Homework 10

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Problem 7.5.4

Original Solution

If $\int_a^x f = F(x) = 0$ then $F'(x) = f(x) = 0$ so there is no rate of change on the interval, or $f(x) = 0$.

If f is not continuous, then it may have a high number of discontinuities such that it fails to be integrable, which is a requirement of the fundamental theorem of calculus.

Self-Evaluation

My answer matches the solution provided.

Problem 7.5.8

Part A

$L(1) = \int_1^1 \frac{1}{t} dt$, $\frac{1}{t}$ is continuous and integrable so L is differentiable.

$$L'(x) = 0$$

Part B

$$L(xy) = \int_1^{xy} \frac{1}{t} dt = \int_1^{x} \frac{1}{t} + \int_x^{xy} \frac{1}{t}$$

The first integral is equivalent to $L(x)$. On the second integral, use u-substitution to produce an integral of the form

$$\int_1^y \frac{1}{u} = L(y)$$

where $u = \frac{t}{x}$ and $du = \frac{1}{x}dt$

so

$$L(xy) = L(x) + L(y)$$

Part C

Using the previous result,

$$L(y * \frac{x}{y}) = L(y) + L(\frac{x}{y})$$

which simplifies to

$$L(x) - L(y) = L(\frac{x}{y})$$

Part D

Attempting to apply the Monotone Convergence Theorem …

> Show that the sequence is bounded

The components of γ_n are each monotone.

$$\sum_{n=1}^{\infty}(\frac{1}{n})$$

is monotonically increasing.

Similarly,

$$\int_1^{n+1} \frac{1}{t} > \int_1^{n} \frac{1}{t}$$

So γ_n is monotonically increasing.

> Show that the sequence is bounded

The sequence is bounded because the expression $\int_1^x \frac{1}{t}$ is accumulating faster than $\sum_{n=1}^{\infty}(\frac{1}{n})$

In other words,

$\sum_{n=1}^{\infty}(\frac{1}{n}) < \int_{1}^{n} \frac{1}{t}$

so γ_n is bounded.

By the Monotone Convergence Theorem, γ_n converges.

Part E

Original Solution

I had trouble on this one. I couldn't quite see how to manipulate the sequence to get it to match the desired solution.

Self-Evaluation

Part B

My approach was different, and I used a u-sub to get the integral to work out. Also I had a typo in the final line which is fixed.

Part D

Just a different style of explanation, but I believe our solutions are both on the same page.

Part E

Thinking of γ_{2n} as a subsequence really helped see why the limit would converge. However I do not think I would have thought to use *Part C* in the way it is presented. Also I think the solution has a typo and it should be $L(2n) - L(n)$

External References

I had some help with the u-substitution on 7.5.8, Part C