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STABILITY AND CONTROL OF ROLL MOTIONS OF SHIPS

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ABSTRACT
In this paper, the dynamics of roll motions of ships are studied by means of modern nonlinear techniques to exemplify the behaviour of nonlinear systems with multiple equilibria. The main objective is to analyze and control this system. A nonlinear recursive backstepping controller is proposed and the transient performance is investigated. Systematic following of a reference model is introduced. Robustness problems as well as ways to tune the controller parameters are examined. Simulation results are submitted for the uncontrolled and controlled cases, verifying the effectiveness of the proposed controller. Finally a discussion and conclusions are given with possible future extensions.

KEY WORDS
Nonlinear Systems, Backstepping Control, and Chaos.

1. Introduction

The study of nonlinear systems has always been a challenge. Unlike linear systems, a complete theory for nonlinear systems does not exist, and hence stability analysis and control law designs are difficult. Usually the first step in analyzing any nonlinear system is to linearize it, if possible, about some nominal operating point and analyze the resulting linear system [1]. This is not always successful as it only provides local solutions, besides nonlinear dynamics are much richer than linear systems'. Unlike linear systems, nonlinear systems can have multiple isolated equilibrium points for which their states may converge depending on initial conditions.

Nonlinear system can exhibit limit cycles, subharmonic, harmonic, or almost-periodic oscillations. They can also go chaotic and some of these chaotic behaviours exhibit randomness despite the deterministic nature of the system. Finally, nonlinear systems can have multiple modes of behaviour that are strongly dependent on the input and initial conditions [2].

A phase portrait is perhaps one on the most powerful tools in analyzing second order nonlinear systems as it provides an easy visualization of the qualitative behaviour of the system. Stability theory plays a central role in analyzing nonlinear systems. Lyapunov techniques have proven to be the most efficient for investigating stability, asymptotic stability, and can be also used to show boundedness of the solutions even when the system has no equilibrium point. Other approaches to investigate stability exist, such as passivity, center manifold, perturbation and others, however, Lyapunov method is the one used in this paper.

Traditional adaptive nonlinear schemes are classified as direct or indirect and as Lyapunov-based or estimation-based. They involve parameter identification [3] and adaptation algorithm. The direct-indirect classification reflects the fact that updated parameters are either those of the controller or the plant respectively. The distinction between Lyapunov-based and estimation-based schemes is more substantial and is indicated in part by proof of asymptotic stability and convergence. Recursive design procedures, referred to as backstepping, can extend the applicability of Lyapunov-based deigns to non-linear systems [3, 4]. When the true parameters of the systems are unknown, the controller parameters are either estimated directly (direct scheme) or computed by solving the same design equations with plant parameters estimates (indirect scheme). The resulting controller is called a certainty equivalence controller. Backstepping designs are flexible and do not force the designed system to appear linear. They also avoid cancellation of perhaps useful nonlinearities and often introduce additional nonlinear terms to improve transient performance [5-7]. The idea of backstepping is to recursively de-sign a nonlinear controller by considering some of the state variables as "virtual controls". When trying to deal with unknown parameters a conflict will exist between virtual controls and parameter update laws that can be sorted out using adaptation [8].

In this paper, a new controller based on backstepping techniques [9] is presented. The paper is organized as follows; in Sec. 2, the mathematical model is presented. In Sec. 3, the stability analysis is discussed. The methodology of the controller is discussed in Sec. 4. Finally, discussions and conclusions are presented in Sec. 5.

2. Mathematical Model of Roll Motions

Roll motion is an undesirable feature of the behaviour of a ship in rough seas, and so it is natural to consider ways of reducing it. The most common devices for increasing roll damping are bilge keels. However, the effectiveness of keels is limited, and anti-roll tanks and fins are used when more control is required. Moreover, unlike keels, anti-roll tanks can be used when the ship is not underway [10]. A simplified mathematical model of the roll motions of ships is given by [11-18]:

$$\ddot{\theta} + \left(2\mu_1\dot{\theta} + \mu_3\dot{\theta}^3\right) + \left(\omega_0^2\theta + \alpha_3\theta^3 + \alpha_5\theta^5\right) = u \tag{1}$$

where the α_i are nonlinear coefficients, μ_1 is the linear damping coefficient, and μ_3 is the nonlinear damping coefficient. The nominal system parameters have the following values:

$\omega_0 = 5.278$, $\alpha_3 = -1.402\,\omega_0^2$, $\alpha_5 = 0.271\,\omega_0^2$.

The following control law is proposed:

$$u = f\left(x_1, x_2\right) \tag{2}$$

where f is a nonlinear function that needs to be found to stabilize the system. Using state space formulation:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ g(x_1, x_2) + h(x_1, x_2)u \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3}$$

where

$g(x_1, x_2) = -\left(\omega_0^2 x_1 + \alpha_3 x_1^3 + \alpha_5 x_1^5\right) - \left(2\mu_1 x_2 + \mu_3 x_2^3\right)$, and

$h(x_1, x_2) = 1$

y is the system output, θ.

At equilibrium, we have:

$$x_{2e} = 0,$$

$$x_{1e} = 0, \left(\frac{-\alpha_3 \pm \sqrt{\alpha_3^2 - 4\omega_0^2\alpha_5}}{2\alpha_5}\right)^{\frac{1}{2}} = 0, \pm 0.924, \pm 2.078 \tag{4}$$

Using linearization techniques, the system dynamics near equilibrium points are given by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}_e \cong \begin{bmatrix} 0 & 1 \\ -\left(\omega_0^2 + 3\alpha_3 x_{1e}^2 + 5\alpha_5 x_{1e}^4\right) & \left(2\mu_1 + 3\mu_3 x_{2e}^2\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_e$$

where the subscript e stands for equilibrium, thus:

$\dot{X}_e \cong A_e X_e + B_e U_e$

resulting in the following Lyapunov exponents: (5)

$$\lambda_e = -\left(\mu_1 + 1.5\mu_3 x_{2e}^2\right) \pm \sqrt{\left(\mu_1 + 1.5\mu_3 x_{2e}^2\right)^2 - \left(\omega_0^2 + 3\alpha_3 x_{1e}^2 + 5\alpha_5 x_{1e}^4\right)}$$

which reduces to:

$$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 - \left(\omega_0^2 + 3\alpha_3 x_{1e}^2 + 5\alpha_5 x_{1e}^4\right)}$$

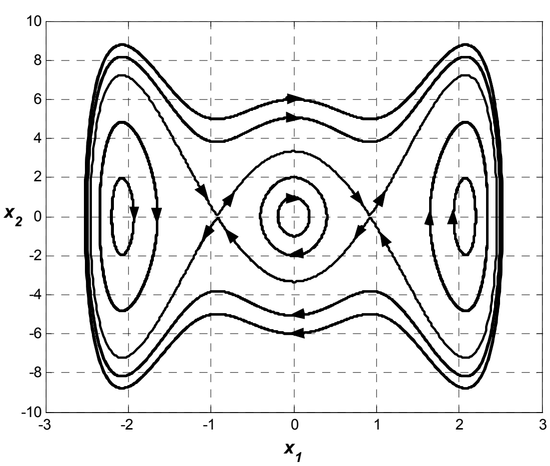
Table I. illustrates all the equilibrium points, and their dependence on μ_1 as well as their phase portrait types. Since $\mu_1 < 1$, points 1, 3, and 5 will have conjugate complex Eigen values with negative real parts indicating stable foci, while points 2 and 4 will have real positive and negative Eigen values resulting in saddle points.

TABLE I: Eigen values at equilibrium.

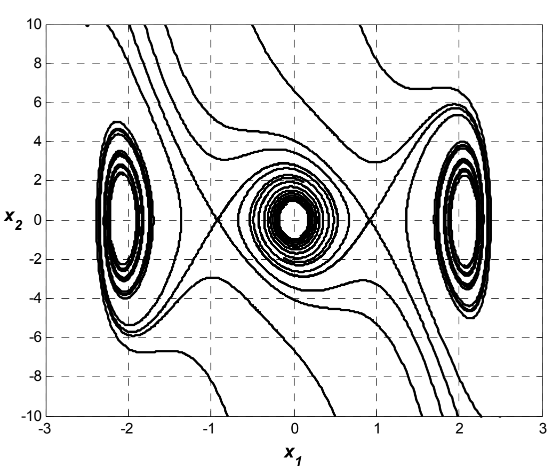
Point	$\left(x_{1e}, x_{2e}\right)$	λ_e	Type
1	(-2.078 , 0)	$-\mu_1 \pm \sqrt{\mu_1^2 - 225.735}$	focus
2	(-0.924 , 0)	$-\mu_1 \pm \sqrt{\mu_1^2 + 44.663}$	Saddle
3	(0 , 0)	$-\mu_1 \pm \sqrt{\mu_1^2 - 27.857}$	focus
4	(+0.924 , 0)	$-\mu_1 \pm \sqrt{\mu_1^2 + 44.663}$	Saddle
5	(+2.078 , 0)	$-\mu_1 \pm \sqrt{\mu_1^2 - 225.735}$	focus

3. Stability Analysis

The case without control signal, i.e. when u in (1) is equal to zero, is now investigated. Assuming zero damping, and using the nominal values of the system parameters, the system will exhibit the response shown in Fig. 1-a, where different trajectories corresponding to different initial conditions are plotted. The points (±2.078 , 0) and (0 , 0) will have pure imaginary Eigen values, thus the system will have a limit cycle with a frequency that depends on the initial conditions.



(a)



(b)

Fig. 1. Uncontrolled simulations.

The two saddle points at (±0.924 , 0) are shown to repel two positive orbits and attract two positive orbits, and the global manifolds of each saddle point are determined by these orbits.

The stable and unstable manifolds of each saddle point intersect transversely at the saddle point. At each saddle point, the tangent space to each manifold is a one-dimensional space, and the tangent spaces to the stable and unstable manifolds, when taken together, span the two-dimensional space of x_1-x_2. Also, it is seen that stable manifold of each addle point forms a nontransversal intersection with the unstable manifold of the other saddle point.

Homoclinic orbits are formed at each saddle point, where one of the ends of the global stable manifold merges with the one of the ends of the global unstable manifold for the same saddle point. Also, heteroclinic orbits are formed by the outermost orbits that run between the saddle points in both forward and reverse time. Both the homoclinic and heteroclinic orbits separate regions of qualitatively different motions, hence there are called separatrices.

Referring to Fig. 1-b, all the foci now have complex eigen values with negative real parts, so any orbit in the neighborhood of these points will spiral in and eventually reach an equilibrium state. Also, it is quite clear that damping has the effect of forcing both homoclinic and heteroclinic orbits to disappear.

The global stable manifolds of the saddle points in Fig 1-b separate the stability boundaries of the stable foci. The region to the right of the stable manifold of (+0.924 , 0) is the basin of attraction of the stable focus (+2.078 , 0), and the region to the left of the stable manifold of (-0.924 , 0) is the basin of attraction of the stable focus (-2.078 , 0), while the region between the stable manifold of the two saddle points is the basin of attraction for the stable focus (0 , 0).

Many nonlinear dynamic systems produced similar kind of results. In fact, in many autonomous systems one can determine the global stable manifolds of the unstable fixed points to define the stability boundaries of the stable fixed points of the system [19].

Fig. 2. further illustrates the effect of damping on the transient response of the system. In Fig. 2-a, the system starts at (x_1 = 3 , x_2 = 0) and because of the absence of damping the system oscillates with a limit cycle of frequency equal to approximately 0.6 seconds and circles the equilibrium point (0 , 0), and after introducing the damping, the oscillation dies out, and the system settles to the new equilibrium point (+2.078 , 0).

In Fig. 2-b, the system starts from (x_1 = 1 , x_2 = 0) and oscillates around the (+2.078 , 0) with a frequency equal to approximately 1.1 seconds, and when damping is introduced, the system settles to the same equilibrium point, unlike the behaviour observed in Fig. 2-a.

Finally in Fig. 2-c, without damping the system passes through the two saddle points at (±0.924 , 0) and encircles the five equilibrium points with a limit cycle of frequency equal to approximately 6.6 seconds, then settles to the equilibrium point (0 , 0) in the presence of damping.

Having investigated the behaviour of the system and the effect o damping, it is obvious that the nonlinear damping coefficient, μ_3, has no effect near equilibrium, and that it will only affect the transient response of the system. The system is shown to settle down to different equilibrium points depending on the initial conditions; hence even with the presence of damping the system is locally stable.

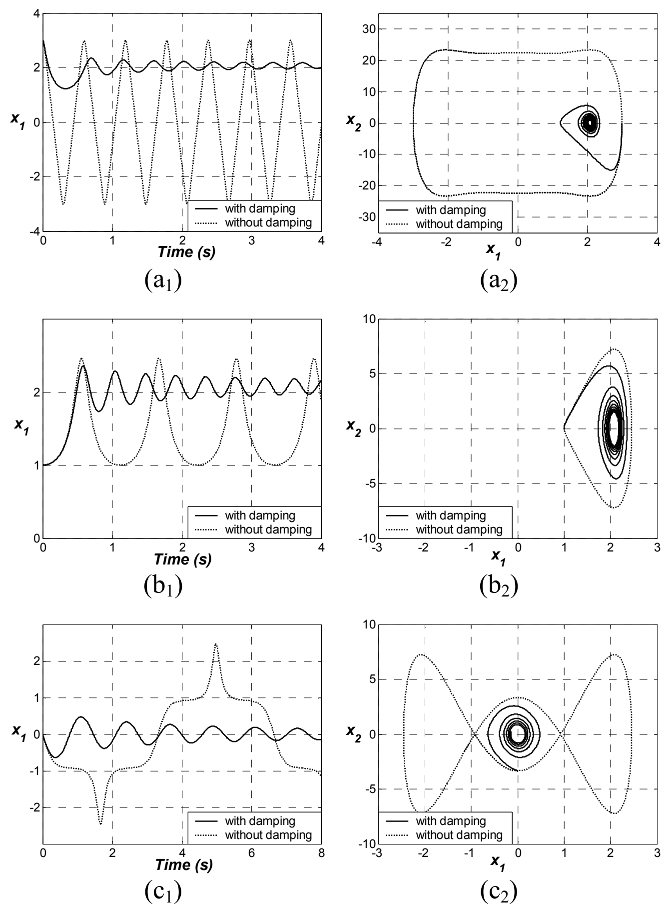


Fig. 2. Effect of damping on uncontrolled response.

The objective of this paper is to introduce a control law that will have the origin as a globally stable equilibrium point and to force the system to approach the origin from any initial condition while exhibiting a satisfactory transient performance. The following section introduces a strategy to design a nonlinear recursive control law that dictates a predefined transient behaviour, thus implementing a model-reference-like action. Also the problem of having an ill-defined model in which some of the parameters are uncertain, or some parasitic dynamics are ignored will be studied.

4. Controller Design

A backstepping controller is designed such that the system is stabilized to the origin. The second state of the system, x_2, is used as a virtual control signal to the system output, x_1. The following nonlinear dynamics will now be assumed for the desired virtual control signal:

$$x_{2_{des}} = -c_1 x_1 - c_2 x_1^3 \tag{6}$$

where c_1 and c_2 are design parameters that need to be carefully chosen such that the closed loop system is stable. The nonlinear term c_2 introduces a nonlinear spring-like action with little effect for small output deviations, and a considerable effect when the deviation from the origin is large. It should be noted that other nonlinear forms could have been proposed, but one must choose the best compromise between complexity and causality. The signal x_{2des} acts as a reference model for the system state x_2 and the difference between them will be derived to zero in a finite time that depends on the control parameters. The virtual error is given by:

$$e_v = x_2 - x_{2_{des}} = x_2 + c_1 x_1 + c_2 x_1^3 \tag{7}$$

Using (6) and (7), the system is now transferred to the following new state space (with the same order):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{e}_v \end{bmatrix} = \begin{bmatrix} e_v - c_1 x_1 - c_2 x_1^3 \\ \dot{x}_2 + x_2\left(c_1 + 3c_2 x_1^2\right) \end{bmatrix} \tag{8}$$

Lyapunov second method will now be used to test for the system stability. Introducing the following augmented positive definite Lyapunov function:

$$V_a = \tfrac{1}{2} x_1^2 + \tfrac{1}{2} e_v^2 \tag{9}$$

results in:

$$\begin{aligned} \dot{V}_a &= x_1 \dot{x}_1 + e_v \dot{e}_v \\ &= -c_1 x_1^2 - c_2 x_1^4 \\ &\quad + e_v \left\{ x_1 + g(x_1,x_2) + h(x_1,x_2)u + x_2\left(c_1 + 3c_2 x_1^2\right) \right\} \\ &= -c_1 x_1^2 - c_2 x_1^4 - c_3 e_v^{\;2} \end{aligned} \tag{10}$$

where c_3 is a design parameter, and the closed loop control law, u, is chosen such that (10) is guaranteed to be negative definite. As seen from (10), this can be easily achieved by letting the term between the curly brackets equal to $-c_3 e_v$. Using (7) and (10), this is equivalent to:

$$\begin{aligned} & x_1 + g(x_1,x_2) + h(x_1,x_2)u + x_2\left(c_1 + 3c_2 x_1^2\right) \\ &= -c_3\left(x_2 + c_1 x_1 + c_2 x_1^3\right) \end{aligned} \tag{11}$$

which can be put in the more compact form:

$$\begin{aligned} & g(x_1,x_2) + h(x_1,x_2)u \\ &= -x_1\left(1 + c_1 c_3\right) - x_2\left(c_1 + c_3\right) - c_2 x_1^2\left(c_3 x_1 + 3x_2\right) \\ &= w(x_1,x_2) \end{aligned} \tag{12}$$

from which, the following control low is deduced:

$$u = \frac{w(x_1,x_2) - g(x_1,x_2)}{h(x_1,x_2)} \tag{13}$$

Because of the special dynamic structure of the system, the designed control, u, is causal and can be easily implemented by choice of the design parameters. The resulting closed loop system is consequently given by:

$$\begin{bmatrix} \dot{y} \\ \dot{e}_v \end{bmatrix} = \begin{bmatrix} -c_1 - c_2 y^2 & 1 \\ -1 & -c_3 \end{bmatrix} \begin{bmatrix} y \\ e_v \end{bmatrix} \tag{14}$$

Eliminating e and its derivative from (14), the output dynamics is given by:

$$\begin{aligned} & \ddot{y} + \dot{y}(c_1 + 3c_2 y^2 + c_3) + y(1 + c_1 c_3 + c_2 c_3 y^2) = 0, \text{ or} \\ & \left\{\ddot{y} + (c_1 + c_3)\dot{y} + (1 + c_1 c_3)y\right\} + c_2\left\{3y^2 \dot{y} + c_3 y^3\right\} = 0 \end{aligned} \tag{15}$$

From (15), it is seen that the closed loop system can be made linear by choosing $c_2 = 0$, but this will be on the expense of loosing the *useful* cubic nonlinearity in (6).

4.1 Choosing the control parameters

With reference to (10), there are no constraints on choosing the design parameters other than: $c_i > 0$, $i = 1, 2$, and 3. This offers more flexibility in the design and enables controlling the transient behaviour of the system to satisfy a certain performance criterion. However, when choosing the best vales for the control parameters, we should also consider the nonlinearities of the system at hand, and the maximum control effort that can used to avoid having a poor performance.

To illustrate this idea, a linear reference model is used by letting $c_2 = 0$, which can be made a standard linear second order system whose performance is characterized by two parameters, damping ratio, ξ, and natural damping frequency, ω_n. This translates to:

$$\ddot{y} + (c_1 + c_3)\dot{y} + (1 + c_1 c_3)y = \ddot{y}_{des} + 2\xi\omega_n \dot{y}_{des} + \omega_n^2 y = 0 \tag{16}$$

Hence more constraints are added to the choice of the remaining control parameters, c_1 and c_3, depending on the required values of both ξ and ω_n. This is represented graphically in Fig. 3., where it is seen that a unique set of design parameters is obtained for each desired performance.

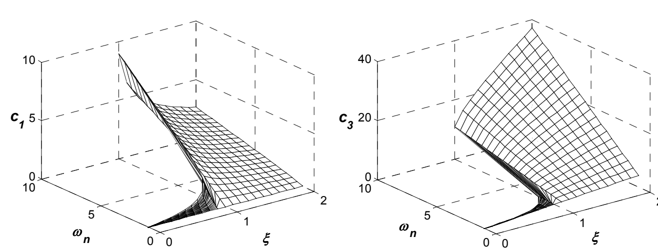


Fig. 3. Design parameters vs. performance parameters.

The control law given by (13) assumes a deterministic model for the nonlinear system, and attempts to cancel its dynamics $g(x_1,x_2)$. If the modeling process is not accurate, nonexact cancellation can lead to instability or even chaos. This problem may be resolved by implicitly implementing an online estimator for the uncertain (unknown) parameters of the system with the drawback of increasing the overall order of the closed loop system [20]. Another feasible and simple solution is to try to mimic the behavior of the well-known PID controllers. Since the required equilibrium point is the origin, this reduces to a simple regulation problem for which a PD controller is sufficient. Using (3), (12) and (13), an explicit form for the control law is given by:

$$\begin{aligned} u = &\left\{ \left(\omega_0^2 - c_1 c_3 - 1\right) x_1 + \left(2\mu_1 - c_1 - c_3\right) x_2 \right\} \\ &+ \left\{ \alpha_3 x_1^3 + \alpha_5 x_1^5 - c_2 c_3 x_1^3 + \mu_3 x_2^3 - 3 c_2 x_1^2 x_2 \right\} \\ = &\; u_{PD} + u_{NL} \end{aligned} \tag{17}$$

where u_{PD} the linear PD controller with $K_P = (\omega_0^2 - c_1c_3 - 1)$, and $K_D = (2\mu_1 - c_1 - c_3)$. The second term, u_{NL} , is the nonlinear term that has a negligible effect provided that $\|x_i\| \le 1$, i = 1 and 2. For a given region of initial conditions, the control parameters can be chosen such that u_{PD} will always dominates u_{NL}, thus ensuring a self-correction action that will add robustness to the control law design when the system dynamics are partially known.

4.2 Simulation Results

The deterministic case is considered first, where the system has the nominal values with no uncertainties. If c_2 is made zero, the closed loop system becomes linear with standard second order dynamics characterized by the two parameters $\omega_n = \sqrt{1 + c_1 c_3}$ and $\xi = (c_1 + c_3)/2\sqrt{1 + c_1 c_3}$.

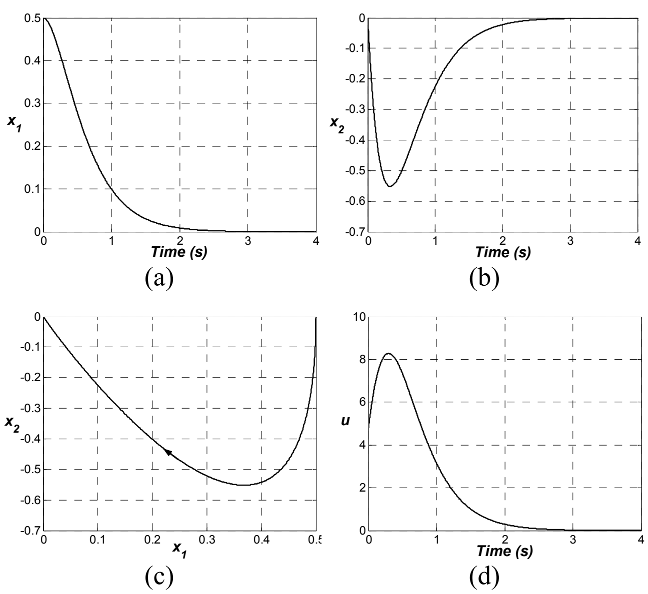


Fig. 4. Controlled simulations (deterministic model).

The results, shown in Fig. 4 illustrates such response with c_1 = 4 and c_3 = 2, which implies a critically damped response.

When c_2 is put into action, as expected it will smooth the response of the system by slowing it down. This is depicted in Fig. 5. where different values for c_2 were used.

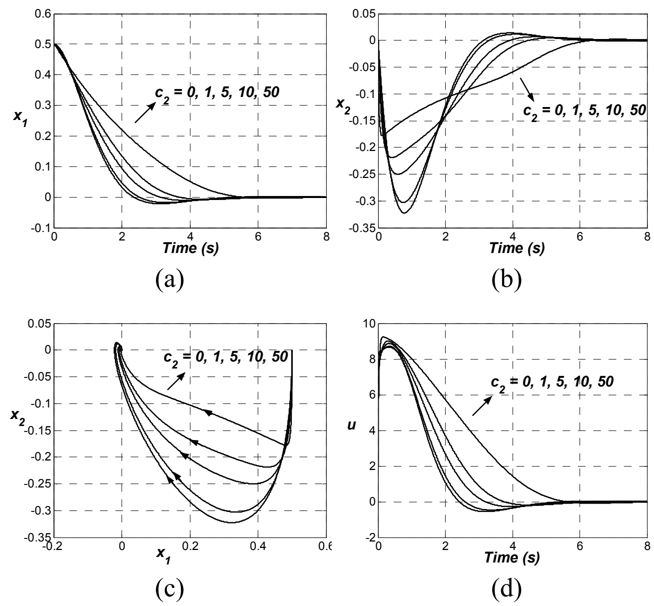


Fig. 5. Effect of c_2 on the controlled response.

Finally, the case when the system dynamics are perturbed is considered. The nonlinear function $g(x_1,x_2)$ is assumed to undergo a multiplicative uncertainty such that:

$$g(x_1, x_2) = g_{nom} * (1 + \Delta g) \tag{18}$$

Fig. 6. illustrates the robustness of the control law design by showing the response of the system under the conditions: $\Delta g\%$ = 1, 2, 5, 10, and 20.

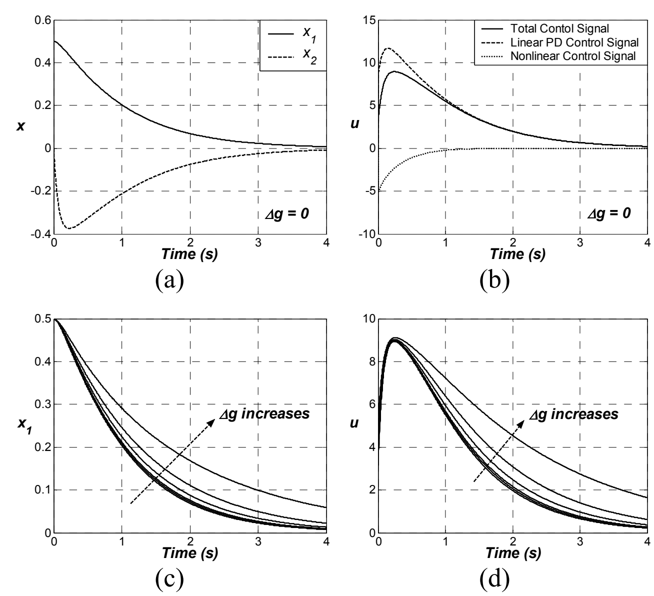


Fig. 6. Controlled simulations (perturbed model).

The control parameters were assumed: $c_1 = 10$, $c_2 = 5$, and $c_3 = 1$ as this choice ensures a satisfactory performance as indicated in Fig 6-a, as well as minimizing the contribution of the nonlinear part in the control signal as shown in Fig. 6-b. Fig. 6-c and 6-d show that the closed loop response is still satisfactory, even when the model is under perturbation, a byproduct of the self-correction action of the designed control law.

5. Discussion and Conclusion

Modern nonlinear control was applied to a nonlinear system that describes the dynamics of roll motions of ships. Nonlinear recursive backstepping controlled the undesirable oscillatory behaviour. The study shows the effectiveness of the technique in all operating regions. The designed controller achieves two goals, first stabilizing the system, and second providing means of shaping the transient performance of the system. Usually a conflict occurs if these two goals are to be achieved simultaneously, but the designed controller resolves such conflict by introducing a flexible set of controller parameters that adds more freedom in the design. The dynamic structure of the virtual control can be carefully chosen to satisfy some design criteria; it need not be as given in (5). This adds more versatility to the designed controller. During the analysis and simulation, it was assumed that the system parameters were deterministic, a crucial prerequisite for the use of (2). If, however, this is not the case, robustness can be added to the system by incorporating a PID-like behaviour, states estimator, parameters estimator, or both. The design of the update law mechanism for these estimators can be integrated into the design by introducing more virtual control parameters and changing the corresponding Lyapunov function, (9), accordingly [20, 21].

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