Uncertainty Modeling of Nonlinear 2-DOF Helicopter Model

Conference Paper · January 2014

DOI: 10.1109/ICCCI.2014.6921840

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Abstract—**This paper proposes the modeling of uncertainty design of a two degree of freedom (2-DOF) nonlinear helicopter model with nine uncertain parameters. This approach is developed based on a robust *μ*-synthesis control tool for a laboratory helicopter model known as twin rotor multi-input multi-output system (TRMS). The goal of control is robust stabilization of position of TRMS by considering nonlinearity and uncertainties of the system. The percentage of uncertainty is introduced in the model and motor parameters for the robust control of TRMS. Simulation results show the frequency response analysis of open loop interconnection of TRMS. The singular value of the frequency response is evaluated for the dynamic TRMS plant. The position response of uncertain linearized model of TRMS is also analyzed.**

Keywords-Uncertainty design, 2-DOF nonlinear helicopter, multi-input multi-output system (MIMO), twin rotor MIMO system (TRMS), linearized model, robust μ-synthesis control

I. INTRODUCTION

The TRMS is a laboratory setup designed by Quancer Instruments limited for control experiments. Behavior of TRMS resembles that of a helicopter. It's posses a strong cross coupling between the main rotor and tail rotor. An estimated model has been obtained by Quancer 2-DOF helicopter user and control manual [1]. Most of the industrial plants can almost be controlled through PID controller and the parameters are partially tuned by trial and error process [4]. Fuzzy controller for the TRMS trajectory tracking performance is superior to the system performance with conventional PID or LQR controller. The drawback is that system performance depends on the number of the fuzzy membership functions [5]. System performance index as part of the fitness function in an RGA is more efficient in finding the parameters of the PID controller. These controllers can reduce the trajectory tracking of a desired path in 2-DOF oscillations for several seconds [2]. The performance of PID controller depends on the gain of the controller. The disadvantage of GA is that the searching process is time consuming for multiple parameter with wider search range [6]. The LQR comparator good tracking capability requires high control effect but has inadequate authority over residual vibration of the system. These vibration appear in the system response as oscillation with long settling time [7]. Terminal sliding mode control law which is estimated for the driving subsystem and n^{th} order tracking problem can be transformed into an equivalent 1^{st} order stabilization problem. The chattering problem arising from the discontinuous control nature of an ideal sliding mode controller [8]. The nonlinear modeling of TRMS by radial basis function network model has verified using time and frequency domain tests [9]. Although there is no reliance on mathematical model and estimated actual model, the careful selection of excitation signal is an important part of nonlinear system identification. Without consideration to this issue, the obtained model would not able to capture the system dynamics, resulting in a poor model. The strong coupling between inputs and outputs are modeled as linearized and then controllers are designed to control the position of level bar [10]. The model uncertainty arises when the system gain and the parameters are not precisely known, or may vary over a given range. The linear fractional transformation (LFT) is object is developed in [11]. In this correspondence, we investigate a control problem involving a cross coupled TRMS.

The system description for both linear and nonlinear system is developed in section II. The modeling of uncertainty, controller synthesis and selection of weighting function is discussed in section III.

II. SYSTEM DESCRIPTION

A. Twin rotor MIMO system

TRMS is shown in Fig. 1 is a complex and highly nonlinear system with some inaccessible parameters for measurements [4]. There are two propellers at the both ends of a beam pivoted on its fixed base in such a way that it can freely rotate both the horizontal and vertical axis. The two rotors are driven by DC motor. The front propeller controls the elevation of the helicopter more about the pitch axis, while the back or tail propeller controls the side to side motion of the helicopter about the yaw axis [5]. The pitch and yaw angles are measured using high resolution encoders. The pitch encoder and motor signals are transmitted via slipring. This eliminates the possibility of wires tangling on the yaw axis and allows the yaw angle to rotate freely about 360° degrees. The system is balanced in such a way that when motors are switched off, the main rotor end of beam is lowered [6]. The measured system outputs are the two angles of pitch and yaw. In normal helicopter, the aerodynamic force is controlled by changing the angle of attack. The laboratory TRMS setup shown in Fig. 1 is

[978-1-4799-2352-6/14/$31.00 ©2014 IEEE]

so constructed that the angle of attack is fixed. The aerodynamic force is controlled by varying the speed of the rotors. Therefore, the control inputs are the supply voltages of the dc motors. A change in the voltage value results in a change in the rotation speed of the propeller. This further results in a change of the corresponding position of the beam [2].

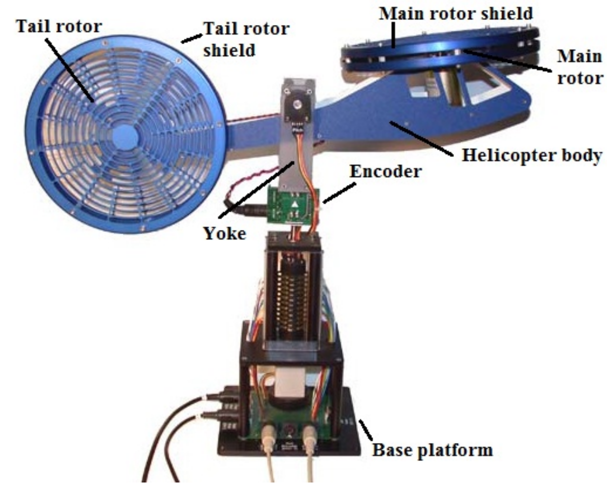

Fig. 1. Laboratory setup of TRMS

The TRMS control consists in stabilization of the beam in arbitrary desired position or tracking of a desired trajectory [7]. In this paper deals with the design of robust control system is implemented in μ-synthesis.

B. Nonlinear system model

The Euler-Lagrange method is used to derive the non-linear equation describes the motions of the helicopter [1]. The potential energy due to gravity is

$$V = m_{heli} g l_{cm} \sin\theta \tag{2.1}$$

The total kinetic energy is

$$T = T_{r,p} + T_{r,y} + T_t \tag{2.2}$$

Equation (2.2), is the sum of the rotational kinetic energies acting from the pitch, $T_{r,p}$ and from the yaw, $T_{r,y}$ along with the translational kinetic energy generated by the moving center of mass T_t .

The potential and kinetic energy expressed here are used to derive the equations of motions. Nonlinear equation of motion for the 2-DOF Helicopter, the Euler-Lagrange equations are

$$\begin{aligned} &\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}1} - \frac{\partial L}{\partial q1} = Q1 \\ &\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}2} - \frac{\partial L}{\partial q2} = Q2 \end{aligned} \tag{2.3}$$

Where L is the Lagrange variable and is the difference between the kinetic and potential energy of the system, $L = T - V$.

The generalized coordinates are

$$\begin{aligned} q &= [q1 \quad q2 \quad q3 \quad q4]^T \\ &= [\,\theta \quad \dot{\psi} \quad \dot{\theta} \quad \dot{\psi}\,] \end{aligned} \tag{2.4}$$

And the generalized forces are

$$\begin{aligned} Q1 &= \lambda p(Vm,p,\ Vm,y) - Bp\dot{\theta} \\ Q2 &= \lambda y(Vm,p,\ Vm,y) - By\dot{\psi} \end{aligned} \tag{2.5}$$

Equation (2.5) includes the viscous rotary friction acting about the pitch and yaw axes B_p and B_y . The torques applied at the pitch and yaw axis from the motors is

$$\begin{aligned} \tau_p(V_{m,p} + V_{m,y}) &= K_{pp}V_{m,p} + K_{py}V_{m,y} \\ \tau_y(V_{m,p} + V_{m,y}) &= K_{yp}V_{m,p} + K_{yy}V_{m,y} \end{aligned} \tag{2.6}$$

Evaluating the Euler-Lagrange expressions in Equation (2.3) using the coordinates defined in Equation (2.4) and the forces in Equation (2.5) results in the nonlinear equations of motion (2.7).

$$\begin{aligned} (J_{eq,p} + m_{heli}l_{cm}^2)\,\ddot{\theta} &= (K_{pp}V_{m,p}) + (K_{py}V_{m,y}) - (m_{heli}gl_{cm}\cos\theta) \\ &\quad - (B_p\dot{\theta}) - (m_{heli}\,l_{cm}^2 \sin\theta\cos\theta\dot{\psi}^2) \\ (J_{eq,y} + m_{heli}l_{cm}^2\cos^2\theta)\ddot{\psi} &= (K_{yy}V_{m,y}) + (K_{yp}V_{m,p}) - (B_y\dot{\psi}) + \\ &\quad (2m_{heli}l_{cm}^2 \sin\theta\cos\theta\dot{\psi}\dot{\theta}) \end{aligned} \tag{2.7}$$

C. Linearized system model

The linear state-space model of the helicopter is used to design the position controller [8]. Linearizing the nonlinear equations of motion in Equation (2.7) about quiecent point [1] (i.e) $\theta_0 = 0, \psi_0 = 0, \dot{\theta}_0 = 0, \dot{\psi}_0 = 0$ and substituting the states, the output variable $y = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ is implies that all the states are being measured. The nominal values of the model parameters are given in Table I. Nominal values of the TRMS are taken from the Quanser instrument manuals [1].

TABLE I. UNCERTAIN PARAMETERS WITH NOMINAL VALUES

Symbol (unit)	**Value**	**Symbol (unit)**	**Value**
$B_{eq,p}$ (N/V)	0.8000	K_{pp} (Nm/V)	0.2040
$B_{eq,y}$ (N/V)	0.3180	K_{py} (Nm/V)	0.0068
$J_{eq,p}$ $(Kg-m^2)$	0.0384	K_{yp} (Nm/V)	0.0219
$J_{eq,y}$ $(Kg-m^2)$	0.0432	K_{yy} (Nm/V)	0.0720
m_{heli} (Kg)	1.3872		

The linearized mathematical model of TRMS represented in the form of state space A B and C matrix given in Equation (2.8).

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9.26 & 0 \\ 0 & 0 & 0 & -3.487 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2.361 & 0.07871 \\ 0.07457 & 0.7895 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.8}$$

The linear model derived from conventional linearization about the operating point i.e. 10° in vertical and 10° horizontal plane [10]. In Fig. 3, the validation of linear model is performed by giving square signal as the input to the vertical system of TRMS in open loop. The validated model is then analyzed for control attributes. In Fig. 3, the result shows poor tracking and poor robustness of the system due to low gain at lower frequencies. It is also indicating the nonlinear and oscillatory behavior of the TRMS.

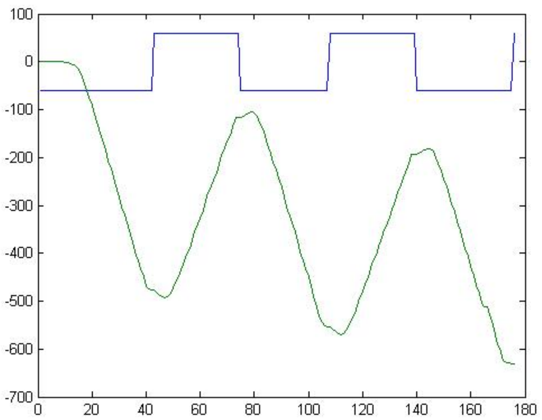

Fig. 3. Validation of linear model in vertical plane

III. MODELING OF UNCERTAINTY

For designing the robust controller, generally it is advisable to use the linearized models since they provide an easy way to set the uncertainty in specific parameters [9]. The model obtained by analytic linearization is very appropriate for this purpose. In general, the model of a TRMS is linearized analytically describing the system behavior around their trim values by assuming small deviations for the variables in the system [11].

The input variables are the voltages $V_{m,p}$ and $V_{m,y}$ of the main rotor and tail rotor motors and output variables are the pitch angle and yaw angle. The plant is two-channel and there is an interaction between the two channels. The plant should be considered as a multivariable plant rather than the conventional consideration of independent channel methods. By describing the plant in a multivariable form, the full dynamic behavior of the plant is analyzed and effectively modeled. The parameters of the linearized model are defined in Table II.

TABLE II. PARAMETERS OF LINEARIZED MODEL

θ, ψ	Pitch and yaw angle
$\dot{\theta}, \dot{\psi}$	Vertical and horizontal angular velocity
$J_{eq,p}$, $J_{eq,y}$	Equivalent moment of inertia of pitch and yaw
$B_{eq,p}$, $B_{eq,y}$	Equivalent viscous damping of pitch and yaw
K_{pp} , K_{yy}	Thrust torque constant acting on pitch and yaw
K_{py} , K_{yp}	Thrust torque constant acting on pitch/yaw axis from yaw/pitch
$V_{m,p}$, $V_{m,y}$	Control voltages of pitch and yaw motor
m_{heli}	Total moving mass of helicopter
l_{cm}	Length of helicopter

The uncertainty parameter in the mathematical description of TRMS which are considered are the moment of inertia $J_{eq,p}$ and $J_{eq,y}$ in respect to the pitch and yaw axis, $B_{eq,p}$ and $B_{eq,y}$ in respect to equivalent viscous damping about pitch and yaw axis, K_{pp} and K_{yy} in respect to thrust torque constant acting on pitch axis from pitch and yaw motor, K_{py} and K_{yp} in respect to thrust torque constant acting on pitch/yaw axis from yaw/pitch motor, m_{heli} is the total moving mass all together nine parameters.

The uncertainties in the moment of inertia $J_{eq,p}$ is due to it depends on the pitch angle, similarly $J_{eq,y}$ is due to it depends on the yaw angle, the uncertainties in the coefficients K_{pp}, K_{yy} , K_{py}, and K_{yp} are introduced as a result of the measuring and approximation of the static characteristics of the rotors, and the uncertainties in the coefficients K_{py} and K_{yp} result from simplification of the aerodynamic interaction between the two channels. Further on, we assume that the moment of inertia and all the coefficients are known with errors up to 10 % while the rest of total moving mass of helicopter is error up to 5 % due to small variation of air disturbance [3].

The *ureal* function from the robust control toolbox in MATLAB is used to set the nine real uncertain parameters. The *sysic* function from the robust toolbox is used to obtain the uncertain TRMS model for plant interconnection [3].

A. Controller Synthesis

The objective of the controller is to achieve efficient set point tracking given by the operator and achieve effective performance with less control effort and also by overcoming the cross coupling impacts. For this purpose a μ-synthesis controller was employed.

The system labeled P is the open-loop interconnection and contains all of the known elements including the nominal plant model and performance and uncertainty weighting functions. The Δ_{pert} block is the uncertain element from the set Δ_{pert}, which parameterizes all of the assumed model uncertainty in the problem. The controller is K. Three sets of inputs enter P: perturbation inputs w, disturbances d, and controls u. Three sets of outputs are generated: perturbation outputs z, errors e, and measurements y [14].

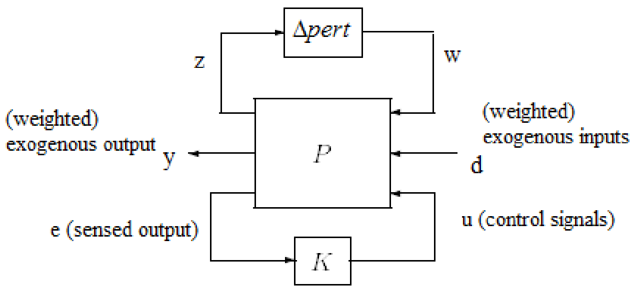

Fig. 4. μ-synthesis

Let P be the transfer function matrix of the discredited nine-input nine-output open-loop system, that consists of the plant model plus the weighting functions and let the block-structure Δ_p is defined as

$$\Delta_p := \begin{bmatrix} \Delta_{pert} & 0 \\ 0 & \Delta_F \end{bmatrix} : \Delta \in R^{9X9}, \Delta_F \in C^{4X2} \tag{3.1}$$

The first block of the matrix Δ_p, the block Δ_{pert}, corresponds to the parametric uncertainties, included in the model of the TRMS system. The second block Δ_F is a fictitious uncertainty block, used to include the performance requirements into the framework of the μ-approach. The inputs of this block are the weighted error signals e_y and e_u, and the outputs are the exogenous signals r and d.

The μ-synthesis is to be done for this sampling frequency of 100Hz. The aim of the μ-synthesis is to find a stabilizing controller K, such that for each frequency ω □ $[0, \pi/T_s]$, where $T_s = 2\pi / f_s$, the structured singular value μ -satisfies the condition.

$$\mu\Delta_p[F_L(P,K)(j\omega)] < 1 \tag{3.2}$$

Where $F_L(P,K)$ is the closed-loop transfer function matrix. The fulfillment of this condition guarantees the robust performance of the closed-loop system,

B. Weighting Function

The μ-synthesis is done for several performances weighting function that ensure a good balance system performance and robustness. The frequency responses of pitch and yaw motor shown in Fig. 5 and Fig. 6 respectively. The performance weighting function and control action weighting function selected on the basis of TRMS frequency response [12] given in the Equations (3.3) and (3.4). The weighting function $W_p(s)$ and $W_u(s)$ are used to penalize the sensitivity function to make the sensitivity to disturbances as small as possible. Since disturbances usually happen in the low frequency range, the weighting function $W_p(s)$ and $W_u(s)$ should be so shaped that it has small magnitude in the low frequency range and large magnitude in the higher frequency range. Lower the magnitudes of $W_p(s)$ the less sensitive of the system to disturbances [14]. This result shows the better robustness of the plant.

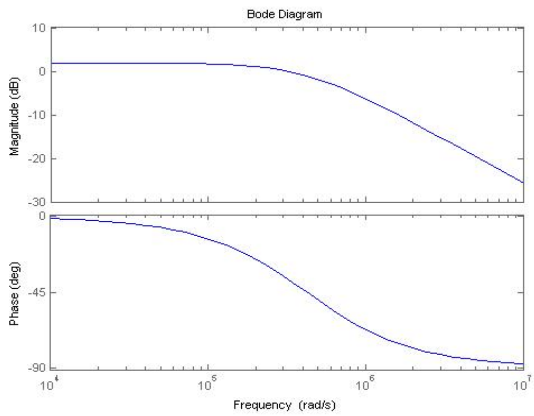

Fig. 5. Frequency response of pitch motor

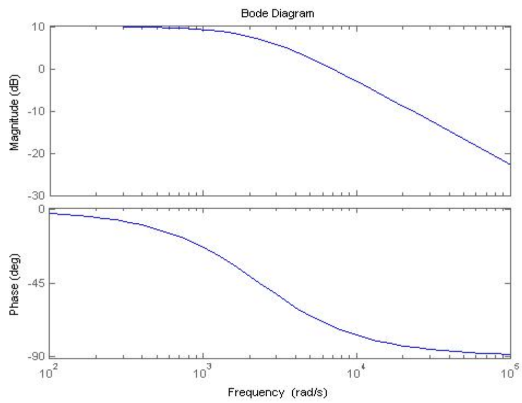

Fig. 6. Frequency response of yaw motor

The performance weighting function

$$W_p(s) = \begin{bmatrix} 8.2\,X\,10^{-2}\,\dfrac{540\,s+1}{540\,s+10^{-2}} & -0.195 \\ 0.125 & \dfrac{520\,s+1}{520\,s+10^{-3}} \end{bmatrix} \quad (3.3)$$

And the control action weighting function

$$W_u(s) = \begin{bmatrix} 4.3X10^{-5}\,\dfrac{0.55s+1}{0.015s+1} & 0 \\ 0 & 2.314X10^{-4}\,\dfrac{0.15s+1}{0.015s+1} \end{bmatrix} \quad (3.4)$$

IV. RESULTS AND DISCUSSION

The bode plot of pitch interconnection with uncertainty is shown in Fig. 7. The first column shows bode plot from disturbance to output theta and theta dot, the second column shows bode plot from omega to output theta and theta dot.

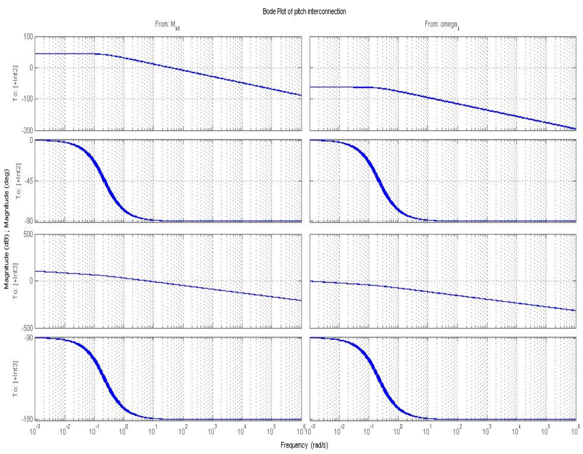

Fig. 7. Frequency response of pitch interconnection with uncertainty

The bode plot of yaw interconnection with uncertainty is shown in Fig. 8. The first column shows bode plot from disturbance to output psi and psi dot, the second column shows bode plot from omega to output psi and psi dot.

The frequency response of pitch and yaw interconnections have stable operation in all frequencies. Since obtained plots crosses the 0dB line. From the obtained results the selected uncertain parameters are stable in operation.

Singular value plot is a useful tool that shows the gain and phase response of a given LTI system for different frequencies. Magnitude plot measures the system Input/Output ratio in special units. The singular value plot for the random values of an uncertain plant parameters shown in Fig. 9. It is seen that results of achieving robust performance, the obtained frequency response are close to the model.

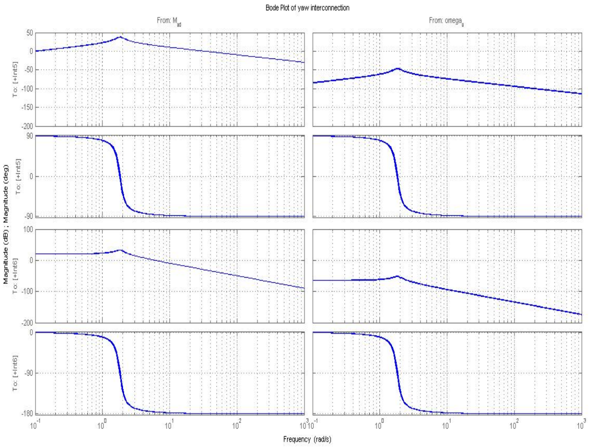

Fig. 8. Frequency response of yaw interconnection with uncertainty

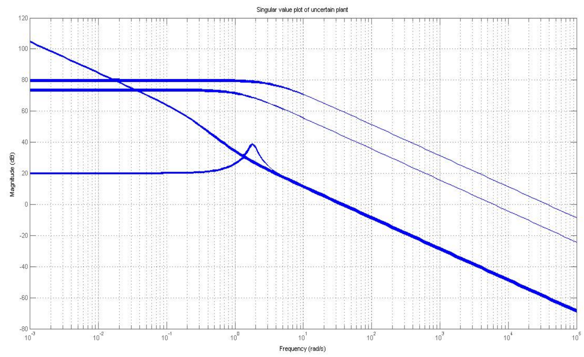

Fig. 9. Singular value plot of uncertain plant

The μ-synthesis optimization is used to shape the singular values of specified transfer functions over frequency. The μ-synthesis optimization is performed by *DK*-iteration method using the function *dksyn* from Robust Control Toolbox [14]. Five iterations are performed that decrease the maximum value of μ to 0.9681. This result shows the better robustness of the plant.

The square response is obtained using a square wave input of magnitude 10 degree for vertical plane and a square wave of magnitude 30 degree for horizontal plane with both having a frequency of 0.02 Hz. The square responses of the horizontal and vertical planes are given in Fig. 10 and Fig. 11.

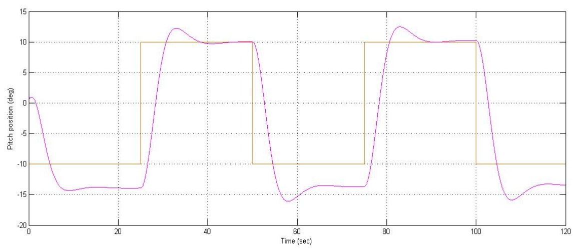

Fig. 10. Output response of vertical plane

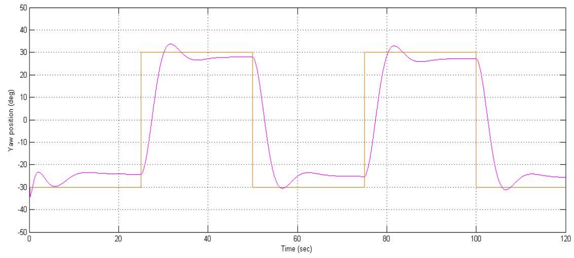

Fig. 11. Output response of horizontal plane

V. CONCLUSION

This paper contributes the methodology to represent the parameter (percentage) uncertainties due to the difference between the linearized mathematical model and the actual system. The uncertainty is developed in plant model and motor parameters for the robust control of TRMS. Simulation result shows the frequency response analysis of open loop interconnection of TRMS. The position response of uncertain linearized model of TRMS is also analyzed.

The singular value of the frequency response is evaluated for the dynamic TRMS plant. The obtained results shown, the bounds on the modeling uncertainty over which we can ensure closed loop stability and performance. These problems are often referred to as the robust stability and robust performance.

In future, Optimization technique like GA, PSO can be used for selection of weighting function in the design of robust controller. The particle swarm optimization algorithm (PSO) can be used to tune the performance and control weighting functions by minimizing the infinity norm of the transfer function matrix of the nominal closed loop system [13].

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