MATH.APP.450

Project: Locating

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Contents

1 Abstract

The aim is to tackle the issue of localization in a windowless environment and predict the location of the vehicle on a colorful checkered map with limited memory available. The colors can be blue, green, red and white and the observation of the current cell is recorded each time a step is taken. However, the initial location of the vehicle is unknown. There is also a constraint on the orientation of the movement of the vehicle and the probability of getting an incorrect observation is 0.001. We propose a probabilistic Bayesian model which predicts the location of the vehicle on the map based on the observed colors of the cells. The model will taken into account the uncertainty at the beginning of the motion, the memory capacity available and the chances of getting incorrect observations. The model makes some assumption at the beginning about the accuracy of the motion, the orientation of the motion and the precision of calculation for the probability.We go through the process of building the model, where we consider the color observation model which discusses the probability of the perception of color. Then we have the motion model which considers the movement of the vehicle around the map. The initial distribution before any observation is made is also considered. And lastly, the derivation of the model is done based on Bayes' theorem. After that, we proceed to discuss the solutions to our model in all possible scenarios. We consider the working of the model when the assumptions we made at the beginning are removed and try to give as practical solutions as possible. Finally, we present a simulation of the model which is implemented using Matlab.

2 Introduction

The purpose of this project is to model a situation where we predict the location of a windowless vehicle on a map where each cell is colored either blue, green, red or white.

In this problem situation, the vehicle can only move forward, backward, left or right and steering is not allowed. There is no knowledge of the location of the vehicle at the beginning. For each step the vehicle takes, an observation of the color of the current cell is recorded. Another layer of complexity is added when the memory is limited to only 10 steps. Lastly, the probability of getting an incorrect color is given as 0.001.

In this report we go through the process of building a probabilistic model based on Bayes' Theorem to formulate a solution for this Localization problem. Some assumptions and considerations are made at the beginning of the project based on which the solutions is derived. The goal is to provide a practical model and solution keeping in mind the conditions given.

3 Model

3.1 Assumptions

Based on the available information given by the project, we have made some assumptions:
- Each step is taken with the 100% accuracy, meaning that the vehicle is always in the middle of the cells.

- The possibility that you will come to the edge of the area does not need to be considered.
- You don't know the orientation of your movement, meaning the direction of your movement doesn't match the way you view the map.
- If there's one cell that reaches 0.999 probability, we can conclude that the vehicle location is in that cell.

3.2 Building the model

The problem has many uncertainties: where you are and your perception of the correct color of the cell; therefore, we proceed to see further if the probabilistic approach will be used to solve the problem, more specifically Bayesian method. With this method, we don't have to worry the memory limit, as the knowledge of the previous steps is already updated in the current step.
We have observation as O_t, the color of each tile as T_t, and x_t be the state of the whole map at step t.

3.2.1 Color observation model

Given the information of the probability of observing the color correctly, we created a model of the color observed each time:
Let $P_{\text{incorrect}}$ be the probability of incorrect color perception, and it equals $= 0.001$. The probability of correct color perception is then $P_{\text{correct}} = 1 - P_{\text{incorrect}} = 0.999$.
We have four different colors. If one of them is observed, there is a $P_{\text{correct}} = 0.999$ chance of correctly perceiving the color. For each of the other three colors, the probability of correctly perceiving that specific color is $P_{\text{other}} = P_{\text{incorrect}} \times \frac{1}{3}$.

3.2.2 Motion model

As the possibility of coming to the edge of the area does not need to be considered, each cell in the map can always be reached from all four directions: left, right, up, down; therefore, $\frac{1}{4}$ possibility that each adjacent cell has to step to the considered cell in the map.

3.2.3 Initial distribution

The initial probability of each cell in the map is predicted to be distributed equally before any observations made: $\frac{1}{n}$ with n is the number of cells in the map.

3.2.4 Derivation of the model

Starting with applying the definition of Bayes' theorem to the map system, we have:

$$p(x_t|O_t) = \frac{p(O_t|x_t) \cdot p(x_t)}{p(O_t)} \tag{1}$$

In words, Bayes' theorem allows to update the prior probability in the probability of an observation of color based on new observation after a new movement (one step away of the current cell) is made.
That is the state of the whole map. However, to create and update the probability state for it, probability of each cell in the map at any step t needs to be considered. From this point

on, the probability of the state will be "deducted" to only the probability of an arbitrary cell, which also represents each of every cell in the map:

$$p(C_{a,b}|O_t) = \frac{p(O_t|C_{a,b}) \cdot p(C_{a,b})}{p(O_t)} \tag{2}$$

Using the law of total probability, we can express the denominator in terms of the joint probability:

$$p(C_{a,b}|O_t) = \frac{p(O_t|C_{a,b}) \cdot p(C_{a,b})}{\sum_{i,j=1}^{n} p(O_t|C_{i,j}) \cdot p(C_{i,j})} \tag{3}$$

We have the first update, when there's no step taken yet:

$$\begin{aligned} p(C_{a,b}|O_0) &= \frac{p(O_0|C_{a,b}) \cdot p(C_{a,b})}{\sum_{i,j=1}^{n} p(O_0|C_{i,j}) \cdot p(C_{i,j})} \\ &= \eta \cdot p(O_0|C_{a,b}) \cdot p(C_{a,b}) \end{aligned} \tag{4}$$

As we proceed step by step, we update the probabilities of the cells accordingly. For each cell, there are four neighboring cells. This implies that if one cell is set to be the current location , there is a $p_{\text{prev cell}} = \frac{1}{4}$ chance for each of these adjacent cells to be the previous location. Therefore, when re-evaluating our prior probability of the any cell, we have to take into account the sum of all the probabilities of surrounding cells in the previous update. Let continue with cell $C_{a,b}$ after the first step:

$$\begin{aligned} p(C_{a,b}|O_t) &= \frac{p(O_t|C_{a,b}) \cdot p(C_{a,b})}{\sum_{i,j=1}^{n} p(O_{t-1}|C_{i,j}) \cdot p(C_{i,j})} \\ &= \frac{p(O_t|C_{a,b}) \cdot \sum_{m=1}^{4} p(C_m|O_{t-1}) \cdot p_{\text{prev cell}}}{\sum_{i,j=1}^{n} \sum_{m=1}^{4} p(C_m|O_{t-1}) \cdot p_{\text{prev cell}} \cdot p(O_t|C_{i,j})} \\ &= \eta \cdot p(O_t|C_{a,b}) \cdot \sum_{m=1}^{4} p(C_m|O_{t-1}) \cdot p_{\text{prev cell}} \end{aligned} \tag{5}$$

Where:

- $p(x_t|O_t)$: Probability of the map x_t given observation O_t at step t. This is in the model is referred as probability of an arbitrary cell $p(C_{a,b}|O_t)$.
- $p(O_t|x_t)$ or $p(O_t|C_{a,b})$: Likelihood of observing O_t given the map/ arbitrary cell $C_{a,b}$. The values given are either 0.999 if the cell in the map has the same color as the observation, else $0.001 \cdot \frac{1}{3}$ for 3 remaining color cells.
- $p(x_t)$ or $p(C_{a,b}$: Prior probability of the map x_t or arbitrary cell $C_{a,b}$.
- $p(O_t)$: Probability of observation O_t at step t given both the evaluated cell and other cells that are not it.
- $\sum_{i,j=1}^{n} p(O_t|C_{i,j}) \cdot p(C_{i,j})$ and $\sum_{i,j=1}^{n} \sum_{m=1}^{4} p(C_m|O_{t-1}) \cdot p_{\text{prev cell}} \cdot p(O_t|C_{i,j})$: normalized factor at each step, which is calculated by total probability of all the cells in

the map. Later is referred to as η.

- $\sum_{m=1}^{4} p(C_m|O_{t-1}) \cdot p_{\text{prev cell}}$: Prior probability of cell $C_{a,b}$ location, calculated by total sum of probability of the location, which can be anywhere of four adjacent cells, in the previous step.

- $p_{\text{prev cell}}$: The probability of each adjacent cell was the previous cell of the current cell. That equals to $\frac{1}{4}$.

Iterate equation (5) for all cells in the map after taking each step until we find a cell that has probability that is equal or more than 0.999.
* Note: At each update of the state after a step, the denominator remains a constant as a normalized factor, so we only need to do it once when all the probabilities of all cells are calculated. For this reason, we will shorten it to η in references.

4 Results

In real life, our model will need remodelling to suit the uncertainty of the movement, since each step is also made with imperfect metric. In that case, we can remodel the motion model with, e.g, Gaussian distribution.

Another thing is the edge of the map, as in real life, the map can't have infinite size. So we need to modify the iteration formula a bit. Instead of generalization that there are always four neighboring cells, we can have three for side cells, and two for corner cells, which can be easily cooperated in our model in equation (5). With this additional requirement, it helps improve the margin error caused by the four-edge generation in simulation.

Our model is robust and is able to give the location with equal or more than 99.99% accuracy, without considering the memory limit or known orientation of the movement.

However, when conducting with simulation of the model, we find the performance, though works well (evaluated by the fact that when tracking the probability of the cells and compare with the actual locations), can't seem to find the location within 10 steps/iterations as expected, even for a map as small as 3x3. Therefore, our model may require many iterations to be able to find the location with equal or more than 99.99% accuracy.

As the number of cells increases, the computation load is larger, and therefore, it takes longer to find the location. In the continuation of this project, we will consider to eliminate areas of the map that constantly have low probabilities after certain amount of iterations to reduce the computations.

5 Summary

Our model gives an assumption based solution where we consider an Uniform distribution and apply the Bayes' Theorem to find the location of the cell.

The model is well-defined and can reach 99.99% accuracy for the situation where the orientation of the movement of the vehicle is unknown and there is a memory constraint. However, while simulating, the model cannot find the location within 10 steps as expected.

Also, in real-life scenario, we are bound to encounter an edge. The iteration formula will be modified then to take into account the fact that the edge cells will have three neighbouring cells and the corner cells will have two neighbouring cells.

In further developments of the project, efforts can be made to eliminate areas of the map that have constantly low probabilities after certain amount of iterations to reduce the computation time.

6 References

6.1 Books

Thrun, Sebastian and Burgard, Wolfram and Fox, Dieter. (2005). *Probabilistic Robotics.* MIT Press. Cambridge, Mass.

7 Attachments

The code and simulation of the model with 10x10 grid as an example.

Observation + motion model

```
% Define the colors
colors = {'green', 'red', 'white', 'blue'};

% Perception probability =  [correct incorrect]
p_O_given_C = [0.999 0.001*(1/3)];

% The probability of each adjacent cell was the previous cell of the
% current cell
p_prev_cell = 1/4;
```

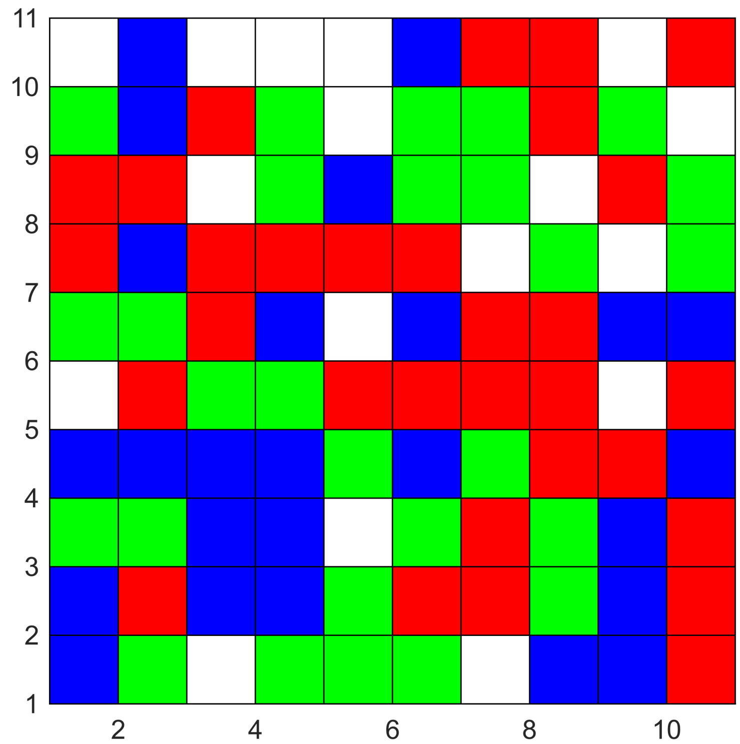
Making the map

```
% Define the size of the map
length = 10;
width = 10;

% Initialize the map with probabilities
map = ones(width, length) * 1/(length*width);
cells = cell(width, length);

% Plot each grid with its color
for i = 1:width
    for j = 1:length
        color_index = randi([1,4]); % Generate a random index between 1 and 4
        rectangle('Position', [j, i, 1, 1], 'FaceColor', colors{color_index}, 'EdgeColor', 'k'
        cells{i,j} = colors{color_index};
    end
end

% So the stored data match the figure
%cells = flipud(cells)
cells = rot90(cells, 3);
axis([1, length+1, 1, width+1]);
axis square;
```




```
% We generate ten real color sequence called O_n from the map, representing
% the observations we make as the vehicle moves. This is to verify the
% algorithm, and stay unknown to us in real life.
O_n = {cells{4,5}, cells{5,5}, cells{5,6}, cells{5,7}, cells{6,7}, cells{6,8}, cells{7,8}, cel
```

Simulating the model

```
format long
% Assume the initial observation is blue
O = 'blue';
step = 0;
% Init normalized factor
eta = 0;

% Before any movement, calculate the map probability.
for i = 1:width
    for j = 1:length
        if strcmp(cells{j,i}, O)
            map(i,j) = p_O_given_C(1).*map(i,j);
        else
            map(i,j) = p_O_given_C(2).*map(i,j);
        end
        eta = eta + map(i,j);
    end
end
```

```
disp(['Current cell color: ', O]);
```

```
Current cell color: blue
```

```
disp(['Current step: ', num2str(step)]);
```

```
Current step: 0
```

```
% Normalize
map = map./eta
```

```
map = 10×10
   0.000013333333333   0.039960000000000   0.000013333333333   0.000013333333333 ···
   0.000013333333333   0.039960000000000   0.000013333333333   0.000013333333333
   0.000013333333333   0.000013333333333   0.000013333333333   0.000013333333333
   0.000013333333333   0.039960000000000   0.000013333333333   0.000013333333333
   0.000013333333333   0.000013333333333   0.000013333333333   0.039960000000000
   0.000013333333333   0.000013333333333   0.000013333333333   0.000013333333333
   0.039960000000000   0.039960000000000   0.039960000000000   0.039960000000000
   0.000013333333333   0.000013333333333   0.039960000000000   0.039960000000000
   0.039960000000000   0.000013333333333   0.039960000000000   0.039960000000000
   0.039960000000000   0.000013333333333   0.000013333333333   0.000013333333333
```

```
% Once the moving starts, iterate until a cell has probability >= 0.9999
%O_n = ['r', 'w', 'g', 'r', 'r', 'r', 'r', 'b', 'g', 'w'];

for n = 1:size(O_n, 2)
    disp(['Current cell color: ', O_n{n}]);
    disp(['Current step: ', num2str(n)]);

    eta = 0;
    for i = 1:width
        for j = 1:length
            sum_previous_steps = 0;
            % Compare the color of the cell and observation to choose
            % proper probability for color perception
            if strcmp(cells{j,i}, O_n{n})
                map(i,j) = p_O_given_C(1);
            else
                map(i,j) = p_O_given_C(2);
            end

            % Calculate p_C for cells inside the map
            if 1 < j && j < length && 1 < i && i < width
                sum_previous_steps = map(j-1,i)+map(j+1,i) + map(j,i-1) + map(j,i+1) ;
            end

            % Calculate p_C for cells at the edges
            if 1 < j && j < length
```

```
                if i == 1
                    sum_previous_steps = map(j-1,i)+map(j+1,i) + map(j,i+1);
                elseif i == width
                    sum_previous_steps = map(j-1,i)+map(j+1,i) + map(j,i-1);
                end
            end
            if 1 < i && i < width
                if j == 1
                    sum_previous_steps = map(j,i-1) + map(j,i+1) + map(j+1,i);
                elseif j == length
                    sum_previous_steps = map(j,i-1) + map(j,i+1) + map(j-1,i);
                end
            end

            % Calculate p_C for cells in the corners
            if i == 1
                if j == 1
                    sum_previous_steps = map(j+1,i) + map(j,i+1);
                elseif j == length
                    sum_previous_steps = map(j-1,i) + map(j,i+1);
                end
            end

            if i == width
                if j == 1
                    sum_previous_steps = map(j+1,i) + map(j,i-1);
                elseif j == length
                    sum_previous_steps = map(j-1,i) + map(j,i-1);
                end
            end
            % Update the probability of the cell
            map(i,j) = map(i,j).*p_prev_cell*sum_previous_steps;
            eta = eta + map(i,j);
        end
    end


    map = map./eta
    if max(map(:)) >= 0.9999
        % Set the flag to true
        break;
    end
end
```

```
Current cell color: blue
Current step: 1
map = 10×10
   0.000031320852195   0.093876416409899   0.000000031341747   0.000031331299443 ···
   0.000031313017628   0.023516076234416   0.000033290972463   0.000000041788996
   0.000007828419890   0.000001970126363   0.000000023832923   0.000031331301430
   0.000000000008706   0.000031792006177   0.000007838053954   0.000031324116307
   0.000007830214790   0.000007830219143   0.000000007837396   0.023514099274194
```

```
   0.000000005225366   0.000007835435802   0.000007843261663   0.000000016355032
   0.023482788054036   0.000054759289050   0.000062579062685   0.000078213391554
   0.000000010442025   0.000000023488027   0.000078213391554   0.000062579062685
   0.000039109304978   0.000000015662167   0.000062576453484   0.023506252593342
   0.000015649984072   0.000000010441155   0.000000010442025   0.000000015658685
Current cell color: white
Current step: 2
map = 10×10
   0.987837032913196   0.000164886776648   0.000350113148372   0.000499144219116 ···
   0.000164888475616   0.000000151244908   0.000000139160782   0.000000061909402
   0.000000055347513   0.000000029277510   0.000412073537491   0.000000255899512
   0.000000036049833   0.000000083236645   0.000000151174358   0.000082532760897
   0.000000090076865   0.000000013778934   0.000000041660614   0.000000137476633
   0.000020694890475   0.000000055391278   0.000000013872157   0.000000109790164
   0.000000020642308   0.000000000139590   0.000000281241562   0.000000013933920
   0.000000013796621   0.000000178388115   0.000000013924743   0.000000384089210
   0.000000006921200   0.000000034420605   0.000000274376357   0.000000027581789
   0.000000027483397   0.000000000059505   0.000082470609411   0.000000109759709
Current cell color: red
Current step: 3
map = 10×10
   0.000020146328363   0.000000014299816   0.000010077206996   0.000000013969150 ···
   0.000000011758344   0.000000010292250   0.075471913882227   0.000000014068094
   0.018849117552261   0.075449060806656   0.000012601616476   0.000005050696254
   0.000002836808482   0.000006289749589   0.015114077576251   0.003823834671062
   0.000000000003386   0.000000000213890   0.049440967183878   0.000066048052222
   0.000000316176005   0.000000022279347   0.000000005780751   0.000016496403803
   0.000000003145879   0.000000317020869   0.000000000015409   0.000000643580814
   0.000000316915544   0.000000003153066   0.000001260616091   0.000000010062927
   0.000000003251155   0.003778691007029   0.000000000014598   0.000001260297579
   0.000000000420552   0.000000000110805   0.000001260293909   0.000000000008067
Current cell color: green
Current step: 4
map = 10×10
   0.000000000017525   0.000012676696346   0.000050744042464   0.000012680919501 ···
   0.000012694073908   0.000101501737128   0.000012697853004   0.182543611798533
   0.000000003171289   0.000015233129825   0.000010170469031   0.107375065158431
   0.000015216196497   0.000008952808246   0.000017785328228   0.000142126636184
   0.000011472505430   0.045593139080560   0.000017260622966   0.000000023923736
   0.000000000107592   0.000008309931938   0.000004335085337   0.024904947402629
   0.000000000004805   0.000000000693356   0.000008307161612   0.000000001344635
   0.000001269911249   0.000000017684661   0.000000001543481   0.000000000001495
   0.000000000001097   0.000000000847507   0.000000000001121   0.000000002992525
   0.000000000423639   0.000000003354719   0.000000000926620   0.000000000072262
Current cell color: blue
Current step: 5
map = 10×10
   0.000000013628451   0.000163416416783   0.000000023175399   0.000000010973597 ···
   0.000000054526893   0.000085793029263   0.000000017423600   0.000024517238886
   0.000000013620401   0.000000014657738   0.000057688366422   0.000000095234827
   0.000000004086660   0.000000037047837   0.000006202876999   0.000000018417351
   0.000000002656957   0.000006125806039   0.000000002242048   0.018342826932860
   0.000000004082820   0.000000003773156   0.000006121721715   0.000003344616504
   0.000007956223251   0.000006118214346   0.000006691443462   0.000000001261602
   0.000000000000171   0.000000000001119   0.000000000756116   0.000003347692349
   0.000000000005646   0.000000000000114   0.000000001799790   0.000004316035351
   0.000000000170835   0.000000000000001   0.000000000000479   0.000000002038495
Current cell color: green
Current step: 6
map = 10×10
   0.000050038628875   0.000026269676165   0.000000022428420   0.000000016322899 ···
   0.078717729916902   0.000006571820098   0.000017674527271   0.011327343137636
   0.000000003663377   0.000018602964801   0.000001930881820   0.000053421982799
   0.000000944103599   0.000000005927236   0.000001418110941   0.005618861453878
```

```
   0.000000616306692   0.008442006209292   0.000000772320407   0.000001873007358
   0.000001404662595   0.000001887992461   0.009820125583499   0.002303603089450
   0.000001120490259   0.000002340544146   0.000001888471581   0.000023854805563
   0.000000935842595   0.003358682099768   0.000001266368061   0.000000000940881
   0.000000000341060   0.000000000266209   0.000000000773214   0.000000660505878
   0.000000000000009   0.000001293329377   0.000000330175749   0.000001136364460
Current cell color: red
Current step: 7
map = 10×10
   0.000066791243846   0.000000011143311   0.000066785541680   0.000000000530891 ⋯
   0.000000016705665   0.000000030773158   0.000004940517109   0.000007178355032
   0.000001236809780   0.000023388620231   0.000000048876535   0.000004766624102
   0.000020261333676   0.000000000809205   0.014289256205931   0.003579678709615
   0.000000000000532   0.000000003777253   0.009480025968742   0.000012800772927
   0.000000002879617   0.000001202961596   0.000002094240373   0.000003181871348
   0.000000711977989   0.000000003941196   0.000000000802399   0.000002130128228
   0.000000001954091   0.000000712452768   0.000000002141621   0.000000006069512
   0.000000712251529   0.000004317692596   0.000000000475342   0.000000000627882
   0.000000000000090   0.000000000511719   0.000000000548227   0.000000000000464
Current cell color: green
Current step: 8
map = 10×10
   0.000000000209154   0.000000009519951   0.000000327949900   0.000000009294899 ⋯
   0.000000774611724   0.000000212825786   0.000000009679708   0.322610611044496
   0.000000000002375   0.000026884629729   0.000107354702049   0.293979650855302
   0.000026884257601   0.000024504239729   0.000053786592530   0.000309624995907
   0.000017789841598   0.080610700689736   0.000042300074006   0.000000485166451
   0.000000003960733   0.000017800761374   0.001340570787052   0.053654200860293
   0.000000004932237   0.000000011885806   0.000017794830972   0.000000865015761
   0.000000016232578   0.000022794150519   0.000000012083907   0.000000000009242
   0.000000002675950   0.000000000010818   0.000000002676478   0.000000005064174
   0.000000000005406   0.000008021367174   0.000000000409864   0.000000005584890
Current cell color: green
Current step: 9
map = 10×10
   0.000000000891577   0.000000000242065   0.000000062017285   0.000000048089402 ⋯
   0.000000740949807   0.000000030641250   0.000000149929380   0.275060525107157
   0.000000000016522   0.000023043783054   0.000334325102044   0.001366241505728
   0.000022929356169   0.000000113879663   0.000023309392193   0.000000197385008
   0.000000051195728   0.138488306236495   0.000015377837522   0.000005828701049
   0.000023279497109   0.000015274996969   0.070911454032899   0.045776882517797
   0.000000016587976   0.000023652793316   0.000015280103603   0.000000381680264
   0.000000000025143   0.000075946101132   0.000000381309940   0.000000005150123
   0.000000008755180   0.000000000000007   0.000000011036307   0.000000001593629
   0.000000000000002   0.000033066522154   0.000000001220024   0.000006833873788
Current cell color: blue
Current step: 10
map = 10×10
   0.000000000802942   0.000000099736742   0.000000050605957   0.000000000178846 ⋯
   0.000000000037478   0.000075327657415   0.000000368579257   0.000150101170107
   0.000000000039041   0.000000380968520   0.000001505380342   0.000000017121649
   0.000000025018996   0.000000104711479   0.000056675365177   0.000000083485105
   0.000000004153898   0.000056706440266   0.000000008275120   0.169846257597850
   0.000000031418603   0.000000008300202   0.000056694231496   0.000012407458844
   0.000012490224472   0.000075878749782   0.000024850091454   0.000038384480075
   0.000000000006407   0.000000000034554   0.000019199031008   0.000012436126432
   0.000000059002903   0.000000000002204   0.000000066862240   0.000002969910807
   0.000000000000337   0.000000000016432   0.000000000000365   0.000000001878072
Current cell color: blue
Current step: 11
map = 10×10
   0.000000000156364   0.000353802092795   0.000000000636314   0.000000000170673 ⋯
   0.000000118052075   0.000091912061329   0.000000010182704   0.000000178325969
   0.000000029484371   0.000000010033454   0.000000088849024   0.000000000315311
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   0.000000000022327   0.000000002713602   0.000066545096992   0.000266262976041
   0.000000004893620   0.000066544990888   0.000000000010239   0.199635025413579
   0.000000000042051   0.000000004903351   0.000066567187507   0.000000019791016
   0.000014685616155   0.000000215449987   0.000000068131078   0.000066653162353
   0.000000000022331   0.000000000011411   0.000000144019813   0.000000053540028
   0.000000004984206   0.000000000015312   0.000000019591506   0.000003520844470
   0.000000000000005   0.000000000000033   0.000000000003283   0.000000000003846
Current cell color: blue
Current step: 12
map = 10×10
   0.000000471667225   0.000367340025557   0.000000000170729   0.000000000049432 ···
   0.000000122529965   0.000091854518267   0.000000007815862   0.000000177381752
   0.000000030612324   0.000000007787748   0.000000088685967   0.000000354858069
   0.000000000022186   0.000000090594327   0.000066867906195   0.000266058071435
   0.000000004891231   0.000066513073953   0.000000000059169   0.200469991616636
   0.000000000022221   0.000000004892877   0.000066535252450   0.000000000436212
   0.000014668805641   0.000000089028961   0.000000005043868   0.000066581055947
   0.000000000000064   0.000000000003289   0.000000069205298   0.000000000149446
   0.000000004899470   0.000000000000447   0.000000000086072   0.000003541438182
   0.000000000000019   0.000000000000005   0.000000000000798   0.000000000000015
Current cell color: white
Current step: 13
map = 10×10
   0.507206336720392   0.000084585676031   0.000179907372336   0.000174052142973 ···
   0.000084586569545   0.000000021284063   0.000000096669611   0.000061433345995
   0.000000021561207   0.000000060958579   0.092787213175033   0.000130309335107
   0.000007670749549   0.000000036924297   0.000137974389907   0.092328351890688
   0.000000026620492   0.000015362553660   0.000000023706793   0.000030934539458
   0.022944285856793   0.000000012679916   0.000015411035188   0.000000002036334
   0.000000001689906   0.000000010234348   0.000000001865711   0.000007709489991
   0.000000000009552   0.000000001734372   0.000000007953704   0.000000001329058
   0.000000000563555   0.000000000144855   0.000000000634926   0.000000003104591
   0.000000000000206   0.000000000000383   0.000000823235467   0.000000000027098
Current cell color: red
Current step: 14
map = 10×10
   0.000000155472784   0.000000000052332   0.000000084842402   0.000000000078214 ···
   0.000000000039587   0.000000000144871   0.255564431524533   0.000000148025414
   0.063827216805856   0.255564328596363   0.000042840621623   0.000084855286282
   0.000005329741932   0.000021304107248   0.254311208791580   0.063663405408944
   0.000000000000017   0.000000003546128   0.008655485322509   0.000011549745711
   0.000005269817318   0.000000000045717   0.000000016081081   0.000002880978564
   0.000000000000920   0.000005268064347   0.000000000000005   0.000000021682123
   0.000005266303028   0.000000000000917   0.000000000199335   0.000000000000483
   0.000000000001045   0.000000586806580   0.000000000000001   0.000000000194653
   0.000000000000063   0.000000000000130   0.000000000191651   0.000000000000000
Current cell color: red
Current step: 15
map = 10×10
   0.000000000000138   0.000095932677892   0.000384122681318   0.000095964697871 ···
   0.000000032010441   0.000768237493009   0.287991042774564   0.000766353910866
   0.071973689302544   0.000672174783233   0.000406414030396   0.000108761600951
   0.000287335509046   0.000024017400862   0.287160710559079   0.227526281370393
   0.000000009083830   0.000000095719527   0.023969558282793   0.000019025741805
   0.000000005935157   0.000006507196303   0.000000060876570   0.000007987703208
   0.000000000000001   0.000000010109187   0.000000001084825   0.000000053709036
   0.000000010107933   0.000000000000001   0.000000004395855   0.000000000000620
   0.000000000000002   0.000013174790083   0.000000000000000   0.000000000221891
   0.000000000000807   0.000000000000000   0.000000000220566   0.000000000000000
Current cell color: green
Current step: 16
map = 10×10
   0.000000049789859   0.000037741076669   0.000000497844328   0.000037354949648 ···
   0.001194706977775   0.000149871343736   0.000037578296121   0.448012429867466
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   0.000000009389527   0.000037557761828   0.000149051776890   0.391137803592027
   0.000037334423070   0.000032602099078   0.000155407851619   0.000082342488893
   0.000012440153185   0.111928421159819   0.000035704724248   0.000000023485345
   0.000000000034799   0.000003109713280   0.000037362871554   0.009317294359514
   0.000000000843619   0.000000000039670   0.000003108675669   0.000000000049893
   0.000000003422243   0.000002527481619   0.000000000029710   0.000000000000346
   0.000000000000000   0.000000000002280   0.000000000000000   0.000000000015722
   0.000000000001140   0.000000000000001   0.000000000001738   0.000000000000000
Current cell color: blue
Current step: 17
map = 10×10
   0.000002739695370   0.000999225934540   0.000002822281178   0.000000100148460 ···
   0.000000333623038   0.000750214590205   0.000000466349919   0.000249325501392
   0.000000083316036   0.000000414867865   0.000869833244622   0.000000407375044
   0.000000041753694   0.000000289781016   0.000062524529364   0.000000032975742
   0.000000008657429   0.000062310481119   0.000000003515897   0.186558774138768
   0.000000041482763   0.000000010384476   0.000062268976470   0.000005178367999
   0.000025881007036   0.000062182675164   0.000010378671070   0.000000041544822
   0.000000000000002   0.000000000003601   0.000000020745477   0.000005177708461
   0.000000002806917   0.000000000000001   0.000000004532347   0.000000030134476
   0.000000000001905   0.000000000000468   0.000000000000003   0.000000000000004
Current cell color: blue
Current step: 18
map = 10×10
   0.000001421417990   0.003198024753479   0.000000001123763   0.000000000542872 ···
   0.000001066956941   0.000802728787512   0.000001304366050   0.000000178170254
   0.000000266610805   0.000001303850431   0.000000089709309   0.000000000471450
   0.000000000022335   0.000000326129677   0.000066323907750   0.000265329938663
   0.000000009198629   0.000066323882912   0.000000000006221   0.198971355331478
   0.000000000036905   0.000000009204727   0.000066345990714   0.000000007705896
   0.000027590118486   0.000000154757312   0.000000033140129   0.000066509082750
   0.000000000014745   0.000000000009210   0.000000066364610   0.000000020245806
   0.000000009196387   0.000000000007377   0.000000007371265   0.000000032168901
   0.000000000000001   0.000000000000002   0.000000000000010   0.000000000000615
```