Robustly Optimal Mechanisms for Selling Multiple Goods (Yeon-koo Che & Weijie Zhong)

Presenter: Renjie Zhong

Theory Reading Group @ Renmin

Multi-dimensional Screening Sucks

Why? Greater Complexity!

economic answer: inclusion, exclusion, extraction & correlated value distribution

mathematical answer: IC analysis & optimize multiple functional

Make Life Easier

various ways to reduce dimensions $\Rightarrow$ everybody is happy with one-dimensional thing

- distributional assumptions sufficient for some certain bundling policy

  Haghpanah and Hartline, 2021; Ghili, 2023; Frank Yang; ...

- robustness

  Carroll, 2017; Deb and Roesler, 2023; Che and Zhong, 2024; ...

- convergence rate

  Hart and Nisan, 2017; Cai, Devanur, and Weinberg, 2019; Frick, Ijima, and Ishii, 2024 ; ...

Bundling Policy & Well-known Arguments

pure bundling: negatively correlated values (Adams and Yellen,1976)

full separation: perfectly correlated (comonotonic) item values (Carroll,2017)

bundling within genres: comonotonic but asymmetrical genres (this paper)

Robustness = Perfectly Correlateion

Robustness is perfectly correlated distributional assumption.

"robustness game"

- nature chooses a value distribution to minimize the expected revenue
  1. pure bundling vs. asymmetric distribution
  2. separate bundling vs. "negatively-correlated" counterfactual distribution
- designer chooses a pricing strategy to maximize the expected revenue
- perfectly correlated: nature "levels" the seller's virtual valuation

Setup

- one seller, one buyer, sell n items
- private values $v = (v_1, v_2, ..., v_n)$
- product category $\mathcal{K}$ be an arbitrary partition of the goods, with $K \in \mathcal{K}$
- $\mathcal{F} \subseteq \Delta(\mathbb{R}_+^n)$ with $\mu_i(F) := \mathbb{E}_F[v_i]$ and $\sigma_K(F) := \mathbb{E}_F\left[\varphi_K\left(\sum_{i \in K} v_i\right)\right]$, $\forall K \in \mathcal{K}$

Remark: the seller has some knowledge about *mean value of item i* and the the *dispersion of category K's value under F*, where $\varphi_K : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying $\varphi_K'' \geq \epsilon$ for some $\epsilon > 0$

Mechanisms & K-bundled sales

direct mechanisms $\mathcal{M} = (q(v), t(v))$: *allocation* $q : \mathbf{v} \mapsto [0,1]^n$ & *payment* $t : \mathbf{v} \mapsto \mathbb{R}_+$

$$\mathbf{v} \cdot \mathbf{q}(\mathbf{v}) - t(\mathbf{v}) \geq \sup_{\mathbf{v}' \in \mathbb{R}^n_+} \mathbf{v} \cdot \mathbf{q}(\mathbf{v}') - t(\mathbf{v}') \tag{IC}$$

$$\mathbf{v} \cdot \mathbf{q}(\mathbf{v}) - t(\mathbf{v}) \geq 0 \tag{IR}$$

for each $\mathbf{v} \in \mathbb{R}^n_+$.

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for each $\mathbf{v} \in \mathbb{R}_+^n$. $K(i)$ be the category $K \in \mathcal{K}$ containing item i

K-bundled sales mechanism $\mathcal{M}_{\mathcal{K}}$: for each $K \in \mathcal{K}$, $q_K : \mathbb{R}_+ \to [0,1]$ and $t_K : \mathbb{R}_+ \to \mathbb{R}$

$$t(v) = \sum_{K \in \mathcal{K}} t_K \left(\sum_{j \in K} v_j \right) \quad \text{and} \quad q_i(v) = q_{K(i)} \left(\sum_{j \in K(i)} v_j \right).$$

sells each bundle K with probability q_K and collects expected payment t_K (also IC IR)

Robustness

The seller's revenue from a mechanism $M \in \mathcal{M}$ given value distribution F is

$$R(M, F) := \int t(v)F(dv).$$

The seller's objective is to maximize the revenue guarantee.

$$R^* := \sup_{M \in \mathcal{M}} \inf_{F \in \mathcal{F}} R(M, F)$$

Main Result

Theorem 1: It is robustly optimal for the seller to use a K-bundled sales mechanism.

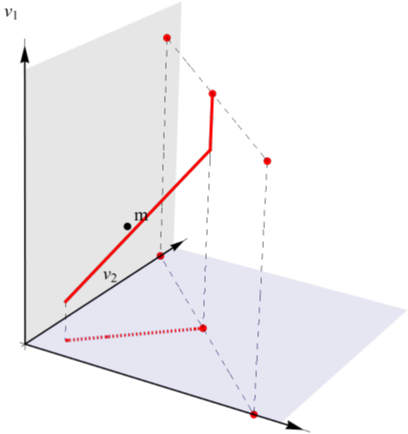
Theorem 2: K-bundled sales is essential for achieving that optimal revenue.

1. for nonempty $J, J' \subseteq K$, $J \cap J' = \Phi$ mechanism separating J and J' is not robustly optimal
2. mechanism bundling K and K' is generically not robustly optimal

Why is full separation not robustly optimal?

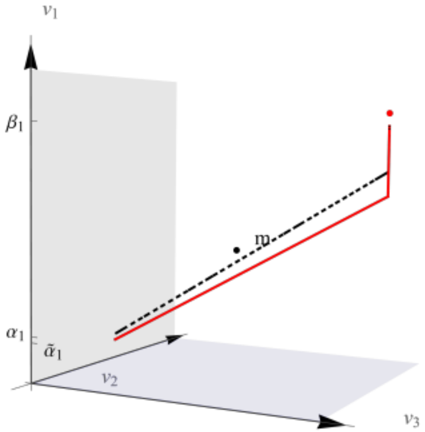
three goods world: $\mathcal{K} = \{\{1\}, \{2,3\}\}$, valuation (v_1, v_2, v_3)

$\mathbb{E}[v_1] = 0.5, \mathbb{E}[v_2] = \mathbb{E}[v_3] = 0.3$, $\mathbb{V}[v_1] = \mathbb{V}[v_2 + v_3] = 0.1$, where $\varphi_{\{1\}}(v) = \varphi_{\{2,3\}}(v) = v^2$

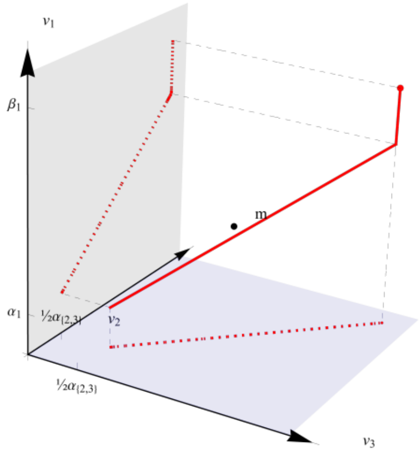


Why is pure bundling not robustly optimal?

bundling entails inefficient screening (the fear of asymmetric distribution)



Robust Optimality for K Sale Bundling



Construction of F^*: X distributed from $[1, \infty)$ according to cdf:

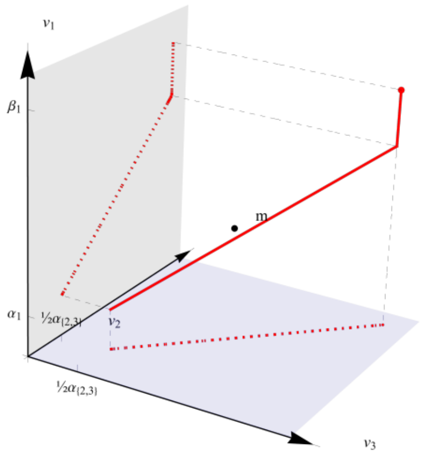
$$H(x) := 1 - \frac{1}{x},$$

valuation is then determined as

$$v_i = \min\{\alpha_i X, \beta_i\},$$

where (α_i, β_i)'s are chosen (uniquely) to satisfy the moment conditions

$\Rightarrow$ one-dimensional screening



virtual value of each item is

$$V_i(x) - V_i'(x)\frac{1-H(x)}{h(x)} = \begin{cases} 0 & \text{if } x < \beta_i/\alpha_i, \\ \beta_i & \text{if } x \geq \beta_i/\alpha_i, \end{cases}$$

Mechanism M^*:

$$g_1(p) = 2\lambda_1 \cdot \frac{\beta_1 - p}{p}, \quad g_{23}(p) = 2\lambda_{23} \cdot \frac{\beta_{23} - p}{p}$$

on support $[\alpha_1, \beta_1]$, and $[\alpha_{23}, \beta_{23}]$, where $\alpha_{23} = \alpha_2 + \alpha_3$ and $\beta_{23} = \beta_2 + \beta_3$

$\lambda = (\lambda_1, \lambda_{23})$: densities integrate to ones

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robustness game: seller hedges against nature

1. bundling $\{2,3\}$: no incentive to redistribute values within the $\{2,3\}$ bundle
2. separating 1 and $\{2,3\}$: no incentive to manipulate correlations of values across bundles
3. randomization: no incentive to redistribute value within each bundle

Application I: Generalizing Deb and Roesler (2023)

ambiguity set

$$\mathcal{F}_G := \left\{ F \in \Delta(\mathbb{R}^n_+) \middle| \mathbb{E}_F[\varphi(\mathbf{v})] \leq \mathbb{E}_G[\varphi(\mathbf{v})], \forall \varphi \text{ convex} \right\}.$$

single moment condition $\Rightarrow$ multiple moment conditions & $\mathcal{K} = \{N\}$

Assumption 1 (Stochastic Comonotonicity): There exists $(\xi_1, \ldots, \xi_n) \in \mathbb{R}^n_+$ with $\sum_i \xi_i = 1$

$$\mathbb{E}\left[v_i \middle| \sum_{j=1}^n v_j \right] = \xi_i \left(\sum_{j=1}^n v_j \right), \forall i \in \{1, 2, ..., n\}.$$

Theorem 3: Under A-1, pure bundling is informationally robust.

Application II: Generalizing Carroll (2017)

Carroll (2017): only know the marginal distribution within each dimension

this paper: only know the marginal distribution within each genre

technical assumptions

Theorem 4: Under these assumptions, K-bundled sales mechanism is robustly optimal.

Concluding Remarks: If you are Interested in Mathematical Details

this paper generalizes several results and shows their mathematical "limits"

- Carroll (2017)
- Roesler and Szentes (2017)
- Deb and Roesler (2023)