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Designing Robust Pole Placement Control For Roll Motions Of Ships via LMIs

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Abstract: This paper addresses the problem of robust pole placement control of nonlinear systems having multiple equilibrium points and low order dynamics (Roll Motion Of Ships) via a dynamic output feedback in which the controller has dynamic states. The design technique is described using linear matrix inequalities (LMIs).pole placement by using LMIs places the close loop poles in appropriate region to have the best response by considering a tradeoff between overshoot of the response, settling time and steady state error. The effectiveness of the proposed technique in control of Roll Motion of Ships is illustrated by simulation results.

Keywords: Dynamic Output Feedback, LMI, Nonlinear Systems, Pole Placement, Roll Motion Of Ships.

1. INTRODUCTION

The study of nonlinear systems has always been a challenge task. Unlike linear systems, a complete theory for nonlinear systems does not exist, and hence stability analysis and control law designs are complicated. Usually the first step in analyzing a nonlinear system is to linearize it, if possible, around some nominal operating point and analyze the achieved linear system [1].

Pole placement has been considered as an objective that places the poles of a system in a desired area of complex plane. Using this objective, the behavior of system in transient time such as rise time and overshoot can be controlled. In [2] and [3], the authors have considered pole placement in classical form. We refer to this technique as regional pole placement, by contrast with point wise pole placement, where the poles are assigned to specific locations in the complex plane, For example, fast decay, good damping, and reasonable controller dynamics can be imposed by confining the poles in the intersection of a shifted half-plane, a sector, or a disk.

Linear matrix inequality (LMIs) has emerged as a powerful formulation and design technique for a variety of linear control problems. Since solving LMIs is a convex optimization problem, such formulation offer a numerically Tractable means of attacking problems that lack an analytical Solution. Consequently, reducing a control design problem to an LMI can be considered as a practical solution to this problem. [4]

Roll motion is an undesirable feature of the behavior of a ship in rough seas, and so it is natural to consider ways of reducing it. The most common devices for increasing roll damping are bilge keels. However, the effectiveness of keels is limited, and anti-roll tanks and fins are used when more control is required. Moreover, unlike keels, anti-roll tanks can be used when the ship is not underway. [5]

Roll motion of ships has nonlinear equations with five equilibrium points, three focus and two saddle points. So if we linearize our system around the origin, we can design pole placement controller using LMIs through dynamic output feedback to have robust control on nonlinear system with existence of parameter uncertainty and external disturbance.

In this paper, we consider the robust pole placement for a Roll motion of ships system through dynamic output feedback controller and fulfill the conditions of the problem using LMIs. This paper is organized as follow: section 2, includes description of system and required lemmas. In section 3, theorems for pole placement LMI via dynamic output feedback are presented and application of design in Roll motion of ships, our results are proposed and in section 4, the simulation results are demonstrated. Finally, some concluding remarks are given in section 5.

2. MODELING

A good modeling can help us to design an appropriate controller, A simplified mathematical model of the roll motions of ships is given by: [6]

$$\ddot{\theta} + \left(2\mu_1\dot{\theta} + \mu_3\dot{\theta}^3\right) + (w_0^2\theta + \alpha_3\theta^3 + \alpha_5\theta^5) = u \quad (1)$$

where θ is the roll angle, the α_i are nonlinear coefficients, μ_1 is the linear damping coefficient, and μ_3 is the nonlinear damping coefficient and u is the control input. The nominal system parameters have the following values.

$$w_0 = 5.278 \quad \alpha_3 = -1.402w_0^2 \qquad \alpha_5 = 0.271w_0^2 \quad (2)$$

Therefore the equilibrium point will be the same as below:

$$x_{2e} = 0,$$
$$x_{1e} = 0, \left(\frac{-\alpha_3 \pm \sqrt{\alpha_3^2 - 4\omega_0^2\alpha_5}}{2\alpha_5}\right)^{\frac{1}{2}} \tag{3}$$
$$= 0, \pm 0.924, \pm 2.078$$

By using linearization techniques the linearized model will be as (4)

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -(w_0^2 + 3\alpha_3 x_{1e}^2 + 5\alpha_5 x_{2e}^4) & 2\mu_1 + 3\mu_3 x_{2e}^2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{4}$$
$$y(t) = [1 \quad 0]X$$

Resulting in the following eigenvalues :

$$\lambda_e = -(\mu_1 + 1.5\mu_3 x_{2e}^2) \pm \sqrt{(\mu_1 + 1.5\mu_3 x_{2e}^2)^2 - (\omega_0^2 + 3\alpha_3 x_{1e}^2 + 5\alpha_5 x_{1e}^4)} \tag{5}$$

So eigenvalues at equilibrium points will be as it has shown in table.1.

Table.1. Eigenvalues at equilibrium point

Type	λ_e	(x_{1e}, x_{2e})
Focus	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 - 225.735}$	(-2.078,0)
Saddle	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 + 44.663}$	(-0.924,0)
Focus	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 - 27.857}$	(0,0)
Saddle	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 + 44.663}$	(0.924,0)
Focus	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 - 225.735}$	(2.078,0)

Types of equilibrium points show effect of initial value and external disturbance in determining final value. Fig. 1 illustrates the behavior of uncontrolled system.

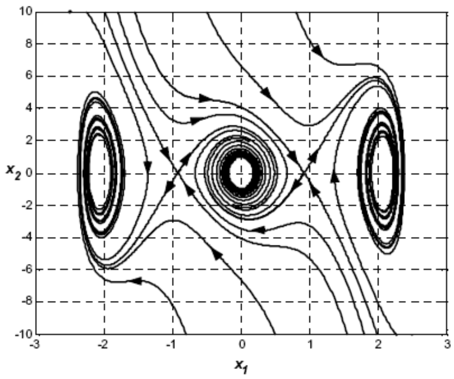

Fig.1. uncontrolled system

The objective of this paper is to introduce a control law that will have the origin as a globally stable equilibrium point to force the system to approach the origin from any initial condition while exhibiting a satisfactory transient performance. The following section introduces a strategy to design a controller that dictates a predefined transient behavior.

3. CONTROLLER DESIGN

A. PRELEMINARIS

Let the perturbed system be described by the following equation,

$$\dot{X}(t) = (A + \Delta A)X(t) + (B + \Delta B)u(t) \tag{6}$$
$$y(t) = CX(t)$$

Where A, B and C are illustrated in (4).

$u(t)\epsilon R$, is the control input. Parameter uncertainties $\Delta A(t), \Delta B(t)$ are norm bounded and are in the form of $[\Delta A(t), \Delta B(t)] = H.F(t)[E_1 \; E_1]$,

Where $H^{n\times n}, E_1{}^{n\times n}, E_1{}^{n\times p}$ are known real constant matrices, and $F(t)$ is an unknown matrix that belongs to the following set

$\Xi := \{F(t) | F^T(t)F(t) \le I, F(t) \textit{ is Lebesgue measurable}\}$

With dynamic output feedback we can use the dynamic of the controller with following structure

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c y(t) \\ u_c(t) = C_c x_c(t) \end{cases} \tag{7}$$

With $x_{cl}(t) = [x^T(t) \; x_c{}^T(t)]^T$ and using (6) and (7), the closed loop equation is

$$\dot{x}_{cl}(t) = (A_{cl} + \Delta A_{cl})x_{cl}(t)$$

Where

$$A_{cl} = \begin{bmatrix} A & BC_c \\ B_c C & A_c \end{bmatrix}, \Delta A_{cl} = H_{cl} F_{cl} E_{cl} \tag{8}$$

And

$$H_{cl} = \begin{bmatrix} H & 0 \\ 0 & 0 \end{bmatrix}, F_{cl} = \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix}, E_{cl} = \begin{bmatrix} E_1 & E_2 C_c \\ 0 & 0 \end{bmatrix}$$

B. DEFINITION AND LEMMAS

Definition 1 (LMI region [7]): A subset $\mathcal{D}$ of the complex plane c is called an LMI region if there exist a symmetric matrix $L \in \mathcal{R}^{k\times k}$ and a matrix $Q \in \mathcal{R}^{k\times k}$ such that

$$\mathcal{D} = \{z \in c \; \therefore L + zQ + \bar{z}Q^T < 0\} \tag{9}$$

Lemma 1 [7]: Definition 1 for dynamical system $\dot{x} = Ax$ is satisfied if and only if there exist a symmetric matrix $P > 0$ such that

$$L \otimes P + Q \otimes (AP) + Q^T \otimes (AP)^T < 0 \tag{10}$$

Where $\otimes$ stands for Kronecker product, which has the following properties

$(A \otimes B)(C \otimes D) = AC \otimes BD$

$(A \otimes B)^T = A^T \otimes B^T$

Lemma 2 (Schur Complement [8]): The linear inequality $\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0$ with $Q = Q^T,\ R = R^T$ and with S as an affine function of X, is equivalent to

$$\begin{cases} Q(X) - S(X)R^{-1}(X)S^T(X) > 0 \\ R(X) > 0 \end{cases} \tag{11}$$

Lemma 3 [9]: Let Ω, Γ, Σ be matrices with appropriate dimensions, and Ω a symmetrical matrix, then for every matrix F with $F^T F \leq I$, we have $\Omega + \Gamma F \Sigma + (\Gamma F \Sigma)^T \leq 0$, if and only if there exist a constant $\varepsilon > 0$, such that

$$\Omega + \varepsilon \Gamma \Gamma^T + \varepsilon^{-1} \Sigma^T \Sigma \leq 0, \tag{12}$$

C. Theorem

We describe the proposed method for the pole placement as following theorem.

Theorem 1: With prescribed scalar $\varepsilon > 0$, the system described by (9) is D-stable through controller of (11) if there exist symmetric matrices S, R and X and matrices $\hat{A}_c, \hat{B}_c, \hat{C}_c$ such that the following inequalities satisfied

$$\Lambda > 0$$
$$\Psi + X < 0 \tag{13}$$
$$X > 0$$

Where

$$\Lambda = \begin{bmatrix} R & I \\ I & S \end{bmatrix}$$

And

$$\Psi = \begin{bmatrix} \begin{pmatrix} L \otimes \Lambda + Q \otimes \Phi_A^T \\ + Q^T \otimes \Phi_A \end{pmatrix} & \varepsilon (Q_2^T \otimes \Phi_H) & Q_1^T \otimes \Phi_E^T \\ \varepsilon (Q^T \otimes \Phi_H^T) & -\varepsilon I & 0 \\ Q_1 \otimes \Phi_E & 0 & -\varepsilon I \end{bmatrix} \tag{14}$$

The matrices Φ_A, Φ_E, Φ_H are as follow

$$\Phi_A = \begin{bmatrix} RA + \hat{B}_c C & \hat{A}_c \\ A & AS + B\hat{C}_c \end{bmatrix} \tag{15}$$

$$\Phi_E = \begin{bmatrix} E_1 & E_1 S + E_2 \hat{C}_c \\ 0 & 0 \end{bmatrix}, \Phi_E = \begin{bmatrix} RH & 0 \\ H & 0 \end{bmatrix}$$

Invertible matrices M and N satisfy

$MN^T = I - RS$

The matrices $\hat{A}_c, \hat{B}_c, \hat{C}_c$ are defined as

$\hat{A}_c = RAS + MB_c C_2 S + RB_2 C_c N^T + MA_c N^T$

$\hat{B}_c = MB_c \quad \hat{C}_c = C_c N^T$

As the matrices $\hat{A}_c, \hat{B}_c, \hat{C}_c$ are achieved, the controller matrices can be obtained by,

$$C_c = \hat{C}_c (N^{-1})^T \tag{16}$$
$$B_c = M^{-1} \hat{B}_c$$
$$A_c = M^{-1}(\hat{A}_c - RAS - MB_c C_2 S - RB_2 C_c N^T)(N^{-1})^T$$

To see Proof of this theory you can refer to [8].

4. SIMULATION RESULTS

This section shows the simulation results. We have considered three different cases. First case shows results of implementing controller on nonlinear model with nominal values, without external disturbance. Second one describes results of implementing controller on nonlinear model with considering uncertainty in nominal parameters and finally third one shows results of implementing controller on nonlinear model with considering uncertainty in nominal parameters and sinusoidal external disturbance.

In this part, external disturbance is equal to sin(t) and initial value is $X_0 = [5\ , -12]^T$. Uncertainties are +20% on g(x_1, x_2) and -25% on h(x_1, x_2).

Figure 2 shows both X_1 and X_2 in time domain, it is almost the same in all three conditions. There is no overshoot or undershoot, settling time is less than one second and steady state errors are negligible.

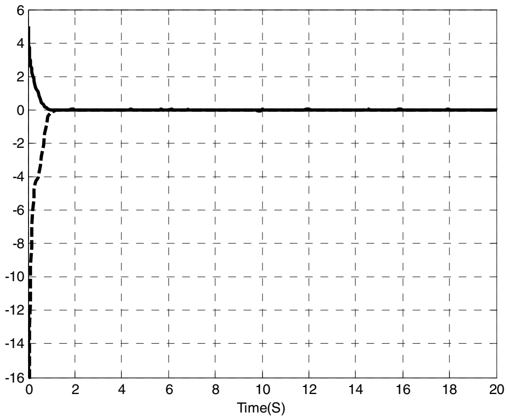

Fig.2. x_1 (line) , x_2 (dashed line)

Figure 3 shows phase portrait of three cases. As it has shown differences are negligible.

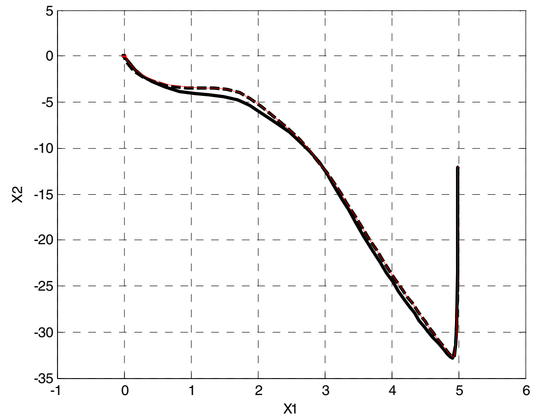

Fig.3. nominal parameters without external disturbance (black line), uncertain parameters without external disturbance (dashed line), uncertain parameters with external disturbance (red line).

Differences in control signals, which cause similarity in outputs, have shown in fig.4.

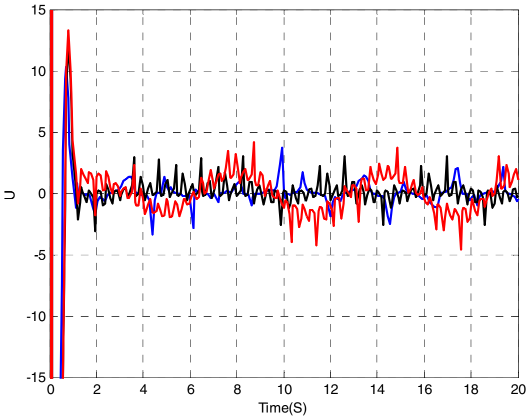

Fig.4. nominal parameters without external disturbance (black line), uncertain parameters without external disturbance (blue line), uncertain parameters with external disturbance (red line).

Although conditions are different in a wide range but our robust controller has produced a stable output which can handle uncertainty and external disturbance.

5. CONCOLUTION

Robust pole placement control via LMIs was applied to a nonlinear system that describes the dynamics of roll motions of ships.

Ships face uncertainty and disturbance in rough seas a lot and simulations show that this controller can handle inaccuracy and external disturbance beside small settling time, without any overshoot or undershoot in output. These properties beside comfortable designing controller make it appropriate to use practically.

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