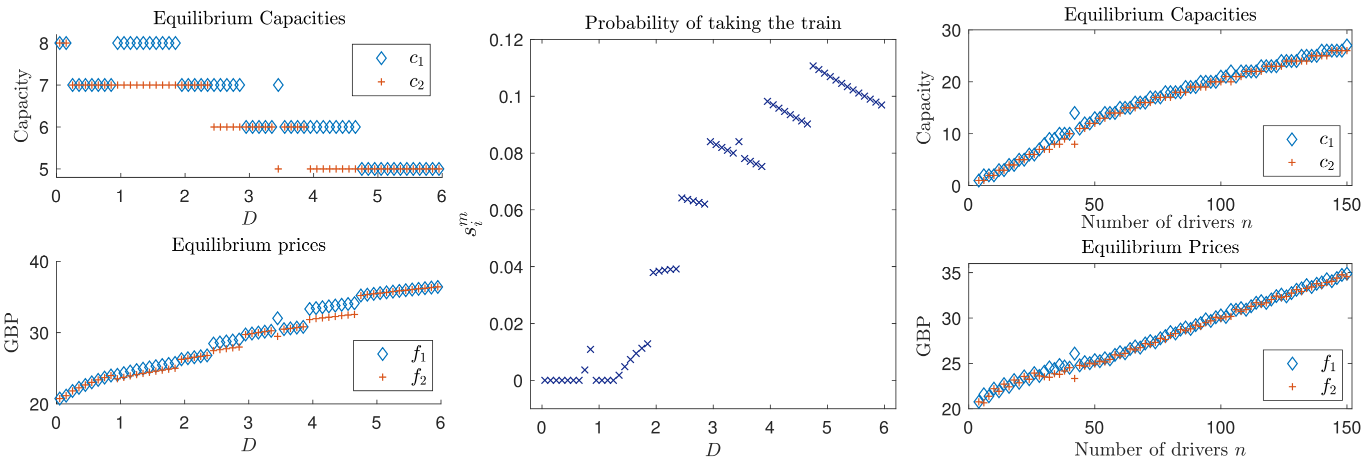
To measure the system-wide efficiency of SPEs, we use the ratio of maximum social welfare over the worst-case social welfare in SPE. This is similar to the Price of Anarchy where utility in equilibrium is compared with utility in a centralised optimum strategy [38]. However, both investors and drivers make decisions, thus the optimal strategy (i.e. where drivers also follow an optimal routing policy) will offer little insight. More meaningful is to allocate stations optimally, *given that drivers will play a mixed NE.*

If X is the set of all SPEs, worst-case social welfare is defined as the sum of utilities in the SPE in which the sum is minimum. The maximum social welfare is defined as the maximum sum of utilities of all players across capacity and price strategies. However, note that the drivers' expected utility in (3) and (4) is negative, and all drivers definitely lose more utility than the investors gain (drivers pay other travel costs, in addition to the fees). Therefore to get the correct Optimum/SPE proportion we need to reverse the actual fraction, as both optimal and SPE social welfare are negative. SPE efficiency is then defined as:

$$SWR = \frac{\min_{\{c^*, f^*, s^*\}_\rho \in X} \left(\sum_{j \in I} \mathbb{E}[r^j(c^*, f^*)|s^*] + nw\mathbb{E}[u_i(x)|s^*, f^*, c^*] \right)}{\max_{c \in C, f \in F} \left(\sum_{j \in I} \mathbb{E}[r^j(c, f)|s^*] + nw\mathbb{E}[u_i(x)|s^*, f, c] \right)}$$

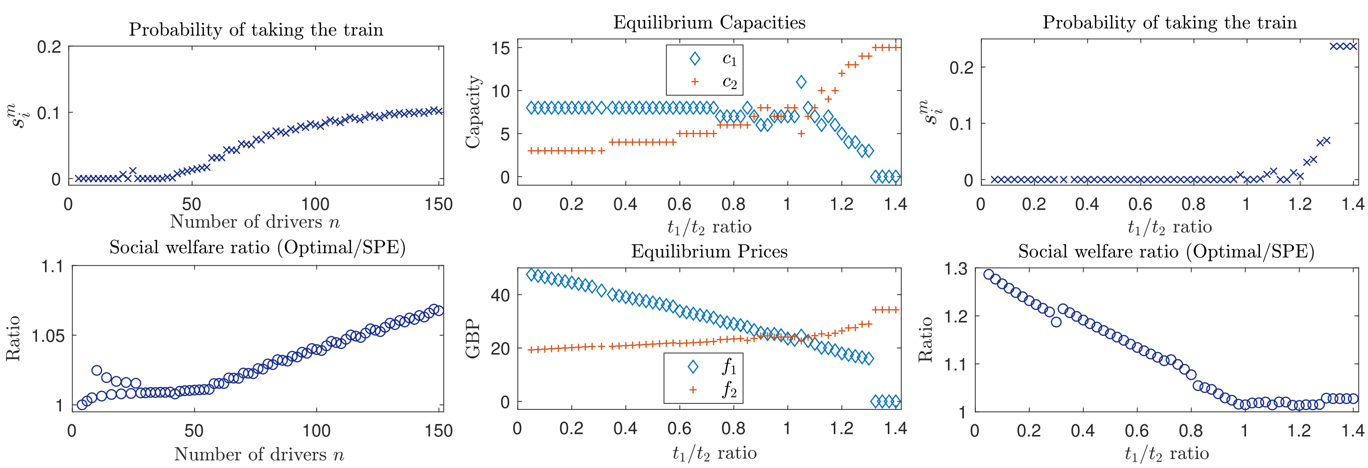
In Section V-A it was hypothesised that an increasing number of drivers beyond $n = 30$, which this model instance has been calibrated for, will use the train increasingly. This is now tested using the reference settings. Results show that SPE capacities (Fig. 2c top) and prices (Fig. 2c bottom) will increase with an increasing n. It is noticed from Fig. 3a top, that at $n = 60$ the probability of taking the train also rises more rapidly, and the data shows that it has already started rising from $n = 39$ onward. The chosen value $D = 0.95$ thus provides headroom for a 30% increase in peak traffic before drivers consider using the train. In terms of robustness of the solution, reasonable fluctuations in peak traffic do not cause behavioural anomalies from investors. That is stations can micro-adjust prices daily to account for small changes in expected traffic, rather than having to adjust capacity. This is also reflected in the SWR (Fig. 3a bottom), which remains largely unaffected around $n = 30$. The SPE solution is found to be very efficient, with optimal station allocation yielding at most 2.4% better system-wide utility compared to the worst SPE, up to a peak traffic of 56 drivers. Efficiency decreases more rapidly after $n = 56$, which is nearly double the peak traffic the model was calibrated for, but still is over 93% even for $n = 150$ drivers.

Interesting is also the situation where the travel time for only station 1 varies, because this can conceptually represent situations where travel time can vary e.g. due to traffic congestion, which this model does not address explicitly. Results show that station 2 will start from a much lower capacity than station 1 (Fig. 3b top) as it is heavily disadvantaged when t_1 is very small. At the same time, station 1, who is also on a very favourable route to start with, will ask for a very high charging price (Fig. 3b bottom), which is in line with findings in Section IV-B. As t_1 approaches t_2, station 2 becomes more competitive and station 1 maintains capacity and reduces price, while station 2 increases capacity and price. When t_1/t_2 enters more realistic levels around 1, stations seem to alter capacities more frequently. However, only the worst SPE shown here. There is also a SPE at $t_1/t_2 = 0.9, 0.925$ for both stations to have a capacity of 7, that is the same capacities as in $t_1/t_2 = 0.875, 0.95, 0.975$. At $t_1/t_2 = 1.2$, the travel time for station 1 equals the travel time for the train. Beyond that, station 1 loses customers to the train (as seen in Fig. 3c top) and reduces capacity. Station 2 increases capacity until station 1 cannot compete anymore and does not open at all, and station 2 then maximises against the train. The model is quite robust to fluctuations in travel time. When the two routes have comparable travel time, SPEs are very efficient (Fig. 3c bottom). For example, in the travel time ratio range $0.8 - 1.2$, that is for a 20% variation in travel



(a) (b) (c)

Fig. 2. SPE capacities and prices (a) and probability of taking the train (b) for an increasing D. When stations can decide capacity, an increasing D leads to a deterioration in station services, and a generally increasing tendency to use the train. Capacities and prices for an increasing n (c) show an increasing trend. After $n = 60$, investors become less inclined to invest and increase prices more.



(a) (b) (c)

Fig. 3. Probability of using the train and SPE efficiency for an increasing n (a). The probability increases with an increasing peak traffic, and SPE efficiency decreases although it is generally very good. SPE capacities and prices (b), and probability of using the train and SPE efficiency (c) for an increasing t_1/t_2 ratio. Station 1 has a very high advantage when the ratio is low, but is forced to not open soon after it becomes less competitive than the train.

time, SPEs are at least 92.85% efficient. Larger travel time differences reduce efficiency, albeit they represent less realistic station competition, because in reality stations only compete with others within a certain range.

The parameter ranges within which SPEs remain highly efficient for the given reference settings and calibration are summarised in Table I. Note that sensitivity analysis has considered all parameters, but only n and t_j showed a noteworthy effect on the efficiency of SPEs. An additional aspect to be noted here is that the probability distribution for the drivers' choices is calculated theoretically and not from observation, hence no dispersion around solutions is included. In practice, we have found that when $n = 30$ drivers are sampled from that theoretical distribution, there is a standard deviation of $\pm 20\%$ for the expected traffic flow which is within the 30% SPE efficiency limit identified above. This dispersion reduces significantly with an increasing number of drivers. Also because the model considers several peak traffic incidences per day over a period of 365 days, this dispersion is expected to even out across available choices hence average station traffic flows are not going to be significantly different from the theoretical distribution should one choose to simulate the driver model than solve it theoretically.

TABLE I

RANGE OF PARAMETERS WITHIN WHICH SPES REMAIN VERY EFFICIENT WITH A GIVEN CALIBRATION FOR THE REFERENCE SETTINGS

Parameter	Range
Number of drivers (n)	$n \leq 1.3n$
Travel time (t_1)	$0.8t_2 \leq t_1 \leq 1.2t_2$

C. Equilibrium Prices

In Section IV-B it was shown that when station capacities are constant, equilibrium charging prices will converge

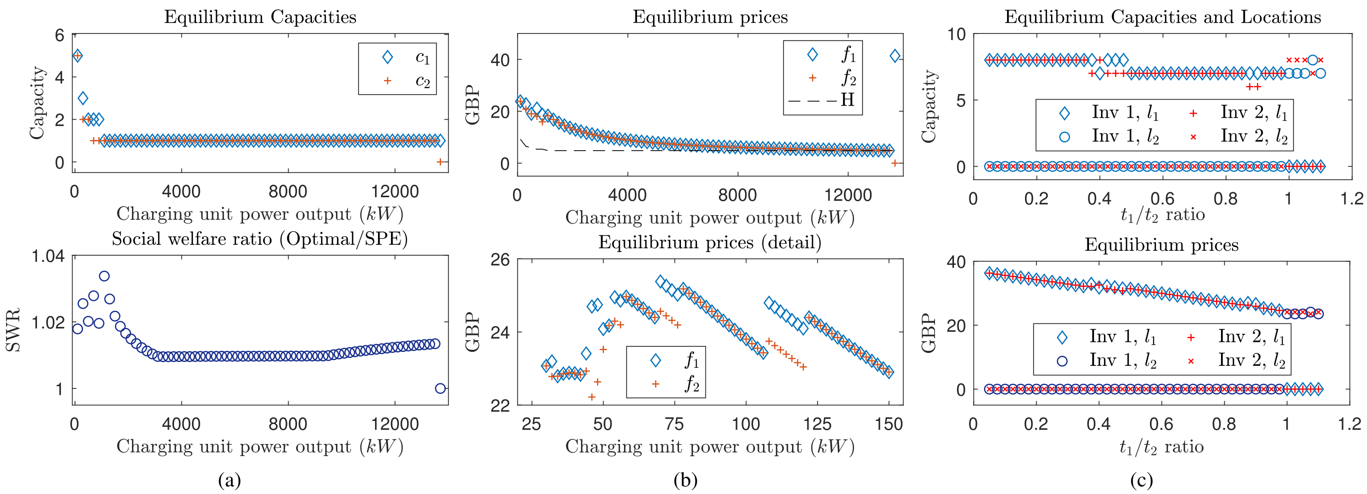


Fig. 4. SPE capacities and efficiency (a), and prices (b) for an increasing charging unit power output. As output increases, investors lose incentive to build more units and prices converge to marginal cost. SPE capacities and prices when investors can build in multiple locations (c). Investors build on location 1 until it less competitive than location 2 when they switch to 2. Notice both choose the same location, which makes the problem symmetric for investors.

asymptotically to marginal cost h as charging time decreases. To investigate this further with stations that choose capacities, we employ a duopoly with reference settings where the power output of the charging units is varied symmetrically. It is hypothesised that investors will show the same behaviour, only now prices will converge to the marginal charging cost H. This should be $h = \$2.8235$ in this case, plus the cost of building the stations shared among the drivers that will recharge ($H = h + \frac{c_1 b_1 + c_2 b_2 + o_1 + o_2}{nw}$).

Indeed, experiments (Fig. 4a top) showed that capacity decreases with an increasing output. At $700kW$ station 2 reduces capacity to 1, and at $1000kW$ station 1 also does the same. From there, prices (Fig. 4b top) start an asymptotic movement toward $H = 4.8326$. At $13575kW$, charging prices are at $\$4.8334$. Immediately after, station 2 closes and station 1 maximises price. Note that in the end, it is also a SPE for station 1 to close and for station 2 to maximise instead. SPEs are very efficient regardless of charging unit power output as seen in Fig. 4a bottom, with optimum allocation being at most 3.8% more efficient than SPEs. By keeping the cost of charging units constant, this experiment essentially demonstrates a technological time-line, as better technology will become more accessible. Even then, it is certain that prices will converge toward the marginal cost at the time, which is difficult to estimate given this may be many decades away. This experiment is also equivalent to the situation where the value of time approaches zero, and the situation where drivers are increasingly indifferent to queues. In these cases, stations do not have incentive to increase capacity beyond 1 which reduces the problem to Bertrand-like competition.

A more microscopic look into prices (Fig. 4b bottom) within our current mainstream technological window, reveals that we are considerably far away from a significant reduction in prices due to quicker satisfaction of demand. In fact, if investors are able to purchase charging units of up to $75kW$ at the price of $50kW$ units, prices (Fig. 4b bottom) will still rise. However driver utility (not shown here), will keep improving as better technology becomes more available due to a reduction in queuing times. After prices peak around $75kW$ they start reducing again. However, even for a power output of $150kW$, prices are still not significantly lower than for $40-50kW$ charging units, and that is assuming all units cost the same.

D. Location and Power Output Competition

To evaluate location choice we consider the reference settings, only now each of the two investors can build one station in each route if they wish to. Results have shown that location choice is independent of peak traffic n. Furthermore, as was hypothesised in Section IV-B, investors do prefer the same route when travel times are different. When the t_1/t_2 ratio is varied for the two locations, both investors prefer location 1 when it offers a faster route (Fig. 4c top). At $t_1/t_2 = 1$, it is an equilibrium for each investor to build in either route, and past that investors prefer location 2. It is interesting that given identical building costs for each location, an investor will never choose to build on multiple locations, even when $o_J = 0$, which is reasonable by everything observed so far. Stations at longer routes have to offer lower prices to be competitive, which means they need a larger investment to produce the same revenue as in shorter routes. Of course, investors will prefer longer routes if building costs at those are low enough.

Returning to two stations with reference settings, we now evaluate the choice of charging unit power output. Two charging unit options are given to investors, a $50kW$ option which costs £36000 as before, and an $80kW$ option the cost of which will vary. Investors both choose the $80kW$ charging units (Fig. 5b) when their cost is low, and at $\$92000$ station 1 switches to the 'slower', $50kW$ units. Up to that point, as the cost of $80kW$ units increases, first station 2 reduces capacity (Fig. 5a top) and increases price (Fig. 5a bottom) at $\$47000$. In response, station 1 asks for a higher price now that it offers better service than 2. After $\$52000$ and up to $\$92000$ stations play the same strategy. Then, station 1 chooses the $50kW$ charging units and reduces price, and station 2 further increases price. It is noteworthy that the expected utility for driver i (Fig. 5c) when both stations are using $50kW$ units is

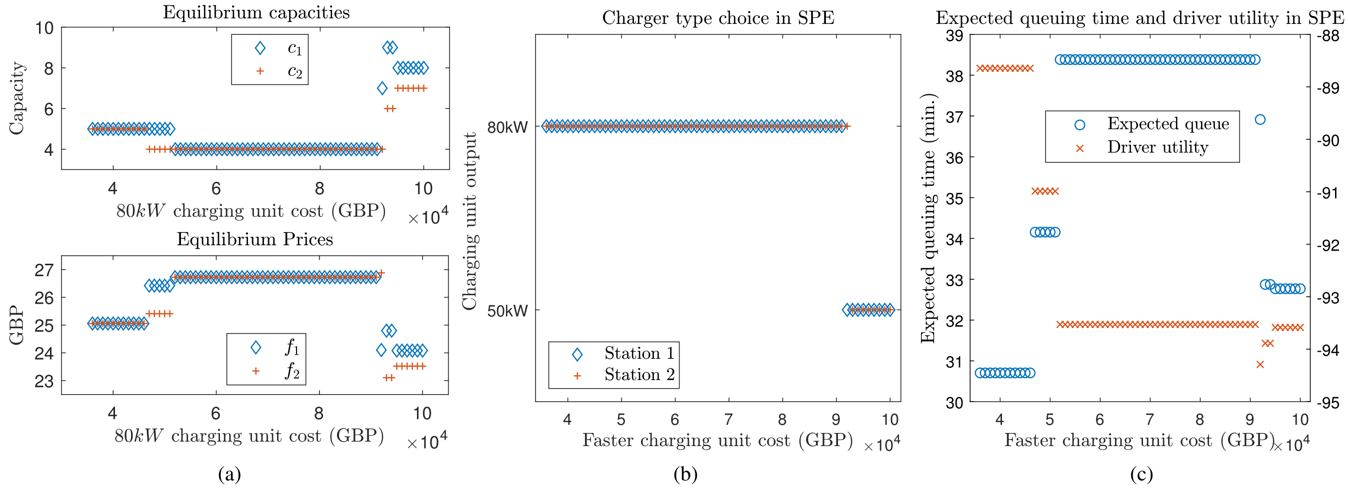


Fig. 5. SPE capacities and prices (a), charging unit output (b), and driver expected queue and utility (c) for an increasing cost of the $80kW$ unit. Investors will choose to build the 'slower' but more affordable units when the $80kW$ units become too expensive. Notice that driver utility is not necessarily higher with the faster units, as prices can be considerably higher and stations will reduce capacity which increases queues.

highly comparable to the utility before they switched. Average expected queuing time for i is worse from \$52000 and on with faster charging units, because investors take advantage of faster charging to build less charging units and ask for higher fees.

E. Subsidies

The model has also been applied to examine how subsidies to charging stations can affect the utility for stations and drivers. This is in essence a sensitivity analysis where the cost of building a charging unit, or the cost of electricity is varied with the addition of relevant metrics to evaluate subsidies. In order to determine the efficiency of a subsidy, the difference between utility with the subsidy and without it, is divided by the total cost of the subsidy. This is done for all investors and all drivers separately. Having used v_d to convert time costs in driver i's utility, the total monetary gain for drivers can be found, provided no drivers use the train as D does not represent a monetary quantity. First, considering a subsidy level σ for each charging unit, we examine the efficiency of subsidies on the purchase cost of charging units. In this case, the efficiency for all investors is:

$$\epsilon_I = \frac{\sum_{k=1}^{z} \mathbb{E}_{su}[r^k(c)|s] - \sum_{k=1}^{z} \mathbb{E}_0[r^k(c)|s]}{\sigma \sum_{k=1}^{z} \sum_{j=1}^{\mu} c_j^k} \tag{12}$$

and for all drivers is:

$$\epsilon_N = nw \frac{\mathbb{E}_{su}[u_i(x)|s] - \mathbb{E}_0[u_i(x)|s]}{\sigma \sum_{k=1}^{z} \sum_{j=1}^{\mu} c_j^k} \tag{13}$$

Using reference settings, stations initially absorb the entire subsidy up to £10500 (Fig. 6a), with no change in capacity or prices. From there, however the subsidy begins to take effect. Capacity subsidies have been found to be very beneficial for both drivers and stations, and can often generate more than one pound in system-wide utility for each pound spent in the subsidy. This is because they provide incentive to investors to both increase capacity and reduce prices, which significantly improves utility for drivers. Additional results show the effectiveness of capacity subsidies increases with an increasing number of drivers. It is also straightforward to determine optimal subsidy levels. For example, optimal system-wide utility is generated at a subsidy level $\sigma = \$16500$, where each pound spent in subsidies generates $\epsilon_I + \epsilon_N = 1.143$ pounds in utility. Peak efficiency for the drivers is at \$23500 and for investors at \$15000.

In a similar way, we analyse subsidising the price per kWh at which stations buy electricity. Efficiency is as in (12) and (13), only now the total cost of the subsidy in the denominator is $w(1 - s_i^m)nE\sigma$. It is noticeable that very small subsidies can result in an improvement for drivers and stations (Fig. 6b), but the subsidy is generally absorbed by stations mostly, who increase profits at the expense of drivers. At a subsidy level of \$0.012, station 1 reduces capacity to 7 and both stations increase prices. Furthermore, at 0.018 there is an increase in average capacity back to the initial levels, but this is followed again by a spike in prices. This results in drivers losing considerable utility overall. Additional results for a varying number of drivers show similar behaviour, with the subsidy being mostly absorbed by stations and minimal, if any, benefit for drivers.

F. Complexity

The drivers' equilibrium in (7) is quick to solve for many stations, and is independent of the number of drivers as the equilibrium is symmetric. As for the equilibrium in prices in (10), even though each of the $z\mu$ partial derivatives is linear with respect to that location's price, solving it symbolically shows an upper limit of 8 stations. This is because each derivative includes μ capacity terms, each of which to the power $\mu - 1$, that is a solution complexity between $O(z^2\mu^{\mu+1})$

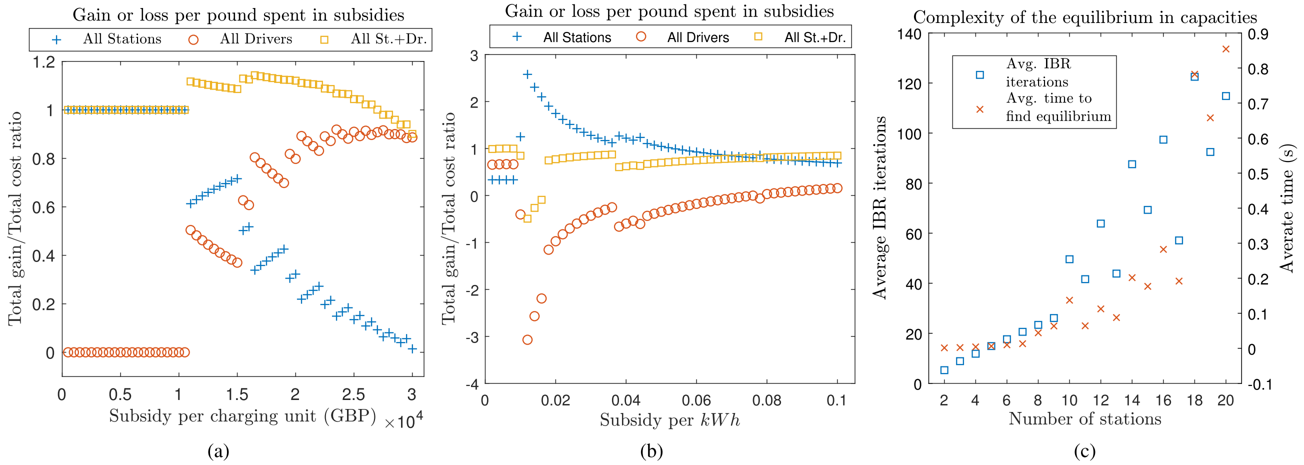


Fig. 6. Capacity subsidy efficiency (a) and electricity price subsidy efficiency (b), iterated best response complexity (c). Capacity subsidies can generate more than £1 in utility for drivers and stations, for each pound spent in the subsidy, and are quite efficient. *kWh* subsidies are not as efficient and can even result is drivers losing utility. The IBR algorithm simplifies calculating the pure NE, but complexity still increases exponentially with the number of stations.

and $O(z^3\mu^{\mu+2})$. For larger settings it is better to solve the equilibrium in prices numerically, which takes about $0.05s$ for 12 stations. Complexity increase comes with calculating the capacity, location and output equilibria. Each investor's utility function is a $(\Theta+1)^{\mu z}\psi^z$ table, and checking whether any investor can deviate through each strategy profile requires $O(z^2(\Theta+1)^{\mu z}\psi^z)$ time. The IBR algorithm reduces this significantly, by calculating the utility of the current player only for a given state of the other players. However, complexity still increases exponentially with the number of stations (Fig. 6c), and the IBR needs to be run many times to locate all SPEs. In practice, complexity is more of an issue in generic scenarios and parameter exploration. In reality there are several constraints which reduce complexity significantly. For example, it is unlikely for many investors to decide across many locations simultaneously, and stations practically compete with other stations within a certain range. More realistic is running the model for a few new investors given existing competition, or to use the model to consult investors on adjusting their prices daily which is very quick to compute.

VI. Conclusion

This paper presented an extensive form game, as a model for deploying an en-route EV charging station network, and has proposed subgame-perfect equilibria (SPEs) to decide competing stations' locations, capacities, charging unit power outputs and prices, given that congestion may occur at stations. The model extends the state of the art in firm competition significantly, by combining several aspects of network pricing games, sequential games and spatial competition, and extends other models of competing EV charging stations by considering several investor decisions and extraneous competition.

This approach has enabled answering substantive questions on charging station competition. Specifically, charging stations' fees will be significantly higher than the cost for stations to recharge EVs (Sections IV-B and V-C), and convergence of these two requires vastly superior technology. In addition, longer routes impose a handicap in charging price and may be undesired by investors (Sections IV-B and V-D). SPEs are highly efficient for the drivers and investors, with worst-case social welfare within 92.85% of optimal, and in many cases within 2.4%. It is important that within the current technological window, switching to faster charging units does not necessarily translate to lower travel costs—or even queues—for drivers, as investors can take advantage of faster charging times to increase prices and decrease capacities (Section V-D). Last, metrics for the efficiency of subsidies to charging stations were defined (Section V-E). Subsidising charging units helps increase capacities and decrease prices, with an increasing portion of the subsidy absorbed by the drivers as the subsidy increases. Capacity subsidies can generate value for drivers and investors in excess of the subsidy's cost (up to 14.3% in the example here). In contrast, subsidising the kWh price of electricity entails a significant risk of stations reducing average capacity and increasing prices, leading to increased travel cost for drivers and abuse of the subsidy by investors.

Although experimental settings were based on real-world parameters and additional results show that the model is generally robust, some realism was sacrificed for abstraction to allow for game-theoretic analysis. However, this also enables using the model for other similar problems in which customers minimise expected costs and demand is uncertain, and firms can set prices and the speed of service. Future work will firstly involve improving computational complexity to promote large-scale application, which includes approximating pure strategy Nash equilibria in the third stage. A second major step will be to consider drivers who have different utilities, perform different trips and have different choices available. Using closed-form solutions for these in larger settings will not be straightforward, but it is also possible to simulate heterogeneous settings and asymmetric stochastic behaviour, or to use techniques such as evolutionary learning.

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