On the Existence of Fully Informative Experiment in Optimal Menu

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Abstract

In their analysis of screening with information products, Bergemann et al. (2022) showed the existence of fully informative information in optimal menu, which constitutes a core foundation for the proof of almost all key theorems in the paper. We provide an additional proof to its existence in the direct mechanism framework and reemphasize the key difference between screening with tradition products and information products lies in the non-congruent preference order for information products due to different priors and the existence of common most-preferred products, i.e. fully informative experiment, thus emphasizing the rents extraction balancing vertical and horizontal values of information.

1 Introduction

How mechanism design with information products differs from tradition products? Bergemann et al. (2018), the seminal paper in screening with information products, emphasizes that the conventional rent extraction versus efficiency trade-off can be resolved due to the information's inherent richness dependent on priors of information, which they call the horizontal differences in information production. Precisely speaking, differences in the buyers' priors may induce non-congruent preference order, thus introducing a novel aspect of horizontal differentiation that widens the seller's scope for price discrimination.

Like conventional mechanism design, they explored the "revelation principle" and some structural properties in the optimal menu. One remarkable property they discovered is that, in their general model of seller's design and price of information products, the fully informative experiment $\bar{E}$ is part of every optimal menu[1]. This property greatly helps uncover the structure of optimal information menu and constitutes a core foundation for the proof of almost all key theorems. And

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[1]More precisely, the fully informative experiment is part of every optimal menu if all types assign positive probability to all states(Bergemann et al. (2018)).

Bergemann et al. (2022) also focuses on the revenue of selling of only fully informative experiment and finds that this mechanism is also the optimal one in some special situations.

In Bergemann et al. (2022), a single data buyer with private signals faces a decision problem under uncertainty, and a information seller maximizes his profits by (unrestrictively) designing the optimal menu of statistical experiments with prices per buyer type. The value of statistical experiments is weighted by its incremental value in the decision problem.

In the model, the state of nature ω is drawn from a finite set $\Omega = \{\omega_1, \ldots, \omega_i, \ldots, \omega_I\}$. The data buyer chooses an action a from a finite set $A = \{a_1, \ldots, a_j, \ldots, a_J\}$. The ex post utility is denoted by $u(\omega_i, a_j) \triangleq u_{ij} \in \mathbb{R}_+$. Following Bergemann et al. (2022), we assume $I \geq J$ and $u_{ii} > u_{ij}$, $\forall j \neq i$.

The interim belief θ about the state is the type of the data buyer $\theta \in \Theta \triangleq \Delta\Omega$, where θ_i denotes the interim probability that type θ assigns to state ω_i, with $i = 1, \ldots, I$. The interim beliefs of the data buyer are his private information. From the perspective of the data seller, these beliefs are distributed according to a distribution $F \in \Delta\Theta$, which we take as a primitive of our model.

The seller designs statistical experiments with prices to maximize his profits. A statistical experiment (equivalently, an information structure) $E = (S, \pi)$ consists of a set S of signals s and a likelihood function $\pi : \Omega \to \Delta S$, which is independent, conditional on the state ω, from the buyer's private signal. Denote $K \triangleq |S|$ and $\pi_{ik} \triangleq \Pr[s_k \mid \omega_i] \geq 0$. Note that $\sum_{k=1}^{K} \pi_{ik} = 1$ for all i.

The data buyer augments his initial private information by obtaining additional information from the experiments, thus improving the quality of his decision making. Denote $u(\theta) \triangleq \max_{a_j \in A} \left\{\sum_{i=1}^{I} \theta_i u_{ij}\right\}$, the (net) value of an experiment E for type θ is given by $V(E, \theta)$, where

$$V(E, \theta) \triangleq E[u(s \mid \theta)] - u(\theta) = \sum_{k=1}^{K} \max_j \left\{\sum_{i=1}^{I} \theta_i \pi_{ik} u_{ij}\right\} - u(\theta)$$

By the "revelation principle", Bergemann et al. (2022) restricts attention to direct mechanisms $\mathcal{M} = \{E(\theta), t(\theta)\}$ that assign an experiment $E(\theta) = (S(\theta), \pi(\theta))$ and a price $t(\theta) \in \mathbb{R}$ to each type θ of data buyer.

The seller's problem consists of maximizing the expected transfers

$$\max_{\{E(\theta), t(\theta)\}} \int_{\theta \in \Theta} t(\theta) dF(\theta)$$

subject to incentive-compatibility constraints (IC)

$$V(E(\theta), \theta) - t(\theta) \geq V(E(\theta'), \theta) - t(\theta'), \quad \forall \theta, \theta' \in \Theta,$$

and individual-rationality constraints (IR)

$$V(E(\theta), \theta) - t(\theta) \geq 0, \quad \forall \theta \in \Theta.$$

Bergemann et al. (2022) presents the proposition 2, discovering structure on the experiments that are part of an optimal menu.

Proposition 2. *The fully informative experiment $\bar{E}$, with $\pi_{ii} = 1$ for all i, is part of an optimal menu.*

Proposition 2 constitutes a core foundation for the proof of almost all key theorems. For example, it provides two key constraints in the sellers$'$ design problem. Because they apply the Fundamental Theorem of Linear Programming to derive their core result (Proposition 4 and Proposition 5), the cardinality of constraints plays a fundamental role in their proof.

This note provides an additional proof to this proposition in the direct mechanism framework and reemphasize the key difference between screening with tradition products and information products. The preference for information products of different priors is non-congruent, while all agents commonly preferr fully informative experiment most.Thus the rents extraction should balance vertical and horizontal values of information.

2 Our Proof

Our proof uses a technique we call vertically informative experiment. The original menu without full informative experiment may take advantage of the more relaxed IC conditions due to the horizontal differentiation information preferences among buyers. Adjusting directly to full informative experiment which destroys this horizontal difference may therefore break IC conditions. To overcome this difficulty, we consider all convex combinations of the buyer's original experiment and the full informative experiment. These combined information structures also increase his willingness to pay, while partly preserves the horizontal difference and less likely to break the IC conditions.

Based on this perspective, we proves that if there is no full informative experiment in an optimal menu, then the seller's expected transfers can be improved by adjusting one of the agents' experiment into more informative one and charging more on it, which is contradictory to the optimality.

Meanwhile, we will show that whose menu can be adjusted to a full informative experiment or a vertically informative experiment does not simply depend on the magnitude of his transfer in the original menu, but on the following two properties of his original IC conditions. If his original IC condition of not pretending to be some type θ is a loose constraint, then an adjustment to a slightly vertically informative experiment will maintain the adjusted IC condition on this type θ due to continuity. Or, if for some type θ, his incremental willingness to pay on adjusting to the full informative experiment is larger than that of θ on the same information struction adjustment, then the full informative experiment, as well as all kinds of convex combined information structure, will not break the adjusted IC conditions on this type θ due to convexity.

In the final part of the proof, we show that there is at least one buyer such that each of his IC conditions about pretending all types of other buyers has at least one of the above two properties.

The failure of this statement will induce to the absence of a maximum value for a continuous function over the entire type space, which contradicts the compactness of the space.

2.1 Vertically Informative Experiment

To reduce the size of the signal space, Bergemann et al. (2018) utilizes the ”revelation principle” in information products screening. They define the items in direct revelation mechanisms, called responsive menus. We further demonstrate that responsiveness property can be maintained under convex combinations for given θ. As the full informative experiment is always responsive, we construct the vertically informative experiment whose value can be expressed as the convex combinations of the original experiment and the full informative one, which plays an important role in our proof of Proposition 2.

Definition 1. *Experiment $E(\theta)$ is responsive if every signal $s \in S(\theta)$ leads type θ to a different optimal choice of action and, in particular,*

$$a\left(s_k \mid \theta\right)=a_k \text{ for all } s_k \in S(\theta).$$

Note that the responsiveness of $E(\theta)$ is only required for its corresponding type θ. It may not hold for any other type $\theta' \neq \theta$, unless as in the following Lemma 1. Lemma 2, which has proved in Bergemann et al. (2018), show that we can restrict attention to menus in which every experiment $E(\theta)$ is responsive without loss of generality.

Lemma 1. *Fully informative experiment $\bar{E}$ is responsive for any θ.*

Lemma 2 (Bergemann et al. (2018) Proposition 1)**.** *The outcome of every menu $\mathcal{M}$ can be attained by a responsive menu, in which every experiment $E(\theta)$ is responsive.*

We now define the convex combination of two (or more) responsive experiments for given θ. Suppose $E=(S,\pi^E)$ and $E'=(S',\pi^{E'})$ are both responsive experiments. By Lemma 2, we can suppose that they share the same signal space $S=S'$=A, which is the action space.

Definition 2. *Experiment $\hat{E}=(S,\pi^{\hat{E}})$ is a convex combination of two responsive experiments if there exists $\alpha \in [0,1]$ such that $\pi^{\hat{E}}(s_k|\omega_i)=\alpha\pi^E(s_k|\omega_i)+(1-\alpha)\pi^{E'}(s_k|\omega_i)$ for any $s_k \in S$, where $E=(S,\pi^E)$ and $E'=(S,\pi^{E'})$ are both responsive experiments. Denote $\hat{E}=\alpha E+(1-\alpha)E'$.*

Lemma 3. *The responsiveness property (for given θ) holds under convex combination.*

Proof. Since E is responsive for θ, we have $a_k=a\left(s_k \mid \theta\right) \in \underset{a_j=A}{\arg\max}\left\{\sum_{i=1}^{I}\left(\frac{\theta_i\pi^E(s_k|\omega_i)}{\sum_{i'=1}^{I}\theta_{i'}\pi^E(s_k|\omega_{i'})}\right)u_{ij}\right\}$.

So we have:

$$\sum_{i=1}^{I}\left(\frac{\theta_i\pi^E(s_k|\omega_i)}{\sum_{i'=1}^{I}\theta_{i'}\pi^E(s_k|\omega_{i'})}\right)u_{ik} \geq \sum_{i=1}^{I}\left(\frac{\theta_i\pi^E(s_k|\omega_i)}{\sum_{i'=1}^{I}\theta_{i'}\pi^E(s_k|\omega_{i'})}\right)u_{ij}$$

i.e.

$$\sum_{i=1}^{I} \theta_i \pi^E(s_k|\omega_i) u_{ik} \geq \sum_{i=1}^{I} \theta_i \pi^E(s_k|\omega_i) u_{ij} \tag{1}$$

Similarly, we have

$$\sum_{i=1}^{I} \theta_i \pi^{E'}(s_k|\omega_i) u_{ik} \geq \sum_{i=1}^{I} \theta_i \pi^{E'}(s_k|\omega_i) u_{ij} \tag{2}$$

With inequality (1) and (2), we have

$$\begin{gathered}\textstyle\sum_{i=1}^{I} \theta_i \pi^{\hat{E}}(s_k|\omega_i) u_{ik} = \sum_{i=1}^{I} \theta_i (\alpha \pi^E(s_k|\omega_i) + (1-\alpha)\pi^{E'}(s_k|\omega_i)) u_{ik} \geq \\ \textstyle\sum_{i=1}^{I} \theta_i (\alpha \pi^E(s_k|\omega_i) + (1-\alpha)\pi^{E'}(s_k|\omega_i) u_{ij} = \sum_{i=1}^{I} \theta_i \pi^{\hat{E}}(s_k|\omega_i) u_{ij}\end{gathered}$$

which implies that in $\hat{E}$, $a_k = a\left(s_k \mid \theta\right) = \underset{a_j \in A}{\arg\max} \left\{ \sum_{i=1}^{I} \left(\frac{\theta_i \pi^{\hat{E}}(s_k|\omega_i)}{\sum_{i'=1}^{I} \theta_{i'} \pi^{\hat{E}}(s_k|\omega_i)} \right) u_{ij} \right\}$.

Therefore the convex combination of E and E' is responsive for θ. □

We construct the vertically more informative experiment as the convex combination of a responsive $E(\theta)$ and $\bar{E}$. This kind of information goods enhance the buyer's willingness to pay, while also maintains the horizontal element of $E(\theta)$ to a certain extent. Corollary 1 is obvious under Lemma 1 and Lemma 3.

Corollary 1. *The convex combination of an responsive experiment for some type θ and the fully informative experiment $\bar{E}$ is still the responsive experiment for θ.*

Finally, we show that the convex combination of responsive experiments implies the convex combination of information values, which enables us to establish a connection on information value comparison before and after our adjustment.

Lemma 4. *The convex combination of two responsive experiments has the information value which is a corresponding convex combination of the information value of these two responsive experiments.*

Proof. We aim to proof that $V(\hat{E}, \theta) = \alpha V(E, \theta) + (1-\alpha) V(E', \theta)$.

i.e.

$$\begin{gathered}\textstyle\sum_{k=1}^{K} \sum_{i=1}^{I} \theta_i \pi^{\hat{E}}(s_k|\omega_i) u_{ik} - u(\theta) = \alpha(\sum_{k=1}^{K} \sum_{i=1}^{I} \theta_i \pi^E(s_k|\omega_i) u_{ik} - u(\theta)) \\ \textstyle+(1-\alpha)(\sum_{k=1}^{K} \sum_{i=1}^{I} \theta_i \pi^{E'}(s_k|\omega_i) u_{ik} - u(\theta))\end{gathered}$$

It is not hard to substitute $\pi^{\hat{E}}(s_k|\omega_i) = \alpha \pi^E(s_k|\omega_i) + (1-\alpha)\pi^{E'}(s_k|\omega_i)$ from the definition of $\hat{E}$ to verify it. □

If there is no responsiveness, as for another type $\theta' \neq \theta$, by the convexity of max operator, we can get the following Lemma 5.

Lemma 5. *The convex combination of two responsive experiments (for given θ) has the information value which is weakly lower than the corresponding convex combination of the information value of these two responsive experiments for any other type $\theta' \neq \theta$.*

Proof. We aim to proof that $V(\hat{E}, \theta') \leq \alpha V(E, \theta') + (1-\alpha)V(E', \theta')$ for any other type $\theta' \neq \theta$.

i.e.

$$\begin{aligned}&\textstyle\sum_{k=1}^{K} \max_j \left\{(1-\alpha)\sum_{i=1}^{I} \theta_i \pi_{ik}^{E}(\theta')u_{ij} + \alpha \sum_{i=1}^{I} \theta_i \pi_{ik}^{E'}(\theta')u_{ij}\right\}\\ \leq\ &\textstyle(1-\alpha)\sum_{k=1}^{K} \max_j \left\{\sum_{i=1}^{I} \theta_i \pi_{ik}^{E}(\theta')u_{ij}\right\} + \alpha \sum_{k=1}^{K} \max_j \left\{\sum_{i=1}^{I} \theta_i \pi_{ik}^{E'}(\theta')u_{ij}\right\}\end{aligned}$$

which holds under the convexity of max operator. □

Moreover, we get the following corollary that all non-trivial vertically informative experiments enhance the information value.

Corollary 2. *For $\hat{E} = \alpha\bar{E} + (1-\alpha)E(\theta)$ where $E(\theta) \neq \bar{E}$ is responsive for θ, we have $V(\hat{E}, \theta) \geq V(E(\theta), \theta)$, where the equality holds if and only if $\alpha = 0$.*

Proof. Only need to notice that $V(\bar{E}, \theta) > V(E(\theta), \theta)$, and $V(\hat{E}, \theta) = \alpha V(\bar{E}, \theta) + (1-\alpha)V(E(\theta), \theta)$ by Lemma 4. □

2.2 Continuity of Value and Surplus

Our final proof relies on the compactness of the type space to ensure that there is at least one buyer who is able to make vertically informative enhancement adjustment. In order to apply Weierstrass Theorem, we need the following two continuities. Here, we use standard Euclidean metric in the metric space $\Delta(\Omega)$ and denote $d(\theta, \theta')$ as the distance between θ and $\theta' \in \Delta(\Omega)$.

Lemma 6. *Given information structure E, the information value $V(E, \theta)$ of an experiment E for type θ is continuous with respect to θ.*

Proof. Notice that

$$V(E, \theta) = \textstyle\sum_{k=1}^{K} \max_j \left\{\sum_{i=1}^{I} \theta_i \pi_{ik} u_{ij}\right\} - \max_j \left\{\sum_{i=1}^{I} \theta_i u_{ij}\right\}$$

It is easy to check the continuity of $V(E, \theta)$ with respect to θ. □

Lemma 7. *The net utility $V(E(\theta), \theta) - t(\theta)$ in a feasible menu (satisfied IC and IR conditions), is lower semi-continuous with respect to θ.*

Proof. We prove it via contradiction.

If the net utility in the optimal menu is not lower semi-continuous. Then at some $\theta \in \Theta$, there exists $\delta > 0$ and for any $\epsilon > 0$, there always exists θ' such that $d(\theta, \theta') < \epsilon$ and $[V(E(\theta'), \theta') - t(\theta')] - [V(E(\theta), \theta) - t(\theta)] \leq -\delta$, i.e.

$$t(\theta) - t(\theta') \leq V(E(\theta), \theta) - V(E(\theta'), \theta') - \delta \tag{3}$$

Now considering the IC conditions that θ' is unwilling to imitiate θ, i.e.

$$t(\theta) - t(\theta') \geq V(E(\theta), \theta') - V(E(\theta'), \theta') \tag{4}$$

With inequality (3) and (4), we can derive that

$$V(E(\theta), \theta) - V(E(\theta), \theta') \geq \delta$$

which contradicts with the Lemma 6. □

2.3 Proof of Proposition 2

With the lemmas and corollaries above, we now prove the existence of the fully informative experiment in the optimal menu via adjustment on a carefully selective buyer.

Proof. Suppose $\bar{E}$ is not in the original menu $\mathcal{M} = \{E(\theta), t(\theta)\}$. For every type θ, considering the adjusted menu $\hat{\mathcal{M}} = \{\hat{E}(\theta), \hat{t}(\theta)\}$, $\alpha(\theta) \in [0, 1]$, where $\hat{E}(\theta) = \alpha(\theta)\bar{E} + (1 - \alpha(\theta))E(\theta)$, $\hat{t}(\theta) = t(\theta) + V(\hat{E}(\theta), \theta) - V(E(\theta), \theta)$. From Corollary 2, $\hat{t}(\theta) \geq t(\theta)$. Therefore all this kind of adjustment is a (weakly) better adjustment (if feasible). According to Corollary 2, $\hat{t}(\theta) > t(\theta)$ if and only if $\alpha(\theta) > 0$.To get the seller better off, we can focus on finding a feasible adjustment with exactly one $\theta' \in \Theta$ such that $\alpha(\theta') > 0$ while for all $\theta \neq \theta'$, $\alpha(\theta') = 0$.

Because the seller precisely extracts the incremental information value of each buyer in menu changing. It is easy to verify that in $\hat{\mathcal{M}}$, $\forall \theta \in \Theta$ (IR-θ) holds. Meanwhile, the IC condition that θ' will not pretend to have another type $\theta \neq \theta'$ holds because the buyer gets the same surplus between the old and new menus and the IC condition holds in the original one.

Therefore, we can only pay attention to the possibility of constructing $\alpha(\theta') > 0$ to prevent any other $\theta \neq \theta'$ to pretend to be θ', which means $V(\hat{E}(\theta), \theta) - \hat{t}(\theta) \geq V(\hat{E}(\theta'), \theta) - \hat{t}(\theta')$, i.e.

$$V(E(\theta), \theta) - t(\theta) \geq V(\hat{E}(\theta'), \theta) - t(\theta') - V(\hat{E}(\theta'), \theta') + V(E(\theta'), \theta')$$

or,

$$V(E(\theta), \theta) - t(\theta) - V(E(\theta'), \theta') + t(\theta') \geq V(\hat{E}(\theta'), \theta) - V(\hat{E}(\theta'), \theta')$$

Then by Lemma 4 and Lemma 5, a sufficient condition for this IC condition to hold is:

$$V(E(\theta),\theta)-t(\theta)-V(E(\theta'),\theta')+t(\theta')\geq \alpha(\theta')V(\bar{E},\theta)+(1-\alpha(\theta'))V(E(\theta'),\theta)$$
$$-\alpha(\theta')V(\bar{E},\theta')-(1-\alpha(\theta'))V(E(\theta'),\theta')$$

and rearrange it to get

$$\alpha(\theta')[(V(\bar{E},\theta)-V(E(\theta'),\theta))-(V(\bar{E},\theta')-V(E(\theta'),\theta'))]\leq V(E(\theta),\theta)-t(\theta)+t(\theta')-V(E(\theta'),\theta) \quad (5)$$

Denote $DS(\theta,\theta')=V(E(\theta),\theta)-t(\theta)+t(\theta')-V(E(\theta'),\theta)$ and denote $DA(\theta,\theta')=(V(\bar{E},\theta)-V(E(\theta'),\theta))-(V(\bar{E},\theta')-V(E(\theta'),\theta'))$. By equation (5), the sufficient condition for an adjustment on θ' is feasible is that for any $\theta\neq\theta'$, $\alpha(\theta')DA(\theta,\theta')\leq DS(\theta,\theta')$. Moreover, by the feasibility of the original menu, we know that $DS(\theta,\theta')$ is weakly higher than 0.

Case 1: If there exists a buyer θ' such that for any $\theta\neq\theta'$, $DS(\theta,\theta')\geq DA(\theta,\theta')>0$ or $DA(\theta,\theta')\leq 0$ holds, then we can set $\alpha(\theta')=1$ to get a feasible adjustment, which means we find a new menu which strictly increases the seller's revenue and contains $\bar{E}$.

Case 2: If for any buyer θ', there exists a θ such that $DA(\theta,\theta')>DS(\theta,\theta')\geq 0$, denote $f(\theta)\triangleq V(E(\theta),\theta)-t(\theta)-V(\bar{E},\theta)$, then we can derive that

$$\begin{aligned}
&f(\theta)-f(\theta')\\
=&[V(E(\theta),\theta)-t(\theta)-V(\bar{E},\theta)]-[V(E(\theta'),\theta')-t(\theta')-V(\bar{E},\theta')]\\
=&[V(E(\theta),\theta)-t(\theta)-V(E(\theta'),\theta)+t(\theta')]\\
&-[V(\bar{E},\theta)-V(\bar{E},\theta')+V(E(\theta'),\theta')-V(E(\theta'),\theta)]\\
=&DS(\theta,\theta')-DA(\theta,\theta')<0
\end{aligned}$$

,which means for any θ', there exists a $\theta\neq\theta'$ having a strictly lower value of $f(\theta)$ than that of θ'.

However, by Lemma 6 and Lemma 7, we know that $f(\theta)$ is a lower semi-continuous function over $\theta\in\Theta$[2]. Since Θ as a simplex space is compact, by Weierstrass Theorem, there exists a θ', for any $\theta\in\Theta$, $f(\theta)\geq f(\theta')$, a contradiction. □

$DS(\theta,\theta')$ in our proof can be interpreted as the surplus increment for buyer θ to pretend to be of type θ' in the original menu, while $DA(\theta,\theta')$ can be interpreted as the difference of incremental willingness to pay from $E(\theta')$ to $\bar{E}$ between θ and θ'. If buyer θ is weakly less willing to pay for the information improvement on θ' than buyer θ' (i.e. $DA(\theta,\theta')\leq 0$), then buyer θ have no incentive to pretend to be θ' in the new menu. Or even if buyer θ is more willing to pay for that information improvement than buyer θ' (i.e. $DA(\theta,\theta')>0$), as long as the corresponding IC condition is loose enough in the original menu (i.e. $DS(\theta,\theta')\geq DA(\theta,\theta')>0$), the corresponding IC condition is still holds when the adjustment to the new menu.

Although from a proof point of view, the technique of vertically informative enhancement is not essential, because we can always find a buyer θ' such that it is feasible to set $\alpha(\theta')=1$ eventually. However, the construct of vertically informative enhancement helps us to understand

[2] The sum of two lower semi-continuous function is still lower semi-continuous.

the trade off in the adjustment by distinguish $DS(\theta, \theta')$ and $DA(\theta, \theta')$. Meanwhile, we indicates not only that whose experiment can be adjusted but also for each of them to what extent can it be adjusted. For each buyer θ', denote $\Theta_{\theta'} \triangleq \{\theta \in \Theta - \{\theta'\} | DA(\theta, \theta') > 0\}$. Then, we could set $\alpha(\theta') = \inf\{1, \frac{DS(\theta,\theta')}{DA(\theta,\theta')} | \theta \in \Theta_{\theta'}\}$ to get a feasible adjustment.

Notice that our proof still holds under a global adjustment, which means it can be a quicker way to search for the extreme menus (the optimal menu depends further on F).

Finally, it is worth mentioned that our proof requires Θ to be a compact set. If $\Theta = \triangle(\Omega)$ as the entire prior space or Θ is a finite set, the compactness holds. Otherwise, we need a weak assumption that Θ is compact. Moreover, some difficulty in our proof due to the infinity of Θ. If Θ is a finite set, there is no need to prove the continuity and use the Weierstrass Theorem, and we can set $\alpha(\theta') = \min\{1, \frac{DS(\theta,\theta')}{DA(\theta,\theta')} | \theta \in \Theta_{\theta'}\}$ for there are only finite θs.

References

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