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Adaptive Sliding Mode Control For Roll Motions Of Ships

Elham Amini Boroujeni[1] and Hamid Reza Momeni[2]

[1] Department of Engineering, Tarbiat Modares University, Tehran, Iran
(E-mail: e.amini@modares.ac.ir)
[2] Department of Engineering, Tarbiat Modares University, Tehran, Iran
(Tel : +98-21-8288-3375; E-mail: momeni_h@modares.ac.ir)

Abstract: This paper is concerned with the problem of adaptive sliding mode control for roll motions of ships as an uncertain nonlinear system with five equilibrium points which faces external disturbance .The purpose is designing a robust controller for this model with these properties, under the influence of external sinusoidal disturbances. The capabilities of the proposed controller are verified by the simulation results.

Keywords: Adaptive Sliding Mode Control, Roll Motions Of Ships, Uncertain nonlinear systems, External disturbance.

1. INTRODUCTION

Nonlinear systems may have multiple modes of behavior that are highly dependent on the input and initial conditions. Roll motions of ships is an example of nonlinear systems with five equilibrium points, three focus and two saddle points. Its model has uncertain parameters and it faces external disturbance a lot.

Roll motion is an undesirable feature of the behavior of a ship in rough seas, and so it is natural to consider ways of reducing it. The most common devices for increasing roll damping are bilge keels. However, the effectiveness of keels is limited, and anti-roll tanks and fins are used when more control is required. Moreover, unlike keels, anti-roll tanks can be used when the ship is not underway. [1]

At present, many control schemes for roll reducing have been studied. The objective of this study is, to design a stabilizing feedback controller that takes into account model uncertainties. Robustness is a major consideration since it is virtually impossible to develop accurate models for ship motions.[2]

Sliding mode control (SMC) has been developed and applied widely in industry. SMC is commonly favored as a powerful robust control method for its independence from parametric uncertainties and external disturbances under matching conditions. However, control input chattering in the sliding mode is a shortcoming in practice. [3]

Adaptive sliding mode control (ASMC) is a solution to guarantee the occurrence of the sliding motion even when the parameters of the model are fully unknown. Robust control of roll motions of ships has an important role in sea industrial, so we want to investigate ASMC for roll motions of ships in this paper and verify its capabilities.

This paper is organized as follow: section 2, includes description of system. In section 3, designed controller for Roll motion of ships and stability prove has come, our results are proposed in section 4, the simulation results are demonstrated. Finally, some concluding remarks are given in section 5.

2. MODELING

A good modeling can help to design an appropriate controller; a simplified mathematical model of the roll motions of ships is given by: [1]

$$\ddot{\theta}+(2\mu_1\dot{\theta}+\mu_3\dot{\theta}^3)+(\omega_0^2\theta+\alpha_3\theta^3+\alpha_5\theta^5)=u \tag{1}$$

Where θ is the roll angle, the α_i are nonlinear coefficients, μ_1 is the linear damping coefficient, and μ_3 is the nonlinear damping coefficient and u is the control input. The nominal system parameters have the following values.

$$\omega_0=5.278 \quad \alpha_3=-1.402\omega_0^2 \quad \alpha_5=0.271\omega_0^2$$
$$\mu_1 \ll 1 \tag{2}$$

Therefore the equilibrium point will be the same as below:

$$x_{2e}=0$$
$$x_{1e}=0,\left(\frac{-\alpha_3\pm\sqrt{\alpha_3^2-4\omega_0^2\alpha_5}}{2\alpha_5}\right)^{\frac{1}{2}}=0,\pm0.924,\pm2.078 \tag{3}$$

By using linearization techniques the linearized model will be as (4)

$$\dot{X}=\begin{bmatrix}0 & 1\\ -(\omega_0^2+3\alpha_3x_{1e}^4) & 2\mu_1+3\mu_3x_{2e}^2\end{bmatrix}X+\begin{bmatrix}0\\1\end{bmatrix}u(t) \tag{4}$$
$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}X$$

Resulting in the following eigenvalues :

$$\lambda_e=-(\mu_1+1.5\mu_3x_{2e}^2)\pm\sqrt{(\mu_1+1.5\mu_3x_{2e}^2)^2-(\omega_0^2+3\alpha_3x_{1e}^2+5\alpha_5x_{1e}^4)} \tag{5}$$

So eigenvalues at equilibrium points will be as it has shown in table.1.

Table.1. Eigenvalues at equilibrium point

Type	λ_e	(x_{1e}, x_{2e})
Focus	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 - 225.735}$	(-2.078,0)
Saddle	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 + 44.663}$	(-0.924,0)
Focus	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 - 27.857}$	(0,0)
Saddle	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 + 44.663}$	(0.924,0)
Focus	$\lambda_e = -\mu_1 \pm \sqrt{\mu_1^2 - 225.735}$	(2.078,0)

Types of equilibrium points show effect of initial value and external disturbance in determining final value. The objective of this paper is to introduce a control law that will have the origin as a globally stable equilibrium point to force the system to approach the origin from any initial condition while exhibiting a satisfactory transient performance. The following section introduces a strategy to design a controller that dictates a predefined transient behavior.

Fig. 1 illustrates the behavior of uncontrolled system.

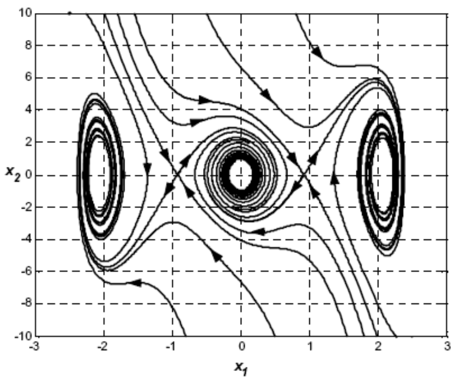

Fig.1. uncontrolled system

3. CONTROLLER

In this section an adaptive sliding mode controller is designed to have the origin as a globally stable equilibrium point to force the system to approach the origin from any initial condition besides the existence of uncertainty in parameters and external disturbance.

A switching surface, which makes it easy to guarantee the stability of the error dynamics in the sliding mode, is first proposed. And then, based on this switching surface, an adaptive sliding mode controller is derived to guarantee the occurrence of the sliding motion.

3.1 Sliding mode controller

The design of sliding mode control [4-6] involves two phases: The first phase is to select the switching surface S(t) to prescribe the desired dynamic characteristics of the controlled system. The second phase is to design control input combined of two parts: equivalent control or linear part of input that it is linear feedback of states and computed to stabilize the nominal linear system. This part makes the states move toward and reach to the sliding surface. Another part is discontinuous control such that the system remains on the sliding surface forever.

To ensure the asymptotical stability of the sliding mode, a switching surface S(t) in the error space is defined as (4),

$$S(t) = e_2 + \lambda e_1 \tag{4}$$

e_1 and e_2 has defined as

$$e_1 = x_1 - x_{1des} \qquad e_2 = \dot{e}_1 \tag{5}$$

By using this definition in (4) we have

$$S(t) = e_2 + \lambda e_1 = 0 \Rightarrow e_2 = -\lambda e_1 \tag{6}$$

$$\dot{e}_1 + \lambda e_1 = 0 \tag{7}$$

Obviously, if the design parameter $\lambda \succ 0$ is specified, the stability of (7) is surely guaranteed, that is $\lim_{t\to\infty} e_1(t) = 0$. Furthermore, by Eq. (6), $e_2(t)$ is also stable, that is $\lim_{t\to\infty} e_2(t) = 0$. Meanwhile, it is worthy of note that the value λ is also relative to the speed of error system response in the sliding mode. And controller will be defined as below,

$$u(t) = -Ce_2 - \xi \,\mathrm{sgn}(S(t)) \tag{8}$$

This controller demonstrates a discontinuous control law and the phenomenon of chattering will appear. In order to eliminate the chattering, the controller (8) can be modified as:

$$|\varepsilon| \ll 1$$

$$\mathrm{sgn}(S) = \frac{S}{|S| + \varepsilon} \tag{9}$$

Where ε is a sufficiently small positive constant.

3.2 Adaptive sliding mode controller

In this part we want to use adaptive rules [7] for ξ .As it has shown in (8) it is a coefficient in control signal. By considering adaptive rules on ξ and defining ξ as bellow

$$\xi = \hat{\omega}_0 X_1^2 + \hat{\alpha}_3 X_1^3 + \hat{\alpha}_5 X_1^5 + 2\hat{\mu}_1 X_2 + \hat{\mu}_3 X_2^3 + \hat{\delta} \tag{11}$$

And by using adaptive rules as (12)

$$\begin{aligned} \dot{\hat{\omega}}_0 &= |X_1^2|.|S| & \hat{\omega}_0(0) &= 5.278 \\ \dot{\hat{\alpha}}_3 &= |X_1^3|.|S| & \hat{\alpha}_3(0) &= -1.402\omega_0^2 \\ \dot{\hat{\alpha}}_5 &= |X_1^5|.|S| & \hat{\alpha}_5(0) &= 0.271\omega_0^2 \end{aligned} \tag{12}$$

$$\dot{\hat{\mu}}_1 = |2X_2|.|S| \qquad \hat{\mu}_1(0) = 0.75$$

$$\dot{\hat{\mu}}_3 = |X_2^3|.|S| \qquad \hat{\mu}_3(0) = 0.5$$

$$\dot{\hat{\delta}} = |S| \qquad \hat{\delta}(0) = 0.1$$

We have an ASMC for roll motions of ships.

The proposed adaptive control scheme above will guarantee the globally asymptotical stability for the error system (5), and is proven in the following.

Prove of stability:

Let:

$$\theta_1 = \hat{\omega}_0 - |\omega_0| \qquad \theta_2 = \hat{\alpha}_3 - |\alpha_3|$$
$$\theta_3 = \hat{\alpha}_5 - |\alpha_5| \qquad \theta_4 = \hat{\mu}_1 - |\mu_1|$$
$$\theta_5 = \hat{\mu}_3 - |\mu_3| \qquad \theta_6 = \hat{\delta} - |\delta| \tag{13}$$

It is assumed that $|\omega_0|$, $|\alpha_3|$, $|\alpha_5|$, $|\mu_1|$, $|\mu_3|$ and $|\delta|$ are unknown constants. Thus the following expression holds.

$$\dot{\theta}_1 = \dot{\hat{\omega}}_0 \qquad \dot{\theta}_2 = \dot{\hat{\alpha}}_3$$
$$\dot{\theta}_3 = \dot{\hat{\alpha}}_5 \qquad \dot{\theta}_4 = \dot{\hat{\mu}}_1$$
$$\dot{\theta}_5 = \dot{\hat{\mu}}_3 \qquad \dot{\theta}_6 = \dot{\hat{\delta}} \tag{14}$$

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}(S^2 + \sum_{i=1}^{5} \theta_i^2) \tag{15}$$

It is clear that V is a positive definite function, then taking the derivative of V(t) with respect to time t

$$\dot{V}(t) = S\dot{S} + \sum_{i=1}^{5} \theta_i \dot{\theta}_i \tag{16}$$

so by (1),(4) and (8) we have

$$\dot{V}(t) = S(\dot{e}_2 + \lambda e_2) + \sum_{i=1}^{5} \theta_i \dot{\theta}_i$$
$$= S(\omega_0 X_1^2 + \alpha_3 X_1^3 + \alpha_5 X_1^5 + 2\mu_1 X_2 + \hat{\mu}_3 X_2^3 + \gamma \operatorname{sgn}(S)) + \sum_{i=1}^{5} \theta_i \dot{\theta}_i \tag{17}$$

Where

$$\gamma = \frac{-\xi}{C} \tag{18}$$

So

$$\dot{V}(t) \le |S||\omega_0||X_1^2| + |S||\alpha_3||X_1^3| + |S||\alpha_5||X_1^5| + 2|S||\mu_1||X_2| + |S||\mu_3||X_2^3| + \delta|S| + \gamma|S| + \sum_{i=1}^{5} \theta_i \dot{\theta}_i$$

$$= |S||X_1^2|(\underbrace{|\omega_0| - \hat{\omega}_0}_{-\theta_1}) + |S||X_1^3|(\underbrace{|\alpha_3| - \hat{\alpha}_3}_{-\theta_2}) + |S||X_1^5|(\underbrace{|\alpha_5| - \hat{\alpha}_5}_{-\theta_3}) + 2|S||X_2|(\underbrace{|\mu_1| - \hat{\mu}_1}_{-\theta_4}) + |S||X_2^3|(\underbrace{|\mu_3| - \hat{\mu}_3}_{-\theta_5})$$
$$+ |S|(\underbrace{|\delta| - \hat{\delta}}_{-\theta_6}) + \gamma|S| + \xi|S| + \sum_{i=1}^{5} \theta_i \dot{\theta}_i$$
$$= (\gamma + \xi)|S| \tag{19}$$

The condition of negative $\dot{V}(t)$ is $(\gamma + \xi) < 0$. So by (17), since $\xi > 0$ for all $t > 0$, if we choose $C < 1$ we have negative $\dot{V}(t)$ which will guarantee zero errors in finite time.

4. SIMULATION RESULTS

This section shows the simulation results. We have considered three different cases. First case shows results of implementing controller on nonlinear model with nominal values, without external disturbance. Second one describes results of implementing controller on nonlinear model with considering uncertainty in nominal parameters and finally third one shows results of implementing controller on nonlinear model with considering uncertainty in nominal parameters and sinusoidal external disturbance.

In this part, external disturbance is equal to 10sin(t) and initial value is $X_0 = [3, -2]^T$ .Uncertainties are +20% on $g(x_1, x_2)$ and -25% on $h(x_1, x_2)$.

As it has shown in fig 1 this initial value in our model causes going to the point (2.078, 0) but here controller guide it to the origin.

Figure 2 shows both X_1 and X_2 in time domain, it is almost the same in all three conditions. There is no overshoot or undershoot in output(X_1), settling time is little and steady state errors are negligible.

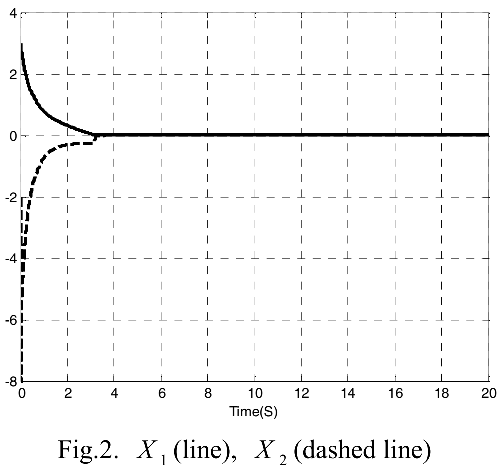

Fig.2. X_1 (line), X_2 (dashed line)

Figure 3 shows phase portrait of three cases. As it has shown the results are the same.X_2 is derivation of X_1 and fig 3 shows how negative X_2 causes decreasing X_1 to reach origin.

Figure 4 shows differences in control signals, which cause similarity in outputs.

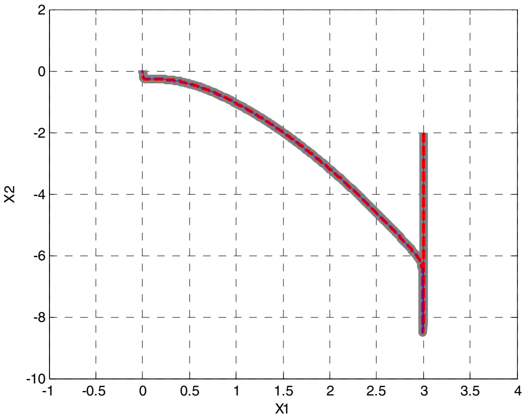

Fig.3. nominal parameters without external disturbance (gray line), uncertain parameters without external disturbance (red dashed line) and uncertain parameters with external disturbance (blue line).

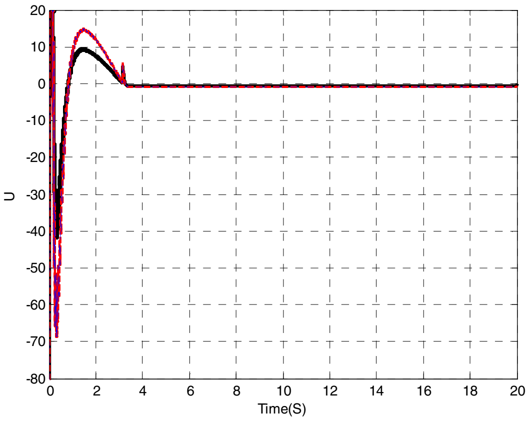

Fig.4. nominal parameters without external disturbance (black line), uncertain parameters without external disturbance (blue doted line) and uncertain parameters with external disturbance (red dashed line).

Although conditions are different in a wide range but our robust adaptive controller has produced a stable output which can handle uncertainty and external disturbance.

5. CONCOLUTION

In this paper adaptive sliding mode control is applied to a nonlinear system that describes the dynamics of roll motions of ships. Ships face uncertainty and disturbance in rough seas a lot and simulations show that not only controller guides the response to the origin as a global equilibrium point but also external disturbance can't disturb convergence. So this controller can handle inaccuracy and external disturbance beside small settling time, without any overshoot or undershoot in output.

This controller adapts itself with different conditions –with and without existence of disturbance and uncertainty- to keep its output constant by changing its control input.

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